

```

> restart, with(grtensor);
      "GRTensor III v2.1.8 Aug 20, 2017"
      "Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"
      "Latest version is at http://github.com/grtensor/grtensor"
      "For help ?grtensor"
      "Support/contact grtensor3@gmail.com"
libname := "/Users/peter/maple/gitlab/GRTensorIII/lib",
          "/Library/Frameworks/Maple.framework/Versions/2017/lib"
grOptionMetricPath := "/Users/peter/maple/gitlab/grtensor/metrics"
      "GRTensor III v2.1.8 Aug 20, 2017"
      "Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"
      "Latest version is at http://github.com/grtensor/grtensor"
      "For help ?grtensor"
      "Support/contact grtensor3@gmail.com"
[Asym, KillingCoords, PetrovReport, Sym, autoAlias, difftool, grDalias, grF_strToDef, gralter,
  gralterd, grapply, grarray, grcalc, grcalc1, grcalcalter, grcalcd, grclear, grcomponent,
  grconstraint, grdata, grdebug, grdef, grdisplay, grdump, greqn2set, grinit, grload,
  grload_maplet, grmap, grmetric, grnewmetric, grnormalize, groptions, grsaveg, grtestinput,
  grtransform, grundef, hypersurf, join, kdelta, makeg, nprotate, nptetrad, qload, spacetime]

```

(1)

```

> grload( );
Loading as schw from
/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics/schw.
mplCalculated ds for schw (0.002000 sec.)
      Default spacetime = schw
      For the schw spacetime:
      Coordinates
      x(up)
      
$$x^a = \begin{bmatrix} r & \theta & \phi & t \end{bmatrix}$$

      Line element
      
$$ds^2 = \frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\theta^2 + r^2 \sin(\theta)^2 d\phi^2 + \left(-1 + \frac{2m}{r}\right) dt^2$$


```

(2)

```

> grdef("G2{a b} := R{a b} - 1/2*Ricciscalar*g{a b} + lambda*g{a b}");
Created definition for G2(dn,dn)
> grcalc(G2(dn,dn));
Calculated g(dn,dn,pdn) for schw (0.002000 sec.)
Calculated Chr(dn,dn,dn) for schw (0.001000 sec.)
Calculated detg for schw (0.005000 sec.)
Calculated g(up,up) for schw (0.010000 sec.)
Calculated Chr(dn,dn,up) for schw (0.002000 sec.)

```

```

Calculated R(dn,dn) for schw (0.002000 sec.)
Calculated Ricciscalar for schw (0.001000 sec.)
Calculated G2(dn,dn) for schw (0.000000 sec.)
CPU Time = 0.020

```

(3)

```
> grdisplay(_);
```

For the schw spacetime:

$$G2_{ab} = \begin{bmatrix} -\frac{\lambda r}{-r+2m} & 0 & 0 & 0 \\ 0 & \lambda r^2 & 0 & 0 \\ 0 & 0 & \lambda r^2 \sin(\theta)^2 & 0 \\ 0 & 0 & 0 & \frac{\lambda (-r+2m)}{r} \end{bmatrix}$$

(4)

```

> grdef("X := R^{a b}*Box[R{a b}]");
Created definition for R(up,up)
Created definition for X
> grdef("T1{a b} := R{a c d b}*R^{c d}");
Created definition for T1(dn,dn)
> grdef("T{a b} := Box[T{a b}]");
Created definition for T(dn,dn)
> grcalc(Box[T1(dn,dn)]);
Created a definition for T1(dn,dn,cdn)
Created a definition for T1(dn,dn,cdn,cdn)
Calculated R(up,up) for schw (0.001000 sec.)
Calculated R(dn,dn,dn,dn) for schw (0.004000 sec.)
Calculated T1(dn,dn) for schw (0.001000 sec.)
Calculated T1(dn,dn,cdn) for schw (0.003000 sec.)
Calculated T1(dn,dn,cdn,cdn) for schw (0.012000 sec.)
Calculated Box[T1(dn,dn)] for schw (0.002000 sec.)
CPU Time = 0.078

```

(5)

```
> grdisplay(Box[T1(dn,dn)]);
```

For the schw spacetime:

