```
> restart; with(grtensor);
```

libname := "/Users/peter/maple/gitlab/GRTensorIII/lib",
"/Library/Frameworks/Maple.framework/Versions/2017/lib"

"GRTensor III v2.0.2+ Debug"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*]

grOptionqloadPath := "/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics"

grOptionMetricPath := "/Users/peter/maple/gitlab/grtensor/metrics"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*] **(1)**

```
> grOptionMetricPath := "/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics/";
> qload(schw);
```

```
Calculated ds for schw (0.001000 sec.)
```

Default spacetime = schw

For the schw spacetime:

Coordinates

$x(up)$

$$x^a = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array}\right]$$

Line element

$$ds^2 = \frac{dr^2}{1-\frac{2m}{r}} + r^2\, d\theta^2 + r^2 \sin(\theta)^2\, d\phi^2 + \left(-1+\frac{2m}{r}\right) dt^2$$

The Schwarzschild metric in curvature coordinates **(2)**

```
> qload(kerr);
```

```
Calculated ds for kerr (0.001000 sec.)
```

Default spacetime = kerr

For the kerr spacetime:

Coordinates

$x(up)$

$$x^a = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array}\right]$$

Line element

$$ds^2 = \frac{\left(r^2 + a^2\cos(\theta)^2\right)\,dr^2}{a^2 - 2\,m\,r + r^2} + \left(r^2 + a^2\cos(\theta)^2\right)\,d\theta^2 + \sin(\theta)^2\left(r^2 + a^2 + \frac{2\,m\,r\,a^2\sin(\theta)^2}{r^2 + a^2\cos(\theta)^2}\right)\,d\phi^2 - \frac{4\,m\,a\,r\sin(\theta)^2\,d\phi\,``\,d\,t\,``}{r^2 + a^2\cos(\theta)^2} + \left(-1 + \frac{2\,m\,r}{r^2 + a^2\cos(\theta)^2}\right)\,dt^2$$

Kerr metric in Boyer-Lindquist coordinates. **(3)**

```
> grcalc(R[schw](dn, dn));
Calculated g(dn,dn,pdn) for schw (0.001000 sec.)
Calculated Chr(dn,dn,dn) for schw (0.001000 sec.)
Calculated detg for schw (0.003000 sec.)
Calculated g(up,up) for schw (0.006000 sec.)
Calculated Chr(dn,dn,up) for schw (0.001000 sec.)
Calculated R(dn,dn) for schw (0.001000 sec.)
```

CPU Time = 0.013 **(4)**

```
> grcalc1(R(dn, dn, dn, dn), [r, theta, r, theta]);
Calculated g(dn,dn,pdn) for kerr (0.022000 sec.)
Calculated Chr(dn,dn,dn) for kerr (0.001000 sec.)
Calculated detg for kerr (0.003000 sec.)
Calculated g(up,up) for kerr (0.004000 sec.)
Calculated the [r, theta, r, theta] component of R(dn,dn,dn,dn)
for kerr.
```

```
> grcomponent(R(dn, dn, dn, dn), [r, theta, r, theta]);
```

$$\frac{1}{\left(r^2 + a^2\cos(\theta)^2\right)\left(a^2 - 2\,m\,r + r^2\right)}\left(\cos(\theta)^4 a^4 + a^4\cos(\theta)^2\sin(\theta)^2 - \cos(\theta)^2 a^4 + 3\cos(\theta)^2 a^2 m\,r - \cos(\theta)^2 a^2 r^2 - \sin(\theta)^2 a^2 r^2 + a^2 r^2 - m\,r^3\right) \quad \textbf{(5)}$$

```
> grdef("v{^a} := [0,0,0,1]");
Components assigned for metric: kerr
Created definition for v(up)
> grcalc(RayEqn[v]);
Created a definition for v(up,cdn)
Created definition for shear(up,dn)
Created definition for v(dn)
Created a definition for v(dn,cdn)
Created definition for acc(dn)
Created definition for vor(up,dn)
Created a definition for acc(up,cdn)
Calculated Chr(dn,dn,up) for kerr (0.026000 sec.)
Calculated v(up,cdn) for kerr (0.001000 sec.)
Calculated expsc[v] for kerr (0.000000 sec.)
Calculated v(dn) for kerr (0.000000 sec.)
Calculated v(dn,cdn) for kerr (0.002000 sec.)
Calculated acc(up) || `[v]` for kerr (0.000000 sec.)
Calculated acc(dn) || `[v]` for kerr (0.000000 sec.)
Calculated vnorm[v] for kerr (0.000000 sec.)
Calculated h(dn,dn) || `[v]` for kerr (0.003000 sec.)
```

```
Calculated shear(dn,dn) || `[v]` for kerr (0.000000 sec.)
Calculated shear(up,dn) || `[v]` for kerr (0.001000 sec.)
Calculated shear[v] for kerr (0.056000 sec.)
Calculated vor(dn,dn) || `[v]` for kerr (0.003000 sec.)
Calculated vor(up,dn) || `[v]` for kerr (0.012000 sec.)
Calculated vor[v] for kerr (0.171000 sec.)
Calculated R(dn,dn) for kerr (0.023000 sec.)
Calculated acc(up) || `[v]` for kerr (0.000000 sec.)
Calculated acc(up,cdn) || `[v]` for kerr (0.009000 sec.)
Calculated RayEqn[v] for kerr (0.003000 sec.)
```

$$CPU\ Time = 0.360 \tag{6}$$

```
> grdisplay(_);
```

For the kerr spacetime:

Raychaudhuri Equation

$$RayEqn[v] = \left(-\left(m a^2 \left(\cos(\theta)^6 a^6 m - 6 \cos(\theta)^6 a^6 r + 3 a^2 m r^4 - 2 a^2 r^5 - 4 m^2 r^5 \right.\right.\right. \tag{7}$$

$$+ 5 m r^6 - 6 \cos(\theta)^6 a^4 r^3 - \cos(\theta)^4 a^6 m + 6 \cos(\theta)^4 a^6 r + 2 \cos(\theta)^4 a^4 r^3$$

$$- 4 \cos(\theta)^4 a^2 r^5 + 4 \cos(\theta)^2 a^4 r^3 + 6 \cos(\theta)^2 a^2 r^5 + 4 \cos(\theta)^2 m^2 r^5$$

$$- 5 \cos(\theta)^2 m r^6 + 2 \sin(\theta)^2 a^2 r^5 + 4 \sin(\theta)^2 m^2 r^5 - 5 \sin(\theta)^2 m r^6 + 2 \cos(\theta)^2 r^7$$

$$+ 2 \sin(\theta)^2 r^7 - 2 r^7 + 11 \cos(\theta)^6 a^4 m r^2 + \cos(\theta)^4 \sin(\theta)^2 a^6 m$$

$$- 6 \cos(\theta)^4 \sin(\theta)^2 a^6 r - 6 \cos(\theta)^4 \sin(\theta)^2 a^4 r^3 - \cos(\theta)^4 a^4 m r^2$$

$$- 20 \cos(\theta)^4 a^2 m^2 r^3 + 18 \cos(\theta)^4 a^2 m r^4 - 4 \cos(\theta)^2 \sin(\theta)^2 a^4 r^3$$

$$- 4 \cos(\theta)^2 \sin(\theta)^2 a^2 r^5 + 4 \sin(\theta)^4 a^2 m r^4 - 10 \cos(\theta)^2 a^4 m r^2$$

$$+ 20 \cos(\theta)^2 a^2 m^2 r^3 - 21 \cos(\theta)^2 a^2 m r^4 - 7 \sin(\theta)^2 a^2 m r^4$$

$$+ 3 \cos(\theta)^4 \sin(\theta)^2 a^4 m r^2 - 8 \cos(\theta)^2 \sin(\theta)^4 a^4 m r^2$$

$$+ 18 \cos(\theta)^2 \sin(\theta)^2 a^4 m r^2 - 20 \cos(\theta)^2 \sin(\theta)^2 a^2 m^2 r^3$$

