PERGAMON

Computers & Operations Research 29 (2002) 33–51

computers & operations research

www.elsevier.com/locate/dsw

A tabu search method for the truck and trailer routing problem

I-Ming Chao

Department of Management Science, Chinese Military Academy, P.O. Box 90602-6, Feng-Shan, Taiwan, ROC

Received 1 November 1999; received in revised form 1 April 2000

Abstract

A solution construction method and a tabu search improvement heuristic coupled with the deviation concept found in deterministic annealing is developed to solve the truck and trailer routing problem. We test our tabu search method on 21 problems that have been converted from the basic vehicle routing problem. Our construction method always solves a problem (it always finds a feasible solution) and the tabu search improvement heuristic significantly improves an initial solution.

Scope and purpose

The vehicle routing problem holds a central place in distribution management and logistics, and its practical significance has been well documented in the literature. The truck and trailer routing problem is a variant of the vehicle routing problem to take into account some real-life applications in which fleet of m_k trucks and m_l trailers ($m_k \geqslant m_l$) services a set of customers. Some customers can be serviced by a complete vehicle (that is, a truck pulling a trailer) or by a truck alone, whereas others can be serviced only a truck alone. There are three types of routes in a solution to the problem: (1) a pure truck route traveled by a truck alone, (2) a pure vehicle route without any sub-tours traveled by a complete vehicle, and (3) a complete vehicle route consisting of a main tour traveled by a complete vehicle, and one or more sub-tours traveled by a truck alone. A sub-tour begins and finishes at a customer on the main tour where the truck uncouples, parks, and re-couples its pulling trailer and continues to service the remaining customers on the sub-tour. The objective is to minimize the total distance traveled, or total cost incurred by the fleet. The problem is more difficult to solve than the basic vehicle routing problem, but it occurs in many real-life applications. The purpose of this article is to develop a solution method that generates an initial solution and improves the solution using tabu search. The tabu search procedure uses the deviation concept found in deterministic annealing to further improve the initial solution. Our heuristic solves the truck and trailer routing problem efficiently and effectively.

Keywords: Tabu search; Heuristic; Truck and trailer routing; Vehicle routing; Deterministic annealing

E-mail address: iming@cc.cma.edu.tw (I.M. Chao).

0305-0548/01/$ - see front matter
PII: S0305-0548(00)00056-3

1. Introduction

In the basic vehicle routing problem (VRP), routes are constructed to dispatch a fleet of homogenous or heterogeneous vehicles to service a set of customers from a single distribution depot. Each vehicle has a fixed capacity and each customer has a known demand that must be fully satisfied. Each customer must be serviced by exactly one visit of a single vehicle and each vehicle must depart from and return to the depot. Each route has a route length constraint that limits the distance traveled by each vehicle. The objective is to provide each vehicle with a sequence of visits so that all customers are serviced and the total distance traveled by the fleet (or the total travel cost incurred by the fleet) is minimized. The basic VRP and its many variants are NP-hard optimization problems [1] and have received a great deal of attention in the literature [2–5].

The truck and trailer routing problem (TTRP) extends the basic VRP to take a real-life application into account. In the TTRP, a fleet of m_k trucks and m_l trailers ($m_k \geqslant m_l$) services a set of customers from a central depot. Each truck and each trailer has a fixed capacity of Q_k and Q_l, respectively. A *complete vehicle* (a truck pulling a trailer) has a total capacity equaling of $Q_k + Q_l$. The locations of the customers vary from a city center to a mountain village and some of these locations make maneuvering a complete vehicle unlikely. Some customers, called vehicle customers (v.c.), are reachable either by a complete vehicle or by a truck alone, but others, called truck customers (t.c.), are reachable only by a truck alone. There are three types of routes in a solution to the TTRP: a pure truck route, a pure vehicle route, and a complete vehicle route. A pure truck route contains v.c. or t.c. customers serviced by a truck alone. A pure vehicle route contains only v.c. customers serviced by a complete vehicle without any sub-tours. A complete vehicle route consists of a main tour traveled by a complete vehicle and one or more sub-tours traveled by a truck alone. A sub-tour starts from and returns to a customer found on the main tour (the trailer is parked at this customer). At the parking place, the truck uncouples and parks the trailer that it is pulling, departs to service the customers on the sub-tour, returns to re-couple the trailer, and then continues to service the remaining customers on the main tour. In Fig. 1, we give two solutions to a TTRP with 38 v.c. customers and 12 t.c. customers. The solid segments, passing only v.c. customers which are depicted by circles, nodes, are edges traveled by a complete vehicle. The dashed segments, passing v.c. customers or t.c. customers that are depicted by dots, are edges traveled by a truck alone on a pure truck route or on a sub-tour. Solution 1 with an objective function value of 600.35 has two pure truck routes and three complete vehicle routes (two have one sub-tour and one has two sub-tours). Solution 2 with an objective function value of 565.02 has two pure truck routes, one pure vehicle route, and two complete vehicle routes with one sub-tour each.

In the TTRP cost can be incurred in several different ways including the different traveling costs by a complete vehicle or by a truck alone, the cost of parking a trailer, the cost of shifting demands between a truck and its pulling trailer, and the fixed cost of maintaining the fleet. For simplicity, we assume that the cost in the TTRP is proportional to the distance traveled by the fleet; therefore, the objective of the TTRP is to minimize the total distance traveled by the fleet on all three types of routes. In addition, we assume that the numbers of trucks and trailers are known in advance and all trucks and trailers are identical. Therefore, each truck is able to pull a trailer. Finally, we assume that the depot and every v.c. customer location can be selected as a trailer-parking place.

It is worth noting that the total demand loads in a pure truck route or in a sub-tour cannot exceed the truck capacity Q_k; however, the summation of the demand loads of all sub-tours in

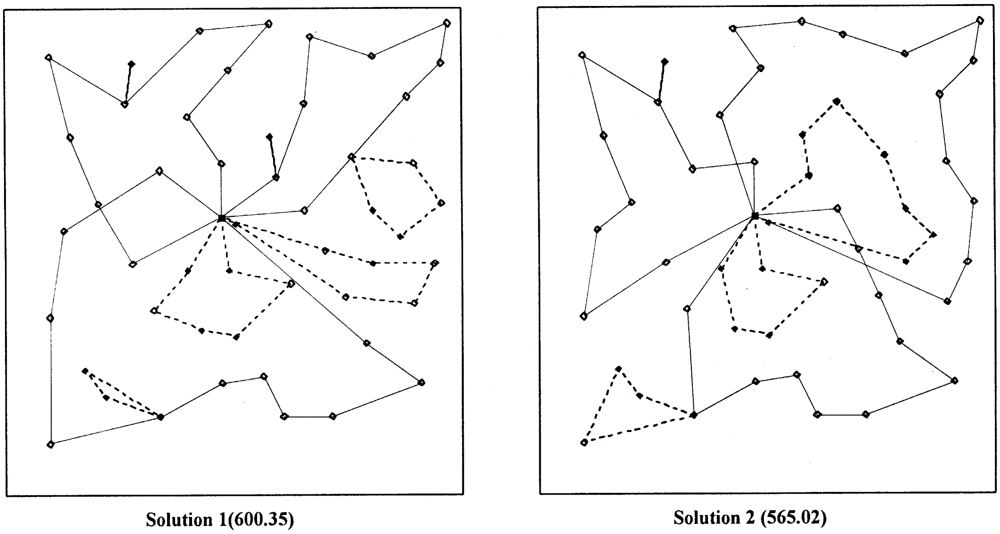

Fig. 1. Two solutions to the TTRP example with 38 vehicle and 12 truck customers.

a complete route can exceed Q_k because shifting demand loads from a truck to its pulling trailer is allowed. The total demand loads in a pure vehicle route and a complete vehicle route, including the main tour and all its associated sub-tours, cannot exceed the sum of the capacities of the truck and its pulling trailer, that is, $Q_k + Q_l$. Just as in the basic VRP, each customer's demand must be fully satisfied and be serviced exactly once, that is, there is no splitting of demand. For every route, there is a restriction on tour length.