CoD{a} CoD^{a} <Object>

Box_{ab} = All components are zero

(6)

```

> grtestinput([seq(i,i=1..100)]);
Added 100 test entries
> grdef("A{(a b)(c d)}");
Created definition for A(dn,dn,dn,dn)
> grcalc(A(dn,dn,dn,dn));

Enter components for object A(dn,dn,dn,dn)
If you wish to quit at any point and leave this object
uninitialized, enter the string exit.
Arr rr
GRTEST: For prompt: Enter component [r, r, r, r]:GRTEST: Entered:

```

1

$$A_{rr}{}^{rr}{}_{\theta}$$

GRTEST: For prompt: Enter component [r, r, r, theta]:GRTEST:
Entered: 2

$$A_{rr}{}^{rr}{}_{\phi}$$

GRTEST: For prompt: Enter component [r, r, r, phi]:GRTEST:
Entered: 3

$$A_{rr}{}^{rr}{}_{rt}$$

GRTEST: For prompt: Enter component [r, r, r, t]:GRTEST: Entered:
4

$$A_{rr}{}^{rr}{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [r, r, theta, theta]:GRTEST:
Entered: 5

$$A_{rr}{}^{rr}{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [r, r, theta, phi]:GRTEST:
Entered: 6

$$A_{rr}{}^{rr}{}_{\theta t}$$

GRTEST: For prompt: Enter component [r, r, theta, t]:GRTEST:
Entered: 7

$$A_{rr}{}^{rr}{}_{\phi\phi}$$

GRTEST: For prompt: Enter component [r, r, phi, phi]:GRTEST:
Entered: 8

$$A_{rr}{}^{rr}{}_{\phi t}$$

GRTEST: For prompt: Enter component [r, r, phi, t]:GRTEST:
Entered: 9

$$A_{rr}{}^{rr}{}_{tt}$$

GRTEST: For prompt: Enter component [r, r, t, t]:GRTEST: Entered:
10

$$A_{r\theta}{}^{rr}{}_{rr}$$

GRTEST: For prompt: Enter component [r, theta, r, r]:GRTEST:
Entered: 11

$$A_{r\theta}{}^{rr}{}_{r\theta}$$

GRTEST: For prompt: Enter component [r, theta, r, theta]:GRTEST:
Entered: 12

$$A_{r\theta}{}^{rr}{}_{r\phi}$$

GRTEST: For prompt: Enter component [r, theta, r, phi]:GRTEST:
Entered: 13

$$A_{r\theta}{}^{rr}{}_{rt}$$

GRTEST: For prompt: Enter component [r, theta, r, t]:GRTEST:
Entered: 14

$$A_{r\theta}{}^{rr}{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [r, theta, theta, theta]
:GRTEST: Entered: 15

$$A_{r\theta}{}^{rr}{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [r, theta, theta, phi]:GRTEST:
Entered: 16

$$A_{r\theta}{}^{\theta}{}_{t}$$

GRTEST: For prompt: Enter component [r, theta, theta, t]:GRTEST:
Entered: 17

$$A_{r\theta}{}^{\phi}{}_{\phi}$$

GRTEST: For prompt: Enter component [r, theta, phi, phi]:GRTEST:
Entered: 18

$$A_{r\theta}{}^{\phi}{}_{t}$$

GRTEST: For prompt: Enter component [r, theta, phi, t]:GRTEST:
Entered: 19

$$A_{r\theta}{}^{\theta}{}_{t}$$

GRTEST: For prompt: Enter component [r, theta, t, t]:GRTEST:
Entered: 20

$$A_{r\phi}{}^{\theta}{}_{r}$$

GRTEST: For prompt: Enter component [r, phi, r, r]:GRTEST:
Entered: 21

$$A_{r\phi}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [r, phi, r, theta]:GRTEST:
Entered: 22

$$A_{r\phi}{}^{\theta}{}_{\phi}$$

GRTEST: For prompt: Enter component [r, phi, r, phi]:GRTEST:
Entered: 23

$$A_{r\phi}{}^{\theta}{}_{t}$$

GRTEST: For prompt: Enter component [r, phi, r, t]:GRTEST:
Entered: 24

$$A_{r\phi}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [r, phi, theta, theta]:GRTEST:
Entered: 25

