```
> restart;
```

libname := "/Users/peter/maple/gitlab/GRTensorIII/lib",
"/Library/Frameworks/Maple.framework/Versions/2017/lib"

grOptionMetricPath := "/Users/peter/maple/gitlab/grtensor/metrics" **(1)**

```
> with(grtensor); grOptionMetricPath := "/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics";
    qload(rw);
```

"GRTensor III v2.1.10 Oct 3, 2017"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *cmcompare*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *gralterd*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grt2DG*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*]

grOptionMetricPath := "/Users/peter/maple/gitlab/GRTensorIII/kayll/metrics"

```
Calculated ds for rw (0.001000 sec.)
```

Default spacetime = rw

For the rw spacetime:

Coordinates

$x(up)$

$$x^a = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array}\right]$$

Line element

$$ds^2 = \frac{a(t)^2\, dr^2}{-k r^2 + 1} + a(t)^2 r^2\, d\theta^2 + a(t)^2 r^2 \sin(\theta)^2\, d\phi^2 - dt^2$$

Robertson-Walker Metric **(2)**

```
> grdef( `dR{^a} := R{^a ^b ; b}` );
Created definition for R(up,up)
Created a definition for R(up,up,cdn)
Created definition for dR(up)
> grdef( `bR{(a) (b)} := R{c d}*e{(a) ^c}*e{(b) ^d}` );
Created definition for bR(bdn,bdn)
> grdef("RR{a c} := (1/2)*R{a b c d}*R{^b ^d} + R{a c}");
Created definition for RR(dn,dn)
```

Next should fail (repeated index in definition)

```
> grdef("X{a b ^b}");
Created definition for X(dn,dn,up)
```

Next definition should fail (listing index mismatch)

```
> grdef("X{a c} := R{a b c d}");
```

```
Indices in name: [[], [a, c]]
Indices in definition: [[], [a, b, c, d]]
Error, (in grtensor:-grdef) lhs/rhs index conflict.
```

Next fails: Two objects on LHS

```
> grdef("X{a b}*X{c ^b} := X2{a c}");
Error, (in grtensor:-grdef) improper op or subscript selector
```

SYM/ASYM in definition

```
> grdef("A2{a b c d} := R{a (b c) d}");
Created definition for A2(dn,dn,dn,dn)
> grdef("A3{a b c d} := R{[a b c] d}");
Created definition for A3(dn,dn,dn,dn)
> grcalc(A3(dn, dn, dn, dn)); grdisplay(_);
Calculated g(dn,dn,pdn) for rw (0.006000 sec.)
Calculated Chr(dn,dn,dn) for rw (0.001000 sec.)
Calculated detg for rw (0.002000 sec.)
Calculated g(up,up) for rw (0.006000 sec.)
Calculated R(dn,dn,dn,dn) for rw (0.009000 sec.)
Calculated A3(dn,dn,dn,dn) for rw (0.004000 sec.)
                              CPU Time = 0.029
```

For the rw spacetime:

A3(dn,dn,dn,dn)

A3(dn,dn,dn,dn) = All components are zero **(3)**

```
> grdef("A4{a ^b c} := Chr{(a ^b c)}");
Created definition for Chr(dn,up,dn)
Created definition for A4(dn,up,dn)
> grdef("A5{a b c d} := R{(a |b c| d)}");
Created definition for A5(dn,dn,dn,dn)
```

SYMMETRIES in definition

```
> grdef("B{(a c)} := R{a b c d}*R{^b ^d}");
Created definition for B(dn,dn)
> grdef("B2{[a b c] d}");
Created definition for B2(dn,dn,dn,dn)
> grdef("B3{[a b] (c d)}");
Created definition for B3(dn,dn,dn,dn)
> grdef("B4{(a ^b c)}");
Created definition for B4(dn,up,dn)
> grdef("B5{(a |b c| d)}");
Created definition for B5(dn,dn,dn,dn)
> grdef("B6{a b c d}", sym = {[1, 2], [3, 4]});
Created definition for B6(dn,dn,dn,dn)
> grdef("B7{a b c d}", sym = {[3, 4]}, asym = {[1, 2]});
Created definition for B7(dn,dn,dn,dn)
> grdef("myR3{c d} := R{a b c d}*g{^a ^b}", restrict = {c = 2..4, d = 2..4});
Created definition for myR3(dn,dn)
> grdef("v{^a} := g(t)*kdelta{^a $t}");
Created definition for v(up)
> grcalc(v(up)); grdisplay(_);
Calculated grtensor:-kdelta(up,dn) for rw (0.000000 sec.)
Calculated v(up) for rw (0.001000 sec.)
                              CPU Time = 0.001
```

For the rw spacetime:

v(up)

$$v(up)$$

$$v^a = \begin{bmatrix} 0 & 0 & 0 & g(t) \end{bmatrix} \tag{4}$$

```
> grdef("X := R{^a ^b}*Box[R{a b}]");
Created definition for X
> grcalc(X);
Created a definition for R(dn,dn,cdn)
Created a definition for R(dn,dn,cdn,cdn)
Calculated Chr(dn,dn,up) for rw (0.002000 sec.)
Calculated R(dn,dn) for rw (0.002000 sec.)
Calculated R(dn,dn,cdn) for rw (0.003000 sec.)
Calculated R(dn,dn,cdn,cdn) for rw (0.013000 sec.)
Calculated Box[R(dn,dn)] for rw (0.002000 sec.)
Calculated R(up,up) for rw (0.001000 sec.)
Calculated X for rw (0.000000 sec.)
```

CPU Time = 0.055 **(5)**

```
> grdisplay(X);
```

For the rw spacetime:

