```
> restart; with(grtensor); grOptionMetricPath :=
        "/Users/peter/maple/support/NorbertVandenBergh";
```

libname := "/Users/peter/maple/gitlab/GRTensorIII/lib",
"/Library/Frameworks/Maple.framework/Versions/2017/lib"

grOptionMetricPath := "/Users/peter/maple/gitlab/grtensor/metrics"

"GRTensor III v2.1.9 Sept 24, 2017"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *cmcompare*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *gralterd*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grt2DG*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*]

grOptionMetricPath := "/Users/peter/maple/support/NorbertVandenBergh" **(1)**

```
> qload(goodwain);
Calculated ds for goodwain (0.001000 sec.)
```

Default spacetime = goodwain

For the goodwain spacetime:

Coordinates

$x(up)$

$$x^a = \left[\begin{array}{cccc} r & t & x & y \end{array}\right]$$

Line element

$$ds^2 = S^2\ dr^2 - S^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ dt^2 - 4\ S^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ q\,y\ dt``dx`` \quad \textbf{(2)}$$
$$+ 4\ S^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ q\,x\ dt``dy`` + \left(S^2\ h0^2\ \left(e^{-c\,r}\right)^2 - 4\ S^2\ q^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ y^2\right)\ dx^2$$
$$+ 8\ S^2\ q^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ y\,x\ dx``dy`` + \left(S^2\ h0^2\ \left(e^{-c\,r}\right)^2$$
$$- 4\ S^2\ q^2\ f0^2\ \left(e^{-2\,c\,r}\right)^2\ x^2\right)\ dy^2$$

```
> grt2DG( );
coord := [r, t, x, y]
Signature has been flipped!
gexpr := -S^2*`&t`(dr,dr)+S^2*f0^2*exp(-2*c*r)^2*`&t`(dt,dt)+2*
S^2*f0^2*exp(-2*c*r)^2*q*y*`&t`(dt,dx)-2*S^2*f0^2*exp(-2*c*r)^2*q*
x*`&t`(dt,dy)+2*S^2*f0^2*exp(-2*c*r)^2*q*y*`&t`(dx,dt)-(S^2*h0^2*
exp(-c*r)^2-4*S^2*q^2*f0^2*exp(-2*c*r)^2*y^2)*`&t`(dx,dx)-4*S^2*
q^2*f0^2*exp(-2*c*r)^2*y*x*`&t`(dx,dy)-2*S^2*f0^2*exp(-2*c*r)^2*q*
x*`&t`(dy,dt)-4*S^2*q^2*f0^2*exp(-2*c*r)^2*y*x*`&t`(dy,dx)-(S^2*
h0^2*exp(-c*r)^2-4*S^2*q^2*f0^2*exp(-2*c*r)^2*x^2)*`&t`(dy,dy)
```

```
> coord;
```

$$[r, t, x, y] \quad \textbf{(3)}$$

Now use the DG package

```
> with(DifferentialGeometry) : with(Tensor) : with(Tools) :
> DGsetup(coord, M);
```

frame name: M (4)

```
M > gmetric := evalDG(gexpr);
```

$$gmetric := -S^2\,dr\,dr + S^2\,f0^2\,e^{-4\,c\,r}\,dt\,dt + 2\,S^2\,f0^2\,e^{-4\,c\,r}\,q\,y\,dt\,dx - 2\,S^2\,f0^2\,e^{-4\,c\,r}\,q\,x\,dt\,dy + 2\,S^2\,f0^2\,e^{-4\,c\,r}\,q\,y\,dx\,dt + e^{-2\,c\,r}\,S^2\,\left(4\,e^{-2\,c\,r}\,f0^2\,q^2\,y^2 - h0^2\right)dx\,dx - 4\,S^2\,q^2\,f0^2\,e^{-4\,c\,r}\,y\,x\,dx\,dy - 2\,S^2\,f0^2\,e^{-4\,c\,r}\,q\,x\,dy\,dt - 4\,S^2\,q^2\,f0^2\,e^{-4\,c\,r}\,y\,x\,dy\,dx - S^2\,\left(-4\,e^{-4\,c\,r}\,f0^2\,q^2\,x^2 + e^{-2\,c\,r}\,h0^2\right)dy\,dy \quad (5)$$

```
M > cm_dg := simplify( RiemannInvariants(gmetric, author = "CarminatiMcLenaghan"),
      symbolic);
```

$$cm\_dg := \mathrm{table}\left(\left[\right.\right.$$

(6)