$$+ 22 \cos(\theta)^2 \sin(\theta)^2 a^2 m r^4 \Big) \Big) \Big/ \Big(\left(r^2 + a^2 \cos(\theta)^2\right)^4 \left(\cos(\theta)^2 a^4 + \cos(\theta)^2 a^2 r^2\right.$$

$$\left. + 2 m r a^2 \sin(\theta)^2 - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4 \right) \Big) = - \Big(\left(\cos(\theta)^8 a^{10} m\right.$$
$$- 6 \cos(\theta)^8 a^{10} r - 6 \cos(\theta)^8 a^8 r^3 + \cos(\theta)^6 a^{10} m + 6 \cos(\theta)^6 a^{10} r$$
$$- 4 \cos(\theta)^6 a^8 r^3 - 10 \cos(\theta)^6 a^6 r^5 + 10 \cos(\theta)^4 a^8 r^3 + 8 \cos(\theta)^4 a^6 r^5$$
$$- 2 \cos(\theta)^4 a^4 r^7 + 2 \cos(\theta)^2 a^6 r^5 + 4 \cos(\theta)^2 a^4 r^7 + 2 \cos(\theta)^2 a^2 r^9$$
$$+ 2 \sin(\theta)^2 a^4 r^7 + 2 \sin(\theta)^2 a^2 r^9 - 4 a^4 m^2 r^5 + 5 a^4 m r^6 + 8 a^2 m^3 r^6$$
$$- 16 a^2 m^2 r^7 + 9 a^2 m r^8 - 2 a^4 r^7 - 2 a^2 r^9 + 8 m^3 r^8 - 8 m^2 r^9 + 2 m r^{10}$$
$$+ 3 \cos(\theta)^6 \sin(\theta)^2 a^8 m r^2 - 8 \cos(\theta)^4 \sin(\theta)^4 a^8 m r^2 + 4 \cos(\theta)^4 \sin(\theta)^2 a^8 m^2 r$$
$$+ 27 \cos(\theta)^4 \sin(\theta)^2 a^8 m r^2 - 8 \cos(\theta)^4 \sin(\theta)^2 a^6 m^3 r^2$$
$$- 16 \cos(\theta)^4 \sin(\theta)^2 a^6 m^2 r^3 + 33 \cos(\theta)^4 \sin(\theta)^2 a^6 m r^4$$
$$+ 16 \cos(\theta)^2 \sin(\theta)^4 a^6 m^2 r^3 - 4 \cos(\theta)^2 \sin(\theta)^4 a^6 m r^4$$
$$- 24 \cos(\theta)^2 \sin(\theta)^2 a^6 m^2 r^3 + 19 \cos(\theta)^2 \sin(\theta)^2 a^6 m r^4$$
$$+ 16 \cos(\theta)^2 \sin(\theta)^2 a^4 m^3 r^4 - 40 \cos(\theta)^2 \sin(\theta)^2 a^4 m^2 r^5$$
$$+ 25 \cos(\theta)^2 \sin(\theta)^2 a^4 m r^6 + 11 \cos(\theta)^8 a^8 m r^2 + \cos(\theta)^6 \sin(\theta)^2 a^{10} m$$
$$- 6 \cos(\theta)^6 \sin(\theta)^2 a^{10} r - 6 \cos(\theta)^6 \sin(\theta)^2 a^8 r^3 - 4 \cos(\theta)^6 a^8 m^2 r$$
$$+ 4 \cos(\theta)^6 a^8 m r^2 - 24 \cos(\theta)^6 a^6 m^2 r^3 + 31 \cos(\theta)^6 a^6 m r^4$$
$$- 10 \cos(\theta)^4 \sin(\theta)^2 a^8 r^3 - 10 \cos(\theta)^4 \sin(\theta)^2 a^6 r^5 - 4 \cos(\theta)^4 a^8 m^2 r$$
$$- 13 \cos(\theta)^4 a^8 m r^2 + 8 \cos(\theta)^4 a^6 m^3 r^2 + 16 \cos(\theta)^4 a^6 m^2 r^3$$
$$- 26 \cos(\theta)^4 a^6 m r^4 + 8 \cos(\theta)^4 a^4 m^3 r^4 - 16 \cos(\theta)^4 a^4 m^2 r^5$$
$$+ 11 \cos(\theta)^4 a^4 m r^6 - 2 \cos(\theta)^2 \sin(\theta)^2 a^6 r^5 - 2 \cos(\theta)^2 \sin(\theta)^2 a^4 r^7$$
$$+ 4 \sin(\theta)^4 a^4 m r^6 + 8 \cos(\theta)^2 a^6 m^2 r^3 - 9 \cos(\theta)^2 a^6 m r^4 - 16 \cos(\theta)^2 a^4 m^3 r^4$$
$$+ 36 \cos(\theta)^2 a^4 m^2 r^5 - 20 \cos(\theta)^2 a^4 m r^6 - 16 \cos(\theta)^2 a^2 m^3 r^6$$
$$+ 16 \cos(\theta)^2 a^2 m^2 r^7 - 7 \cos(\theta)^2 a^2 m r^8 + 4 \sin(\theta)^2 a^4 m^2 r^5 - 7 \sin(\theta)^2 a^4 m r^6$$

$$- 8\sin(\theta)^2 a^2 m^3 r^6 + 8\sin(\theta)^2 a^2 m^2 r^7 - 5\sin(\theta)^2 a^2 m r^8) m) \Big/ ((r^2 + a^2\cos(\theta)^2)^5 (\cos(\theta)^2 a^4 + \cos(\theta)^2 a^2 r^2 + 2 m r a^2 \sin(\theta)^2 - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4)))$$

```
> grcalc(E[v](dn, dn));
Calculated Ricciscalar for kerr (0.008000 sec.)
Calculated R(dn,dn,dn,dn) for kerr (0.085000 sec.)
Calculated C(dn,dn,dn,dn) for kerr (0.068000 sec.)
Calculated E(dn,dn) || `[v]` for kerr (0.003000 sec.)
```

CPU Time = 0.167 **(8)**

```
> grdisplay(_);
```

For the kerr spacetime:

Electric part of Weyl

$E(dn, dn)$

$$E_{a\,b} = [[(\cos(\theta)^{10} a^{10} r^4 + \cos(\theta)^8 \sin(\theta)^2 a^{14} + 2\cos(\theta)^8 a^{12} m^2 + 3\cos(\theta)^8 a^{10} r^4 \quad \textbf{(9)}$$

$$+ 2\cos(\theta)^8 a^8 r^6 - 2\cos(\theta)^6 a^{12} m^2 - 2\cos(\theta)^6 a^{12} r^2 - 4\cos(\theta)^6 a^{10} r^4$$

$$- 2\cos(\theta)^6 a^8 r^6 - 2\cos(\theta)^4 a^8 r^6 - 4\cos(\theta)^4 a^6 r^8 - 2\cos(\theta)^4 a^4 r^{10}$$

$$+ 2\cos(\theta)^2 a^8 r^6 + 3\cos(\theta)^2 a^6 r^8 - \cos(\theta)^2 a^2 r^{12} - \sin(\theta)^2 a^6 r^8$$

$$- 2\sin(\theta)^2 a^4 r^{10} - \sin(\theta)^2 a^2 r^{12} - 32 a^6 m^3 r^5 + 38 a^6 m^2 r^6 - 13 a^6 m r^7$$

$$+ 48 a^4 m^4 r^6 - 140 a^4 m^3 r^7 + 116 a^4 m^2 r^8 - 33 a^4 m r^9 + 128 a^2 m^4 r^8$$

$$+ 2\cos(\theta)^{10} a^{12} r^2 - 228 a^2 m^3 r^9 + 138 a^2 m^2 r^{10} - 30 a^2 m r^{11}$$

$$+ 37\cos(\theta)^8 \sin(\theta)^2 a^{12} m r + 19\cos(\theta)^8 \sin(\theta)^2 a^{10} m^2 r^2$$