$$A_{r\phi}{}^{\theta}{}_{\phi}$$

GRTEST: For prompt: Enter component [r, phi, theta, phi]:GRTEST:
Entered: 26

$$A_{r\phi}{}^{\theta}{}_{t}$$

GRTEST: For prompt: Enter component [r, phi, theta, t]:GRTEST:
Entered: 27

$$A_{r\phi}{}^{\phi}{}_{\phi}$$

GRTEST: For prompt: Enter component [r, phi, phi, phi]:GRTEST:
Entered: 28

$$A_{r\phi}{}^{\phi}{}_{t}$$

GRTEST: For prompt: Enter component [r, phi, phi, t]:GRTEST:
Entered: 29

$$A_{r\phi}{}^{\theta}{}_{t}$$

GRTEST: For prompt: Enter component [r, phi, t, t]:GRTEST:
Entered: 30

$$A_{rt}{}^{\theta}{}_{r}$$

GRTEST: For prompt: Enter component [r, t, r, r]:GRTEST: Entered:
31

$$A_{rt}{}''{}_{r\theta}$$

GRTEST: For prompt: Enter component [r, t, r, theta]:GRTEST:
Entered: 32

$$A_{rt}{}''{}_{r\phi}$$

GRTEST: For prompt: Enter component [r, t, r, phi]:GRTEST:
Entered: 33

$$A_{rt}{}''{}_{rt}$$

GRTEST: For prompt: Enter component [r, t, r, t]:GRTEST: Entered:
34

$$A_{rt}{}''{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [r, t, theta, theta]:GRTEST:
Entered: 35

$$A_{rt}{}''{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [r, t, theta, phi]:GRTEST:
Entered: 36

$$A_{rt}{}''{}_{\theta t}$$

GRTEST: For prompt: Enter component [r, t, theta, t]:GRTEST:
Entered: 37

$$A_{rt}{}''{}_{\phi\phi}$$

GRTEST: For prompt: Enter component [r, t, phi, phi]:GRTEST:
Entered: 38

$$A_{rt}{}''{}_{\phi t}$$

GRTEST: For prompt: Enter component [r, t, phi, t]:GRTEST:
Entered: 39

$$A_{rt}{}''{}_{tt}$$

GRTEST: For prompt: Enter component [r, t, t, t]:GRTEST: Entered:
40

$$A_{\theta\theta}{}''{}_{rr}$$

GRTEST: For prompt: Enter component [theta, theta, r, r]:GRTEST:
Entered: 41

$$A_{\theta\theta}{}''{}_{r\theta}$$

GRTEST: For prompt: Enter component [theta, theta, r, theta]
:GRTEST: Entered: 42

$$A_{\theta\theta}{}''{}_{r\phi}$$

GRTEST: For prompt: Enter component [theta, theta, r, phi]:GRTEST:
Entered: 43

$$A_{\theta\theta}{}''{}_{rt}$$

GRTEST: For prompt: Enter component [theta, theta, r, t]:GRTEST:
Entered: 44

$$A_{\theta\theta}{}''{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [theta, theta, theta, theta]
:GRTEST: Entered: 45

$$A_{\theta\theta}{}''{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [theta, theta, theta, phi]
:GRTEST: Entered: 46

$$A_{\theta\theta}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, theta, theta, t]
:GRTEST: Entered: 47

$$A_{\theta\theta}{}^{\phi}{}_{\phi}$$

GRTEST: For prompt: Enter component [theta, theta, phi, phi]
:GRTEST: Entered: 48

$$A_{\theta\theta}{}^{\phi}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, theta, phi, t]:GRTEST:
Entered: 49

$$A_{\theta\theta}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, theta, t, t]:GRTEST:
Entered: 50

$$A_{\theta\phi}{}^r{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, r, r]:GRTEST:
Entered: 51

$$A_{\theta\phi}{}^r{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, r, theta]:GRTEST:
Entered: 52

$$A_{\theta\phi}{}^r{}_{\phi}$$

GRTEST: For prompt: Enter component [theta, phi, r, phi]:GRTEST:
Entered: 53

$$A_{\theta\phi}{}^r{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, r, t]:GRTEST:
Entered: 54

$$A_{\theta\phi}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, theta, theta]
:GRTEST: Entered: 55

$$A_{\theta\phi}{}^{\theta}{}_{\phi}$$

GRTEST: For prompt: Enter component [theta, phi, theta, phi]
:GRTEST: Entered: 56

$$A_{\theta\phi}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, theta, t]:GRTEST:
Entered: 57

