MAST90104 - Lecture 1 Extras

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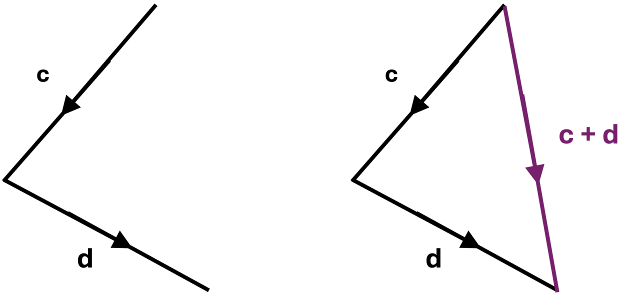
Geometric interpretation of vector additions

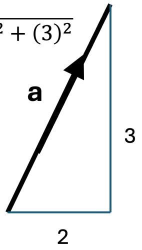
Consider two vector **c** and **d**.



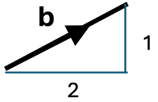
Vector norm = length of vector

The norm of a vector $||\mathbf{a}||$ may be interpreted geometrically as the *length*. For two vectors $\mathbf{a} = (2, 3)$ and $\mathbf{b} = (2, 1)$:

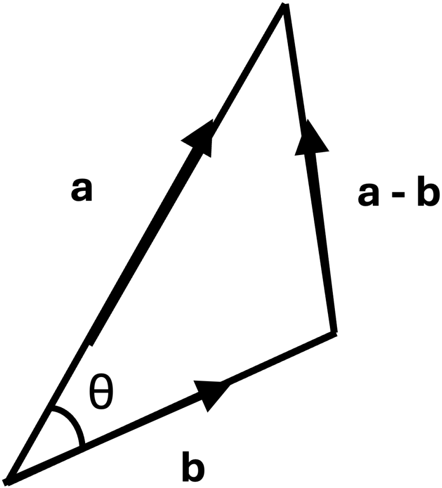
$$||a|| = \sqrt{(2)^2 + (3)^2}$$
$$= \sqrt{13}$$



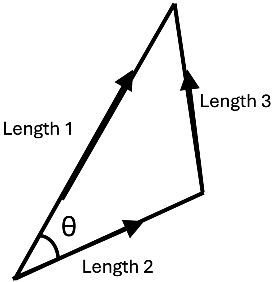
$$||b|| = \sqrt{(2)^2 + (1)^2}$$
$$= \sqrt{5}$$



Angle between two vectors a and b

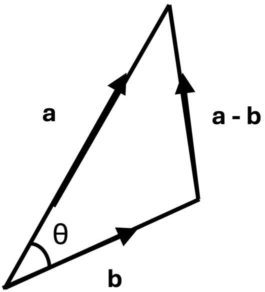


Cosine rule: a great too for trigonometric problems



$$(\text{Length3})^2 = (\text{Length1})^2 + (\text{Length2})^2 - 2\,(\text{Length1})\,(\text{Length2})\cos\theta$$

Cosine rule: a great too for trigonometric problems



$(\text{Length3})^2 = (\text{Length1})^2 + (\text{Length2})^2 - 2\,(\text{Length1})\,(\text{Length2})\cos\theta$

$||a-b||^2 = ||a||^2 + ||b||^2 - 2\,||a||\,||b||\cos\theta$

Further algebraic and vector manipulation

$$
\begin{aligned}
LHS &= \|a - b\|^2 \\
&= \sum_j (a_j - b_j)^2 \\
&= (\mathbf{a} - \mathbf{b})^T(\mathbf{a} - \mathbf{b}) \\
&= \|\mathbf{a}\|^2 - \mathbf{a}^T\mathbf{b} - \mathbf{b}^T\mathbf{a} + \|b\|^2 \\
&= \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\mathbf{a}^T\mathbf{b}
\end{aligned}
$$

Further algebraic and vector manipulation

$$
\begin{aligned}
LHS &= RHS \\
\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\mathbf{a}^T\mathbf{b} &= \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta \\
\Rightarrow -2\mathbf{a}^T\mathbf{b} &= -2\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta \\
\Rightarrow \cos\theta &= \frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} \\
\Rightarrow \cos\theta &= \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}
\end{aligned}
$$