$$+ 35\cos(\theta)^8 \sin(\theta)^2 a^{10} m r^3 + 22\cos(\theta)^6 \sin(\theta)^4 a^{12} m r$$

$$+ 51 \cos(\theta)^6 \sin(\theta)^4 a^{10} m^2 r^2 + 22 \cos(\theta)^6 \sin(\theta)^4 a^{10} m r^3$$

$$+ 32 \cos(\theta)^4 \sin(\theta)^6 a^{10} m^2 r^2 + 11 \cos(\theta)^6 \sin(\theta)^2 a^{12} m r$$

$$+ 4 \cos(\theta)^6 \sin(\theta)^2 a^{10} m^3 r - 142 \cos(\theta)^6 \sin(\theta)^2 a^{10} m^2 r^2$$

$$+ 120 \cos(\theta)^6 \sin(\theta)^2 a^{10} m r^3 - 112 \cos(\theta)^6 \sin(\theta)^2 a^8 m^3 r^3$$

$$- 83 \cos(\theta)^6 \sin(\theta)^2 a^8 m^2 r^4 + 101 \cos(\theta)^6 \sin(\theta)^2 a^8 m r^5$$

$$+ 4 \cos(\theta)^4 \sin(\theta)^4 a^{10} m^3 r + 13 \cos(\theta)^4 \sin(\theta)^4 a^{10} m^2 r^2$$

$$+ 36 \cos(\theta)^4 \sin(\theta)^4 a^{10} m r^3 - 152 \cos(\theta)^4 \sin(\theta)^4 a^8 m^3 r^3$$

$$+ 95 \cos(\theta)^4 \sin(\theta)^4 a^8 m^2 r^4 + 36 \cos(\theta)^4 \sin(\theta)^4 a^8 m r^5$$

$$+ 32 \cos(\theta)^2 \sin(\theta)^6 a^8 m^3 r^3 + 20 \cos(\theta)^2 \sin(\theta)^6 a^8 m^2 r^4$$

$$- 8 \cos(\theta)^4 \sin(\theta)^2 a^{10} m^3 r + 25 \cos(\theta)^4 \sin(\theta)^2 a^{10} m^2 r^2$$

$$+ 25 \cos(\theta)^4 \sin(\theta)^2 a^{10} m r^3 - 8 \cos(\theta)^4 \sin(\theta)^2 a^8 m^3 r^3$$

$$- 180 \cos(\theta)^4 \sin(\theta)^2 a^8 m^2 r^4 + 122 \cos(\theta)^4 \sin(\theta)^2 a^8 m r^5$$

$$+ 160 \cos(\theta)^4 \sin(\theta)^2 a^6 m^4 r^4 - 220 \cos(\theta)^4 \sin(\theta)^2 a^6 m^3 r^5$$

$$- 133 \cos(\theta)^4 \sin(\theta)^2 a^6 m^2 r^6 + 85 \cos(\theta)^4 \sin(\theta)^2 a^6 m r^7$$

$$- 4 \cos(\theta)^2 \sin(\theta)^4 a^8 m^3 r^3 + 23 \cos(\theta)^2 \sin(\theta)^4 a^8 m^2 r^4$$

$$+ 6 \cos(\theta)^2 \sin(\theta)^4 a^8 m r^5 - 128 \cos(\theta)^2 \sin(\theta)^4 a^6 m^4 r^4$$

$$- 100 \cos(\theta)^2 \sin(\theta)^4 a^6 m^3 r^5 + 37 \cos(\theta)^2 \sin(\theta)^4 a^6 m^2 r^6$$

$$+ 6 \cos(\theta)^2 \sin(\theta)^4 a^6 m r^7 - 88 \cos(\theta)^2 \sin(\theta)^2 a^8 m^3 r^3$$

$$- 13 \cos(\theta)^2 \sin(\theta)^2 a^8 m^2 r^4 + 17 \cos(\theta)^2 \sin(\theta)^2 a^8 m r^5$$

$$+ 256 \cos(\theta)^2 \sin(\theta)^2 a^6 m^4 r^4 - 188 \cos(\theta)^2 \sin(\theta)^2 a^6 m^3 r^5$$

$$- 50 \cos(\theta)^2 \sin(\theta)^2 a^6 m^2 r^6 + 32 \cos(\theta)^2 \sin(\theta)^2 a^6 m r^7$$

$$+ 384 \cos(\theta)^2 \sin(\theta)^2 a^4 m^4 r^6 - 192 \cos(\theta)^2 \sin(\theta)^2 a^4 m^3 r^7$$

$$- \cos(\theta)^2 \sin(\theta)^2 a^4 m^2 r^8 + 7 \cos(\theta)^2 \sin(\theta)^2 a^4 m r^9 + 15 \cos(\theta)^{10} a^{12} m r$$

$$+ 13 \cos(\theta)^{10} a^{10} m r^3 + \cos(\theta)^8 \sin(\theta)^2 a^{12} m^2 + 2 \cos(\theta)^8 \sin(\theta)^2 a^{12} r^2$$

$$+ \cos(\theta)^8 \sin(\theta)^2 a^{10} r^4 + \cos(\theta)^6 \sin(\theta)^4 a^{12} m^2 - 96 \cos(\theta)^8 a^{10} m^2 r^2$$

$$+ 57 \cos(\theta)^8 a^{10} m r^3 - 86 \cos(\theta)^8 a^8 m^2 r^4 + 47 \cos(\theta)^8 a^8 m r^5$$

$$+ \cos(\theta)^6 \sin(\theta)^2 a^{12} m^2 + 2 \cos(\theta)^6 \sin(\theta)^2 a^{12} r^2 + 4 \cos(\theta)^6 \sin(\theta)^2 a^{10} r^4$$

$$+ 2 \cos(\theta)^6 \sin(\theta)^2 a^8 r^6 + 21 \cos(\theta)^6 a^{12} m r - 4 \cos(\theta)^6 a^{10} m^3 r$$

$$
\begin{aligned}
&- 50\cos(\theta)^6 a^{10} m^2 r^2 + 69\cos(\theta)^6 a^{10} m r^3 + 212\cos(\theta)^6 a^8 m^3 r^3 \\
&- 374\cos(\theta)^6 a^8 m^2 r^4 + 153\cos(\theta)^6 a^8 m r^5 + 192\cos(\theta)^6 a^6 m^3 r^5 \\
&- 278\cos(\theta)^6 a^6 m^2 r^6 + 85\cos(\theta)^6 a^6 m r^7 - 12\sin(\theta)^6 a^6 m^2 r^6 \\
&+ 4\cos(\theta)^4 a^{10} m^3 r - 70\cos(\theta)^4 a^{10} m^2 r^2 + 29\cos(\theta)^4 a^{10} m r^3 \\
&+ 160\cos(\theta)^4 a^8 m^3 r^3 - 302\cos(\theta)^4 a^8 m^2 r^4 + 105\cos(\theta)^4 a^8 m r^5 \\
&- 160\cos(\theta)^4 a^6 m^4 r^4 + 704\cos(\theta)^4 a^6 m^3 r^5 - 634\cos(\theta)^4 a^6 m^2 r^6 \\
&+ 165\cos(\theta)^4 a^6 m r^7 - 144\cos(\theta)^4 a^4 m^4 r^6 + 476\cos(\theta)^4 a^4 m^3 r^7 \\
&- 330\cos(\theta)^4 a^4 m^2 r^8 + 69\cos(\theta)^4 a^4 m r^9 - 2\cos(\theta)^2 \sin(\theta)^2 a^8 r^6 \\
&- 4\cos(\theta)^2 \sin(\theta)^2 a^6 r^8 - 2\cos(\theta)^2 \sin(\theta)^2 a^4 r^{10} - 32\sin(\theta)^4 a^6 m^3 r^5 \\
&+ 11\sin(\theta)^4 a^6 m^2 r^6 - 8\sin(\theta)^4 a^6 m r^7 + 48\sin(\theta)^4 a^4 m^4 r^6 + 28\sin(\theta)^4 a^4 m^3 r^7 \\
&- 7\sin(\theta)^4 a^4 m^2 r^8 - 8\sin(\theta)^4 a^4 m r^9 + 60\cos(\theta)^2 a^8 m^3 r^3 - 30\cos(\theta)^2 a^8 m^2 r^4 \\
&- 5\cos(\theta)^2 a^8 m r^5 - 128\cos(\theta)^2 a^6 m^4 r^4 + 288\cos(\theta)^2 a^6 m^3 r^5 \\
&- 134\cos(\theta)^2 a^6 m^2 r^6 + 3\cos(\theta)^2 a^6 m r^7 - 384\cos(\theta)^2 a^4 m^4 r^6 \\
&+ 528\cos(\theta)^2 a^4 m^3 r^7 - 218\cos(\theta)^2 a^4 m^2 r^8 + 24\cos(\theta)^2 a^4 m r^9
\end{aligned}
$$

