ELSEVIER

Contents lists available at ScienceDirect

# **Expert Systems with Applications**

journal homepage: www.elsevier.com/locate/eswa



# **Short Communication**

# A note on the truck and trailer routing problem

Shih-Wei Lin a, Vincent F. Yu b,\*, Shuo-Yan Chou b

- <sup>a</sup> Department of Information Management, Chang Gung University, No. 259, Wen-Hwa 1st Road, Kwei-Shan, Tao-Yuan 333, Taiwan
- b Department of Industrial Management, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Road, Taipei 106, Taiwan

#### ARTICLE INFO

#### Keywords: Vehicle routing problem Truck and trailer routing problem Simulated annealing

#### ABSTRACT

This study considers the relaxed truck and trailer routing problem (RTTRP), a relaxation of the truck and trailer routing problem (TTRP). TTRP is a variant of the well studied vehicle routing problem (VRP). In TTRP, a fleet of trucks and trailers are used to service a set of customers with known demands. Some customers may be serviced by a truck pulling a trailer, while the others may only be serviced by a single truck. This is the main difference between TTRP and VRP. The number of available trucks and available trailers is limited in the original TTRP but there are no fixed costs associated with the use of trucks or trailers. Therefore, it is reasonable to relax this fleet size constraint to see if it is possible to further reduce the total routing cost (distance). In addition, the resulting RTTRP can also be used to determine a better fleet mix. We developed a simulated annealing heuristic for solving RTTRP and tested it on 21 existing TTRP benchmark problems and 36 newly generated TTRP instances. Computational results indicate that the solutions for RTTRP are generally better than the best solutions in the literature for TTRP. The proposed SA heuristic is able to find better solutions to 18 of the 21 existing benchmark TTRP instances. The solutions for the remaining three problems are tied with the best so far solutions in the literature. For the 36 newly generated problems, the average percentage improvement of RTTRP solutions over TTRP solutions is about 5%. Considering the ever rising crude oil price, even small reduction in the route length is significant.

© 2009 Elsevier Ltd. All rights reserved.

#### 1. Introduction

In this paper we consider a relaxation of the truck and trailer routing problem (TTRP), called the relaxed truck and trailer routing problem (RTTRP). TTRP is a variant of the well known vehicle routing problem (VRP) which is one of the most studied combinatorial optimization problems in the last few decades (Christofides, Mingozzi, & Toth, 1979; Dantzig & Ramser, 1959; Fisher, 1995; Gillett & Miller, 1974; Laporte, 1992; Laporte, Gendreau, Potvin, & Semet, 2000; Laporte & Nobert, 1987; Laporte & Semet, 2002). In the standard VRP, a set of customers is serviced by a fleet of homogeneous vehicles based at a central depot. The demand of each customer and vehicle capacity are known in advance. The goal is to design vehicle routes originating from and terminating at the central depot to fulfill each customer's demand so that the total cost (or route length) is minimized. The total demand on each route should not exceed the vehicle capacity and each customer can only be serviced once by exactly one vehicle. For a more in depth treatment of VRP, readers are referred to Bodin, Golden, Assad, and Ball (1983), Christofides et al. (1979), Fisher (1995), Golden and Assad (1988) and Toth and Vigo (2002).

TTRP was first proposed by Chao (2002) and subsequently studied by Scheuerer (2006) and Lin, Yu, and Chou (2009). In TTRP, the use of trailers (a commonly neglected feature in the VRP) is considered. Some customers can be serviced by a complete vehicle (i.e., a truck pulling a trailer), while other customers can only be serviced by a single truck due to some practical limitations, such as government regulations, limited maneuvering space at customer site, road conditions. Such limitations exist in many real-world settings (Gerdessen, 1996; Hoff, 2006; Semet & Taillard, 1993). Those customers who can only be serviced by a single truck are referred to as tuck customers (TCs); the other customers that can be serviced by either a single truck or a complete vehicle are called vehicle customers (VCs).

TTRP can be formally defined as follows. Let G = (V, A) be an undirected graph, where  $V = \{0, 1, 2, ..., n\}$  is the set of vertices and  $A = \{(i,j): i,j \in V\}$  is the set of edges. Vertex 0 corresponds to the central depot, while the remaining vertices in  $V \setminus \{0\}$  represent customers. Each vertex i in  $V \setminus \{0\}$  is associated with a known demand  $d_i \ge 0$  and a customer type  $t_i \in \{0,1\}$ , where  $t_i = 1$  indicates that customer i is a TC;  $t_i = 0$  denotes that customer i is a VC. Each edge (i,j) is associated with a non-negative cost  $c_{ij}$  that represents the travel time required on the edge or simply the travel distance between node i and node j.

There are  $m_k$  available trucks and  $m_r$  available trailers ( $m_k \ge m_r$ ) but the number of trucks and trailers that are actually used in the

<sup>\*</sup> Corresponding author. Tel.: +886 2 2730 3276; fax: +886 2 2737 6344. E-mail addresses: vincent@mail.ntust.edu.tw, vince.yu@gmail.com (V.F. Yu).

vehicle routes are not determined *a priori*. Therefore, it is not uncommon that some trucks or trailers are not used at all in a TTRP solution. The capacity of each truck and each trailer is  $Q_k$  and  $Q_t$ , respectively. Once a trailer is assigned to a truck, it may not be assigned to another truck. The goal of the TTRP is to find a set of least cost vehicle routes that originate from and terminate at the central depot so that each customer is serviced exactly once and the cumulated demand on each route does not exceed the total capacity of the vehicle(s) used for that route.

There are three types of routes in a TTRP solutions: (1) a pure truck route (PTR) that is traveled by a single truck; (2) a pure vehicle route (PVR) that is traveled by a complete vehicle without any sub-tour; and (3) a complete vehicle route (CVR) which consists of a main tour traveled by a complete vehicle, and one or more subtours traveled by the truck alone. A sub-tour starts and ends at the depot or the same vehicle customer site on the main tour, i.e. the trailer is dropped-off at the depot, or a VC site while the truck proceeds to service customers on the sub-tour. The trailer drop-off point is called the root of the sub-tour. After all customers on the sub-tour are serviced, the truck must return to the root where its trailer is parked, pick up the trailer and move on to service remaining customers on the same route.

Note that in TTRP, there are no fix costs associated with the vehicles although there are limitations on the number of available trucks and available trailers. Thus, it is possible to construct better vehicle routes by utilizing more vehicles or allowing vehicles to take on multiple trips. Further, if the reduction in costs resulting from such relaxation is significant, it may be worthwhile to acquire or lease extra vehicles provided that the acquisition/lease costs can be justified. In view of this, we relax the fleet size constraint in TTRP and call the resulting problem the relaxed truck and trailer routing problem. This is the only difference between the TTRP and the RTTRP.

TTRP is more difficult to solve than VRP because VRP is a special case of the TTRP. Since the VRP itself is a hard combinatorial optimization problem and is usually solved by heuristics, Chao (2002), Scheuerer (2006), Lin et al. (2009) all applied heuristic approaches to solve TTRP. Both Chao and Scheuerer solved the TTRP by a two-phase approach. They first construct an initial solution with some heuristics. The initial solution is then improved with a tabu search (TS) algorithm. Lin et al. (2009) developed a simulated annealing (SA) based heuristic for the TTRP. Computational results indicate that their SA heuristic performs slightly better than other TS based heuristics in solving the TTRP. Thus, we developed an SA heuristic for the RTTRP and compared the results with the solutions to TTRP obtained from TS based heuristics and SA based heuristics in the literature whenever applicable.

