

Discrete Optimization

A hybrid multi-objective evolutionary algorithm for solving truck and trailer vehicle routing problems

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Abstract

This paper considers a transportation problem for moving empty or laden containers for a logistic company. Owing to the limited resource of its vehicles (trucks and trailers), the company often needs to sub-contract certain job orders to outsourced companies. A model for this truck and trailer vehicle routing problem (TTVRP) is first constructed in the paper. The solution to the TTVRP consists of finding a complete routing schedule for serving the jobs with minimum routing distance and number of trucks, subject to a number of constraints such as time windows and availability of trailers. To solve such a multi-objective and multi-modal combinatorial optimization problem, a hybrid multi-objective evolutionary algorithm (HMOEA) featured with specialized genetic operators, variable-length representation and local search heuristic is applied to find the Pareto optimal routing solutions for the TTVRP. Detailed analysis is performed to extract useful decision-making information from the multi-objective optimization results as well as to examine the correlations among different variables, such as the number of trucks and trailers, the trailer exchange points, and the utilization of trucks in the routing solutions. It has been shown that the HMOEA is effective in solving multi-objective combinatorial optimization problems, such as finding useful trade-off solutions for the TTVRP routing problem.

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1. Introduction

1.1. *The trucks and trailers vehicle routing problem*

Singapore ranks among the top international maritime centers of the world. Its sheltered and deep-water harbor lies strategically at the crossroads of major sea routes in South-east Asia. It is the focal point for some 400 shipping lines with links to more than 740 ports worldwide. The Republic's standing as an international maritime centre rests on its port, which is one of the busiest in the world in terms of container throughput. In 2002, the port handled a total of 16.94 million 20-foot equivalent units (TEUs) (Maritime, 2002). In order to support the port activities in lieu with the extremely high throughput at the port, container related logistic services are very prosperous in Singapore. A general model for vehicle capacity planning system (VCPS) consisting of a number of job orders to be served by trucks and trailers daily was constructed for a logistic company that provides transportation services for container movements within the country (Lee et al., 2003). Due to the limited capacity of vehicles owned by the company, engineers in the company have to decide whether to assign the job orders of container movements to its internal fleet of vehicles or to outsource the jobs to other companies daily. The Tabu search meta-heuristic was applied to find a solution for the VCPS problem, where some new rules on how to assign jobs for outsourcing were derived and shown to be about 8% better than existing rules adopted by the company (Lee et al., 2003).

By analyzing different kinds of job orders received from the company, this paper presents a transportation solution for trucks and trailers vehicle routing problem (TTVRP) containing multiple objectives and constraints, which is extended from the VCPS model with detail maneuver of trailers in a routing plan. In TTVRP, the trailers are resources with certain limitations similar to real world scenarios and the allocation of trailers in different locations could affect the routing plans. The TTVRP is a difficult problem which involves many intricate factors such as time window constraints and availability of trailers. The number of trucks in a fleet regulates the maximum number of jobs that can be handled internally within a certain period of time and all jobs must be serviced within a given time window. Instead of handling jobs by the internal fleet of trucks, the jobs can also be considered for outsourcing, if necessary. The routing plan in TTVRP also needs to determine the number of trailer exchange points (TEPs) that are distributed in the region where different types of trailers can be found. Besides, there are a wide variety of job orders that may have diverse requirements for the types of the trailers, time window constraints as well as locations of the source and destination.

The transportation solution to TTVRP contains useful decision-making information, such as the best fleet size to accommodate a variety of job orders and the trend for different number of trailers available at TEPs, which could be utilized by the management to visualize the complex correlations among different variables in the routing problem. Dynamic resource management is an essential component in a logistic company. Long-term planning in resource management (such as the number of vehicles) is rather tedious especially when the business is in a dynamic environment. In order to maintain efficiency, minimizing the cost and investment and maximizing quality of service, long-term resource planning and day-to-day operations are two crucial factors to ensure an organization's success. In this paper, various test cases for the TTVRP model are generated with random variables simulating the long-term operation of business activities. The management can thus formulate the planning for certain variables, such as the number of trucks (long-term capital cost) so that the day-to-day operational cost could be kept at the minimum.