$$
\begin{aligned}
&- 224\cos(\theta)^2 a^2 m^4 r^8 + 228\cos(\theta)^2 a^2 m^3 r^9 - 66\cos(\theta)^2 a^2 m^2 r^{10} \\
&+ 6\cos(\theta)^2 a^2 m r^{11} + 64\sin(\theta)^2 a^6 m^3 r^5 - 37\sin(\theta)^2 a^6 m^2 r^6 + 3\sin(\theta)^2 a^6 m r^7 \\
&- 96\sin(\theta)^2 a^4 m^4 r^6 + 112\sin(\theta)^2 a^4 m^3 r^7 - 13\sin(\theta)^2 a^4 m^2 r^8 \\
&- 7\sin(\theta)^2 a^4 m r^9 - 128\sin(\theta)^2 a^2 m^4 r^8 + 60\sin(\theta)^2 a^2 m^3 r^9 \\
&+ 30\sin(\theta)^2 a^2 m^2 r^{10} - 12\sin(\theta)^2 a^2 m r^{11} + a^2 r^{12} + 96 m^4 r^{10} - 144 m^3 r^{11} \\
&+ 72 m^2 r^{12} - 12 m r^{13} + a^6 r^8 + 2 a^4 r^{10} + \cos(\theta)^{10} a^{14} - \cos(\theta)^8 a^{14}\Big) \Big/ \Big(6\,(r^2 \\
&+ a^2\cos(\theta)^2)^3\,(a^2 - 2 m r + r^2)\,(\cos(\theta)^2 a^4 + \cos(\theta)^2 a^2 r^2 \\
&+ 2 m r a^2 \sin(\theta)^2 - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4)^2\Big),
\end{aligned}
$$

$$
\begin{aligned}
&-\Big(\cos(\theta)\, a^2\,(\cos(\theta)^{10} a^8 r^3 + 4\cos(\theta)^8 a^6 r^5 + 6\cos(\theta)^6 a^4 r^7 + 4\cos(\theta)^4 a^2 r^9 \\
&- 8\cos(\theta)^2 m^3 r^8 + 12\cos(\theta)^2 m^2 r^9 - 6\cos(\theta)^2 m r^{10} - 80\sin(\theta)^2 m^3 r^8 \\
&+ 84\sin(\theta)^2 m^2 r^9 - 24\sin(\theta)^2 m r^{10} + \cos(\theta)^{10} a^{10} r + \cos(\theta)^2 r^{11} + \sin(\theta)^2 r^{11} \\
&+ 3\cos(\theta)^8 \sin(\theta)^2 a^8 m r^2 + 4\cos(\theta)^6 \sin(\theta)^4 a^8 m^2 r + 3\cos(\theta)^6 \sin(\theta)^4 a^8 m r^2 \\
&+ 4\cos(\theta)^4 \sin(\theta)^6 a^8 m^2 r - 14\cos(\theta)^6 \sin(\theta)^2 a^8 m^2 r \\
&- 14\cos(\theta)^6 \sin(\theta)^2 a^6 m^2 r^3 - 3\cos(\theta)^6 \sin(\theta)^2 a^6 m r^4 \\
&+ 6\cos(\theta)^4 \sin(\theta)^4 a^8 m^2 r - 16\cos(\theta)^4 \sin(\theta)^4 a^6 m^3 r^2 \\
&- 2\cos(\theta)^4 \sin(\theta)^4 a^6 m^2 r^3 - 3\cos(\theta)^4 \sin(\theta)^4 a^6 m r^4 \\
&+ 8\cos(\theta)^2 \sin(\theta)^6 a^6 m^3 r^2 - 12\cos(\theta)^2 \sin(\theta)^6 a^6 m^2 r^3 \\
&+ 16\cos(\theta)^4 \sin(\theta)^2 a^4 m^3 r^4 + 12\cos(\theta)^4 \sin(\theta)^2 a^4 m^2 r^5 \\
&- 39\cos(\theta)^4 \sin(\theta)^2 a^4 m r^6 - 16\cos(\theta)^2 \sin(\theta)^4 a^6 m^3 r^2 \\
&+ 8\cos(\theta)^2 \sin(\theta)^4 a^4 m^3 r^4 - 40\cos(\theta)^2 \sin(\theta)^4 a^4 m^2 r^5
\end{aligned}
$$

$$
\begin{aligned}
&- 15\cos(\theta)^2\sin(\theta)^4 a^4 m r^6 + 8\cos(\theta)^2\sin(\theta)^2 a^6 m^3 r^2 \\
&- 24\cos(\theta)^2\sin(\theta)^2 a^2 m^3 r^6 + 110\cos(\theta)^2\sin(\theta)^2 a^2 m^2 r^7 \\
&- 57\cos(\theta)^2\sin(\theta)^2 a^2 m r^8 - \cos(\theta)^8 a^{10} r + 3\cos(\theta)^8 a^8 r^3 - 4\cos(\theta)^6 a^8 r^3 \\
&+ 2\cos(\theta)^6 a^6 r^5 - 6\cos(\theta)^4 a^6 r^5 - 2\cos(\theta)^4 a^4 r^7 - 4\cos(\theta)^2 a^4 r^7 \\
&- 3\cos(\theta)^2 a^2 r^9 + \sin(\theta)^2 a^2 r^9 + 8 a^2 m^3 r^6 - 12 a^2 m^2 r^7 + 6 a^2 m r^8 - a^2 r^9 \\
&+ 8 m^3 r^8 - 12 m^2 r^9 + 6 m r^{10} + 3\cos(\theta)^8\sin(\theta)^2 a^{10} m + \cos(\theta)^8\sin(\theta)^2 a^{10} r \\
&+ \cos(\theta)^8\sin(\theta)^2 a^8 r^3 + 3\cos(\theta)^6\sin(\theta)^4 a^{10} m - 6\cos(\theta)^8 a^6 m r^4 \\
&+ 4\cos(\theta)^6\sin(\theta)^2 a^6 r^5 + 12\cos(\theta)^6 a^4 m^2 r^5 - 18\cos(\theta)^6 a^4 m r^6 \\
&+ 6\cos(\theta)^4\sin(\theta)^2 a^4 r^7 - 32\sin(\theta)^6 a^4 m^3 r^4 - 16\sin(\theta)^6 a^4 m^2 r^5 \\
&- 8\cos(\theta)^4 a^2 m^3 r^6 + 24\cos(\theta)^4 a^2 m^2 r^7 - 18\cos(\theta)^4 a^2 m r^8 \\
&+ 4\cos(\theta)^2\sin(\theta)^2 a^2 r^9 + 64\sin(\theta)^4 a^4 m^3 r^4 + 104\sin(\theta)^4 a^2 m^3 r^6 \\
&- 34\sin(\theta)^4 a^2 m^2 r^7 - 9\sin(\theta)^4 a^2 m r^8 - 32\sin(\theta)^2 a^4 m^3 r^4 - r^{11} \\
&- 3\cos(\theta)^4\sin(\theta)^4 a^8 m r^2 - 10\cos(\theta)^4\sin(\theta)^2 a^8 m^2 r \\
&- 3\cos(\theta)^4\sin(\theta)^2 a^8 m r^2 + 16\cos(\theta)^4\sin(\theta)^2 a^6 m^3 r^2 \\
&+ 2\cos(\theta)^4\sin(\theta)^2 a^6 m^2 r^3 - 42\cos(\theta)^4\sin(\theta)^2 a^6 m r^4 \\
&- 12\cos(\theta)^2\sin(\theta)^4 a^6 m^2 r^3 - 15\cos(\theta)^2\sin(\theta)^4 a^6 m r^4 \\
&+ 24\cos(\theta)^2\sin(\theta)^2 a^6 m^2 r^3 - 15\cos(\theta)^2\sin(\theta)^2 a^6 m r^4 \\
&- 16\cos(\theta)^2\sin(\theta)^2 a^4 m^3 r^4 + 134\cos(\theta)^2\sin(\theta)^2 a^4 m^2 r^5 \\
&- 72\cos(\theta)^2\sin(\theta)^2 a^4 m r^6 - 6\cos(\theta)^8 a^8 m r^2 + 3\cos(\theta)^6\sin(\theta)^2 a^{10} m \\
&+ 4\cos(\theta)^6\sin(\theta)^2 a^8 r^3 + 6\cos(\theta)^6 a^8 m r^2 + 12\cos(\theta)^6 a^6 m^2 r^3 \\
&- 12\cos(\theta)^6 a^6 m r^4 + 6\cos(\theta)^4\sin(\theta)^2 a^6 r^5 - 12\cos(\theta)^4 a^6 m^2 r^3 \\
&+ 18\cos(\theta)^4 a^6 m r^4 - 8\cos(\theta)^4 a^4 m^3 r^4 + 12\cos(\theta)^4 a^4 m^2 r^5 \\
&+ 4\cos(\theta)^2\sin(\theta)^2 a^4 r^7 - 9\sin(\theta)^4 a^4 m r^6 + 8\cos(\theta)^2 a^4 m^3 r^4 \\
&- 24\cos(\theta)^2 a^4 m^2 r^5 + 18\cos(\theta)^2 a^4 m r^6 - 12\cos(\theta)^2 a^2 m^2 r^7 \\
&+ 12\cos(\theta)^2 a^2 m r^8 + 34\sin(\theta)^2 a^4 m^2 r^5 - 9\sin(\theta)^2 a^4 m r^6 \\
&- 112\sin(\theta)^2 a^2 m^3 r^6 + 118\sin(\theta)^2 a^2 m^2 r^7 - 33\sin(\theta)^2 a^2 m r^8
\end{aligned}
$$