We now describe a network optimization formulation of the TTRP. Let $V = \{0, 1, 2, \ldots, n\}$ be the set of customer nodes where 0 is the depot and E is the set of edges between nodes in V. $G = \{V, E\}$ is a complete graph in which each point $i \in V \backslash 0$ has a positive demand q_i and an index either equaling to 1 indicating a t.c. customer that can be solely serviced by a truck alone, or 0 indicating a v.c. customer that can be serviced by either a complete vehicle or a truck alone. Each edge $ij \in E$ has a symmetric, nonnegative cost c_{ij} associated with it, where c_{ij} is the Euclidean distance between node i and node j. In the TTRP, a set of routes consisting of m_l pure and complete vehicle routes and $m_k - m_l$ pure truck routes are constructed so that the total distance traveled by the fleet over all three types of routes is minimized and all constraints are satisfied.

It is important to point out that the TTRP is a multi-level optimization problem. At the first level, an allowable type of route has to be selected for each customer and no t.c. customer can appear on a pure vehicle route or on the main tour of a complete vehicle route. At the second level, three kinds of routes need to be constructed. The pure truck and the pure vehicle routes can be constructed by using a method for solving the traveling salesman problem. The complete vehicle routes are more difficult to construct since we need to decide on the number of sub-tours, the trailer-parking place for each sub-tour, and the sequence of customers on the main tour and each sub-tour. Other examples of multi-level routing problems are the multiple-depot VRP, the period VRP, and the site-dependent VRP [6–9].

A problem related to the TTRP, called the Vehicle Routing Problem with Trailer (VRPWT), is due to Gerdessen [10]. Gerdessen presents two actual TTRP applications. The first is the distribution of dairy products by the Dutch dairy industry in which many customers are located in crowded cities. Maneuvering a complete vehicle is very difficult, so that the trailer is often parked while only the truck delivers the products. Another application is the delivery of compound animal feed that has to be distributed among farmers. On many roads, there are narrow roads or small bridges, so various types of vehicles are needed to distribute animal feed to farmers. One type of vehicle, called double bottoms, has a truck and a trailer. The trailer is left behind at a parking place while the truck is servicing some part of the route. Another actual application related to the TTRP appears in the article by Semet and Taillard [11]. This application occurred in a major chain store with 45 grocery stores located in the cantons of Vaud and Valais in Switzerland. The stores were serviced by a fleet of 21 trucks and 7 trailers. The site-dependent VRP [6,7,12,13] is related to the TTRP. In the SDVRP, the fleet has many types of vehicles and there are vehicle–site compatibilities between customer sites and vehicle types.

The TTRP can be reduced to the VRP if there are no truck customers. Therefore, the TTRP is at least as difficult as the VRP which is NP-Hard [1]. An exact algorithm that can optimally solve a large-size TTRP problem is unlikely. In this paper, we develop and test a heuristic method for solving the TTRT that is based on tabu search and is coupled with the deviation concept from deterministic annealing. The remainder of this paper is organized as follows. In Section 2, we briefly review solution approaches to the TTRP and related problems. In Section 3, we describe a solution construction method that generates an initial solution to the TTRP. In Section 4, we develop an innovative tabu search improvement heuristic for solving the TTRP. In Section 5, we present the computational results from applying our tabu search method to 21 test problems. In Section 6, we give our conclusions.

2. Reviewing solution approaches to the TTRP and related problems

In the past three decades, the basic VRP and its variants have attracted the attention of many researchers [2–5]. However, to our knowledge, only a few papers describe solution approaches for solving the TTRP and related problems. Semet [14] models a TTRP-related problem called the Partial Accessibility Constraint VRP (PACVRP). The author assumes that all available trucks are used and the number of trailers needed has to be determined. This assumption will increase the total cost for using more drivers since every truck needs a driver (we remove this assumption from our model and assume that the number of available trucks and number of available trailers are known in advance). Semet formulates the PACVRP as an integer program. He adapts the VRP solution approach of Fisher and Jaikumar [15] to construct routes after the trailers have been assigned to the trucks. Most of the paper is devoted to solving the trailer assignment problem by using a Lagrangian relaxation method.

Gerdessen [10] proposes heuristics to solve a TTRP-related problem, known as the VRP with trailer (VRPT), in which two important assumptions are imposed to simplify the problem. First, Gerdessen assumes that every customer possesses unit demand. Thus, the VRPT can be easily solved to obtain a feasible solution since the total of the vehicle capacities is greater than or equal to the total number of customers. Second, Gerdessen assumes each trailer is parked exactly once, so that it is not necessary to take into account the cost of parking, uncoupling, and re-coupling trailers. This assumption leads to only one type of route (a complete vehicle route with exactly one sub-tour in a solution to a problem; recall that, in our model, we do not place any restriction on the number of parking places). Gerdessen develops three construction heuristics that are followed by an improvement heuristic.

Sement and Taillard [11] present a heuristic based on a clustering method that constructs an initial solution and then applies a tabu search method to improve the solution. However, their heuristic does not handle some constraints found in the TTRP where the v.c. customers are serviced on the main tour of a complete vehicle route or on a pure vehicle route. However, in a feasible solution to the TTRP, v.c. customers are allowed to be serviced on a sub-tour or on a truck route.

We point out that other solution approaches that can be considered for the TTRP are designed to solve the site-dependent VRP (for more details, see Nag et al. [12], Chao et al. [6,7], and Rochat and Semet [13]). None of these solution approaches can be used to solve the TTRP directly (each must first be modified in some way to handle the TTRP).

3. A solution construction approach for the TTRP

Our solution construction approach consists of a relaxed generalized assignment, route construction, and descent improvement. We describe each individual step in detail, and then integrate them into a construction approach that generates an initial solution to the TTRP.