$$\text{"m5"} = \frac{35\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^3\,\left(c^2\,h0^4 + 16\,f0^2\,q^2\right)\,\left(6\,\mathrm{I}\,c\,f0\,h0^2\,q + c^2\,h0^4 - 8\,f0^2\,q^2\right)}{54\,S^{10}\,h0^{20}},$$

$$\text{"m4"} = -\frac{1}{8\,S^{10}\,h0^{20}}\left(\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^3\,\left(6\,\mathrm{I}\,c\,f0\,h0^2\,q + c^2\,h0^4 - 8\,f0^2\,q^2\right)\,\left(6\,\mathrm{I}\,c\,f0\,h0^2\,q - c^2\,h0^4 + 8\,f0^2\,q^2\right)\right),$$

$$\text{"r1"} = \frac{11\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^2}{4\,S^4\,h0^8},\ \text{"m3"} = \frac{19\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^3\,\left(c^2\,h0^4 + 16\,f0^2\,q^2\right)}{18\,S^8\,h0^{16}},$$

$$\text{"r2"} = -\frac{9\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^3}{8\,S^6\,h0^{12}},\ \text{"m2"} = \frac{1}{18\,S^8\,h0^{16}}\left(19\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^2\,\left(12\,\mathrm{I}\,c^3\,f0\,h0^6\,q - 96\,\mathrm{I}\,c\,f0^3\,h0^2\,q^3 + c^4\,h0^8 - 52\,c^2\,f0^2\,h0^4\,q^2 + 64\,f0^4\,q^4\right)\right),$$

$$\text{"r3"} = \frac{197\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^4}{64\,S^8\,h0^{16}},\ \text{"m1"} = \frac{4\,\left(c^2\,h0^4 + 4\,f0^2\,q^2\right)^2\,\left(6\,\mathrm{I}\,c\,f0\,h0^2\,q + c^2\,h0^4 - 8\,f0^2\,q^2\right)}{3\,S^6\,h0^{12}},\ \text{"w1"}$$

$$= \frac{2\,\left(12\,\mathrm{I}\,c^3\,f0\,h0^6\,q - 96\,\mathrm{I}\,c\,f0^3\,h0^2\,q^3 + c^4\,h0^8 - 52\,c^2\,f0^2\,h0^4\,q^2 + 64\,f0^4\,q^4\right)}{3\,S^4\,h0^8},\ \text{"w2"}$$

$$= \frac{1}{9\,S^6\,h0^{12}}\left(2\,\left(18\,\mathrm{I}\,c^5\,f0\,h0^{10}\,q - 504\,\mathrm{I}\,c^3\,f0^3\,h0^6\,q^3 + c^6\,h0^{12}\right.\right.$$

$$+ 1152\,\mathrm{I}\,c\,f0^5\,h0^2\,q^5 - 132\,c^4\,f0^2\,h0^8\,q^2 + 1056\,c^2\,f0^4\,h0^4\,q^4 - 512\,f0^6\,q^6)\,)\,\Big]\Big)$$