$$-18\sin(\theta)^4 a^4 m^2 r^5)\Big)\Big/\Big(2\left(r^2+a^2\cos(\theta)^2\right)^3\left(\cos(\theta)^2 a^4+\cos(\theta)^2 a^2 r^2\right.$$
$$\left.+2 m r a^2\sin(\theta)^2-2 a^2 m r+a^2 r^2-2 m r^3+r^4\right)^2\sin(\theta)\Big),\ 0,\ 0\Big],$$
$$\Big[-\Big(\cos(\theta)\, a^2\left(\cos(\theta)^{10} a^8 r^3+4\cos(\theta)^8 a^6 r^5+6\cos(\theta)^6 a^4 r^7\right.$$
$$+4\cos(\theta)^4 a^2 r^9-8\cos(\theta)^2 m^3 r^8+12\cos(\theta)^2 m^2 r^9-6\cos(\theta)^2 m r^{10}$$
$$-80\sin(\theta)^2 m^3 r^8+84\sin(\theta)^2 m^2 r^9-24\sin(\theta)^2 m r^{10}+\cos(\theta)^{10} a^{10} r$$
$$+\cos(\theta)^2 r^{11}+\sin(\theta)^2 r^{11}+3\cos(\theta)^8\sin(\theta)^2 a^8 m r^2+4\cos(\theta)^6\sin(\theta)^4 a^8 m^2 r$$
$$+3\cos(\theta)^6\sin(\theta)^4 a^8 m r^2+4\cos(\theta)^4\sin(\theta)^6 a^8 m^2 r$$
$$-14\cos(\theta)^6\sin(\theta)^2 a^8 m^2 r-14\cos(\theta)^6\sin(\theta)^2 a^6 m^2 r^3$$
$$-3\cos(\theta)^6\sin(\theta)^2 a^6 m r^4+6\cos(\theta)^4\sin(\theta)^4 a^8 m^2 r$$
$$-16\cos(\theta)^4\sin(\theta)^4 a^6 m^3 r^2-2\cos(\theta)^4\sin(\theta)^4 a^6 m^2 r^3$$
$$-3\cos(\theta)^4\sin(\theta)^4 a^6 m r^4+8\cos(\theta)^2\sin(\theta)^6 a^6 m^3 r^2$$
$$-12\cos(\theta)^2\sin(\theta)^6 a^6 m^2 r^3+16\cos(\theta)^4\sin(\theta)^2 a^4 m^3 r^4$$
$$+12\cos(\theta)^4\sin(\theta)^2 a^4 m^2 r^5-39\cos(\theta)^4\sin(\theta)^2 a^4 m r^6$$
$$-16\cos(\theta)^2\sin(\theta)^4 a^6 m^3 r^2+8\cos(\theta)^2\sin(\theta)^4 a^4 m^3 r^4$$
$$-40\cos(\theta)^2\sin(\theta)^4 a^4 m^2 r^5-15\cos(\theta)^2\sin(\theta)^4 a^4 m r^6$$

$$+ 8\cos(\theta)^2 \sin(\theta)^2 a^6 m^3 r^2 - 24\cos(\theta)^2 \sin(\theta)^2 a^2 m^3 r^6$$

$$+ 110\cos(\theta)^2 \sin(\theta)^2 a^2 m^2 r^7 - 57\cos(\theta)^2 \sin(\theta)^2 a^2 m r^8 - \cos(\theta)^8 a^{10} r$$

$$+ 3\cos(\theta)^8 a^8 r^3 - 4\cos(\theta)^6 a^8 r^3 + 2\cos(\theta)^6 a^6 r^5 - 6\cos(\theta)^4 a^6 r^5$$

$$- 2\cos(\theta)^4 a^4 r^7 - 4\cos(\theta)^2 a^4 r^7 - 3\cos(\theta)^2 a^2 r^9 + \sin(\theta)^2 a^2 r^9 + 8 a^2 m^3 r^6$$

$$- 12 a^2 m^2 r^7 + 6 a^2 m r^8 - a^2 r^9 + 8 m^3 r^8 - 12 m^2 r^9 + 6 m r^{10}$$

$$+ 3\cos(\theta)^8 \sin(\theta)^2 a^{10} m + \cos(\theta)^8 \sin(\theta)^2 a^{10} r + \cos(\theta)^8 \sin(\theta)^2 a^8 r^3$$

$$+ 3\cos(\theta)^6 \sin(\theta)^4 a^{10} m - 6\cos(\theta)^8 a^6 m r^4 + 4\cos(\theta)^6 \sin(\theta)^2 a^6 r^5$$

$$+ 12\cos(\theta)^6 a^4 m^2 r^5 - 18\cos(\theta)^6 a^4 m r^6 + 6\cos(\theta)^4 \sin(\theta)^2 a^4 r^7$$

$$- 32\sin(\theta)^6 a^4 m^3 r^4 - 16\sin(\theta)^6 a^4 m^2 r^5 - 8\cos(\theta)^4 a^2 m^3 r^6$$

$$+ 24\cos(\theta)^4 a^2 m^2 r^7 - 18\cos(\theta)^4 a^2 m r^8 + 4\cos(\theta)^2 \sin(\theta)^2 a^2 r^9$$

$$+ 64\sin(\theta)^4 a^4 m^3 r^4 + 104\sin(\theta)^4 a^2 m^3 r^6 - 34\sin(\theta)^4 a^2 m^2 r^7$$