X

$$\begin{aligned} X = -\frac{1}{a(t)^6}\Bigg(6\Bigg(2\left(\frac{d^4}{dt^4}a(t)\right)\left(\frac{d^2}{dt^2}a(t)\right)a(t)^4 + \left(\frac{d^4}{dt^4}a(t)\right)a(t)^3\left(\frac{d}{dt}a(t)\right)^2 \\ + 4\left(\frac{d^3}{dt^3}a(t)\right)\left(\frac{d^2}{dt^2}a(t)\right)a(t)^3\left(\frac{d}{dt}a(t)\right) + 5\left(\frac{d^3}{dt^3}a(t)\right)a(t)^2\left(\frac{d}{dt}a(t)\right)^3 \\ - 7\left(\frac{d^2}{dt^2}a(t)\right)^2 a(t)^2\left(\frac{d}{dt}a(t)\right)^2 - \left(\frac{d^2}{dt^2}a(t)\right)a(t)\left(\frac{d}{dt}a(t)\right)^4 - 4\left(\frac{d}{dt}a(t)\right)^6 \\ + \left(\frac{d^4}{dt^4}a(t)\right)a(t)^3 k + 5\left(\frac{d^3}{dt^3}a(t)\right)a(t)^2\left(\frac{d}{dt}a(t)\right)k + \left(\frac{d^2}{dt^2}a(t)\right)^2 a(t)^2 k \\ - 5\left(\frac{d^2}{dt^2}a(t)\right)a(t)\left(\frac{d}{dt}a(t)\right)^2 k - 8\left(\frac{d}{dt}a(t)\right)^4 k - 4\left(\frac{d^2}{dt^2}a(t)\right)a(t)\,k^2 \\ - 4\left(\frac{d}{dt}a(t)\right)^2 k^2\Bigg)\Bigg) \end{aligned} \tag{6}$$

```
> grdef("T{(^a ^b)} := rho(t)*kdelta{^a $t}*kdelta{^b $t} + PP(t)·(g{^a ^b} + kdelta{^a $t}
      ·kdelta{^b $t})");
Created definition for T(up,up)
> grcalcd(T(up, up));
Calculated detg for rw (0.001000 sec.)
Calculated g(up,up) for rw (0.003000 sec.)
Calculated grtensor:-kdelta(up,dn) for rw (0.000000 sec.)
Calculated T(up,up) for rw (0.001000 sec.)
```

CPU Time = 0.005

For the rw spacetime:

$$T(up,up)$$

$$T(up,up)$$

$$T^{a\ b} = \begin{bmatrix} -\frac{PP(t)\left(k r^2 - 1\right)}{a(t)^2} & 0 & 0 & 0 \\ 0 & \frac{PP(t)}{r^2\, a(t)^2} & 0 & 0 \\ 0 & 0 & \frac{PP(t)}{\sin(\theta)^2\, r^2\, a(t)^2} & 0 \\ 0 & 0 & 0 & \rho(t) \end{bmatrix} \tag{7}$$

```
> grdebug(T(up,up));
[grC_grdefArgs]:
["T{(^a ^b)} := rho(t)*kdelta{^a $t}*kdelta{^b $t} + P(t)*(g{^a ^b} + kdelta{^a $t}*kdelta{^b
    $t})"]
[grC_header]:
```

$$T(up,up)$$

```
[grC_attributes]:
```

$$\{user\_defined\_\}$$

```
[grC_symmetry]:
proc(objectName, root, calcFn)
    global a1_, a2_;
    if grG_calc and assigned(calcFn) then
        for a1_ to Ndim[grG_metricName] do
            for a2_ from a1_ to Ndim[grG_metricName] do
                gr_data[root, grG_metricName, grG_operands, a2_, a1_] := gr_data[root,
                grG_metricName, grG_operands, a1_, a2_];
                gr_data[root, grG_metricName, grG_operands, a1_, a2_
                ] := calcFn(objectName, [a1_, a2_])
            end do
        end do
    end if;
    for a1_ to Ndim[grG_metricName] do
        for a2_ from a1_ to Ndim[grG_metricName] do
            grF_symCore(objectName, [a1_, a2_], root)
        end do
    end do;
    RETURN( )
end proc
[grC_root]:
```

$$Tupup\_$$

```
[grC_indexList]:
```

$$[up, up]$$

```
[grC_calcFn]:
```

proc(*object*, *iList*)

local *s*, *explicit_t*;

explicit_t := *grF_checkExplicitIndex*(*grG_metricName*, *t*);

s := ρ(*t*) * *gr_data*[*kdeltaupdn_*, *grG_metricName*, *a1_*, *explicit_t*] * *gr_data*[*kdeltaupdn_*, *grG_metricName*, *a2_*, *explicit_t*] + *gr_data*[*P_*, *grG_metricName*](*t*) * *gr_data*[*gupup_*, *grG_metricName*, *a1_*, *a2_*] + *gr_data*[*P_*, *grG_metricName*](*t*) * *gr_data*[*kdeltaupdn_*, *grG_metricName*, *a1_*, *explicit_t*] * *gr_data*[*kdeltaupdn_*, *grG_metricName*, *a2_*, *explicit_t*]

end proc

```
[grC_defineStr]:
"T{(^a ^b)} := rho(t)*kdelta{^a $t}*kdelta{^b $t} + P(t)*(g{^a ^b} + kdelta{^a $t}*kdelta{^b $t})"
[grC_rootStr]:
```

$$T$$

```
[grC_depends]:
```

$$\{P, g(up, up), kdelta(up, dn)\}$$

```
[grC_symList]:
```

$$[\{[1, 2]\}, \varnothing]$$ **(8)**

```
>
```