# 2. Simulated annealing heuristic for the RTTRP

Simulated annealing has been applied successfully to a wide variety of highly complicated combinatorial optimization problems (Chwif, Barretto, & Moscato, 1998; Jayaraman & Ross, 2003; Lim, Rodrigues, & Zhang, 2006; McKendall, Shan, & Kuppusamy, 2006), including TTRP (Lin et al., 2009). In the following subsections, we discuss the proposed SA heuristic for RTTRP in detail, including solution representation, generation of the initial solution, calculation of the objective function value, neighborhood structure, parameters used in the computational study, and finally the SA procedure for RTTRP.

## 2.1. Solution representation and initial solution

An RTTRP solution is represented by a string of numbers consisting of a permutation of n customers denoted by the set

 $\{1,2,\ldots,n\}$ ,  $N_{dummy}$  zeros (artificial depot or the root of a sub-tour), and the service vehicle types of individual VCs. The  $N_{dummy}$  zeros serve the purpose of separating routes or terminating sub-tours. The parameter  $N_{dummy}$  is defined by  $\left|\sum_i \frac{d_i}{Q_k}\right|$ , where  $\lfloor \cdot \rfloor$  denotes the largest integer which is smaller than or equal to the enclosed number. The ith non-zero number in the first  $n+N_{dummy}$  positions indicates the ith customer to be serviced.

The service vehicle type of a VC is either 0 or 1. If the VC is serviced by a truck alone, its service vehicle type is set to be 1. Otherwise, it is serviced by a complete vehicle, and its service vehicle type is 0.

The first number in the solution representation indicates the first customer to be serviced in the first route. Other customers are added to the route one at a time from left to right to represent the order in which they are serviced, provided that the capacity of the vehicle currently in use is not violated. Note that the capacity of the vehicle in use varies depending on the type of the route and the portion of the route under consideration. The capacity of the vehicle in use is  $(Q_k + Q_t)$  if the route is a PVR or the vehicle is on the main tour of a CVR; or  $Q_t$  if the vehicle is on a PTR or on a sub-tour of a CVR. When encountering zero in the solution representation, the vehicle will either return to the root of current sub-tour or the depot. More specifically, if the vehicle is currently on a sub-tour of a CVR, it will return to the root of the sub-tour where the trailer was dropped-off and the sub-tour is terminated. Otherwise, it is currently on a PTR, on a PVR, or on a main tour of a CVR. In either case, the vehicle will return to the depot and the route is terminated.

Whenever a route is terminated and there are still customers that have not been serviced, a new route will be generated with the next customer in the solution representation being the first customer of the route. It can be verified that this solution representation always gives a feasible RTTRP solution.

The initial solution is generated at random. It includes a random sequence of the customers and the dummy zeros, and randomly generated service vehicle types of individual VCs.

## 2.2. Neighborhood structure

We adopted a standard SA procedure with a random neighborhood structure that features several move types, namely insertion, swap, and change of service vehicle type, to solve the RTTRP. Let  $\mathcal{N}(X)$  denote the neighborhood of the current solution X. In each iteration, a new solution Y is selected from  $\mathcal{N}(X)$  to be the next solution, either by insertion, swap, or change of service vehicle type of VCs. In the following, we discuss the new solution generations process in detail.

The insertion move is carried out by randomly selecting the *i*th customer in *X* and inserting it into the position immediately before another randomly selected *j*th customer of *X*. The swap move randomly selects two customers in *X*, and then switches their positions. The change of service vehicle type of VCs is carried out by randomly selecting a VC from *X*, and then changing its service vehicle type from 0 to 1 or from 1 to 0. In other words, if the selected VC was serviced by a complete vehicle before the move, it will be serviced by a single truck after the move, and vice versa. We set the probability of choosing the swap move, insertion move, or the change of service vehicle type of VCs move to be 0.2, 0.2, and 0.1, respectively.

To increase the chance of obtaining a better solution, in addition to the aforementioned random moves, we also include in our algorithm the best-of-N-trials moves, in which the best solution among the N trial solutions is chosen as the next solution, where N is a predetermined number of trails. For swap move and insertion move, this number is set to be  $N_{trial}$ , obtained by  $\lfloor (n + N_{dummy})/3 \rfloor$ . For the change of service vehicle type of VCs, each VC's service

vehicle type is changed one at a time, thus the number of trials is the same as the number of VCs.

We set the probabilities of performing the best-of-N-trials moves to be 0.2 and 0.2 for swap move and insertion move, respectively. The probability of performing the best-of-N-trials moves for the change of service vehicle type of VCs is set to be 0.1. Note that the probabilities of performing swap, insertion, and change of service vehicle type of VCs on one or two randomly selected customers total to 0.5, and the probability of performing the best-of-N-trials moves also adds up to 0.5. Thus, the total probability of performing these neighborhood moves is 1.

### 2.3. Parameters and the SA procedure

The SA procedure starts by setting the current temperature T to be  $T_0$ , the initial temperature, and generating an initial solution X at random. Initially, the current best solution  $X_{best}$  and the best objective function value obtained so far are set to be X and obj(X), respectively.

In each iteration, the next solution Y is generated from  $\mathcal{N}(X)$ and its objective function value, obj(Y), is evaluated. Let  $\Delta$  = obj(Y) - obj(X), i.e. the improvement in objective function values of *Y* over *X*. The probability of replacing *X* with *Y* is 1 when  $\Delta \leq 0$ ; otherwise, it is determined by  $\exp(-\Delta/KT)$ , where K is the Boltzmann constant. In our implementation, this is accomplished by first generating a random number  $r \in [0,1]$  and then replacing Xwith *Y* if  $r < \exp(-\Delta/KT)$ .

After running  $I_{iter}$  iterations at the current temperature T, we decrease the current temperature by setting  $T \leftarrow \alpha T$ , where  $0 < \alpha < 1$ . After every three temperature reductions, a local search procedure that performs 2-opt, swap, insertion, and change of service vehicle types sequentially is applied to improve the current best solution.

The algorithm terminates when the current temperature T is lower than  $T_F$ , the predetermined final temperature, or when the current best solution  $X_{best}$  has not been improved for  $N_{non-improving}$ consecutive temperature reductions. A (near) optimal routing plan for the RTTRP can easily be derived from  $X_{best}$  after the SA is terminated

### 3. Computational study

We coded the proposed SA heuristic for RTTRP in C++ and compiled it with Microsoft Visual C++ 6.0. We then applied the program to Chao's 21 TTRP benchmark problems (Chao, 2002) on a Pentium IV 1.5 GHz PC with 1 GB RAM running Microsoft Windows XP operating system.

Except for K, the parameter values for the computational study are adopted from Lin et al. (2009). That is,  $\alpha = 0.965$ ,  $I_{iter} = 150,000$ ,  $T_0$  = 100,  $T_F$  = 1, and  $N_{non-improving}$  = 30. The value of K is set to be 1/6 in this study. The parameter P for penalty cost used in Lin et al. (2009) is not needed since the fleet size constraint is dropped in RTTRP.