$$A_{\theta\phi}{}^{\phi}{}_{\phi}$$

GRTEST: For prompt: Enter component [theta, phi, phi, phi]:GRTEST:
Entered: 58

$$A_{\theta\phi}{}^{\phi}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, phi, t]:GRTEST:
Entered: 59

$$A_{\theta\phi}{}^{\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, phi, t, t]:GRTEST:
Entered: 60

$$A_{\theta t}{}^r{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, t, r, r]:GRTEST:
Entered: 61

$$A_{\theta t}{}^{rr}{}_{\theta}$$

GRTEST: For prompt: Enter component [theta, t, r, theta]:GRTEST:
Entered: 62

$$A_{\theta t}{}^{rr}{}_{\phi}$$

GRTEST: For prompt: Enter component [theta, t, r, phi]:GRTEST:
Entered: 63

$$A_{\theta t}{}^{rr}{}_{tt}$$

GRTEST: For prompt: Enter component [theta, t, r, t]:GRTEST:
Entered: 64

$$A_{\theta t}{}^{rr}{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [theta, t, theta, theta]:
GRTEST: Entered: 65

$$A_{\theta t}{}^{rr}{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [theta, t, theta, phi]:GRTEST:
Entered: 66

$$A_{\theta t}{}^{rr}{}_{\theta t}$$

GRTEST: For prompt: Enter component [theta, t, theta, t]:GRTEST:
Entered: 67

$$A_{\theta t}{}^{rr}{}_{\phi\phi}$$

GRTEST: For prompt: Enter component [theta, t, phi, phi]:GRTEST:
Entered: 68

$$A_{\theta t}{}^{rr}{}_{\phi t}$$

GRTEST: For prompt: Enter component [theta, t, phi, t]:GRTEST:
Entered: 69

$$A_{\theta t}{}^{rr}{}_{ttt}$$

GRTEST: For prompt: Enter component [theta, t, t, t]:GRTEST:
Entered: 70

$$A_{\phi\phi}{}^{rr}{}_{rr}$$

GRTEST: For prompt: Enter component [phi, phi, r, r]:GRTEST:
Entered: 71

$$A_{\phi\phi}{}^{rr}{}_{r\theta}$$

GRTEST: For prompt: Enter component [phi, phi, r, theta]:GRTEST:
Entered: 72

$$A_{\phi\phi}{}^{rr}{}_{r\phi}$$

GRTEST: For prompt: Enter component [phi, phi, r, phi]:GRTEST:
Entered: 73

$$A_{\phi\phi}{}^{rr}{}_{rt}$$

GRTEST: For prompt: Enter component [phi, phi, r, t]:GRTEST:
Entered: 74

$$A_{\phi\phi}{}^{rr}{}_{\theta\theta}$$

GRTEST: For prompt: Enter component [phi, phi, theta, theta]:
GRTEST: Entered: 75