1.2. *Background on vehicle routing problems*

Vehicle routing problem (VRP) is a generic name referred to a class of combinatorial problem in which customers are to be served by a number of vehicles. Some famous models in literature for vehicle routing problems include Dantzig and Ramser (1959), Gendreau et al. (1999a), Laporte et al. (2002), Belenguer

et al. (2000), Yang et al. (2000), Kenyon and Morton (2003), Ichoua et al. (2003), Ghiani and Improta (2000), Swihart and Papastavrou (1999), Salhi and Sari (1997), Min et al. (1998) and Wu et al. (2002). Among these models, there are three types of vehicle routing problems closely related to the TTVRP model presented in this paper, i.e., vehicle routing problem with time windows (VRPTW), vehicle scheduling problem (VSP) and truck and trailer routing problem (TTRP).

The vehicle routing problem with time windows (VRPTW) diverts from the famous vehicle routing problem (VRP). In this problem, a set of vehicles with limited capacity is to be routed from a central depot to a set of geographically dispersed customers with known demands and predefined time window. The time window can be specified in terms of single-sided or double-sided window. In single-sided time window, the pickup points usually specify the deadlines by which they must be serviced. In double-sided time window, however, both the earliest and the latest service times are imposed by the nodes. A vehicle arriving earlier than the earliest service time of a node will incur waiting time. This penalizes the transport management in either the direct waiting cost or the increased number of vehicles, since a vehicle can only service fewer nodes if the waiting time is longer. Some recent publications of VRPTW can be found in Bräysy (2003), Breedam (2001), Caseau and Laburthe (1999), Dullaert (2000), Gezdur and Türkay (2002), Ioannou et al. (2001), Shaw (1998), Li and Lim (2002), Chavalitwongse et al. (2003), Bent and Van Hentenryck (2001) and Berger et al. (2001). Surveys about VRPTW can be found in Desrosier et al. (1995), Desrochers et al. (1992), Golden and Assad (1988), Solomon (1987), Crainic and Laporte (1998), Kilby et al. (2000), Toth and Vigo (2002), Bräysy and Gendreau (2001a,b) etc. In contrast to the TTVRP, the VRPTW neither have any limitation on resources of trailers nor the outsourcing of jobs to external companies.

The vehicle scheduling problem (VSP) (Baita et al., 2000; Brandão and Mercer, 1997; Pretolani, 2000; Boland et al., 2000; Dror, 2000; Hertz and Mittaz, 2001) assumed that the routing to different sites can be completed with multiple trips. Each trip consists of a pair of specified source and destination, each one defined by the starting and ending times. The objective is to minimize the number of vehicles and the cost function based upon deadheading trips (gas, driver etc) and idling time for the vehicle. The constraints for this model include the traveling distance and time for normal service and refueling as well as the restriction that certain tasks can only be handled by specified type of vehicles. In contrast to vehicle routing problem, one customer may be visited more than once or not at all, which is solely depending on the trips data. Although trips in VSP may be analogous to the concept of a job in TTVRP, the VSP does not include the complexity of trailer type constraints.

Chao (2002) presented the problem of TTRP (a variant of VRP), which considers the fleet size of trucks and trailers in the model. In order to provide service to different categories of customers, there are three types of routes in a solution: (1) route that a truck travels alone; (2) route that a truck and trailer are required; (3) route that trailer is only required at certain sub-tour. The objective is to minimize the total traveling distance and the cost incurred by the fleet. Unlike TTRP, the TTVRP requires the trucks to visit trailer exchange points for picking up the correct trailer types depending on the jobs to be serviced. Besides, jobs that are not routed by self-fleets in TTVRP can be outsourced to external companies.