To evaluate the improvement in solution quality of RTTRP over that of TTRP, we compared the solution to RTTRP with the best TTRP solutions reported in the literature (Lin et al., 2009; Scheuerer, 2006). TTRP results obtained by Scheuerer (2006) and Lin et al. (2009), and the RTTRP results by the proposed SA heuristic are presented in Table 1. It can been seen that in 18 out of the 21 benchmark TTRP instances, RTTRP solution is better than the best TTRP solution obtained by Scheuerer (2006) and Lin et al. (2009). For

Table 1 A comparison of RTTRP solutions obtained by the proposed SA heuristic with TTRP solutions reported in the literature for Chao's problem set (Chao, 2002).

ID	Scheuerer	$\lambda = 15,000$	0)	Lin et al.	(K = 1/3)		SA heuris	tic for RTTF	RP(K=1/6	5)		
	$ \begin{array}{c} \text{Min} \\ c(s^*)^a \end{array} $	Avg $c(s^*)^b$	T <sup>c</sup>	$ \begin{array}{c} \text{Min} \\ c(s^*)^{\mathbf{d}} \end{array} $	Avg c(s*)e	T <sup>f</sup>	$ \begin{array}{c} \text{Min} \\ c(s^*)^g \end{array} $	Avg $c(s^*)^h$	T <sup>i</sup>	Trucks used <sup>j</sup>	Trailers used <sup>k</sup>	Improvement in Avg $c(s^*)$ over Lin et al. (%)
1	566.80	567.98	9.51	566.82	568.86	6.80	557.11	559.59	6.77	5	4	1.71
2	615.66	619.35	9.60	612.75	617.48	6.67	608.22	610.38	6.66	5	4	0.74
3	620.78	629.59	11.24	618.04	620.50	5.59	618.04	618.5	5.35	5	3	0.00
4	801.60	809.13	18.49	808.84	817.71	16.32	784.73	790.76	15.53	8	7	2.98
5	839.62	858.98	15.16	839.62	858.95	14.42	839.62	845.89	14.73	9	5	0.00
6	936.01	949.89	18.62	934.11	942.60	13.65	930.64	935.76	12.58	9	5	0.37
7	830.48	832.91	33.60	830.48	838.50	24.96	810.38	815.09	24.63	6	6	2.42
8	878.87	881.26	25.66	875.76	882.70	24.03	873.80	882.55	24.29	8	3	0.22
9	942.31	955.95	30.47	912.64	921.97	21.75	911.49	918.2	21.27	9	2	0.13
10	1039.23	1052.65	60.94	1053.90	1074.38	63.61	1018.62	1027.76	63.55	10	9	3.35
11	1098.84	1107.47	56.17	1093.57	1108.88	60.33	1076.88	1088.84	60.10	10	9	1.53
12	1175.23	1184.58	63.71	1155.44	1166.59	51.70	1154.13	1166.27	51.22	12	6	0.11
13	1288.46	1296.33	165.41	1320.21	1340.98	119.56	1263.62	1284.81	119.51	13	13	4.29
14	1371.42	1384.13	132.06	1351.54	1367.91	113.75	1336.03	1354.1	112.66	15	13	1.15
15	1459.55	1488.71	154.10	1436.78	1454.91	93.87	1422.22	1454.59	92.27	17	10	1.01
16	1002.49	1003.00	43.14	1004.47	1007.26	41.46	975.65	978.22	40.64	6	5	2.87
17	1042.35	1042.79	33.73	1026.88	1035.23	38.81	1006.79	1008.97	37.55	6	5	1.96
18	1129.16	1141.94	31.78	1099.09	1110.13	31.34	1097.56	1103.61	31.09	7	5	0.14
19	813.50	813.98	28.84	814.07	823.01	29.58	797.19	801.51	29.18	8	7	2.07
20	848.93	852.89	24.57	855.14	859.06	28.47	847.21	849.85	29.00	9	6	0.93
21	909.06	914.04	26.84	909.06	915.38	24.03	909.06	914.33	23.94	10	5	0.00
Avg.	962.40	970.84	47.32	958.06	968.24	39.56	944.71	952.84	38.18	-	-	1.33

- Best TTRP solutions from 10 runs by Scheuerer ( $\lambda = 15,000$ ).
- Average TTRP solutions from 10 runs by Scheuerer ( $\lambda$  = 15,000).
- Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for Scheuerer ( $\lambda$  = 15,000).
- Best TTRP solutions from 10 runs by Lin et al. (K = 1/3).
- Average TTRP solutions from 10 runs by Lin et al. (K = 1/3).
- Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for Lin et al. (K = 1/3).
- Best RTTRP solutions from 10 runs by the SA heuristic (K = 1/6).
- Average RTTRP solutions from 10 runs by the SA heuristic (K = 1/6).
- Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for the SA heuristic (K = 1/6).
- Number of Truck used in the best solution that attains Min  $c(s^*)$ .
- Number of Trailer used in the best solution attains Min  $c(s^*)$ .
- Calculated by  $(\operatorname{Min} c(s^*)^{\operatorname{d}} \operatorname{Min} c(s^*)^{\operatorname{g}})/\operatorname{Min} c(s^*)^{\operatorname{d}}$ .

 Table 2

 A comparison of RTTRP solutions obtained by the proposed SA heuristic with TTRP solutions obtained by the SA heuristic of Lin et al. (2009).