M > $grcalc(CM)$:

```
Created definition for S(up,dn)
Created definition for C(up,up,up,up)
Created definition for CS(up,dn)
Created definition for S(up,up)
Created definition for CSstar(up,dn)
Created definition for CS(up,up)
Created definition for CSstar(up,up)
Calculated g(dn,dn,pdn) for goodwain (0.002000 sec.)
Calculated Chr(dn,dn,dn) for goodwain (0.001000 sec.)
Calculated detg for goodwain (0.002000 sec.)
Calculated g(up,up) for goodwain (0.007000 sec.)
Calculated Chr(dn,dn,up) for goodwain (0.003000 sec.)
Calculated R(dn,dn) for goodwain (0.003000 sec.)
Calculated Ricciscalar for goodwain (0.001000 sec.)
Calculated S(dn,dn) for goodwain (0.001000 sec.)
Calculated S(up,dn) for goodwain (0.002000 sec.)
Calculated R1 for goodwain (0.001000 sec.)
Calculated R2 for goodwain (0.001000 sec.)
Calculated R3 for goodwain (0.002000 sec.)
Calculated R(dn,dn,dn,dn) for goodwain (0.010000 sec.)
Calculated C(dn,dn,dn,dn) for goodwain (0.006000 sec.)
Calculated C(dn,dn,up,up) for goodwain (0.006000 sec.)
Calculated WeylSq for goodwain (0.000000 sec.)
Calculated W1R for goodwain (0.001000 sec.)
Calculated C(up,up,up,up) for goodwain (0.045000 sec.)
Calculated detg for goodwain (0.000000 sec.)
Calculated LevC(dn,dn,dn,dn) for goodwain (0.004000 sec.)
Calculated Cstar(dn,dn,up,up) for goodwain (0.002000 sec.)
Calculated W1I for goodwain (0.001000 sec.)
Calculated C2(dn,dn,up,up) for goodwain (0.002000 sec.)
Calculated W2R for goodwain (0.001000 sec.)
Calculated W2I for goodwain (0.001000 sec.)
Calculated S(up,up) for goodwain (0.000000 sec.)
Calculated CS(dn,dn) for goodwain (0.001000 sec.)
Calculated CS(up,dn) for goodwain (0.002000 sec.)
Calculated M1R for goodwain (0.000000 sec.)
Calculated Cstar(dn,dn,dn,dn) for goodwain (0.001000 sec.)
Calculated CSstar(dn,dn) for goodwain (0.001000 sec.)
Calculated CSstar(up,dn) for goodwain (0.001000 sec.)
Calculated M1I for goodwain (0.001000 sec.)
Calculated M2a for goodwain (0.000000 sec.)
Calculated M2b for goodwain (0.000000 sec.)
Calculated M2R for goodwain (0.001000 sec.)
Calculated M2I for goodwain (0.000000 sec.)
Calculated M3 for goodwain (0.001000 sec.)
Calculated M4a for goodwain (0.000000 sec.)
Calculated M4b for goodwain (0.001000 sec.)
Calculated M4 for goodwain (0.000000 sec.)
Calculated CS(up,up) for goodwain (0.001000 sec.)
Calculated M5a for goodwain (0.002000 sec.)
Calculated CSstar(up,up) for goodwain (0.000000 sec.)
Calculated M5b for goodwain (0.002000 sec.)
Calculated M5R for goodwain (0.000000 sec.)
Calculated M5c for goodwain (0.002000 sec.)
Calculated M5d for goodwain (0.002000 sec.)
Calculated M5I for goodwain (0.000000 sec.)
```

CPU Time = 0.156

(7)

```
M > assume(S > 0, c > 0, r > 0);
Error, (in assume) cannot assume on an protected name
M > gralterd(CM, simplify, radsimp);
Component simplification of a GRTensorIII object:

Applying routine simplify to object Ricciscalar
Applying routine simplify to object R1
Applying routine simplify to object R2
Applying routine simplify to object R3
Applying routine simplify to object W1R
Applying routine simplify to object W1I
Applying routine simplify to object W2R
Applying routine simplify to object W2I
Applying routine simplify to object M1R
Applying routine simplify to object M1I
Applying routine simplify to object M2R
Applying routine simplify to object M2I
Applying routine simplify to object M3
Applying routine simplify to object M4
Applying routine simplify to object M5R
Applying routine simplify to object M5I
Applying routine radsimp to object Ricciscalar
Applying routine radsimp to object R1
Applying routine radsimp to object R2
Applying routine radsimp to object R3
Applying routine radsimp to object W1R
Applying routine radsimp to object W1I
Applying routine radsimp to object W2R
Applying routine radsimp to object W2I
Applying routine radsimp to object M1R
Applying routine radsimp to object M1I
Applying routine radsimp to object M2R
Applying routine radsimp to object M2I
Applying routine radsimp to object M3
Applying routine radsimp to object M4
Applying routine radsimp to object M5R
Applying routine radsimp to object M5I
```

$$CPU\ Time = 0.241$$

For the goodwain spacetime:

Ricci scalar

$$R = -\frac{2\left(11\,c\sim^2\,h0^4 - 4\,f0^2\,q^2\right)}{h0^4\,S\sim^2}$$

CM invariant R1

$$R1 = \frac{11\left(c\sim^2\,h0^4 + 4\,f0^2\,q^2\right)^2}{4\,S\sim^4\,h0^8}$$

CM invariant R2

$$R2 = \frac{9\left(c\sim^2\,h0^4 + 4\,f0^2\,q^2\right)^3}{8\,S\sim^6\,h0^{12}}$$

CM invariant R3

$$R3 = \frac{197 \left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^4}{64 S\sim^8 h0^{16}}$$

CM invariant Re(W1)

$$W1R = \frac{2 \left(c\sim^4 h0^8 - 52 c\sim^2 f0^2 h0^4 q^2 + 64 f0^4 q^4\right)}{3 S\sim^4 h0^8}$$

CM invariant Im(W1)

$$W1I = \frac{8 f0 \sqrt{e^{-8 c\sim r}}\, q c\sim e^{4 c\sim r} \left(c\sim^2 h0^4 - 8 f0^2 q^2\right)}{h0^6 S\sim^4}$$