1.3. Meta-heuristic solutions to vehicle routing problems

Most vehicle routing problems are NP-hard and associated with real world transportation problems (Glaab, 2002; Baptista et al., 2002; Mourão and Almeida, 2000; Dillmann et al., 1996; Fölsz et al., 1995; Karkazis and Boffey, 1995; Muyldermans et al., 2002; Doerner et al., 2002; Baita et al., 2000). Due to the inherent variations in real world environment, the solution to each vehicle routing problem is often unique and satisfies an exclusive set of constraints and objectives according to the problem scenario. Generally, vehicle routing problems have been attempted by different approaches ranging from exact algorithms (Applegate et al., 2002; Bard et al., 2002; Mingozzi et al., 1999) to heuristics (Gerdeseen, 1996; Kohl et al., 1999; Beullens et al., 2003; Renaud and Boctor, 2002; Breedam, 2002; Toth and Vigo, 1999; Liu and Shen,

1999; Beasley and Christofides, 1997). Categorized by Fisher (1995) as the third generation approach, a number of meta-heuristics such as Tabu search (Taillard et al., 1997; Kelly and Xu, 1999; Rego, 1998; Gendreau et al., 1999c; Tuzun and Burke, 1999; Amberg et al., 2000; Rego and Roucairol, 1995; Potvin et al., 1996; Cordeau et al., 2001; Cordone and Wolfler-Calvo, 2001; Lee et al., 2003), ant colony optimization (Gambardella et al., 1999; Reimann and Doerner, 2002), simulated annealing (Breedam, 1995; Chiang and Russel, 1996) and genetic algorithms (Gehring and Homberger, 2001; Grefenstette et al., 1985; Homberger and Gehring, 1999; Malmberg, 1996; Poon and Carter, 1995; Tan et al., 2001a,b; Thangiah et al., 1994; Thangiah, 1995) have been applied to find good solutions for large-scale vehicle routing problems. A recent survey on various meta-heuristic algorithms was presented by Ribeiro and Hansen (2002).

The TTVRP problem addressed in this paper is NP-hard, which involves the optimization of routes for multiple trucks in order to meet all given constraints and to minimize multiple objectives of routing distance and number of trucks concurrently. Some of the existing routing approaches that strive to minimize a single criterion of routing cost or number of trucks is not suitable for solving such a multi-modal and multi-objective combinatorial problem. The TTVRP should be best tackled by multi-objective optimization methods, which offer a family of Pareto-optimal routing solutions containing both the minimized routing cost and number of trucks. In this paper, a hybrid multi-objective evolutionary algorithm (HMOEA) that incorporates the heuristic search for local exploitation and the concept of Pareto's optimality for finding the trade-off is applied to solve the problem of TTVRP. The HMOEA optimizes all routing constraints and objectives concurrently, without the need of aggregating multiple criteria into a compromise function. Unlike conventional multi-objective evolutionary algorithms (MOEAs) that are designed with simple coding or genetic operators for parameterized optimization problems (Cvetkovic and Parmee, 2002; Knowles and Corne, 2000; Tan et al., 2001c), the HMOEA is featured with specialized genetic operators and variable-length chromosome representation to accommodate the sequence-oriented optimization problem in TTVRP.

The paper is organized as follows: Section 2 describes the scenario and modeling of the TTVRP with mathematical formulation. Section 3 gives a brief description of multi-objective evolutionary optimization and its applications in a number of domain-specific combinatorial problems. The program flowchart of HMOEA and its various features including variable-length chromosome representation, specialized genetic operators, Pareto fitness ranking and local search heuristics are also described in Section 3. Section 4 presents the extensive simulation results and discussions for the TTVRP problem. Conclusions are drawn in Section 5.