3.1. The relaxed generalized assignment problem

In this step, we try to allocate customers to routes by solving a relaxed generalized assignment problem. At first, a seed point is selected for each of m_l pure or complete vehicle routes and $m_k - m_l$ pure truck routes. Initially, the first seed is selected; then the remaining seeds are customers with the furthest distances from the preceding seeds and the depot. Varying the first seed will generate a different seed set; hence, a different initial solution is generated. Ten initial solutions are generated to run the entire procedure 10 times and the best solution is selected as the final solution. The first initial solution selects the customer furthest from depot as the first seed; the second initial solution selects the second furthest customer from depot as the first seed; the remaining initial solutions are generated in the same way. The cost of assigning customer i to vehicle or truck route j is $d_{ij} = c_{0i} + c_{is_j} - c_{0s_j}$, where s_j is the seed of route j. The integer program for the assignment problem is as follows:

$$\min \quad \sum_{i=1}^{n} \sum_{j=1}^{m_k} d_{ij} x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{m_k} x_{ij} = 1, \quad i = 1, 2, \ldots, n, \tag{2}$$

$$\sum_{i=1}^{n} q_i x_{ij} \leqslant Q_l + Q_k, \quad j = 1,2, \ldots, m_l, \tag{3}$$

$$\sum_{i=1}^{n} q_i x_{ij} \leqslant Q_k, \quad j = m_l + 1, \ldots, m_k, \tag{4}$$

$$x_{ij} = \{0, 1\} \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m_k. \tag{5}$$

The objective function (1) minimizes the total assignment cost. Constraints (2) and (5) ensure that exactly one route is selected for each customer. Constraints (3) force the maximum demand load of each pure or complete vehicle route to be less than or equal to the sum of the capacities of a truck and a trailer. Constraints (4) force the maximum demand load of each truck route to be less than or equal to the truck capacity. To solve the integer program, we relax the integrality constraints (5) with

$$0 \leqslant x_{ij}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m_k. \tag{6}$$

Constraints (2) and (6) will force the x_{ij}'s to have values within the closed interval between 0 and 1. As shown in Lasdon [16] and Chao et al. [9], the solution to the relaxed LP ((1)–(4), (6)) has at most $2m_k$ fractional values; hence at most m_k customers have non-integer x_{ij}'s values. Let n_1 customers have integer x_{ij} values. In an optimal basic solution to a bounded and feasible LP, there are at most $n + m_k$ x_{ij}'s having nonzero values. This implies that

$$n - n_1 \leqslant m_k, \tag{7}$$

and

$$n + m_k - n_1 \leqslant 2m_k. \tag{8}$$

The number of $(n + m_k) - n_1$ in (8) is a bound on the number of fractional-valued x_{ij} variables and $n - n_1$ in (7) is the number of customers having fractional-valued x_{ij} variables. In most TTRPs, m_k is much smaller than n, so that m_k or $2m_k$ is not very large. After solving the relaxed LP, we round the largest fractional-valued x_{ij} to 1 for customer i. The solution to the relaxed LP assigns exactly one route (vehicle or truck route) to every customer, but the roundoff of x_{ij} values might make the total load on some routes exceed the capacity of the truck, thereby leading to an infeasible solution. Regardless of the feasibility, three type routes are constructed at first, and then infeasibility is tackled by using a penalty function in a series of improvement steps.

3.2. Route construction step

Every customer is assigned to one of three types of routes in the assignment program of the preceding step. In this step, we construct the routes. A pure truck route containing v.c. or t.c. customers and a pure vehicle route containing only v.c. customers are treated as traveling salesman problems (TSPs) and the tours are constructed with a cheapest insertion heuristic [17].

A complete vehicle route is constructed in the following way over v.c. or t.c. customers. At first, the main tour is constructed over v.c. customers using a cheapest insertion TSP heuristic. Sub-tours are then constructed over t.c. customers by inserting them into one of the existing sub-tours or connecting them with a v.c. customer of the main tour to start a new sub-tour. Initially, we connect the first t.c. customer to its nearest v.c. customer of the main tour to construct the first sub-tour. The rest of the t.c. customers are either inserted onto the existing sub-tours by checking the truck capacity or are connected to a v.c. customer of the main tour to generate a new sub-tour. A v.c. customer can be serviced on a sub-tour of a complete vehicle route; however, we force it onto the main tour in the construct step for simplicity. The v.c. customers can be moved from the main tour to a sub-tour to form a better route when improvements are considered.

3.3. Descent improvement steps

Descent improvement has four steps in which we try to move customers among routes with the goal of converting infeasible solution to a feasible one (perhaps by increasing the total travel distance or by decreasing the total distance without increasing the penalty). The penalty (θ) of a solution is a measure of the degree of infeasibility. It is the sum of the total over-capacity demand loads in all routes. The penalty of route R (θ_R) can be computed as $\theta_R = \max(0, \eta_R - \gamma_R)$, where η_R is the demand load of R and γ_R is the capacity of R, $\gamma_R = Q_k + Q_l$ if R is a pure or complete route, and $\gamma_R = Q_k$ if R is a pure truck route or a sub-tour. In descent improvement, we try to convert the infeasible solution to a feasible solution by taking into account the objective function value and the penalty term. The four steps of descent improvement are now described.

3.3.1. One-point descent movement

In the one-point descent movement step (OPD), we try to move one customer from a route to another route, and one customer is examined at a time. If a candidate move decreases the penalty with or without increasing distance, or decreases the distance without increasing penalty, then it is executed immediately. The one-point descent movement is presented in Table 1. In performing the movement, two candidate moves have to be excluded: (1) moving a t.c. customer to the main tour of a complete vehicle route or to a pure vehicle route and (2) moving a v.c. customer that is the parking-place of a sub-tour (this is called a root node and moving it will be examined in the sub-tour root-refining step described later in the paper). Customers serviced on the pure truck routes and on the main tours are examined first, and then customers in the sub-tours are examined second. In Table 1, penalties in both routes are re-computed and the objective function is evaluated by using the cheapest insertion when a candidate is examined.

3.3.2. Two-point descent exchange

In the two-point descent exchange step (TPD), two customers in two different routes can be exchanged. We take into account the penalties of two routes and the total incurred distance in the same manner as in the one-point descent movement. We outline the TPD in Table 2. We prohibit moving a t.c. customer to the main tour of a complete vehicle route or to a pure vehicle route, or moving a root node of a sub-tour. When customers are exchanged between routes, all sub-tours are kept feasible with respect to truck capacity.

3.3.3. Sub-tour root-refining step

In the preceding two descent steps, root nodes never change their positions. It might be possible to improve the solution if some root nodes are replaced. In this step, we try to re-select roots or to re-sequence the customers of sub-tours.