$$A_{\phi\phi}{}^{rr}{}_{\theta\phi}$$

GRTEST: For prompt: Enter component [phi, phi, theta, phi]:GRTEST:
Entered: 76

$$A_{\phi\phi\theta t}$$

GRTEST: For prompt: Enter component [phi, phi, theta, t]:GRTEST:
Entered: 77

$$A_{\phi\phi\phi\phi}$$

GRTEST: For prompt: Enter component [phi, phi, phi, phi]:GRTEST:
Entered: 78

$$A_{\phi\phi\phi t}$$

GRTEST: For prompt: Enter component [phi, phi, phi, t]:GRTEST:
Entered: 79

$$A_{\phi\phi t t}$$

GRTEST: For prompt: Enter component [phi, phi, t, t]:GRTEST:
Entered: 80

$$A_{\phi t r r}$$

GRTEST: For prompt: Enter component [phi, t, r, r]:GRTEST:
Entered: 81

$$A_{\phi t r \theta}$$

GRTEST: For prompt: Enter component [phi, t, r, theta]:GRTEST:
Entered: 82

$$A_{\phi t r \phi}$$

GRTEST: For prompt: Enter component [phi, t, r, phi]:GRTEST:
Entered: 83

$$A_{\phi t r t}$$

GRTEST: For prompt: Enter component [phi, t, r, t]:GRTEST:
Entered: 84

$$A_{\phi t \theta \theta}$$

GRTEST: For prompt: Enter component [phi, t, theta, theta]:GRTEST:
Entered: 85

$$A_{\phi t \theta \phi}$$

GRTEST: For prompt: Enter component [phi, t, theta, phi]:GRTEST:
Entered: 86

$$A_{\phi t \theta t}$$

GRTEST: For prompt: Enter component [phi, t, theta, t]:GRTEST:
Entered: 87

$$A_{\phi t \phi \phi}$$

GRTEST: For prompt: Enter component [phi, t, phi, phi]:GRTEST:
Entered: 88

$$A_{\phi t \phi t}$$

GRTEST: For prompt: Enter component [phi, t, phi, t]:GRTEST:
Entered: 89

$$A_{\phi t t t}$$

GRTEST: For prompt: Enter component [phi, t, t, t]:GRTEST:
Entered: 90

$$A_{t t r r}$$

GRTEST: For prompt: Enter component [t, t, r, r]:GRTEST: Entered:
91

$$A_{tt}{}^{rr}{}_{\theta}$$

GRTEST: For prompt: Enter component [t, t, r, theta]:GRTEST:
Entered: 92

$$A_{tt}{}^{rr}{}_{\phi}$$

GRTEST: For prompt: Enter component [t, t, r, phi]:GRTEST:
Entered: 93

$$A_{tt}{}^{rr}{}_{t}$$

GRTEST: For prompt: Enter component [t, t, r, t]:GRTEST: Entered:
94

$$A_{tt}{}^{\theta\theta}{}_{\theta}$$

GRTEST: For prompt: Enter component [t, t, theta, theta]:GRTEST:
Entered: 95

$$A_{tt}{}^{\theta\theta}{}_{\phi}$$

GRTEST: For prompt: Enter component [t, t, theta, phi]:GRTEST:
Entered: 96

$$A_{tt}{}^{\theta\theta}{}_{t}$$

GRTEST: For prompt: Enter component [t, t, theta, t]:GRTEST:
Entered: 97

$$A_{tt}{}^{\phi\phi}{}_{\phi}$$

GRTEST: For prompt: Enter component [t, t, phi, phi]:GRTEST:
Entered: 98

$$A_{tt}{}^{\phi\phi}{}_{t}$$

GRTEST: For prompt: Enter component [t, t, phi, t]:GRTEST:
Entered: 99

$$A_{tt}{}^{tt}{}_{t}$$

GRTEST: For prompt: Enter component [t, t, t, t]:GRTEST: Entered:
100

Calculated A(dn,dn,dn,dn) for schw (0.472000 sec.)
CPU Time = 0.473

(7)

> grdisplay(_);

For the schw spacetime:

$$A(dn,dn,dn,dn)$$

$$A_{rr}{}^{rr}{}_{rr} = 1$$

$$A_{rr}{}^{rr}{}_{r\theta} = 2$$

$$A_{rr}{}^{rr}{}_{r\phi} = 3$$

$$A_{rr}{}^{rr}{}_{rt} = 4$$

$$A_{rr}{}^{\theta\theta}{}_{\theta\theta} = 5$$

$$A_{rr}{}^{\theta\theta}{}_{\theta\phi} = 6$$

$$A_{rr}{}^{\theta\theta}{}_{\theta t} = 7$$

$$A_{rr}{}^{\phi\phi}{}_{\phi\phi} = 8$$

$$A_{rr}{}^{\prime\prime}{}_{\phi}{}^t=9$$

$$A_{rr}{}^{\prime\prime}{}_t{}^t=10$$

$$A_{r\theta}{}^{\prime\prime}{}_r{}^r=11$$

$$A_{r\theta}{}^{\prime\prime}{}_r{}^{\theta}=12$$

$$A_{r\theta}{}^{\prime\prime}{}_r{}^{\phi}=13$$

$$A_{r\theta}{}^{\prime\prime}{}_r{}^t=14$$

$$A_{r\theta}{}^{\prime\prime}{}_{\theta}{}^{\theta}=15$$

$$A_{r\theta}{}^{\prime\prime}{}_{\theta}{}^{\phi}=16$$

$$A_{r\theta}{}^{\prime\prime}{}_{\theta}{}^t=17$$

$$A_{r\theta}{}^{\prime\prime}{}_{\phi}{}^{\phi}=18$$

$$A_{r\theta}{}^{\prime\prime}{}_{\phi}{}^t=19$$

$$A_{r\theta}{}^{\prime\prime}{}_t{}^t=20$$

$$A_{r\phi}{}^{\prime\prime}{}_r{}^r=21$$

$$A_{r\phi}{}^{\prime\prime}{}_r{}^{\theta}=22$$

$$A_{r\phi}{}^{\prime\prime}{}_r{}^{\phi}=23$$

$$A_{r\phi}{}^{\prime\prime}{}_r{}^t=24$$

$$A_{r\phi}{}^{\prime\prime}{}_{\theta}{}^{\theta}=25$$

$$A_{r\phi}{}^{\prime\prime}{}_{\theta}{}^{\phi}=26$$

$$A_{r\phi}{}^{\prime\prime}{}_{\theta}{}^t=27$$

$$A_{r\phi}{}^{\prime\prime}{}_{\phi}{}^{\phi}=28$$

$$A_{r\phi}{}^{\prime\prime}{}_{\phi}{}^t=29$$

$$A_{r\phi}{}^{\prime\prime}{}_t{}^t=30$$

$$A_{rt}{}^{\prime\prime}{}_r{}^r=31$$

$$A_{rt}{}^{\prime\prime}{}_r{}^{\theta}=32$$

$$A_{rt}{}^{\prime\prime}{}_r{}^{\phi}=33$$

$$A_{rt}{}^{\prime\prime}{}_r{}^t=34$$

$$A_{rt}{}^{\prime\prime}{}_{\theta}{}^{\theta}=35$$

$$A_{rt}{}^{\prime\prime}{}_{\theta}{}^{\phi}=36$$

$$A_{rt}{}^{\prime\prime}{}_{\theta}{}^t=37$$

$$A_{rt}{}^{\prime\prime}{}_{\phi}{}^{\phi}=38$$

$$A_{r\,t\,\,\,\phi\,\,t}=39$$

$$A_{r\,t\,\,\,t\,\,t}=40$$

$$A_{\theta\,\theta\,\,\,r\,\,r}=41$$

$$A_{\theta\,\theta\,\,\,r\,\,\theta}=42$$

$$A_{\theta\,\theta\,\,\,r\,\,\phi}=43$$

$$A_{\theta\,\theta\,\,\,r\,\,t}=44$$

$$A_{\theta\,\theta\,\,\,\theta\,\,\theta}=45$$

$$A_{\theta\,\theta\,\,\,\theta\,\,\phi}=46$$

$$A_{\theta\,\theta\,\,\,\theta\,\,t}=47$$

$$A_{\theta\,\theta\,\,\,\phi\,\,\phi}=48$$

$$A_{\theta\,\theta\,\,\,\phi\,\,t}=49$$

$$A_{\theta\,\theta\,\,\,t\,\,t}=50$$

$$A_{\theta\,\phi\,\,\,r\,\,r}=51$$

$$A_{\theta\,\phi\,\,\,r\,\,\theta}=52$$

$$A_{\theta\,\phi\,\,\,r\,\,\phi}=53$$

$$A_{\theta\,\phi\,\,\,r\,\,t}=54$$

$$A_{\theta\,\phi\,\,\,\theta\,\,\theta}=55$$

$$A_{\theta\,\phi\,\,\,\theta\,\,\phi}=56$$

$$A_{\theta\,\phi\,\,\,\theta\,\,t}=57$$

$$A_{\theta\,\phi\,\,\,\phi\,\,\phi}=58$$

$$A_{\theta\,\phi\,\,\,\phi\,\,t}=59$$

$$A_{\theta\,\phi\,\,\,t\,\,t}=60$$

$$A_{\theta\,t\,\,\,r\,\,r}=61$$

$$A_{\theta\,t\,\,\,r\,\,\theta}=62$$

$$A_{\theta\,t\,\,\,r\,\,\phi}=63$$

$$A_{\theta\,t\,\,\,r\,\,t}=64$$

$$A_{\theta\,t\,\,\,\theta\,\,\theta}=65$$

$$A_{\theta\,t\,\,\,\theta\,\,\phi}=66$$

$$A_{\theta\,t\,\,\,\theta\,\,t}=67$$

$$A_{\theta t} \phi \phi = 68$$

$$A_{\theta t} \phi t = 69$$