Source         VC         TC         Avail.         Avail.         Q <sub>4</sub> Avg Influence         Time(s) in Influence         Min         Time(s) in Influence         Min         Time(s) in Influence         Time(s) in Influence         Time(s) in Influence         Influence         Time(s) in Influence         Influence         Influence         Avg         Time(s) in Influence         Influence         Influence         Avg         Time(s) in Influence         Influence         Influence         Influence         Avg         Influence         Influence         Avg         Influence         <	Ω	Characte	eristic (	of test	problem				With fleet	size constraint	(K = 1/3, P =	(200)		Without f	Without fleet size constraint $(K = 1/6)$	int $(K = 1/6)$	0		Improvement rate (%)	ient
u175a         57         18         13         7         75         75         18         30         7         14		Source	VC	TC	Avail. trucks	Avail. trailers			$ \text{Avg} \\ c(s^*) $	Time (s) in Min	Min c(s*)	Trucks used	Trailers used	$\mathop{\rm Avg}_{c(s^*)}$	Time (s) in Min	$\min_{c(s^*)}$	Trucks used	Trailers used	Avg (s)	Min (s)
tai75a         38         37         13         7         750         750         1911.36         1873         1891.64         12         7         144.74           tai75b         54         18         12         6         850         850         1642.89         1994         1455.47         12         7         147.08           tai75b         57         18         12         6         850         850         1642.89         1894         1455.47         12         6         137.38           tai75c         57         18         11         6         600         600         1642.89         1894         1457.44         11         6         1893.32           tai75c         57         18         11         6         600         600         1642.89         1894         1457.44         11         6         1893         850         1894         1457.44         11         6         1893         1894         1457.44         11         6         1893         850         1894         1457.44         11         6         1893         890         1893         890         1893         890         1893         890         1894         1874	1	tai75a	57	18	13	7	750	750	1855.24	20.06	1833.20	12	7	1670.23	17.63	1656.62	11	10	-9.97	-9.63
tai75a         19         56         13         7         750         1996.88         16.79         1991.03         12         7         1917.36           tai75b         38         12         6         850         850         1495.07         1759         1491.30         12         6         1437.38           tai77b         38         37         12         6         850         850         1495.07         1759         1491.30         16         1437.38           tai77c         38         37         12         6         600         600         1465.57         1749         1486.18         11         6         1437.08           tai77c         38         37         11         6         600         600         1465.57         1749         1486.18         11         6         1437.08           tai77cd         39         40         1552.15         1346.11         16         600         600         1495.57         1749         1486.18         11         6         1437.08         1486.18         11         6         1437.08         1486.18         11         6         1437.08         1486.18         11         6         1437.08         14	2	tai75a	38	37	13	7	750	750	1911.36	18.73	1898.60	12	7	1747.46	18.65	1746.25	11	10	-8.58	-8.02
tai75b         57         18         12         6         850         850         1462.89         145.47         12         6         1437.08           tai75b         57         18         12         6         850         850         1462.89         153         12         6         1437.08           tai75c         18         12         6         850         850         155.75         18.3         14.24         1         6         1618.39           tai75c         38         11         6         600         600         1652.15         15.69         1579.16         1         16.83         1           tai75d         38         11         6         600         600         1652.15         15.99         145.44         1         6         1437.08           tai75d         38         11         6         600         600         1552.15         149.49         1437.44         1         6         1437.08           tai75d         38         16         150         160         1457.55         149.44         1         6         1437.88           tai75d         38         16         16         16         16         <	3	tai75a	19	99	13	7	750	750	1996.86	16.79	1991.03	12	7	1923.62	17.63	1918.60	11	10	-3.67	-3.64
tar/75         13         6         85         85         145         149         15         6         149         15         16         18 <t< td=""><th>4</th><td>tai75b</td><td>57</td><td>18</td><td>12</td><td>9</td><td>820</td><td>850</td><td>1462.89</td><td>19.94</td><td>1455.47</td><td>12</td><td>9</td><td>1377.36</td><td>14.15</td><td>1375.08</td><td>11</td><td>∞</td><td>-5.85</td><td>-5.52</td></t<>	4	tai75b	57	18	12	9	820	850	1462.89	19.94	1455.47	12	9	1377.36	14.15	1375.08	11	∞	-5.85	-5.52
tai75b         19         56         12         6         850         156.76         1533         1618.38         12         6         1618.39           tai75c         38         37         11         6         600         600         165.21         1589         14         1         6         1618.39           tai75c         38         37         11         6         600         600         155.15         156.91         1         6         1457.41           tai75d         38         37         12         6         800         600         155.15         15.91         1         6         1506.39           tai75d         38         37         12         6         800         800         154.25         1         1         6         1506.30           tai70d         36         37         12         6         800         800         154.25         1         6         1506.30           tai100a         50         6         800         800         154.37         3.21         1         7         245.08           tai100a         50         14         7         750         243.37         3.21 <t< td=""><th>2</th><td>tai75b</td><td>38</td><td>37</td><td>, .</td><td>9</td><td>850</td><td>850</td><td>1495.07</td><td>17.59</td><td>1491.30</td><td>12</td><td>9</td><td>1437.08</td><td>18.65</td><td>1435.68</td><td>12</td><td>∞</td><td>-3.88</td><td>-3.73</td></t<>	2	tai75b	38	37	, .	9	850	850	1495.07	17.59	1491.30	12	9	1437.08	18.65	1435.68	12	∞	-3.88	-3.73
tai75c         5         18         11         6         600         1465.26         18.73         145.74         11         6         148.71         11         6         148.71         11         6         148.71         11         6         148.71         11         6         148.71         11         6         148.71         11         6         148.23         1         148.11         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         148.23         1         1         6         144.23         1         1         6         144.23         1         1         6         144.23         1         1         6         144.23         1	9	tai75b	19	99	12	9	820	850	1656.76	15.33	1643.83	12	9	1618.39	14.47	1614.21	12	7	-2.32	-1.80
tai75c         38         37         11         6         600         1495.75         17.49         1486.18         11         6         140.50           tai75c         38         37         11         6         600         155.215         15.49         153.91.4         11         6         147.65           tai75c         37         12         6         850         850         158.43         18.99         157.16         1         6         157.05           tai70d         75         12         6         850         850         158.43         18.99         157.16         1         6         157.05           tai10d         75         25         14         7         750         2243.3         1.2         6         157.05         1.0         157.05         1.0         17.0	7	tai75c	57	18	, .	9	009	009	1462.62	18.78	1457.44	11	9	1343.18	17.10	1338.10	10	6	-8.17	-8.19
tai75c         19         56         11         6         600         1552.15         15.69         1539.14         11         6         1407.65           tai75d         35         11         6         850         154.215         15.99         1559.16         12         6         1417.65           tai75d         37         12         6         850         184.35         18.91         1579.16         12         6         1417.65           tai70d         35         12         4         7         750         228.31         33.51         223.42         14         7         208.10           tai10da         50         50         14         7         750         2263.30         27.50         2489.78         14         7         2406.00           tai10db         50         50         14         7         750         2503.20         27.50         14         7         2406.00           tai10db         50         50         14         7         950         950         2589.13         14         7         1407.00           tai10db         50         50         140         7         950         950         2589.14 <th>∞</th> <td>tai75c</td> <td>38</td> <td>37</td> <td>11</td> <td>9</td> <td>009</td> <td>009</td> <td>1495.75</td> <td>17.49</td> <td>1486.18</td> <td>11</td> <td>9</td> <td>1428.24</td> <td>17.10</td> <td>1423.89</td> <td>11</td> <td>7</td> <td>-4.51</td> <td>-4.19</td>	∞	tai75c	38	37	11	9	009	009	1495.75	17.49	1486.18	11	9	1428.24	17.10	1423.89	11	7	-4.51	-4.19
tai75d         37         18         12         6         850         184125         1899         1579.11         12         6         113.74           tai75d         38         37         12         6         850         184135         1899         1579.11         12         6         133.74           tai70da         38         37         12         6         850         1843.55         16.0         173.56         12         6         133.74           tai100a         25         25         14         7         750         228.13         33.11         2324.23         14         7         2251.00           tai100b         25         25         14         7         750         234.37         2324.23         14         7         245.00           tai100b         25         25         14         7         50         203.237.12         2324.23         14         7         245.00           tai100b         25         25         14         7         50         203.237.12         232.43         14         7         245.00           tai100c         25         25         14         7         50         20	6	tai75c	19	26	11	9	009	009	1552.15	15.69	1539.14	11	9	1506.93	12.88	1506.50	11	7	-2.91	-2.12
tai75d         38         37         12         6         850         18435         1899         1579,16         12         6         1534           tai70da         75         75         750         750         178,25         14         7         2231,00           tai10da         75         76         75         750         2541,37         32,11         233,42         14         7         2081,04           tai10da         75         76         75         2241,37         32,10         233,42         14         7         2041,04           tai10db         75         76         75         260         260         2483,71         2483,72         14         7         2460,83           tai10db         75         76         75         2441,37         232,42         14         7         2450,83           tai10db         75         76         76         250,333         27,20         14         7         2441,86           tai10dc         75         76         160         105         1497,65         30.43         14         7         2430,78           tai10dc         75         76         160         105	10	tai75d	57	18	12	9	820	850	1541.25	19.14	1540.11	12	9	1417.65	16.42	1412.72	6	6	-8.02	-8.27