For each sub-tour, we try to remove two edges that link the sub-tour to a main tour, namely its first and last edges, and replace them with two edges that link two consecutive customers in the

Table 1
One-point descent movement (OPD)

For R = the first to the last route in R_{tr} and R_{pv}, the first to the last main tour in R_{cv} (A loop)
  For J = the first to the last customer in R ($J \neq$ a root node) (B loop)
  For S = the first to the last route in R_{tr} and R_{pv}, the first to the last main tour in R_{cv}, the first to the last sub-route of routes in R_{cv} (If $S = R$ or (J is a t.c. and S is in R_{pv} or a main tour in R_{cv}), then skip) (C loop)
      Compute $\theta_R = \max(\theta_R - q_J, 0)$
      Compute $\theta_S = \max(\eta_S + q_J - \gamma_S, 0)$
      If (S is a sub-tour and $\theta_S > 0$), then skip
      Let M = moving J from R to S with the cheapest insertion
      Compute B = the cost of M
    If (θ_S is non-increasing and (θ_R decreases or B is negative)), then execute M
    End C loop
  End B loop
End A loop
For R = the first to the last sub-tours of routes in R_{cv} (D loop)
  For J = the first to the last non-root customer in R (E loop)
    Perform C loop
  End E loop
End D loop

R_{tr}: the set of pure truck routes R_{pv}: the set of pure vehicle routes
R_{cv}: the set of complete vehicle routes q_J: the demand of customer J.

Table 2
Two-point descent exchange (TPD)

For R = the first to the last route in R_{tr} and R_{pv}, the main tours and the sub-tours in R_{cv} (A loop)
  For I = the first to the last customer in R ($I \neq$ a root node) (B loop)
  For S = the first to the last route in R_{tr} and R_{pv}, the main tours and the sub-tours in R_{cv} ($S \neq R$) (C loop)
      (If I is a t.c. and (S is in R_{pv} or a main tour in R_{cv}), then skip)
      For J = the first to the last customer in S ($J \neq$ a root node) (D loop)
      (If J is a t.c. and (R is in R_{pv} or a main tour), then skip)
    Compute $\theta_R = \max(\eta_R + q_J - q_I - \gamma_R, 0)$
      Compute $\theta_S = \max(\eta_S + q_I - q_J - \gamma_S, 0)$
    If ((S is a sub-tour and $\theta_S > 0$) or (R is a sub-tour and $\theta_R > 0$), then skip
      Let M_1 = moving I from R and inserting into S with the cheapest insertion
      M_2 = moving J from S and inserting into R with the cheapest insertion
      Compute B = the cost of M_1 and M_2
    If ((both θ_R and θ_S are non-increasing) and ((θ_R or θ_S decreases) or B is negative)), then execute M_1 and M_2.
      End D loop
    End C loop
  End B loop
End A loop

sub-tour to a v.c. customer on the main tour. If a candidate move that decreases distance exists, the link replacement is executed; otherwise, the sub-tour remains unchanged. After a link replacement is made, the root of the sub-tour is changed and the customers of the sub-tour

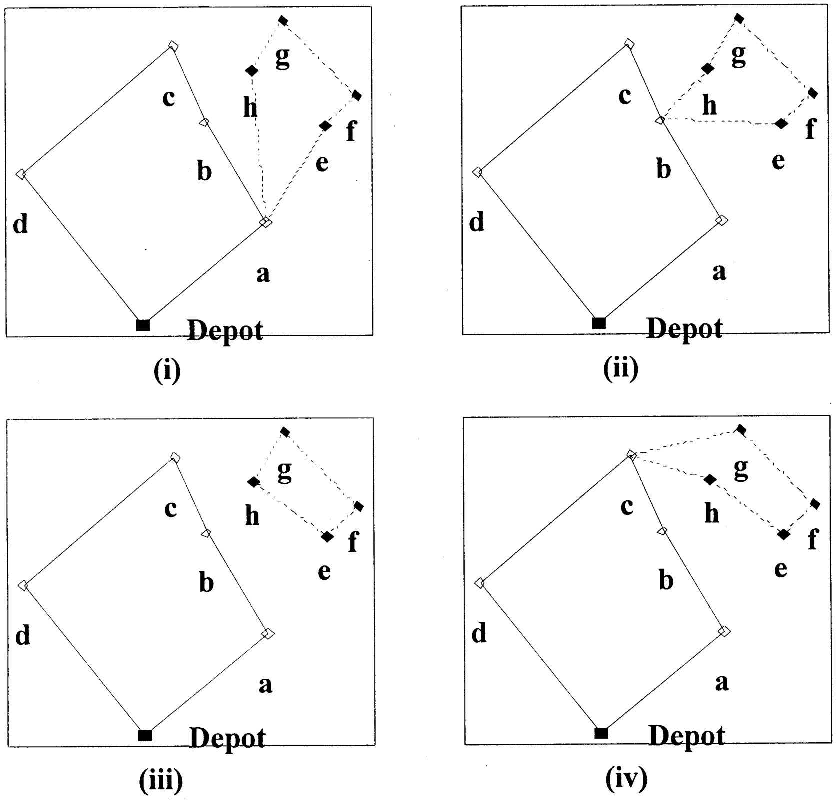

Fig. 2. The sub-tour toot-refining stop for the TTRP heuristic.

are re-sequenced to improve the solution. In Fig. 2, we illustrate this step, where (i) is a complete vehicle route consisting of a main tour (depot–a–b–c–d–depot), and a sub-tour (a–e–f–g–h–a) with customer a as the root node. From (i) to (ii), links a–e and h–a are replaced by links b–e and h–b to form a new sub-tour with customer b as the root node. From (ii) to (iii), links b–e and h–b are temporarily removed and customer e and customer h are linked with an attempt to further refine the sub-tour. From (iii) to (iv), link h–g is removed and links c–h and g–c are connected to form a new sub-tour with customer c as the root node. From (i) through (ii) and (iii) to (iv), the root node is changed twice and the customers on the sub-tour are re-sequenced to refine the sub-tour.

3.3.4. Two-opt clean up

After we apply the construction steps to generate an initial solution, the three descent steps are performed consecutively to improve the solution. The three descent steps are not stopped until none of the candidate moves is made in examining an iteration including three descent steps. A 2-opt procedure [18] is applied to each route in R_{tr}, R_{pv}, and each main tour and sub-tour in R_{cv} to clean up each route and reduce the total distance traveled. Our solution construction approach for generating an initial solution to the TTRP is presented in Table 3.

Table 3
Solution construction approach for the TTRP

Step 1. The route construction steps
Step 1.1. Solve the relaxed assignment problem
Step 1.2. Perform the route construction steps
Step 2. The descent improvement steps
Step 2.1. Perform the OPD
Step 2.2. Perform the TPD
Step 2.3. Perform the sub-tour root refining
Step 3. Repetition or termination steps
If at least one candidate in Steps 2.1, 2.2, or 2.3 is executed, then repeat step 2; otherwise, perform the two-opt clean up and then terminate all steps

4. The tabu search improvement heuristic for the TTRP

In this section, we develop an improvement approach based on tabu search coupled with the deviation concept from deterministic annealing to improve the initial solution generated in the construction step. Tabu search (TS), first presented by Glover [19] and also sketched by Hansen [20] in 1986, is a general improvement heuristic procedure. Although no clean proof of convergence has been presented in the literature, tabu search has been successful in a variety of problem settings like scheduling, transportation, layout and circuit design, and graphs. Procedures based on TS explores the search space by moving from a solution to its best neighbor (the one with the best objective value among all examined candidates), even if this results in a deterioration of the objective function value, in order to increase the likelihood of escaping from a poor local optimum. To avoid cycling in the course of the search, the reverses of the last certain number of moves, formed as a tabu list, are forbidden or declared as tabu restricted for certain number of iterations (this is known as the tabu duration). To prevent a too strict setting the tabu restriction, aspiration criteria are usually introduced to override the tabu restriction and thereby to lead the search to a promising region of the solution space. Intensification and diversification strategies are also applied to accentuate and broaden the search in the solution space, respectively. Glover [21] provides a detailed survey and description of tabu search.