$$A_{\theta t} t t = 70$$

$$A_{\phi \phi} r r = 71$$

$$A_{\phi \phi} r \theta = 72$$

$$A_{\phi \phi} r \phi = 73$$

$$A_{\phi \phi} r t = 74$$

$$A_{\phi \phi} \theta \theta = 75$$

$$A_{\phi \phi} \theta \phi = 76$$

$$A_{\phi \phi} \theta t = 77$$

$$A_{\phi \phi} \phi \phi = 78$$

$$A_{\phi \phi} \phi t = 79$$

$$A_{\phi \phi} t t = 80$$

$$A_{\phi t} r r = 81$$

$$A_{\phi t} r \theta = 82$$

$$A_{\phi t} r \phi = 83$$

$$A_{\phi t} r t = 84$$

$$A_{\phi t} \theta \theta = 85$$

$$A_{\phi t} \theta \phi = 86$$

$$A_{\phi t} \theta t = 87$$

$$A_{\phi t} \phi \phi = 88$$

$$A_{\phi t} \phi t = 89$$

$$A_{\phi t} t t = 90$$

$$A_{t t} r r = 91$$

$$A_{t t} r \theta = 92$$

$$A_{t t} r \phi = 93$$

$$A_{t t} r t = 94$$

$$A_{t t} \theta \theta = 95$$

$$A_{t t} \theta \phi = 96$$

$$A_{t t} \theta t = 97$$

$$A_{t t} \phi \phi = 98$$

$$A_{t t} \phi t = 99$$

$$A_{t t} t t = 100$$

(8)

INDEX SYMMETRIES

```
> grdef("A{a b}", sym = {[1, 2]});
Created definition for A(dn,dn)
> grtestinput([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]);
Added 16 test entries
> grcalc(A(dn, dn));
```

Enter components for object A(dn,dn)
If you wish to quit at any point and leave this object uninitialized, enter the string exit.

$$A_{r r}$$

GRTEST: For prompt: Enter component [r, r]:GRTEST: Entered: 1

$$A_{r \theta}$$

GRTEST: For prompt: Enter component [r, theta]:GRTEST: Entered: 2

$$A_{r \phi}$$

GRTEST: For prompt: Enter component [r, phi]:GRTEST: Entered: 3

$$A_{r t}$$

GRTEST: For prompt: Enter component [r, t]:GRTEST: Entered: 4

$$A_{\theta \theta}$$

GRTEST: For prompt: Enter component [theta, theta]:GRTEST: Entered: 5

$$A_{\theta \phi}$$

GRTEST: For prompt: Enter component [theta, phi]:GRTEST: Entered: 6

$$A_{\theta t}$$

GRTEST: For prompt: Enter component [theta, t]:GRTEST: Entered: 7

$$A_{\phi \phi}$$

GRTEST: For prompt: Enter component [phi, phi]:GRTEST: Entered: 8

$$A_{\phi t}$$

GRTEST: For prompt: Enter component [phi, t]:GRTEST: Entered: 9

$$A_{t t}$$

GRTEST: For prompt: Enter component [t, t]:GRTEST: Entered: 10
Calculated A(dn,dn) for schw (0.043000 sec.)

CPU Time = 0.043

(9)

```
> grdef("B{a b}", asym = {[1, 2]});
Created definition for B(dn,dn)
> grtestinput([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]);
Added 16 test entries
> grcalc(B(dn, dn));
```

Enter components for object B(dn,dn)

If you wish to quit at any point and leave this object uninitialized, enter the string exit.

$$B_{r\theta}$$

GRTEST: For prompt: Enter component [r, theta]:GRTEST: Entered: 1

$$B_{r\phi}$$

GRTEST: For prompt: Enter component [r, phi]:GRTEST: Entered: 2

$$B_{rt}$$

GRTEST: For prompt: Enter component [r, t]:GRTEST: Entered: 3

$$B_{\theta\phi}$$

GRTEST: For prompt: Enter component [theta, phi]:GRTEST: Entered: 4

$$B_{\theta t}$$

GRTEST: For prompt: Enter component [theta, t]:GRTEST: Entered: 5

$$B_{\phi t}$$

GRTEST: For prompt: Enter component [phi, t]:GRTEST: Entered: 6
Calculated B(dn,dn) for schw (0.027000 sec.)