tai175d         19         56         12         6         850         860         1748-56         16.1         1735-56         12         6         1759         12         1759         26.2         1748-56         16.1         1735-56         12         6         1749-02         1741-02         1741-02         1750         26.2         1730         2750         2243-33         14         7         200         2258-13         33.51         2233-42         14         7         2476-08           tai100b         25         75         14         7         750         2276-45         33.11         2217-58         14         7         2476-08           tai100b         55         24         7         950         950         2275-45         14         7         2476-08           tai100b         55         14         7         950         950         235-75         14         7         2476-08           tai100c         56         14         7         1650         1650         1437-6         145-65         14         7         2476-08           tai100c         56         14         7         1750-48         14         7         2476-08 <th>11</th> <td>tai75d</td> <td>38</td> <td>37</td> <td>12</td> <td>9</td> <td>820</td> <td>850</td> <td>1584.35</td> <td>18.99</td> <td>1579.16</td> <td>12</td> <td>9</td> <td>1513.74</td> <td>16.42</td> <td>1506.95</td> <td>10</td> <td>∞</td> <td>-4.46</td> <td>-4.57</td>	11	tai75d	38	37	12	9	820	850	1584.35	18.99	1579.16	12	9	1513.74	16.42	1506.95	10	∞	-4.46	-4.57
tail00a         75         25         14         7         750         750         258.13         33.51         2233.42         14         7         2051.00           tail00a         5         25         14         7         750         750         25.93         23.13         14         7         2551.00           tail00a         5         75         14         7         750         750         250.23         234.33         14         7         2551.00           tail00b         5         25         14         7         950         950         2592         275.04         14         7         251.00           tail00b         5         5         14         7         950         950         2592         239.71         28.25         14         7         251.34         3         250.14         7         251.34         3         4         7         251.34         3         4         7         251.34         3         4         7         251.00         3         3         4         7         251.00         3         3         4         7         251.00         3         3         3         4         7 <th< td=""><th>12</th><td>tai75d</td><td>19</td><td>26</td><td>12</td><td>9</td><td>820</td><td>850</td><td>1748.56</td><td>16.16</td><td>1735.56</td><td>12</td><td>9</td><td>1739.62</td><td>14.90</td><td>1733.86</td><td>12</td><td>9</td><td>-0.51</td><td>-0.10</td></th<>	12	tai75d	19	26	12	9	820	850	1748.56	16.16	1735.56	12	9	1739.62	14.90	1733.86	12	9	-0.51	-0.10
tail 100a         50         64         7         750         750         244137         32.10         2324.23         14         7         750         750         2449.78         14         7         750         750         2249.78         14         7         750         750         2249.78         14         7         245.68           tai100b         50         50         14         7         950         950         2398.71         28.25         2489.78         14         7         2456.88           tai100c         50         50         14         7         950         950         2531.77         2320.43         14         7         2456.88           tai100c         50         50         14         7         1050         1050         1405.65         14         7         2138.88           tai100c         50         51         14         7         1050         1050         1405.65         1449.76         7         1138.87           tai100d         75         15         14         7         1050         1050         1770.67         25.81         14         7         1138.45           tai100d         75         1	13	tai 100a	75	25		7	750	750	2258.13	33.51	2233.42	14	7	2081.04	31.01	2047.43	11	10	-7.84	-8.33
tai100a         25         75         14         7         750         750         3750         750 <th>14</th> <td>tai 100a</td> <td>20</td> <td>20</td> <td>•</td> <td>7</td> <td>750</td> <td>750</td> <td>2341.37</td> <td>32.10</td> <td>2324.23</td> <td>14</td> <td>7</td> <td>2251.00</td> <td>31.01</td> <td>2223.02</td> <td>11</td> <td>10</td> <td>-3.86</td> <td>-4.35</td>	14	tai 100a	20	20	•	7	750	750	2341.37	32.10	2324.23	14	7	2251.00	31.01	2223.02	11	10	-3.86	-4.35
tail00b         75         25         14         7         950         950         2376.45         33.11         2217.58         14         7         1983.52           tail00b         50         14         7         950         950         2398.71         28.22         2230.43         14         7         1983.52           tail00c         25         75         14         7         950         950         2331.27         23.28         1454.65         14         7         1394.56           tail00c         50         14         7         1050         1050         1403.76         3.29         1454.65         14         7         1394.56           tail00c         50         14         7         1050         1650         1706.5         23.94         1439.64         14         7         1394.56           tail00c         50         16         170         160         1706.5         170         1706.5         170         1706.5         170         1706.5         170         1706.5         170         1706.5         170         1706.5         14         7         11394.56           tail00d         50         50         16         8	15	tai 100a	25	75		7	750	750	2503.30	27.50	2489.78	14	7	2476.08	25.06	2470.66	14	∞	-1.09	-0.77
tail00b 50 50 14 7 950 950 2398.71 28.22 230.43 14 7 2138.68 tail00b 25 75 14 7 950 950 2398.71 28.22 230.43 14 7 2331.88 tail00c 75 25 14 7 1050 1050 1497.65 30.99 1492.64 14 7 1394.56 tail00c 25 75 14 7 1050 1050 1497.65 30.99 1492.64 14 7 1493.79 tail00c 25 75 14 7 1050 1050 1497.65 30.99 1492.64 14 7 1709.48 14 8 1593.79 tail00c 25 75 14 7 1050 1050 1497.65 30.99 1492.64 14 7 1709.48 14 8 1683.52 tail00c 25 75 15 8 650 650 1807.50 13.77 1709.48 14 8 1683.52 tail00c 25 75 15 8 650 650 1807.50 13.77 1709.48 14 8 1683.52 tail00c 15 75 20 10 800 800 3443.60 13.72 1824.8 15 8 9 1990.56 tail50a 113 37 20 10 800 800 3743.60 10 3564.45 10 10 12 20 10 800 800 3743.60 10 3642.4 19 10 3526.10 12 18 9 100 1000 1000 1000 1000 1000 1000	16	tai 100b	75	25	•	7	920	950	2276.45	33.11	2217.58	14	7	1983.52	28.83	1957.00	11	10	-12.87	-11.75
tail00b 25 75 14 7 950 950 2531.27 27.38 2508.92 14 7 2331.88 tail00c 25 14 7 1050 1050 1463.76 32.59 1454.65 14 7 1394.56 tail00c 25 14 7 1050 1050 1463.76 32.59 1454.65 14 7 1394.79 1394.56 tail00c 25 25 14 7 1050 1050 1403.76 31.77 1790.48 14 7 1761.74 tail00d 75 25 15 8 650 650 1807.50 31.77 1790.48 15 8 1990.56 tail00d 25 75 15 8 650 650 1807.50 31.77 1790.48 15 8 1990.56 tail00d 25 75 15 8 650 650 1807.50 31.77 1790.48 15 8 1990.56 tail00d 25 75 15 8 650 650 1807.50 31.77 1790.48 15 8 1990.56 tail50 113 37 20 10 800 800 3443.60 72.7 2429.4 19 10 3293.80 tail50 113 37 18 9 1000 1000 2807.7 70.12 2982.58 18 9 2698.36 tail50 110 37 18 9 1000 1000 302.70 70.12 2982.58 18 9 200.046 tail50 110 112 18 9 100 1000 302.70 70.12 2982.58 18 9 3040.46 tail50 110 112 19 10 1050 1050 245.15 72.07 2429.7 18 10 2470.30 tail50 113 37 19 10 1050 1050 245.15 72.07 2429.7 18 10 2470.30 tail50 113 37 19 10 950 950 3102.66 68.38 3066.94 18 10 2948.06 tail50 113 37 19 10 950 950 3261.0 2443.0 18 10 2242.5 18 10 2242.5 18 10 2242.5 18 10 2242.5 19	17	tai 100b	20	20	•	7	950	950	2398.71	28.22	2320.43	14	7	2138.68	28.83	2114.81	11	10	-10.84	-8.86
tail 00c 75 25 14 7 1050 1050 1463.76 32.59 1454.65 14 7 1394.56 tail 00c 25 75 14 7 1050 1050 1050 1497.65 30.40 1492.64 14 7 1439.79 1439.79 tail 00c 25 75 14 7 1050 1050 1050 1770.67 25.81 1755.33 14 7 1761.74 1	18	tai 100b	25	75		7		950	2531.27	27.38	2508.92	14	7	2331.88	24.74	2329.98	12	10	-7.88	-7.13
tailode         50         14         7         1050         1497.65         30.40         1492.64         14         7         1439.79           tailode         25         75         14         7         1050         1050         1770.67         25.81         1755.33         14         7         1761.74           tailode         75         25         15         8         650         650         180.66         31.77         1792.48         15         8         1683.52           tailodd         25         75         15         8         650         650         1770.62         26.81         1882.48         15         8         1690.56           tailodd         25         75         15         8         650         650         2008.42         26.81         18         9         1800.56           tail50a         10         800         800         343.60         7         1434.90         340.24         18         1990.56           tail50b         113         37         18         9         1000         1000         280.74         72.27         2848.54         18         9         160.46           tail50b         13 <th>19</th> <td>tai 100c</td> <td>75</td> <td>25</td> <td></td> <td>7</td> <td>_</td> <td>020</td> <td>1463.76</td> <td>32.59</td> <td>1454.65</td> <td>14</td> <td>7</td> <td>1394.56</td> <td>32.41</td> <td>1383.14</td> <td>12</td> <td>10</td> <td>-4.73</td> <td>-4.92</td>	19	tai 100c	75	25		7	_	020	1463.76	32.59	1454.65	14	7	1394.56	32.41	1383.14	12	10	-4.73	-4.92
tailOoc         25         75         14         7         1050         170.67         25.81         1755.33         14         7         1761.74           tailOod         75         25         15         8         650         650         1807.62         31.77         1790.48         15         8         1683.52           tailOod         50         50         15         8         650         650         650         1806.63         172.48         15         8         1683.52           tailOod         10         800         800         343.60         74.29         3440.24         18         10         3134.90           tailSoa         10         10         800         800         3564.45         70.41         3514.94         18         10         3143.90           tailSoa         10         10         800         800         3713.70         60.05         3694.24         19         10         3293.80           tailSoa         10         100         1000         2880.74         72.27         2848.54         18         9         2698.36           tailSoa         10         100         1000         280.71         70.12	20	tai 100c	20	20		7		020	1497.65	30.40	1492.64	14	7	1439.79	32.41	1436.41	12	6	-3.86	-3.77
tail 00d         75         25         15         8         650         650         1807.50         31.77         1790.48         14         8         1683.52           tail 00d         50         15         8         650         650         180.66         31.72         1872.48         15         8         190.56           tail 00d         25         75         15         8         650         650         180.66         31.72         1872.48         15         8         190.56           tail 50a         13         7         20         10         800         800         344.30         74.29         351.494         18         10         313.490           tail 50a         10         10         800         800         3713.70         60.25         3694.24         19         10         313.490           tail 50a         10         100         1000         2880.74         72.77         2848.54         18         10         2698.36           tail 50b         10         100         1000         2880.74         72.27         2848.54         18         10         2893.50           tail 50b         10         100         1000	21	tai100c	25	75		7	_	020	1770.67	25.81	1755.33	14	7	1761.74	24.97	1755.33	14	7	-0.50	0.00