In our improvement heuristic, we use a frequency-based tabu restriction (FTB) that forbids some backward movements. In addition, we use the deviation concept found in deterministic annealing (DA) [4,22,23] and develop a new type of tabu restriction, called the objective-based tabu restriction (OTB). The objective-based tabu restriction is used to implement intensification and diversification strategies by varying the values of deviations within two different ranges.

Deterministic annealing (as opposed to the stochastic search of simulated annealing; see Golden et al. [4] for more details) first appeared in the OR literature in the early 1990s (threshold accepting was developed by Deuck and Sheurer [22], and the Great Deluge Algorithm and the Record-to-Record Travel was developed by Deuck [23]). In the sections that follow, we present two types of tabu restrictions, an aspiration criterion, and the stopping rules, and then integrate each of them into the tabu search OPD and TPD methods. Finally, all of these steps are integrated into a new tabu search improvement heuristic for solving the TTRP.

4.1. Tabu restrictions

The OPD and TPD methods of the route construction steps allow only candidate moves that either decrease the penalty or distance and no tabu restriction is applied when we are more interested in generating an initial solution, and less interested in generating a high-quality solution to the problem. In this section, two types of tabu restrictions as well as an aspiration criterion are used to transform each of the descent steps to corresponding tabu search improvement steps. The first tabu restriction is FTB that forbids a customer being moved back to a tour from which it was just removed for π iterations. We set the tabu duration π equal to a random number between 5 and 10. The second type of tabu restriction is the objective-based value (OTB) that is the deviation or threshold found in deterministic annealing [4,22,23]. In deterministic annealing, solutions with objective function values that are within a specified amount (deviation) of the best objective function value are considered. We define the deviation as the second type of tabu restriction. A candidate solution with an objective function value greater than the best objective function value plus a deviation will be forbidden as a tabu restriction. In deterministic annealing, the user needs to set the value of the deviation. It is not an easy task since, typically, a prescribed deviation does not work consistently well for all problems. In this article, the deviation is self-adjusted within two ranges of values – smaller values for intensification and larger values for diversification. In the intensification search, we set the initial deviation ratio δ equal to 1% and the deviation equal to δ times the objective function value of the best solution obtained so far. If no candidate solution passes the OTB tabu restriction within the execution of an iteration, then δ is updated with an increment of 1%. The iteration is then executed once more with the updated δ, unless the updated δ exceeds 10%. In the diversification search, δ is set equal to 10% initially, and its increment is set equal to 5%. Diversification stops when at least one new solution is obtained (the search has been led to a new part of the solution space and intensification is restarted by using the solution generated by diversification as the new starting point for the search procedure). The initial δ is reset as 1% and then is self-adjusted with an increment of 1%.

4.1.1. Aspiration criterion

In tabu search, an aspiration criterion (AC) can be used to determine when tabu restrictions can be overridden. We use an aspiration criterion that overrides the tabu restriction whenever a candidate solution produces an objective function value that is less than the current best value. We use two types of movements to generate the neighborhood of a solution: a one-point tabu search movement (OPT) that tries to move a customer from one route to another route feasibly and a two-point tabu search exchange (TPT) that tries to exchange two customers between two different routes feasibly. Both the OPT movement and the TPT movement involve two types of tabu restrictions and an aspiration criterion to convert the descent steps into corresponding TS improvement steps. We outline OPT and TPT in Tables 4 and 5, respectively.

4.1.2. Local clean-up and stopping rules

To improve a solution locally, a 2-opt procedure (see Lin [18]) is applied to every route in the solution in the tabu search improvement. As in the descent steps, 2-opt will not accept any solution that deteriorates the objective function value (the tabu restrictions and aspiration criterion are not

Table 4
One-point tabu search improvement (*OPT*)

For R = the first to the last route in R_{tr} and R_{pv}, the first to the last main tour in R_{cv}	(A loop)
For J = the first to the last customer in R ($J \neq$ a root node)	(B loop)
Set C_J = None (C_J is the valid candidate to move)	
For S = the first to the last route in R_{tr} and R_{pv}, the first to the last main tour in R_{cv}, the first to the last sub-route of routes in R_{cv} (If $S = R$ or (J is a t.c. and S is in R_{pv} or a main tour in R_{cv}), then skip)	(C loop)
Compute $\theta_R = \max(\theta_R - q_J, 0)$ and $\theta_S = \max(\eta_S + q_J - \gamma_S, 0)$	
If (S is a sub-tour and $\theta_S > 0$), then skip	
Let M = moving J from R to S with the cheapest insertion	
Compute B = the objective value if M is executed	
If (θ_S is non-increasing and (θ_R decreases or M is not OTB), then	
If ((M is not a FTB or (M is a FTB, but M meets AC)) and ($B <$ the objective of C_J), then set $C_J = M$	
End C loop	
If C_J exists, perform C_J	
End B loop	
End A loop	
For R = the first to the last sub-tours of routes in R_{cv}	(D loop)
For J = the first to the last non-root customer in R	(E loop)
Perform C loop	
End E loop	
End D loop	

Table 5
Two-point tabu search improvement (*TPT*).

For R = the first to the last route in R_{tr} and R_{pv}, the main tours and the sub-tours in R_{cv}	(A loop)
For I = the first to the last customer in R ($I \neq$ a root node)	(B loop)
Set C_I = None (C_I is the valid candidate exchange for I)	
For S = the first to the last route in R_{tr} and R_{pv}, the main tours and the sub-tours in R_{cv} ($S \neq R$)	(C loop)
(If I is a t.c. and (S is in R_{pv} or a main tour), then skip)	
For J = the first to the last customer in S ($J \neq$ a root node)	(D loop)
If J is a t.c. and (R is in R_{pv} or a main tour), then skip	
Compute $\theta_R = \max(\eta_R + q_J - q_I - \gamma_R, 0)$ and $\theta_S = \max(\eta_S + q_I - q_J - \gamma_S, 0)$	
If ((S is a sub-tour and $\theta_S > 0$) or (R is a sub-tour and $\theta_R > 0$), then skip	
Let M_1 = moving I from R and inserting into S with the cheapest insertion	
M_2 = moving J from S and inserting into R with the cheapest insertion	
Compute B = the objective if both M_1 and M_2 are executed	
If ((both θ_R and θ_S are non-increasing) and ((θ_R or θ_S decreases) or both M_1 and M_2 are not OTB)), then If (((neither M_1 nor M_2 is FTB) or (performing both M_1 and M_2 is AC)) and ($B <$ the objective of C_I)), then set $C_I = M_1$ and M_2	
End D loop	
End C loop	
If C_I exists, perform C_I	
End B loop	
End A loop	

Table 6
Tabu search improvement stages for the *TTRP*

Set initial $\delta = 1\%$ and 10% for the intensification and the diversification stages, respectively
For $k = 1, \ldots, K$
Perform the OPT and the TPT
Set π for the FTB $=$ a random number between 5 and 10 for a candidate is executed in the above steps
If no candidate is executed, increase δ by the increment of 1% and 5% for the intensification and the diversification stages, respectively,
Check the local stopping rule (INS and DIS with respect to the intensification and the diversification stages),
Reduce tabu duration π by 1
End K loop

applied in the step). We apply clean-up only to the best solution obtained in the intensification stage in order to improve the solution locally.