CPU Time = 0.028

(10)

> grdisplay(B(dn,dn));

For the schw spacetime:

$$B(dn,dn)$$

$$B(dn,dn)$$

$$B_{ab} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ -1 & 0 & 4 & 5 \\ -2 & -4 & 0 & 6 \\ -3 & -5 & -6 & 0 \end{bmatrix}$$

(11)

> grundef(B(dn,dn));

Cleared B(dn,dn) for the schw metric.

object B(up,up)

object B(up,dn)

object B(dn,up)

object B(dn,dn)

exec: grG_ObjDef[B(dn,dn)][grC_defineStr] := evaln(grG_ObjDef[B(dn,dn)][grC_defineStr]);

parse done

check assigned grG_ObjDef[B(dn,dn)][grC_defineStr]: false

exec: grG_ObjDef[B(dn,dn)][grC_symList] := evaln(grG_ObjDef[B(dn,dn)][grC_symList]);

parse done

check assigned grG_ObjDef[B(dn,dn)][grC_symList]: false

exec: grG_ObjDef[B(dn,dn)][grC_depends] := evaln(grG_ObjDef[B(dn,dn)][grC_depends]);

parse done

check assigned grG_ObjDef[B(dn,dn)][grC_depends]: false

exec: grG_ObjDef[B(dn,dn)][grC_symmetry] := evaln(grG_ObjDef[B(dn,dn)][grC_symmetry]);

parse done

check assigned grG_ObjDef[B(dn,dn)][grC_symmetry]: false

exec: grG_ObjDef[B(dn,dn)][grC_header] := evaln(grG_ObjDef[B(dn,dn)][grC_header]);

```

dn)][grC_header]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_header]: false
exec: grG_ObjDef[B(dn,dn)][grC_root] := evaln(grG_ObjDef[B(dn,dn)]
[grC_root]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_root]: false
exec: grG_ObjDef[B(dn,dn)][grC_attributes] := evaln(grG_ObjDef[B
(dn,dn)][grC_attributes]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_attributes]: false
exec: grG_ObjDef[B(dn,dn)][grC_preCalcFn] := evaln(grG_ObjDef[B
(dn,dn)][grC_preCalcFn]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_preCalcFn]: false
exec: grG_ObjDef[B(dn,dn)][grC_grdefArgs] := evaln(grG_ObjDef[B
(dn,dn)][grC_grdefArgs]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_grdefArgs]: false
exec: grG_ObjDef[B(dn,dn)][grC_calcFn] := evaln(grG_ObjDef[B(dn,
dn)][grC_calcFn]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_calcFn]: false
exec: grG_ObjDef[B(dn,dn)][grC_indexList] := evaln(grG_ObjDef[B
(dn,dn)][grC_indexList]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_indexList]: false
exec: grG_ObjDef[B(dn,dn)][grC_rootStr] := evaln(grG_ObjDef[B(dn,
dn)][grC_rootStr]);
parse done
check assigned grG_ObjDef[B(dn,dn)][grC_rootStr]: false
Definition for B(dn,dn) has been removed.
done

```

```

> grdef("B{a b}", asym = {[1, 2]});
Created definition for B(dn,dn)
> grtestinput([20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]);
Added 17 test entries
> grcalc(B(dn, dn));

```

Enter components for object B(dn,dn)
If you wish to quit at any point and leave this object
uninitialized, enter the string exit.

$$B_{r\theta}$$

GRTEST: For prompt: Enter component [r, theta]:GRTEST: Entered: 20

$$B_{r\phi}$$

GRTEST: For prompt: Enter component [r, phi]:GRTEST: Entered: 21

$$B_{rt}$$

GRTEST: For prompt: Enter component [r, t]:GRTEST: Entered: 22

$$B_{\theta\phi}$$

GRTEST: For prompt: Enter component [theta, phi]:GRTEST: Entered: 23

$$B_{\theta t}$$

GRTEST: For prompt: Enter component [theta, t]:GRTEST: Entered: 24

$$B_{\phi t}$$

```
GRTEST: For prompt: Enter component [phi, t]:GRTEST: Entered: 25
Calculated B(dn,dn) for schw (0.022000 sec.)
CPU Time = 0.022
```

(12)

```
> grdisplay(B(dn,dn));
```

For the schw spacetime:

$B(dn,dn)$

$B(dn,dn)$

$$B_{a\ b} = \begin{bmatrix} 0 & 20 & 21 & 22 \\ -20 & 0 & 23 & 24 \\ -21 & -23 & 0 & 25 \\ -22 & -24 & -25 & 0 \end{bmatrix}$$

(13)

```
>
```

```
>
```