tail 00d         50         15         8         650         650         180.66         31.72         1872.48         15         8         1810.37           tail 00d         25         75         15         8         650         650         2008.42         26.81         1988.54         15         8         1990.56           tail 50a         113         37         20         10         800         800         354.45         70.41         188.54         18         10         3134.90           tail 50a         10a         800         800         356.42         70.41         369.424         19         10         3293.80           tail 50a         113         37         18         9         1000         1000         2880.74         72.27         2848.54         18         9         2698.36           tail 50b         15         16         100         1000         3027.07         70.12         2882.54         18         9         2698.36           tail 50b         16         100         1000         3027.07         70.12         2882.58         18         9         2893.60           tail 50c         16         100         1000 </td <th>22</th> <td>tai 100d</td> <td>75</td> <td>25</td> <td></td> <td>∞</td> <td>029</td> <td>650</td> <td>1807.50</td> <td>31.77</td> <td>1790.48</td> <td>14</td> <td>∞</td> <td>1683.52</td> <td>29.28</td> <td>1662.28</td> <td>12</td> <td>11</td> <td>98.9-</td> <td>-7.16</td>	22	tai 100d	75	25		∞	029	650	1807.50	31.77	1790.48	14	∞	1683.52	29.28	1662.28	12	11	98.9-	-7.16
tai150d 25 75 15 8 650 650 2008.42 26.81 1988.54 15 8 1990.56 tai150d 13 37 20 10 800 800 804.34 16 18 1024 18 10 3293.80 tai150a 150 12 20 10 800 800 800 3564.45 70.41 3514.94 18 10 3293.80 tai150b 150 172 20 10 800 800 3713.70 60.05 3694.24 18 9 2698.36 tai150b 150 172 18 9 1000 1000 2880.74 72.27 288.54 18 9 2698.36 tai150b 100 112 18 9 1000 1000 3135.74 57.96 3054.24 18 9 2698.36 tai150c 113 37 19 10 1050 1050 2452.15 72.07 2429.72 18 10 2379.57 tai150c 10 112 19 10 1050 1050 2934.65 65.38 2561.72 18 10 2890.23 tai150c 10 112 19 10 950 950 950 3026.12 73.60 2997.12 18 10 2895.43 tai150d 150 175 19 10 950 950 3267.04 65.33 2066.94 18 10 2948.06 tai150d 10 112 19 10 950 950 3267.04 57.54 83.20 2242.54 18 9 2835.03	23	tai 100d	20	20	15	∞	650	650	1880.66	31.72	1872.48	15	8	1810.37	29.28	1799.13	13	10	-3.74	-3.92
tail50a         113         37         20         10         800         3443.60         74.29         3410.24         18         10         3134.90           tail50a         150         10         800         800         356.445         70.41         3514.94         18         10         3134.90           tail50a         150         10         800         800         3713.70         60.05         3645.4         18         10         3293.80           tail50b         13         78         9         1000         1000         3713.70         60.05         288.54         18         9         2683.36           tail50b         150         16         1000         1000         3027.07         70.12         288.24         18         9         2683.36           tail50c         15         18         9         1000         1000         3027.07         70.12         2982.58         18         9         2683.36           tail50c         16         100         1000         313.74         57.07         2429.72         18         9         2895.43           tail50c         13         37         19         10         1050         1050 <th>24</th> <th>tai100d</th> <th>25</th> <th>75</th> <th>15</th> <th>∞</th> <th>650</th> <th>650</th> <th>2008.42</th> <th>26.81</th> <th>1988.54</th> <th>15</th> <th>8</th> <th>1990.56</th> <th>24.38</th> <th>1978.66</th> <th>14</th> <th>6</th> <th>-0.89</th> <th>-0.50</th>	24	tai100d	25	75	15	∞	650	650	2008.42	26.81	1988.54	15	8	1990.56	24.38	1978.66	14	6	-0.89	-0.50
tail50a         15         20         10         800         3564.45         70.41         3514.94         18         10         3293.80           tail50a         10         11         20         10         800         800         3113.70         60.05         3694.24         19         10         3293.80           tail50b         112         20         10         1000         1000         3713.70         60.05         3694.24         19         10         2683.61           tail50b         113         37         18         9         1000         1000         3713.70         60.05         3694.24         19         10         2683.61           tail50b         113         37         19         10         1000         313.74         57.96         305.35         18         9         2693.36           tail50c         113         37         19         10         1050         1050         245.15         7.07         2429.72         18         10         2379.57           tail50d         10         1050         1050         245.15         7.07         2429.72         18         10         2470.30           tail50d         10	25	tai150a	113	37	70	10	800	800	3443.60	74.29	3410.24	18	10	3134.90	70.91	3122.79	14	14	-8.96	-8.43
tail50a         100         112         20         10         800         313.70         60.05         3694.24         19         10         3526.10           tail50b         113         37         18         9         1000         1000         2880.74         72.27         2848.54         18         9         2698.36           tail50b         150         160         1000         1000         3135.74         57.36         3045.35         18         9         2698.36           tail50c         15         18         9         1000         1000         3135.74         57.36         3045.35         18         9         2698.36           tail50c         150         16         1050         1050         245.15         72.07         2429.72         18         10         2379.57           tail50c         16         1650         1050         256.13         253.36         2895.42         10         2470.30           tail50c         11         37         19         10         1050         1050         236.32         259.32         2511.30         18         10         2895.43           tail50d         15         10         950 <th< td=""><th>56</th><td>tai150a</td><td>150</td><td>75</td><td>20</td><td>10</td><td>800</td><td>800</td><td>3564.45</td><td>70.41</td><td>3514.94</td><td>18</td><td>10</td><td>3293.80</td><td>70.91</td><td>3286.83</td><td>15</td><td>14</td><td>-7.59</td><td>-6.49</td></th<>	56	tai150a	150	75	20	10	800	800	3564.45	70.41	3514.94	18	10	3293.80	70.91	3286.83	15	14	-7.59	-6.49
tai150b 113 37 18 9 1000 1000 2880.74 72.27 2848.54 18 9 2688.36 tai150b 113 37 18 9 1000 1000 2880.74 72.27 2848.54 18 9 2688.36 tai150b 150 75 18 9 1000 1000 3135.74 57.96 3075.35 18 9 3040.46 51.00 112 18 9 100 1000 1000 3135.74 57.96 245.21 18 10 2375.77 51.00 112 19 10 1050 1050 2561.24 65.93 2856.72 19 10 2895.43 tai150c 150 75 19 10 1050 1050 2934.67 56.98 2866.72 19 10 2895.43 tai150d 150 75 19 10 950 950 3102.56 68.38 3066.94 18 10 2948.06 tai150d 100 112 19 10 950 3267.00 57.54 3244.30 18 10 3226.23 tai150d 100 112 19 10 950 3267.00 57.54 3244.30 18 10 3226.23 12.14 10 10 10 10 10 10 10 10 10 10 10 10 10	27	tai150a		112	20	10	800	800	3713.70	60.05	3694.24	19	10	3526.10	29.00	3501.39	16	13	-5.05	-5.22
tail50b         150         75         18         9         1000         302707         70.12         2982.58         18         9         2835.00           tail50b         100         1100         1000         30270         457.96         3075.35         18         9         2835.00           tail50c         112         18         10         1050         1050         245.15         72.07         2429.72         18         10         2379.73           tail50c         150         10         1050         1050         2561.24         65.93         2561.30         18         10         2470.30           tail50c         10         12         10         1050         1050         2561.24         65.93         2565.73         19         10         2470.30           tail50c         113         37         19         10         950         950         30.61.12         18         10         2895.43           tail50d         15         10         950         950         3102.56         68.38         3066.94         18         10         2948.06           tail50d         10         11         950         950         3267.00         57.	28	tai150b	_	37	18	6	_	000	2880.74	72.27	2848.54	18	6	2698.36	68.73	2680.50	14	12	-6.33	-5.90
tail50b 100 112 18 9 1000 1000 3135.74 57.96 3075.35 18 9 3040.46 and a subject of tail50c 113 37 19 10 1050 1050 2561.24 65.93 2511.30 18 10 2470.30 and a subject of tail50c 100 112 19 10 1050 1050 293.46 56.93 2856.72 18 10 2895.43 tail50c 100 112 19 10 950 950 3026.12 73.60 2997.12 18 10 2895.43 tail50d 150 75 19 10 950 950 3102.56 68.38 3066.94 18 10 2948.06 tail50d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 226.23 226.24 38.20 2242.54 10 3226.23 3226.23	59	tai150b	150	75		6	_	000	3027.07	70.12	2982.58	18	6	2835.00	68.73	2803.86	15	12	-6.34	-5.99
tail50c         113         37         19         10         1050         1050         2452.15         72.07         2429.72         18         10         2379.57           tail50c         150         75         19         10         1050         1050         2451.24         65.93         2511.30         18         10         2470.30           tail50c         10         10         1050         1050         293.467         56.98         2856.72         19         10         2895.43           tail50c         10         10         950         950         302.61         7360         2997.12         18         10         2895.43           tail50d         15         10         950         950         3102.56         68.38         3066.94         18         10         2948.06           tail50d         10         10         950         950         3267.00         57.54         3244.30         18         10         3226.23           tail50d         10         10         950         950         326.00         25.54         3244.30         18         10         3226.23	30	tai150b	100	112		6	_	000	3135.74	57.96	3075.35	18	6	3040.46	57.14	2999.32	16	12	-3.04	-2.47
tai150c 150 75 19 10 1050 1050 2551.24 65.93 2511.30 18 10 2470.30 tai150c 100 112 19 10 1050 1050 2934.67 56.98 2856.72 19 10 2895.43 tai150d 113 37 19 10 950 950 3102.56 68.38 3066.94 18 10 2948.06 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 2948.06 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 tai150d 100 112 19 10 950 950 950 950 950 950 950 950 950 95	31	tai150c	113	37	19	10	_	020	2452.15	72.07	2429.72	18	10	2379.57	62.69	2360.47	16	14	-2.96	-2.85
tai150c 100 112 19 10 1050 1050 2934.67 56.98 2856.72 19 10 2895.43 tai150d 113 37 19 10 950 950 3026.12 73.60 2997.12 18 10 2802.23 tai150d 150 75 19 10 950 950 3102.56 68.38 3066.94 18 10 2948.06 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 2266.94 38.20 2242.54	32	tai150c	150	75	19	10	_	020	2561.24	65.93	2511.30	18	10	2470.30	62.69	2448.50	16	12	-3.55	-2.50
tail50d 113 37 19 10 950 950 3026.12 73.60 2997.12 18 10 2802.23 0	33	tai150c	100	112	19	10	_	020	2934.67	56.98	2856.72	19	10	2895.43	57.19	2844.30	18	11	-1.34	-0.43
tai150d 150 75 19 10 950 3102.56 68.38 3066.94 18 10 2948.06 0 tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3256.23 2266.94 38.20 2242.54 242.54	34	tai150d	113	37	19	10	920	950	3026.12	73.60	2997.12	18	10	2802.23	89.89	2783.68	14	14	-7.40	-7.12
tai150d 100 112 19 10 950 950 3267.00 57.54 3244.30 18 10 3226.23 2266.94 38.20 2242.54 2147.69	32	tai150d	150	75		10	920	950		68.38	3066.94	18	10	2948.06	89.89	2901.66	14	14	-4.98	-5.39
2266.94 38.20 2242.54 2147.69	36	tai150d	100	112		10	920	950		57.54	3244.30	18	10	3226.23	55.34	3180.48	16	13	-1.25	-1.97
	Avg.								2266.94	38.20	2242.54			2147.69	36.75	2131.67			-5.18	-4.89