$$- 9\sin(\theta)^4 a^2 m r^8 - 32\sin(\theta)^2 a^4 m^3 r^4 - r^{11} - 3\cos(\theta)^4 \sin(\theta)^4 a^8 m r^2$$

$$- 10\cos(\theta)^4 \sin(\theta)^2 a^8 m^2 r - 3\cos(\theta)^4 \sin(\theta)^2 a^8 m r^2$$

$$+ 16\cos(\theta)^4 \sin(\theta)^2 a^6 m^3 r^2 + 2\cos(\theta)^4 \sin(\theta)^2 a^6 m^2 r^3$$

$$- 42 \cos(\theta)^4 \sin(\theta)^2 a^6 m r^4 - 12 \cos(\theta)^2 \sin(\theta)^4 a^6 m^2 r^3$$

$$- 15 \cos(\theta)^2 \sin(\theta)^4 a^6 m r^4 + 24 \cos(\theta)^2 \sin(\theta)^2 a^6 m^2 r^3$$

$$- 15 \cos(\theta)^2 \sin(\theta)^2 a^6 m r^4 - 16 \cos(\theta)^2 \sin(\theta)^2 a^4 m^3 r^4$$

$$+ 134 \cos(\theta)^2 \sin(\theta)^2 a^4 m^2 r^5 - 72 \cos(\theta)^2 \sin(\theta)^2 a^4 m r^6 - 6 \cos(\theta)^8 a^8 m r^2$$

$$+ 3 \cos(\theta)^6 \sin(\theta)^2 a^{10} m + 4 \cos(\theta)^6 \sin(\theta)^2 a^8 r^3 + 6 \cos(\theta)^6 a^8 m r^2$$

$$+ 12 \cos(\theta)^6 a^6 m^2 r^3 - 12 \cos(\theta)^6 a^6 m r^4 + 6 \cos(\theta)^4 \sin(\theta)^2 a^6 r^5$$

$$- 12 \cos(\theta)^4 a^6 m^2 r^3 + 18 \cos(\theta)^4 a^6 m r^4 - 8 \cos(\theta)^4 a^4 m^3 r^4$$

$$+ 12 \cos(\theta)^4 a^4 m^2 r^5 + 4 \cos(\theta)^2 \sin(\theta)^2 a^4 r^7 - 9 \sin(\theta)^4 a^4 m r^6$$

$$+ 8 \cos(\theta)^2 a^4 m^3 r^4 - 24 \cos(\theta)^2 a^4 m^2 r^5 + 18 \cos(\theta)^2 a^4 m r^6$$

$$- 12 \cos(\theta)^2 a^2 m^2 r^7 + 12 \cos(\theta)^2 a^2 m r^8 + 34 \sin(\theta)^2 a^4 m^2 r^5 - 9 \sin(\theta)^2 a^4 m r^6$$

$$- 112 \sin(\theta)^2 a^2 m^3 r^6 + 118 \sin(\theta)^2 a^2 m^2 r^7 - 33 \sin(\theta)^2 a^2 m r^8$$

$$- 18 \sin(\theta)^4 a^4 m^2 r^5 \Big) \Big) \Big/ \Big(2 \left(r^2 + a^2 \cos(\theta)^2 \right)^3 \left(\cos(\theta)^2 a^4 + \cos(\theta)^2 a^2 r^2 \right.$$

$$\left. + 2 m r a^2 \sin(\theta)^2 - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4 \right)^2 \sin(\theta) \Big), \left(\cos(\theta)^{10} a^{10} r^4 \right.$$
$$+ \cos(\theta)^8 \sin(\theta)^2 a^{14} - \cos(\theta)^8 a^{12} m^2 + 3 \cos(\theta)^8 a^{10} r^4 + 2 \cos(\theta)^8 a^8 r^6$$
$$+ \cos(\theta)^6 a^{12} m^2 - 2 \cos(\theta)^6 a^{12} r^2 - 4 \cos(\theta)^6 a^{10} r^4 - 2 \cos(\theta)^6 a^8 r^6$$
$$\left. - 2 \cos(\theta)^4 a^8 r^6 - 4 \cos(\theta)^4 a^6 r^8 - 2 \cos(\theta)^4 a^4 r^{10} + 2 \cos(\theta)^2 a^8 r^6 \right.$$

$$+ 3\cos(\theta)^2 a^6 r^8 - \cos(\theta)^2 a^2 r^{12} - \sin(\theta)^2 a^6 r^8 - 2\sin(\theta)^2 a^4 r^{10}$$
$$- \sin(\theta)^2 a^2 r^{12} + 10 a^6 m^3 r^5 - 7 a^6 m^2 r^6 - a^6 m r^7 - 24 a^4 m^4 r^6 + 52 a^4 m^3 r^7$$
$$- 28 a^4 m^2 r^8 - 40 a^2 m^4 r^8 + 2\cos(\theta)^{10} a^{12} r^2 + 66 a^2 m^3 r^9 - 33 a^2 m^2 r^{10}$$
$$+ 3 a^2 m r^{11} - 3\cos(\theta)^8 a^{12} m r + 24\sin(\theta)^6 a^6 m^3 r^5 - 20\cos(\theta)^8 \sin(\theta)^2 a^{12} m r$$
$$- 14\cos(\theta)^8 \sin(\theta)^2 a^{10} m^2 r^2 - 16\cos(\theta)^8 \sin(\theta)^2 a^{10} m r^3$$
$$- 8\cos(\theta)^6 \sin(\theta)^4 a^{12} m r - 30\cos(\theta)^6 \sin(\theta)^4 a^{10} m^2 r^2$$
$$- 8\cos(\theta)^6 \sin(\theta)^4 a^{10} m r^3 - 16\cos(\theta)^4 \sin(\theta)^6 a^{10} m^2 r^2$$
$$- 43\cos(\theta)^6 \sin(\theta)^2 a^{12} m r - 2\cos(\theta)^6 \sin(\theta)^2 a^{10} m^3 r$$
$$+ 122\cos(\theta)^6 \sin(\theta)^2 a^{10} m^2 r^2 - 144\cos(\theta)^6 \sin(\theta)^2 a^{10} m r^3$$
$$+ 74\cos(\theta)^6 \sin(\theta)^2 a^8 m^3 r^3 + 61\cos(\theta)^6 \sin(\theta)^2 a^8 m^2 r^4$$
$$- 85\cos(\theta)^6 \sin(\theta)^2 a^8 m r^5 - 2\cos(\theta)^4 \sin(\theta)^4 a^{10} m^3 r$$
$$- 92\cos(\theta)^4 \sin(\theta)^4 a^{10} m^2 r^2 - 18\cos(\theta)^4 \sin(\theta)^4 a^{10} m r^3$$
$$+ 154\cos(\theta)^4 \sin(\theta)^4 a^8 m^3 r^3 - 148\cos(\theta)^4 \sin(\theta)^4 a^8 m^2 r^4$$
$$- 18\cos(\theta)^4 \sin(\theta)^4 a^8 m r^5 - 64\cos(\theta)^2 \sin(\theta)^6 a^8 m^3 r^3$$
$$- 16\cos(\theta)^2 \sin(\theta)^6 a^8 m^2 r^4 + 4\cos(\theta)^4 \sin(\theta)^2 a^{10} m^3 r$$
$$+ 85\cos(\theta)^4 \sin(\theta)^2 a^{10} m^2 r^2 - 65\cos(\theta)^4 \sin(\theta)^2 a^{10} m r^3$$
$$- 110\cos(\theta)^4 \sin(\theta)^2 a^8 m^3 r^3 + 390\cos(\theta)^4 \sin(\theta)^2 a^8 m^2 r^4$$
$$- 184\cos(\theta)^4 \sin(\theta)^2 a^8 m r^5 - 104\cos(\theta)^4 \sin(\theta)^2 a^6 m^4 r^4$$
$$+ 74\cos(\theta)^4 \sin(\theta)^2 a^6 m^3 r^5 + 197\cos(\theta)^4 \sin(\theta)^2 a^6 m^2 r^6$$
$$- 95\cos(\theta)^4 \sin(\theta)^2 a^6 m r^7 + 104\cos(\theta)^2 \sin(\theta)^4 a^8 m^3 r^3$$
$$- 46\cos(\theta)^2 \sin(\theta)^4 a^8 m^2 r^4 - 12\cos(\theta)^2 \sin(\theta)^4 a^8 m r^5$$
$$+ 40\cos(\theta)^2 \sin(\theta)^4 a^6 m^4 r^4 + 224\cos(\theta)^2 \sin(\theta)^4 a^6 m^3 r^5$$
$$- 74\cos(\theta)^2 \sin(\theta)^4 a^6 m^2 r^6 - 12\cos(\theta)^2 \sin(\theta)^4 a^6 m r^7$$
$$- 16\cos(\theta)^2 \sin(\theta)^2 a^8 m^3 r^3 + 47\cos(\theta)^2 \sin(\theta)^2 a^8 m^2 r^4$$
$$- \cos(\theta)^2 \sin(\theta)^2 a^8 m r^5 - 80\cos(\theta)^2 \sin(\theta)^2 a^6 m^4 r^4$$
$$- 134\cos(\theta)^2 \sin(\theta)^2 a^6 m^3 r^5 + 166\cos(\theta)^2 \sin(\theta)^2 a^6 m^2 r^6$$