The improvement phase has three stages: the descent stage, the intensification stage, and the diversification stage. The descent stage was described in the preceding section, and the remaining two stages follow the steps given in Table 6. The intensification stage searches the best neighbor by using smaller values of the deviation. Diversification allows the search procedure to explore a new solution region by using larger values of the deviation and by applying the search procedure to a high-quality solution found in the intensification and the descent stages. The three stages are performed consecutively and each terminates according to a different local stopping rule. The intensification stopping rules (INS) have of two parts: either the loop is running full, namely $k > K$ in Table 6, or δ increases up to a certain prescribed threshold, namely $\delta > 10\%$ in Table 6. The diversification-stopping rule (DIS) terminates the stage when candidates of an iteration of diversification have been examined and at least one of them has been executed (that is, a new solution region has been found). The descent-stopping rule (DES) terminates the loop when no improvement solution has been found.

The global stopping rule (GLS) is used to terminate the entire search procedure. When intensification, descent, and diversification have been performed at least 30 consecutive times and no new best solution appears in 10 consecutive iterations, the entire search procedure stops and the best solution found is the final solution. In Table 7, we show new tabu search heuristic (TSTTRP) for solving the TTRP.

5. Computational test

5.1. New test problems for the TTRP

To our knowledge, there are no test problems in the OR literature for the TTRP. In order to test our heuristic, we select seven basic VRPs from the well-known test problems of Christofides, Mingozzi, and Toth (CMT) [24] and convert them into 21 TTRPs. The characteristics of the 21 TTRP problems are shown in Table 8. The 21 TTRPs are generated in the following way. For each customer i in the CMT problem, the distance between i to its nearest neighbor customer

Table 7
Tabu Search Improvement for the it TTRP (TSTTRP)

Step 1. Construction phase
 Step 1.1 Solve the generalized assignment problem
 Step 1.2 Perform the route construction step
 Step 1.3 Perform the descent steps
 Set solution = initial solution
Step 2. TS improvement phase
 Step 2.1. Apply the intensification stage to solution and check INS
 Step 2.2. If a new better solution appears in Step 2.1, perform the descent stage to the new better solution and check DES
 Step 2.3. Apply local clean-up to solution and check GLS
 Step 2.4. Perform the diversification stage solution to the best solution obtained so far, and check DIS
 Step 2.5. Perform Step 2 with the new initial solution

Table 8
Dimensions of the TTRP test problems

Problem number	Original[a]	Customers		Trucks		Trailers		Ratio of demand to capacity
		v.c.	t.c.	Number	Capacity	Number	Capacity	
1		38	12					
2	CMT1	25	25	5	100	3	100	0.971
3		13	37					
4		57	18					
5	CMT2	38	37	9	100	5	100	0.974
6		19	56					
7		75	25					
8	CMT3	50	50	8	150	4	100	0.911
9		25	75					
10		113	37					
11	CMT4	75	75	12	150	6	100	0.931
12		38	112					
13		150	49					
14	CMT5	100	99	17	150	9	100	0.923
15		50	149					
16		90	30					
17	CMT11	60	60	7	150	4	100	0.948
18		30	90					
19		75	25					
20	CMT12	50	50	10	150	5	100	0.903
21		25	75					

[a] CMT_x: Number x in Christofides et al. [7].

is calculated and this is denoted by A_i. For each problem of CMT, three TTRPs are created. In the first problem, 25% of the customers with the smallest A_i values are specified as truck customers; in the second problem, this is increased to 50%, and in the third problem, it is 75%. The numbers and capacities of available trucks and trailers for all problems and the ratio of total demand to total capacity are also presented in Table 8. The data sets are available at http://www.cma.edu.tw/iming/research/ttrp.

5.2. Computational results

Our heuristic (TSTTRP) was coded in Fortran, compiled by Digital Visual FORTRAN 5.0, and run on a 350 MHz Pentium II-based PC. The XMP code of Marsten [25] was used to solve the relaxation of the generalized assignment in the construction phase.

In order to evaluate the performance of TSTTRP, we ran it with five different iterations of the intensification steps, denoted set 1, set 2, set 3, set 4, and set 5 with $K = 10$, 20, 30, 40, and 50 (see Table 6), respectively. Each set was run with 10 different sets of seeds by selecting the 10 customers furthest from the depot as the fist seeds, and the best solution that appeared in 10 runs of the heuristic is specified as the final improvement solution. The computational results are given in Table 9. In Table 9, the first column is the problem number and columns 3, 4, 5, and 6 give the average of the 10 initial solutions obtained in the construction and the descent steps (that is, average objective function values and average penalties). In Table 9, we present the results from the five sets of TSTTRP in two columns; the first column is the objective function value of the solution, and the second column is the computational time in minutes. No penalty is presented since all solutions are feasible after the descent method is applied to the construction solutions. The solutions are compared using the average initial solutions obtained at the end of the descent steps. The improvement percentages are listed in the first portion of Table 10. The solutions are also compared to the best solutions found in the five sets of TSTTRP and the percentages above the best solutions are listed in the second portion of Table 10.

In examining Table 9, we point out that the solutions generated in the construction steps are all infeasible with the average penalties ranging from 9.4 to 57. The descent steps always converted an infeasible solution into a feasible one (only four problems increased the average objective values). In examining the performance of the five sets of TS improvement phases, we point out that all sets improved the initial solutions. Sets 1, 2, 3, 4, and 5 produced 2, 5, 4, 6, and 6 best solutions, respectively (these are the bold numbers in Table 9). Set 1 took an average of 4.26 min (ranging from 1.19 to 9.78 min). Set 2 took an average of 6.84 min (ranging from 1.97 to 16.69 min). Set 3 took an average of 9.41 min (ranging from 2.76 to 22.92 min). Set 4 took an average of 9.41 min (ranging from 3.72 to 26.44 min). Set 5 took an average of 14.51 min (ranging from 4.19 to 42.34 min).

In examining the comparison of the TS improvement solutions (T_i, $i = 1, 2, \ldots, 5$ in Table 9) and their average initial solutions (A) in Table 10, we point out that Set 1 improves the initial solutions an average of 10.79% (ranging from a low of 5.52% to a high of 18.20%). Set 2 improves the initial solutions an average of 11.12% (ranging from 6.50 to 21.11%). Set 3 improves the initial solutions an average of 11.17% (ranging from 6.61 to 18.59%). Set 4 improves the initial solutions an average of 11.77% (ranging from 6.55 to 17.77%. Set 5 improves the initial solutions an average of 11.87% (ranging from 6.74 to 18.91%).