CM invariant Re(W2)

$$W2R = \frac{2 \left(c\sim^6 h0^{12} - 132 c\sim^4 f0^2 h0^8 q^2 + 1056 c\sim^2 f0^4 h0^4 q^4 - 512 f0^6 q^6\right)}{9 S\sim^6 h0^{12}}$$

CM invariant Im(W2)

$$W2I = \frac{4 q c\sim \left(c\sim^4 h0^8 - 28 c\sim^2 f0^2 h0^4 q^2 + 64 f0^4 q^4\right) e^{4 c\sim r} f0 \sqrt{e^{-8 c\sim r}}}{h0^{10} S\sim^6}$$

CM invariant Re(M1)

$$M1R = \frac{4 \left(c\sim^6 h0^{12} - 48 c\sim^2 f0^4 h0^4 q^4 - 128 f0^6 q^6\right)}{3 S\sim^6 h0^{12}}$$

CM invariant Im(M1)

$$M1I = \frac{8 f0 \sqrt{e^{-8 c\sim r}}\, q c\sim e^{4 c\sim r} \left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^2}{S\sim^6 h0^{10}}$$

CM invariant Re(M2)

$$M2R = \frac{19 \left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^2 \left(c\sim^4 h0^8 - 52 c\sim^2 f0^2 h0^4 q^2 + 64 f0^4 q^4\right)}{18 S\sim^8 h0^{16}}$$

CM invariant Im(M2)

$$M2I = -\frac{38 e^{4 c\sim r} \left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^2 \left(c\sim^2 h0^4 - 8 f0^2 q^2\right) f0 \sqrt{e^{-8 c\sim r}}\, q c\sim}{3 h0^{14} S\sim^8}$$

CM invariant M3

$$M3 = \frac{19 \left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^3 \left(c\sim^2 h0^4 + 16 f0^2 q^2\right)}{18 S\sim^8 h0^{16}}$$

CM invariant M4

$$M4 = -\frac{\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^4 \left(c\sim^2 h0^4 + 16 f0^2 q^2\right)}{8 S\sim^{10} h0^{20}}$$

CM invariant Re(M5)

$$M5R = \frac{35\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^3 \left(c\sim^2 h0^4 + 16 f0^2 q^2\right)\left(c\sim^2 h0^4 - 8 f0^2 q^2\right)}{54\, S\sim^{10} h0^{20}}$$

CM invariant Im(M5)

$$M5I = \frac{35\, f0 \sqrt{e^{-8\, c\sim r}}\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^3 q\, e^{4\, c\sim r}\, c\sim \left(c\sim^2 h0^4 + 16 f0^2 q^2\right)}{18\, h0^{18}\, S\sim^{10}} \quad \textbf{(8)}$$

M >

M >

Use a special function written to compare the CM from DG and GRIII.
This first does a simple difference. This shows that there are some discrepencies:
Sign differences in R2, M4 and M2I.
A factor of 2 difference in M5I.

M > cmcompare(cm_dg);

$$R1 = 0$$

$$R2 = -\frac{9\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^3}{4\, S\sim^6 h0^{12}}$$

$$R3 = 0$$

$$M3 = 0$$

$$M4 = \frac{\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^4 \left(c\sim^2 h0^4 + 16 f0^2 q^2\right)}{4\, S\sim^{10} h0^{20}}$$

$$W1R = 0$$

$$W1I = 0$$

$$W2R = 0$$

$$W2I = 0$$

$$M1R = 0$$

$$M1I = 0$$

$$M2R = 0$$

$$M2I = \frac{76\, c\sim f0\, q\left(c\sim^6 h0^{12} - 48\, c\sim^2 f0^4 h0^4 q^4 - 128 f0^6 q^6\right)}{3\, h0^{14}\, S\sim^8}$$

$$M5R = 0$$

$$M5I = \frac{35\left(c\sim^2 h0^4 + 4 f0^2 q^2\right)^3 \left(c\sim^2 h0^4 + 16 f0^2 q^2\right) c\sim q\, f0}{18\, h0^{18}\, S\sim^{10}}$$

Sum of invariants:

$$\text{"r2"} = 0$$

$$\text{"m4"} = 0$$

$$M2I = 0$$

Special case M5I: CM(M5I) - 2*GRIII(M5I)

$$M5I = 0 \quad \textbf{(9)}$$

```
M > cm_dg["r1"];
```

$$\frac{11\left(c\sim^2 h0^4 + 4\, f0^2\, q^2\right)^2}{4\, S\sim^4 h0^8} \qquad \textbf{(10)}$$

```
M > #make diff a full blown object, keep real and Im ??
M >
```