$$
\begin{aligned}
&- 16\cos(\theta)^2\sin(\theta)^2 a^6 m r^7 - 192\cos(\theta)^2\sin(\theta)^2 a^4 m^4 r^6 \\
&+ 30\cos(\theta)^2\sin(\theta)^2 a^4 m^3 r^7 + 35\cos(\theta)^2\sin(\theta)^2 a^4 m^2 r^8 \\
&+ \cos(\theta)^2\sin(\theta)^2 a^4 m r^9 - 12\cos(\theta)^{10} a^{12} m r - 8\cos(\theta)^{10} a^{10} m r^3 \\
&- 2\cos(\theta)^8\sin(\theta)^2 a^{12} m^2 + 2\cos(\theta)^8\sin(\theta)^2 a^{12} r^2 + \cos(\theta)^8\sin(\theta)^2 a^{10} r^4 \\
&- 2\cos(\theta)^6\sin(\theta)^4 a^{12} m^2 + 72\cos(\theta)^8 a^{10} m^2 r^2 - 54\cos(\theta)^8 a^{10} m r^3 \\
&+ 49\cos(\theta)^8 a^8 m^2 r^4 - 31\cos(\theta)^8 a^8 m r^5 + \cos(\theta)^6\sin(\theta)^2 a^{12} m^2 \\
&+ 2\cos(\theta)^6\sin(\theta)^2 a^{12} r^2 + 4\cos(\theta)^6\sin(\theta)^2 a^{10} r^4 + 2\cos(\theta)^6\sin(\theta)^2 a^8 r^6 \\
&- 3\cos(\theta)^6 a^{12} m r + 2\cos(\theta)^6 a^{10} m^3 r + 13\cos(\theta)^6 a^{10} m^2 r^2 \\
&- 15\cos(\theta)^6 a^{10} m r^3 - 148\cos(\theta)^6 a^8 m^3 r^3 + 247\cos(\theta)^6 a^8 m^2 r^4 \\
&- 93\cos(\theta)^6 a^8 m r^5 - 102\cos(\theta)^6 a^6 m^3 r^5 + 139\cos(\theta)^6 a^6 m^2 r^6 \\
&- 41\cos(\theta)^6 a^6 m r^7 - 2\cos(\theta)^4 a^{10} m^3 r + 23\cos(\theta)^4 a^{10} m^2 r^2 \\
&- 7\cos(\theta)^4 a^{10} m r^3 - 44\cos(\theta)^4 a^8 m^3 r^3 + 85\cos(\theta)^4 a^8 m^2 r^4 \\
&- 21\cos(\theta)^4 a^8 m r^5 + 104\cos(\theta)^4 a^6 m^4 r^4 - 394\cos(\theta)^4 a^6 m^3 r^5 \\
&+ 329\cos(\theta)^4 a^6 m^2 r^6 - 69\cos(\theta)^4 a^6 m r^7 + 72\cos(\theta)^4 a^4 m^4 r^6 \\
&- 208\cos(\theta)^4 a^4 m^3 r^7 + 123\cos(\theta)^4 a^4 m^2 r^8 - 15\cos(\theta)^4 a^4 m r^9 \\
&- 2\cos(\theta)^2\sin(\theta)^2 a^8 r^6 - 4\cos(\theta)^2\sin(\theta)^2 a^6 r^8 - 2\cos(\theta)^2\sin(\theta)^2 a^4 r^{10} \\
&- 38\sin(\theta)^4 a^6 m^3 r^5 + 44\sin(\theta)^4 a^6 m^2 r^6 - 2\sin(\theta)^4 a^6 m r^7 - 24\sin(\theta)^4 a^4 m^4 r^6 \\
&- 74\sin(\theta)^4 a^4 m^3 r^7 + 44\sin(\theta)^4 a^4 m^2 r^8 - 2\sin(\theta)^4 a^4 m r^9 \\
&- 24\cos(\theta)^2 a^8 m^3 r^3 + 15\cos(\theta)^2 a^8 m^2 r^4 - 5\cos(\theta)^2 a^8 m r^5 \\
&+ 40\cos(\theta)^2 a^6 m^4 r^4 - 90\cos(\theta)^2 a^6 m^3 r^5 + 43\cos(\theta)^2 a^6 m^2 r^6 \\
&- 9\cos(\theta)^2 a^6 m r^7 + 192\cos(\theta)^2 a^4 m^4 r^6 - 276\cos(\theta)^2 a^4 m^3 r^7 \\
&+ 121\cos(\theta)^2 a^4 m^2 r^8 - 15\cos(\theta)^2 a^4 m r^9 + 88\cos(\theta)^2 a^2 m^4 r^8 \\
&- 66\cos(\theta)^2 a^2 m^3 r^9 - 3\cos(\theta)^2 a^2 m^2 r^{10} + 9\cos(\theta)^2 a^2 m r^{11} \\
&+ 4\sin(\theta)^2 a^6 m^3 r^5 - 37\sin(\theta)^2 a^6 m^2 r^6 + 21\sin(\theta)^2 a^6 m r^7 + 48\sin(\theta)^2 a^4 m^4 r^6 \\
&+ 22\sin(\theta)^2 a^4 m^3 r^7 - 100\sin(\theta)^2 a^4 m^2 r^8 + 44\sin(\theta)^2 a^4 m r^9 \\
&+ 40\sin(\theta)^2 a^2 m^4 r^8 + 54\sin(\theta)^2 a^2 m^3 r^9 - 87\sin(\theta)^2 a^2 m^2 r^{10}
\end{aligned}
$$

