Revision Solution

2) (a)

$$\mu = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

(b) Since $\Omega \underset{\sim}{y}$ is a linear combination of an MVN vector $\underset{\sim}{y}$, $\Omega y \sim MVN$ with mean

$$\Omega \mu = \begin{pmatrix} -2 & 1 \\ 0.5 & 2 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 0.5 \end{pmatrix}$$

and covariance matrix

$$\Omega \Omega^T = \begin{pmatrix} 5 & 1 \\ 1 & 4.25 \end{pmatrix}$$

(c) $\cancel{\psi^T B}$ = Use theorem 3.14:

$$\psi^T I B = \psi^T B$$

$$= \begin{pmatrix} -3.5 & -7 \end{pmatrix} \begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -5.25 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$\therefore$ $y^T B y$ and $\psi^T y$ are NOT indept.

Q3 (a) A full rank matrix X corresponds to assumption II in Lect 4 Part 1.

By Gauss-Markov theorem (thm 4.4), we also need

(I) The true relationship between X and y is $y = X\beta + \varepsilon$, where X is an $n \times (k+1)$ matrix and β is a $(k+1)$-dimensional vector for any $k \geq 0$.

(III) The random error are zero-centered, i.e, $E(\varepsilon) = 0$

(IV) The random errors are uncorrelated and have homogeneous variance, i.e.,

$$Var(\varepsilon) = \sigma^2 I_n$$

(b) $$y_i \overset{ind}{\sim} N(x_i^T\beta, d_i), \quad i = 1, \ldots, n$$

$$\Rightarrow \quad \underset{\sim}{y} \sim MVN(X\beta, D) \text{ where } D = \begin{bmatrix} d_1 & & & 0 \\ & d_2 & & \\ & & \ddots & \\ 0 & & & d_n \end{bmatrix}$$

Let $\tilde{y} = D^{-\frac{1}{2}} \underset{\sim}{y}$ and $\tilde{X} = D^{-\frac{1}{2}} X$

Then $\tilde{y} \sim MVN(\tilde{X}\beta, I)$

By theorem 4.11,

$$\tilde{\beta} = (\tilde{X}^T\tilde{X})^{-1}\tilde{X}^T\tilde{y} \text{ is UMVUE for } \beta$$

But....

$$\tilde{\beta} = (\tilde{X}^T\tilde{X})^{-1}\tilde{X}^T\tilde{y}$$

$$= (X^T D^{-1} X)^{-1} X^T D^{-1} y$$

$$\neq \hat{\beta}$$

$\therefore$ $\hat{\beta}$ is not UMVUE for β.

Q4) (a) $p=6$ df of $SS_{res} = 16$

$\Rightarrow n-p=16$

$\Rightarrow n = 16+p = 22$

(b)

$$F = \frac{y^T H y / p}{SS_{res}/(n-p)} \sim F_{p,n-p}$$

$\therefore$ null dist is F- distribution with 6 and 16 degrees of freedom.

Assumptions required: By theorems 5.2 and 5.3 we need: $\underset{22\times1}{y}$ = vector of responses $\underset{22\times6}{X}$ = design matrix

(I) The true relationship between X and y is
$y = X\beta + \varepsilon$ ......

(II) X is full rank

(V) $\varepsilon \sim MVN(0, \sigma^2 I)$, for some $\sigma > 0$.

(c) $H_0: \beta_{wing} = 0$ vs. $H_1: \beta_{wing} \neq 0$.

$$T = \frac{\hat{\beta}_{wing}}{SE(\hat{\beta}_{wing})} = -1.325 \sim t_{16}$$

p-value = 0.20378 > 0.1

since p-value > 0.1, we don't reject H_0 and we conclude that β_{wing} is plausibly zero.

(d) Multiple $R^2 = 1 - \frac{SS_{res}}{\text{Corrected } SS_{total}}$

$$\text{Adj-}R^2 = 1 - \frac{n-1}{n-p}(1-R^2)$$

Since R^2 never decreases as we add new predictors in the model, it is unsuitable for model selection. On the other hand, adj-R^2 accounts for model complexity by penalising models based on number of coefficients. Hence, unlike R^2, the adj-R^2 is suitable for model selection.

(f) Since $\beta_{beaktonotch}$ has the smallest absolute t-statistic value (equivalent to smallest F-statistic), we should drop it first.

(e) $$\text{adj-}R^2 = 1 - \frac{n-1}{n-1-k}(1-R^2)$$

$$= 1 - \frac{22-1}{22-1-5}(1-0.9633)$$

$$= 0.9518313$$