Table 9
Comparison of the Computational Results of 21 TTRP test problems

Problem	Initial solution				TS improvement solution									
	Construction		Descent		Set 1($K = 10$)		Set 2($K = 20$)		Set 3($K = 30$)		Set 4($K = 40$)		Set 5($K = 50$)	
	Objective	Penalty	Objective	Penalty	Objective (T_1)	Time (min)	Objective (T_2)	Time (min)	Objective (T_3)	Time (min)	Objective (T_4)	Time (min)	Objective (T_5)	Time (min)
1	657.15	9.6	646.02	0	586.07	1.19	591.38	2.02	**565.02**	2.76	**565.02**	3.72	**565.02**	4.19
2	739.04	13.9	739.90	0	671.72	1.14	658.83	1.97	**658.07**	2.94	662.84	4.38	662.84	5.22
3	785.54	16.8	774.78	0	677.57	1.27	674.17	2.54	680.88	3.57	**648.74**	4.76	664.73	6.50
4	937.82	26.0	943.47	0	867.53	1.59	**856.20**	4.01	860.95	5.40	858.66	5.50	857.84	7.53
5	1108.87	22.9	1130.85	0	988.92	2.30	1011.54	3.36	999.56	4.53	1012.84	4.64	**949.98**	7.06
6	1174.17	32.1	1236.69	0	1057.36	1.93	1083.05	3.62	**1053.23**	4.26	1086.70	5.37	1084.82	7.96
7	937.31	14.1	906.31	0	834.43	4.88	842.06	7.48	839.07	9.32	**832.56**	11.28	837.80	16.43
8	1004.45	18.5	971.60	0	918.00	3.03	**900.54**	5.63	901.46	7.58	901.34	9.64	906.16	11.11
9	1156.50	45.6	1106.66	0	994.00	3.35	979.83	4.33	981.72	6.67	**971.62**	9.10	1000.27	10.18
10	1232.10	33.6	1159.78	0	1073.75	7.32	1081.06	8.19	**1073.50**	15.40	1083.31	19.63	1076.88	21.72
11	1422.41	38.0	1288.74	0	1185.55	4.76	1178.00	7.12	1173.84	11.36	1178.00	15.60	**1170.17**	17.10
12	1578.79	34.0	1453.82	0	1290.66	5.59	1293.54	9.04	1257.38	10.36	1230.93	13.31	**1217.01**	20.27
13	1624.16	35.3	1481.40	0	1382.71	9.78	1385.15	16.69	1383.42	22.92	1384.42	26.44	**1364.50**	42.34
14	1760.51	37.1	1624.96	0	1493.07	8.92	1519.36	14.62	1486.77	18.81	1482.68	20.70	**1464.20**	25.96
15	2105.02	33.6	1858.87	0	**1540.25**	8.97	1580.50	14.14	1564.83	19.89	1586.84	20.48	1544.21	24.62
16	1288.48	10.3	1267.87	0	1170.57	3.21	1068.23	7.27	1146.06	10.21	**1041.36**	12.52	1064.89	14.56
17	1314.09	9.4	1261.17	0	1143.25	3.61	**1090.46**	6.31	1118.30	8.54	1099.99	11.24	1104.67	13.74
18	1383.19	10.8	1366.21	0	1176.26	4.95	1194.29	6.55	1178.23	8.07	**1141.36**	10.71	1202.00	12.52
19	1146.74	22.0	969.96	0	**854.02**	4.26	888.62	6.63	894.25	8.62	896.09	12.11	887.22	16.13
20	1144.96	24.0	1140.47	0	943.94	3.84	**942.39**	6.23	959.30	7.52	958.30	10.09	963.06	10.09
21	1263.70	57.0	1174.43	0	960.68	3.56	**926.47**	5.88	956.13	8.95	965.71	9.86	952.29	9.57
Average	1226.90	25.93	1166.86	0	1038.59	4.26	1035.51	6.84	1034.86	9.41	1028.06	11.48	1025.74	14.51

Table 10
Comparison of solutions

Problem	Initial (A)	Best (B)	Percentage improvement of the initial solution (%)					Percentage above the best solution (%)				
			Set 1 $(A-T_1)/A$	Set 2 $(A-T_2)/A$	Set 3 $(A-T_3)/A$	Set 4 $(A-T_4)/A$	Set 5 $(A-T_5)/A$	Set 1 $(T_1-B)/B$	Set 2 $(T_2-B)/B$	Set 3 $(T_3-B)/B$	Set 4 $(T_4-B)/B$	Set 5 $(T_5-B)/B$
1	646.02	565.02	9.28	8.46	12.54	12.54	12.54	3.73	4.67	0.00	0.00	0.00
2	739.90	658.07	9.21	10.96	11.06	10.41	10.41	2.07	0.12	0.00	0.72	0.72
3	774.78	648.74	12.55	12.99	12.12	16.27	14.20	4.44	3.92	4.95	0.00	2.46
4	943.47	856.20	8.05	9.25	8.75	8.99	9.08	1.32	0.00	0.55	0.29	0.19
5	1130.85	949.98	12.55	10.55	11.61	10.44	15.99	4.10	6.48	5.22	6.62	0.00
6	1236.69	1053.23	14.50	12.42	14.83	12.13	12.28	0.39	2.83	0.00	3.18	3.00
7	906.31	832.56	7.93	7.09	7.42	8.14	7.56	0.22	1.14	0.78	0.00	0.63
8	971.6	900.54	5.52	7.31	7.22	7.23	6.74	1.94	0.00	0.10	0.09	0.62
9	1106.66	971.62	10.18	11.46	11.29	12.20	9.61	2.3	0.84	1.04	0.00	2.95
10	1159.78	1073.50	7.42	6.79	7.44	6.59	7.15	0.02	0.7	0.00	0.91	0.31
11	1288.74	1170.17	8.01	8.59	8.92	8.59	9.20	1.31	0.67	0.31	0.67	0.00
12	1453.82	1217.01	11.22	11.02	13.51	15.33	16.29	6.05	6.29	3.32	1.14	0.00
13	1481.4	1364.50	6.66	6.50	6.61	6.55	7.89	1.33	1.51	1.39	1.46	0.00
14	1624.96	1464.20	8.12	6.50	8.50	8.76	9.89	1.97	3.77	1.54	1.26	0.00
15	1858.87	1540.25	17.14	14.98	15.82	14.63	16.93	0.00	2.61	1.60	3.02	0.26
16	1267.87	1041.36	7.67	15.75	9.61	17.87	16.01	12.41	2.58	10.05	0.00	2.26
17	1261.17	1090.46	9.35	13.54	11.33	12.78	12.41	4.84	0.00	2.55	0.87	1.30
18	1366.21	1141.36	13.90	12.58	13.76	16.46	12.02	3.06	4.64	3.23	0.00	5.31
19	969.96	854.02	11.95	8.39	7.81	7.62	8.53	0.00	4.05	4.71	4.93	3.89
20	1140.47	942.39	17.23	17.37	15.89	15.97	15.56	0.16	0.00	1.79	1.69	2.19
21	1174.43	926.47	18.20	21.11	18.59	17.77	18.91	3.69	0.00	3.20	4.24	2.79
Average			10.79	11.12	11.17	11.77	11.87	2.64	2.23	2.21	1.48	1.38