each the remaining three problems, the objective function value of the RTTRP solution is the same as that of the best TTRP solution. On average, the objective function value is improved by 1.33% when the fleet size constraint is dropped. Note that we only compared the average results of RTTRP with the TTRP results of Lin et al. (2009) since their solutions are slightly better than Scheuerer's results (2006) on average.

To gain more insights into the difference of solution quality between RTTRP solutions and TTRP solutions, we conducted more computational experiments on a second set of 36 newly generated TTRP instances. These instances are converted from 12 basic VRP problems given by Rochat and Taillard (1995) in a manner similar to what Chao used to generate TTRP benchmark problems (Chao, 2002). For each customer i in an original problem, the distance between i and its nearest neighbor customer is calculated and denoted by  $A_i$ . Each problem is then converted into three TTRP problems. In the first problem, 25% of the customers with the smallest  $A_i$  values are specified as truck customers. This percentage was increased to 50% and 75% in the second and third problem respectively.

The SA parameters used in the computational study for the second set of TTRP problems remain the same except for the penalty cost P, which is increased to 200. The RTTRP solutions obtained by the proposed SA heuristic are compared with the TTRP solutions obtained by the SA heuristic of Lin et al. (2009). Each problem is run 10 times. The best and average solutions from 10 runs, as well as the required computational time for each problem are presented in Table 2. It can be seen that in 35 out of the 36 new TTRP instances, the minimum RTTRP solutions from 10 runs are better that of TTRP solutions (tied on other one). Moreover, the average RTTRP solutions from 10 runs are better than that of TTRP solutions for all instances. The percentage of improvement of average and minimum RTTRP solutions over those of TTRP solutions ranges from 0.5% to 12.87% and 0% to 11.75% respectively. On average the improvement rate over TTRP solutions is about 5% for average and minimum RTTRP solutions.

