```
> restart; with(grtensor);
```

libname := "/Users/peter/maple/gitlab/GRTensorIII/lib",
"/Library/Frameworks/Maple.framework/Versions/2017/lib"

"GRTensor III v2.0.2+ Debug"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*]

grOptionqloadPath := "/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics"

grOptionMetricPath := "/Users/peter/maple/gitlab/grtensor/metrics"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*] **(1)**

```
> qload(npschw);
```

Default spacetime = npschw

For the npschw spacetime:

Coordinates

$x(up)$

$$x^{a} = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array}\right]$$

Basis inner product

$\eta(bup, bup)$

$$\eta^{(a)\,(b)} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

Null tetrad (contravaraint components)

$NPl(up)$

$$l^{a} = \left[\begin{array}{cccc} \dfrac{\sqrt{2}}{2\sqrt{\dfrac{r}{r-2m}}} & 0 & 0 & \dfrac{\sqrt{2}}{2\sqrt{\dfrac{r-2m}{r}}} \end{array}\right]$$

$$NPn(up)$$

$$n^a = \left[-\frac{\sqrt{2}}{2\sqrt{\frac{r}{r-2m}}} \quad 0 \quad 0 \quad \frac{\sqrt{2}}{2\sqrt{\frac{r-2m}{r}}} \right]$$

$$NPm(up)$$

$$m^a = \left[0 \quad \frac{\sqrt{2}}{2r} \quad \frac{\frac{I}{2}\sqrt{2}}{r\sin(\theta)} \quad 0 \right]$$

$$NPmbar(up)$$

$$mbar^a = \left[0 \quad \frac{\sqrt{2}}{2r} \quad \frac{-\frac{I}{2}\sqrt{2}}{r\sin(\theta)} \quad 0 \right]$$

Contravariant NPtetrad for the Schwarzschild metric in curvature coordinates **(2)**

```
> grcalc(WeylSc);
Created a definition for e(bdn,dn,pdn)
Calculated e(bup,up) for npschw (0.018000 sec.)
Calculated g(up,up) for npschw (0.020000 sec.)
Calculated detg for npschw (0.000000 sec.)
Calculated g(dn,dn) for npschw (0.021000 sec.)
Calculated e(bdn,dn) for npschw (0.000000 sec.)
Calculated e(bdn,dn,pdn) for npschw (0.003000 sec.)
Calculated lambda(bdn,bdn,bdn) for npschw (0.003000 sec.)
Calculated rot(bdn,bdn,bdn) for npschw (0.001000 sec.)
Calculated NPbeta for npschw (0.000000 sec.)
Calculated NPkappa for npschw (0.000000 sec.)
Calculated NPpi for npschw (0.000000 sec.)
Calculated NPpibar for npschw (0.001000 sec.)
Calculated NPrho for npschw (0.000000 sec.)
Calculated NPsigma for npschw (0.000000 sec.)
Calculated NPtau for npschw (0.000000 sec.)
Calculated NPalpha for npschw (0.000000 sec.)
Calculated NPalphabar for npschw (0.007000 sec.)
Calculated NPepsilon for npschw (0.000000 sec.)
Calculated NPepsilonbar for npschw (0.000000 sec.)
Calculated NPrhobar for npschw (0.001000 sec.)
Calculated Psi0 for npschw (0.000000 sec.)
Calculated NPgamma for npschw (0.000000 sec.)
Calculated NPmu for npschw (0.000000 sec.)
Calculated Psi1 for npschw (0.000000 sec.)
Calculated NPmubar for npschw (0.001000 sec.)
Calculated NPnu for npschw (0.000000 sec.)
Calculated NPbetabar for npschw (0.000000 sec.)
Calculated NPgammabar for npschw (0.001000 sec.)
Calculated NPlambda for npschw (0.000000 sec.)
Calculated NPtaubar for npschw (0.000000 sec.)
Calculated Psi2 for npschw (0.001000 sec.)
Calculated Psi3 for npschw (0.000000 sec.)
Calculated Psi4 for npschw (0.000000 sec.)
```

CPU Time = 0.065 **(3)**

```
> grdisplay(_);
```

For the npschw spacetime:

Weyl Scalar, NP Psi0

$$\Psi 0 = 0$$

Weyl Scalar, NP Psi1

$$\Psi 1 = 0$$

Weyl Scalar, NP Psi2

$$\Psi 2 = -\frac{m}{r^3}$$

Weyl Scalar, NP Psi3

$$\Psi 3 = 0$$

Weyl Scalar, NP Psi4

$$\Psi 4 = 0 \qquad \textbf{(4)}$$

```
> grcalc(testNP(bdn, bdn));
Calculated testNP(bdn,bdn) for npschw (0.001000 sec.)
```

$$CPU\ Time = 0.002 \qquad \textbf{(5)}$$

```
> grdisplay(_);
```

For the npschw spacetime:

Test NP inner product

$$testNP(bdn, bdn)$$

$$testNP_{(a)\ (b)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \textbf{(6)}$$

```
> grapply(g(dn, dn), latex, 'x');
# For the npschw metric
%a g(dn,dn)
{\frac {r}{-r+2\,m}}
%a g(dn,dn)
-{r}^{2}
%a g(dn,dn)
-{r}^{2} \left( \sin \left( \theta \right)  \right) ^{2}
%a g(dn,dn)
-{\frac {-r+2\,m}{r}}
>
```