In examining Table 10, we point out that the TS solutions generated by sets 1, 2, 3, 4, and 5 exceed the best solutions (B in Table 10) an average of 2.64, 2.23, 2.21, 1.48, and 1.38%, respectively. The routes of the best solution to each problem are available at http://www.cma.edu.tw/iming/research/ttrp. Compared with the average initial solutions obtained in the construction phase, the solutions generated by the five sets of TS improvement phases improve all solutions an average of 11.34% (ranging from 10.79 to 11.87%). Compared with the best solutions obtained so far, solutions generated by the five sets of TS improvement phases exceed the best solutions an average of 1.98% (ranging from 1.38 to 2.64%). With respect to computational time, the five sets of TSTTRP took an average of 9.3 min (ranging from 4.26 to 14.51 min). Among the five sets of TSTTRP, Set 5 produced the best sets of solutions. The performance of the TSTTRP did not appear to be very sensitive to the different settings of parameters and the computational times were reasonable.

6. Conclusions

In this article, we developed a new tabu search heuristic for solving the truck and trailer routing problem. Our new heuristic always generated a feasible solution to the test problems (the ratio of the total demand to the total capacity of each test problem was above 90%). The construction step roughly allocated customers to routes and three types of routes were constructed by using a cheapest insertion heuristic. In the descent step, two criteria were used to convert an initial solution (feasible or infeasible) into a solution that either decreased the penalty with or without increasing distance, or decreased the distance without increasing the penalty. In the TS improvement steps, besides the frequency-based tabu restriction, we used the deviation concept from deterministic annealing heuristic as a new type of tabu restriction. We varied the values of deviations to perform intensification and diversification in order to accentuate and broaden the search in the solution space. Compared with deterministic annealing, our new method involved two types of tabu restrictions that avoided cycling and escaped poor local optima. The values of the deviation were set in a flexible way to overcome the drawback of threshold acceptance. Our computational tests on 21 test problems using five sets of TSTTRP showed that our tabu search method coupled with the deviation concept from deterministic annealing can consistently, effectively, and efficiently solve the TTRP.

Acknowledgements

The support of the National Science Council of the Republic of China in Taiwan under grant number NSC 87-2416-h-145-001 is gratefully acknowledged. I am indebted to two referees for their helpful comments and to Edward Wasil for his editing assistance.

References

[1] Garey MR, Johnson DS. Computers and intractability: a guide to the theory of NP-completeness. San Francisco, USA: Freeman, 1979.

[2] Ball M, Magnanti T, Monma C, Nemhauser G editors. Network routing, handbooks in operations research and management science, vol. 8. Amsterdam, The Netherlands: Elsevier, 1995.
[3] Golden BL, Assad A editors. Vehicle routing: methods and studies, studies in management science and systems, vol. 16. Amsterdam, The Netherlands: North-Holland, 1988.
[4] Golden BL, Wasil E, Kelly J, Chao IM. The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results. In: Crainic T, Laporte G, editors. Fleet management and logistics. Dordrecht, The Netherlands: Kluwer Academic Publishers, 1998. p. 33–56.
[5] Laporte G. The vehicle routing problem: an overview of exact and approximate algorithms. European Journal of Operational Research 1992;59:345–58.
[6] Chao IM, Golden BL, Wasil EA. A computational study of a new heuristic for the site-dependent vehicle routing problem. Information Systems and Operational Research 1999;37(3):319–36.
[7] Chao IM, Golden BL, Wasil EA. A new algorithm for the site-dependent vehicle routing problem. In: Woodruff DL, editor. Advances in computational and stochastic optimization, logic programming, and heuristic search. Norwell, MA: Kluwer Academic Publishers, 1998. p. 301–12.
[8] Chao IM, Golden BL, Wasil EA. A new heuristic for the multi-depot vehicle routing problem that improves upon best-known solutions. American Journal of Mathematical and Management Sciences 1993;13:371–406.
[9] Chao IM, Golden BL, Wasil EA. An improved heuristic for the period vehicle routing problem. Networks 1995;26:25–44.
[10] Gerdessen JC. Vehicle routing problem with trailers. European Journal of Operational Research 1996;93:135–47.
[11] Semet F, Tailard E. Solving real-life vehicle routing problems efficiently using tabu search. Annals of Operations Research 1993;41:469–88.
[12] Nag B, Golden BL, Assad A. Vehicle routing with site dependencies. In: Golden BL, Assad A, editors. Vehicle routing: methods and studies, studies in management science and systems, vol. 16. Amsterdam, The Netherlands: North-Holland, 1988. p. 149–59.
[13] Rochat Y, Semet F. A tabu search approach for delivering pet food and flour in Switzerland. Journal of Operational Research Society 1994;45(11):1233–46.
[14] Semet F. A two-phase algorithm for the partial accessibility constrained vehicle routing problem. Annals of Operations Research 1995;61:45–65.
[15] Fisher ML, Jaikumar R. A generalized assignment heuristic for the vehicle routing. Networks 1981;11:109–24.
[16] Lasdon LS. Optimization theory for large systems. London: Macmillan, 1970.
[17] Bodin L, Golden BL, Assad A, Ball M. Routing and scheduling of vehicles and crews: the state of the art. Computers & Operations Research 1983;10:62–212.
[18] Lin S. Computer solutions of the traveling salesman problem. Bell System Technical Journal 1965;44:2245–69.
[19] Glover F. Future paths for integer programming and links to artificial intelligence. Computers & Operations Research 1986;41:3–28.
[20] Hansen P. The steepest ascent mildest descent heuristic for combinatorial programming. Talk presented at the Congress on numerical methods in combinatorial optimization, Capri, Italy, 1986.
[21] Glover F. Tabu search. Boston: Kluwer Academic Publishers, 1997.
[22] Dueck G. New optimization heuristics: the great deluge algorithm and the record-to-record travel. Journal of Computational Physics 1993;104:86–92.
[23] Dueck G, Scheurer T. Threshold accepting: a general purpose optimization algorithm. Journal of Computational Physics 1990;90:161–75.
[24] Christofides N, Mingozzi A, Toth P. The vehicle routing problem. In: Christofides N, Mingozzi A, Toth P, Sandi C, editors. Combinatorial optimization. Chichester, UK: Wiley, 1979. p. 315–38.
[25] Marsten R. The design of the XMP linear programming library. ACM Transactions on Mathematical Software 1981;7:481–97.

I-Ming Chao received his M.S. from the University of New Mexico at Albuquerque and Ph.D. from the University of Maryland at College Park. He is now an associate professor and the chairman of the Department of Management Science at Chinese Military Academy in Taiwan. His research interests include network optimization, heuristic search, and applied operations research.