It is interesting to note that for most test problems, the number of trucks used in RTTRP solutions is less than or equal to that used in TTRP solutions, while the number of trailers used in RTTRP solutions is greater than or equal to that used in TTRP solutions. In other words, the trailer/truck ratio is closer to 1 in RTTRP solutions which is more consistent with the common practice in less than truck load industry. Since the acquisition costs and maintenance costs for trailers are generally less than those of trucks, it may prove worthwhile to use more trailers and fewer trucks in the fleet mix. In addition, using fewer trucks also implies that fewer drivers are needed to service all customers which could result in more savings in addition to routing costs.

# 4. Conclusions

In this paper, we proposed an SA heuristic for the RTTRP, a relaxation of TTRP without the constraint on fleet size. The RTTRP solutions obtained by the SA and the best TTRP solutions reported in prior studies are compared on the 21 benchmark TTRP instances. The RTTRP solution is better than the best TTRP solution in 18 of

the 21 problems. The solution quality of the RTTRP solutions and the best TTRP solutions are the same for the remaining three problems. Further, the average RTTRP results are slightly better than the TTRP results of Lin et al. (2009), which is by far the best solutions to the TTRP. Further experiments are conducted on 36 new TTRP instances generated by this research and obtained similar results. The percentage of improvement of RTTRP solutions over TTRP solutions are about 5%. Consider the rising crude oil price and the low margin of trucking companies, it may be worthwhile for trucking companies to consider leasing or acquiring additional vehicles, especially trailers as they are inexpensive compared to trucks, provided that the resulting savings is greater than the leasing or acquisition costs of the additional vehicles.

#### References

- Bodin, L., Golden, B. L., Assad, A., & Ball, M. O. (1983). Routing and scheduling of vehicles and crews: The state of the art. Computers and Operations Research, 10, 62–212
- Chao, I. M. (2002). A tabu search method for the truck and trailer routing problem. Computers and Operations Research, 29(1), 33–51.
- Christofides, N., Mingozzi, A., & Toth, P. (1979). The vehicle routing problem. In N. Christofides, A. Mingozzi, P. Toth, & C. Sandi (Eds.), *Combinatorial optimization* (pp. 315–338). Chichester, UK; Wiley.
- Chwif, L., Barretto, M. R. P., & Moscato, L. A. (1998). A solution to the facility layout problem using simulated annealing. *Computers in Industry*, 36, 125–132
- Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science*. 6, 80–91.
- Fisher, M. L. (1995). Vehicle routing. In M. O. Ball, T. L. Magnanti, C. L. Monma, & G. L. Nemhauser (Eds.), *Network routing* (pp. 1–33). Amsterdam: North-Holland.
- Gerdessen, J. C. (1996). Vehicle routing problem with trailers. European Journal of Operational Research, 93(1), 135–147.
- Gillett, B., & Miller, L. (1974). A heuristic algorithm for the vehicle dispatch problem. Operations Research, 22(2), 340–349.
- Golden, B. L., & Assad, A. (1988). Vehicle routing: Methods and studies. Amsterdam: North-Holland.
- Hoff, A. (2006). Heuristics for rich vehicle routing problems. Ph.D. thesis, Molde University College, Norway.
- Jayaraman, V., & Ross, A. (2003). A simulated annealing methodology to distribution network design and management. European Journal of Operational Research, 144, 620–645.
- Laporte, G. (1992). The vehicle routing problem: An overview of exact and approximate algorithms. European Journal of Operational Research, 59, 345–358.
- Laporte, G., Gendreau, M., Potvin, J. Y., & Semet, F. (2000). Classical and modern heuristics for the vehicle routing problem. *International Transactions in Operational Research*, 7(4–5), 285–300.
- Laporte, G., & Nobert, Y. (1987). Exact algorithms for the vehicle routing problem. Surveys in Combinatorial Optimization, 31, 147–184.
- Laporte, G., & Semet, F. (2002). Classical heuristics for the capacitated VRP. The Vehicle Routing Problem, 109–128.
- Lim, A., Rodrigues, B., & Zhang, X. (2006). A simulated annealing and hill-climbing algorithm for the traveling tournament problem. European Journal of Operational Research, 174, 1459–1478.
- Lin, S.-W., Yu, V. F., & Chou, S.-Y. (2009). Solving the truck and trailer routing problem based on a simulated annealing heuristic. *Computers and Operations Research*, 36(5), 1683–1692.
- McKendall, A. R., Jr., Shan, J., & Kuppusamy, S. (2006). Simulated annealing heuristics for the dynamic facility layout problem. *Computers and Operations Research*, 33, 2431–2444.
- Rochat, Y., & Taillard, É. D. (1995). Probabilistic diversification and intensification in local search for vehicle routing. *Journal of Heuristics*, 1(1), 147–167.
- Scheuerer, S. (2006). A tabu search heuristic for the truck and trailer routing problem. Computers and Operations Research, 33, 894–909.
- Semet, F., & Taillard, E. (1993). Solving real-life vehicle routing problems efficiently using tabu search. *Annals of Operations Research*, 41, 469–488.
- Toth, P., & Vigo, D. (2002). The vehicle routing problem. Philadelphia, PA: SIAM Monographs on Discrete Mathematics and Applications.