$$+ 27 \sin(\theta)^2 a^2 m r^{11} + a^2 r^{12} - 48 m^4 r^{10} + 72 m^3 r^{11} - 36 m^2 r^{12} + 6 m r^{13} + a^6 r^8$$
$$+ 2 a^4 r^{10} + \cos(\theta)^{10} a^{14} - \cos(\theta)^8 a^{14}) \Big/ \left(6 \left(r^2 + a^2 \cos(\theta)^2\right)^3 \left(\cos(\theta)^2 a^4\right.\right.$$
$$\left.\left. + \cos(\theta)^2 a^2 r^2 + 2 m r a^2 \sin(\theta)^2 - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4\right)^2\right), 0, 0\Big],$$
$$\Big[0, 0, -\left(\sin(\theta)^2 \left(3 \cos(\theta)^8 a^{10} m r + 5 \cos(\theta)^8 a^8 m r^3\right.\right.$$
$$+ 4 \cos(\theta)^6 \sin(\theta)^2 a^{10} r^2 + 2 \cos(\theta)^6 \sin(\theta)^2 a^8 r^4 + \cos(\theta)^6 a^{10} m r$$
$$- 18 \cos(\theta)^6 a^8 m^2 r^2 + 8 \cos(\theta)^6 a^8 m r^3 - 27 \cos(\theta)^6 a^6 m^2 r^4$$
$$+ 15 \cos(\theta)^6 a^6 m r^5 + 4 \cos(\theta)^4 \sin(\theta)^2 a^8 r^4 + 2 \cos(\theta)^4 \sin(\theta)^2 a^6 r^6$$
$$- 36 \cos(\theta)^4 a^8 m^2 r^2 + 49 \cos(\theta)^4 a^8 m r^3 + 28 \cos(\theta)^4 a^6 m^3 r^3$$
$$- 93 \cos(\theta)^4 a^6 m^2 r^4 + 56 \cos(\theta)^4 a^6 m r^5 + 36 \cos(\theta)^4 a^4 m^3 r^5$$
$$- 82 \cos(\theta)^4 a^4 m^2 r^6 + 33 \cos(\theta)^4 a^4 m r^7 - 4 \cos(\theta)^2 \sin(\theta)^2 a^6 r^6$$
$$- 2 \cos(\theta)^2 \sin(\theta)^2 a^4 r^8 - 83 \cos(\theta)^2 a^6 m^2 r^4 + 23 \cos(\theta)^2 a^6 m r^5$$
$$+ 96 \cos(\theta)^2 a^4 m^3 r^5 - 100 \cos(\theta)^2 a^4 m^2 r^6 + 28 \cos(\theta)^2 a^4 m r^7$$
$$+ 68 \cos(\theta)^2 a^2 m^3 r^7 + 17 \cos(\theta)^2 a^2 m r^9 - 55 \sin(\theta)^2 a^4 m^2 r^6$$
$$+ 29 \sin(\theta)^2 a^4 m r^7 + 44 \sin(\theta)^2 a^2 m^3 r^7 - 35 \sin(\theta)^2 a^2 m^2 r^8 + 11 \sin(\theta)^2 a^2 m r^9$$
$$+ 2 \cos(\theta)^8 a^{12} + 4 a^4 r^8 + 2 a^2 r^{10} - 24 m^3 r^9 + 24 m^2 r^{10} - 6 m r^{11}$$
$$+ 15 \cos(\theta)^4 \sin(\theta)^2 a^8 m^2 r^2 - 33 \cos(\theta)^4 \sin(\theta)^2 a^8 m r^3$$
$$- 28 \cos(\theta)^4 \sin(\theta)^2 a^6 m^3 r^3 + 11 \cos(\theta)^4 \sin(\theta)^2 a^6 m^2 r^4$$
$$+ \cos(\theta)^4 \sin(\theta)^2 a^6 m r^5 - 46 \cos(\theta)^2 \sin(\theta)^4 a^6 m^2 r^4 + 4 \cos(\theta)^2 \sin(\theta)^4 a^6 m r^5$$
$$+ 129 \cos(\theta)^2 \sin(\theta)^2 a^6 m^2 r^4 - 21 \cos(\theta)^2 \sin(\theta)^2 a^6 m r^5$$
$$- 96 \cos(\theta)^2 \sin(\theta)^2 a^4 m^3 r^5 + 55 \cos(\theta)^2 \sin(\theta)^2 a^4 m^2 r^6$$
$$- 7 \cos(\theta)^2 \sin(\theta)^2 a^4 m r^7 + 17 \cos(\theta)^6 \sin(\theta)^2 a^{10} m r$$
$$+ 5 \cos(\theta)^6 \sin(\theta)^2 a^8 m^2 r^2 + 19 \cos(\theta)^6 \sin(\theta)^2 a^8 m r^3$$
$$+ 14 \cos(\theta)^4 \sin(\theta)^4 a^8 m r^3 + 4 \cos(\theta)^8 a^{10} r^2 + 2 \cos(\theta)^8 a^8 r^4 + 2 \cos(\theta)^6 a^6 r^6$$
$$- 6 \cos(\theta)^4 a^8 r^4 - 6 \cos(\theta)^4 a^6 r^6 - 2 \cos(\theta)^4 a^4 r^8 - 2 \cos(\theta)^2 a^2 r^{10}$$
$$- 4 \sin(\theta)^2 a^4 r^8 - 2 \sin(\theta)^2 a^2 r^{10} + 38 a^4 m^2 r^6 - 25 a^4 m r^7 - 44 a^2 m^3 r^7$$
$$- 23 a^2 m r^9 + 2 \cos(\theta)^6 \sin(\theta)^2 a^{12} + \cos(\theta)^6 a^{10} m^2 - 2 \cos(\theta)^6 a^{10} r^2$$

$$
\begin{aligned}
&+ 2\cos(\theta)^6 a^8 r^4 - \cos(\theta)^4 a^{10} m^2 - 2\cos(\theta)^4 a^{10} r^2 + 2\cos(\theta)^2 a^8 r^4 \\
&+ 2\cos(\theta)^2 a^6 r^6 - 2\cos(\theta)^2 a^4 r^8 - 2\sin(\theta)^2 a^6 r^6 + 11 a^6 m^2 r^4 - 10 a^6 m r^5 \\
&- 12 a^4 m^3 r^5 + 59 a^2 m^2 r^8 + 2 a^6 r^6 - 2\cos(\theta)^6 a^{12} + 14\cos(\theta)^4 \sin(\theta)^4 a^{10} m r \\
&+ 21\cos(\theta)^4 \sin(\theta)^4 a^8 m^2 r^2 + 16\cos(\theta)^2 \sin(\theta)^6 a^8 m^2 r^2 \\
&- 28\cos(\theta)^4 \sin(\theta)^2 a^{10} m r - 50\cos(\theta)^2 \sin(\theta)^4 a^8 m^2 r^2 \\
&+ 4\cos(\theta)^2 \sin(\theta)^4 a^8 m r^3 + 44\cos(\theta)^2 \sin(\theta)^4 a^6 m^3 r^3 \\
&+ 52\cos(\theta)^2 \sin(\theta)^2 a^8 m^2 r^2 - 8\cos(\theta)^2 \sin(\theta)^2 a^8 m r^3 \\
&- 88\cos(\theta)^2 \sin(\theta)^2 a^6 m^3 r^3 - \cos(\theta)^6 \sin(\theta)^2 a^{10} m^2 - \cos(\theta)^4 \sin(\theta)^4 a^{10} m^2 \\
&+ 2\cos(\theta)^4 \sin(\theta)^2 a^{10} m^2 + 2\cos(\theta)^4 \sin(\theta)^2 a^{10} r^2 - 12\sin(\theta)^6 a^6 m^2 r^4 \\
&+ 14\cos(\theta)^4 a^{10} m r - 2\cos(\theta)^2 \sin(\theta)^2 a^8 r^4 + 35\sin(\theta)^4 a^6 m^2 r^4 \\
&- 10\sin(\theta)^4 a^6 m r^5 - 12\sin(\theta)^4 a^4 m^3 r^5 + 17\sin(\theta)^4 a^4 m^2 r^6 - 10\sin(\theta)^4 a^4 m r^7 \\
&- 18\cos(\theta)^2 a^8 m^2 r^2 + 4\cos(\theta)^2 a^8 m r^3 + 44\cos(\theta)^2 a^6 m^3 r^3 \\
&- 59\cos(\theta)^2 a^2 m^2 r^8 - 34\sin(\theta)^2 a^6 m^2 r^4 + 20\sin(\theta)^2 a^6 m r^5 \\
&+ 24\sin(\theta)^2 a^4 m^3 r^5 \Big) \Big/ \Big(6 \left(\cos(\theta)^2 a^4 + \cos(\theta)^2 a^2 r^2 + 2 m r a^2 \sin(\theta)^2\right. \\
&\left. - 2 a^2 m r + a^2 r^2 - 2 m r^3 + r^4\right) \left(r^2 + a^2 \cos(\theta)^2\right)^4 \Big), 0 \Big], \\
&\left[\begin{matrix} 0, 0, 0, 0 \end{matrix}\right] \Big]
\end{aligned}
$$

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