MAST90104 - Lecture 3 Extras

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Consider the random vector

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \sim \text{MVN}\left(\boldsymbol{\mu}, \sigma^2 \mathbf{I}\right)$$

where $\boldsymbol{\mu} \in \mathbb{R}^2$ and $\sigma^2 > 0$. Let

$$h(\ell_1, \ell_2) = \ell_1 y_1 - 2\ell_2 y_2.$$

Find a pair of values for ℓ_1 and ℓ_2 such $h(\ell_1, \ell_2)$ and $2y_1^2 - 3y_1y_2 + y_2^2$ are independent.

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Solution: The two random quantities are actually NOT independent for all non-zero values of ℓ_1 and ℓ_2!!!

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where $\boldsymbol{\mu} = (\mu_1, \mu_2)^T$. Let

$$\mathbf{A} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$$

Prove that $\mathbf{y}^T\mathbf{A}\mathbf{y}$ and $\mathbf{y}^T\mathbf{B}\mathbf{y}$ are independent.

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Two solutions:

1. Use Theorem 3.15
2. Use Theorem 3.16 (Cochran-Fisher Theorem)