2. The problem scenario

The TTVRP model with detail maneuver of the trailers in a routing plan is extended from a real world VCPS system proposed by Lee et al. (2003). Both of the problems are variants of vehicle routing problem with time windows constraints (VRPTW). The additional constraints and conditions apply in TTVRP indicate that the problem is fundamentally more difficult than a simple VRPTW, and thus it is essentially another NP hard problem. In solving the TTVRP, the movement of containers among customers, depots and the port are major transportation job orders considered. A container load is handled like a normal truck-load but these loads use containers with a possible chassis instead of trailers only. From the equipment assignment point of view, a correct trailer type is essential for the routing. For an inbound job, a loaded container is taken from a vessel to a customer and returned empty to the depot. For an outbound job, however, an empty container is picked up from the depot and taken to the customer before returning loaded to the vessel. Every job order contains the location of source and destination as well as other customers' information. Other specification such as load requirement and time windows are specified as hard constraints in the model. There are a total of six types of job orders which are varied according to the source and desti-

nation (port, warehouse, depot or trailer exchange), time windows (tight or loose), loaded trip (or empty) and type of trailers (20 or 40) as follows:

1. Import with trailer type 20.
2. Import with trailer type 40.
3. Export with trailer type 20.
4. Export with trailer type 40.
5. Empty container movement with trailer type 20.
6. Empty container movement with trailer type 40.

The logistic company owns a maximum of 40 trucks and a number of trailers that are larger than the number of trucks. A truck must be accompanied with a trailer when servicing a customer, i.e., the routing needs to consider both the locations of truck and trailer. An “export” job order works as follows: a truck first picks up a correct trailer at a trailer exchange point and a container at the depot. It then proceeds to the warehouse and leaves the trailer and container there for about two days where the container is filled. A truck (which may not be the same truck as earlier) will later be allocated to move the loaded container using the earlier assigned trailer and leaves the container at the port before departing with the trailer. In contrast, an “import” job order works as follows: a truck picks up a correct trailer at a TEP before it proceeds to the port. The trailer is used to carry loaded container at the port. The truck then moves the container to the warehouse and leaves it there for about two days. A truck (which may not be the same truck as earlier) will later move this empty container from the warehouse to the depot (using a trailer) and leaves the depot with its trailer unloaded. Intuitively, there are times when a truck has a correct trailer type and thus can serve a job without going to a trailer exchange point. Otherwise, a truck is required to pick up a trailer (from the nearest TEP where the trailer is available to be picked up or exchanged) when it has mismatch trailer type or does not carry a trailer. The number of trailers available at an exchange point depends on how many trailers were picked up and returned to the TEP. The constraint imposed on the model is the time windows at the source and destination of job orders. An assumption is made such that all trailer exchange points have similar operating hours as the truck drivers’ working hours, i.e., from 8:00 am to 8:00 pm.

2.1. Modeling the problem scenarios

Based on the scenarios described, some refinements have been made to the model proposed by Lee et al. (2003). The problem is modeled here on a daily basis where the planning horizon spans only one day. All import and export jobs consist of two sub-trips and a two-day interval at the customer warehouses. Therefore the two-day interval at customer warehouses divides a job nicely into two separate planning horizons (one day each). The import and export jobs can be broken into two independent tasks, where each of them falls into a different planning horizon. In this way, job orders are broken into sub-job type precisely (Hereinafter this is referred as sub-job or a task). Generally a task involves traveling from a point (source) to another point (destination) as listed in Table 1.

The number of trailers at TEPs depends on the trailers that are left over from the previous planning horizon. All the pickup, return and exchange activities can also change the number of trailers available. Besides, a number of trailers could also be parked at the customer warehouses instead of the TEPs. All these undetermined factors suggest that the resource of trailers available at each TEP at the initial of planning horizon is random. Therefore the daily number of trailers at each trailer exchange point is randomly generated in our model. A truck has to pick up a correct trailer from the nearest TEP if it serves task type 1, 2, 5, 6, 9, 10, 11 or 12 and does not have a trailer or has an incorrect trailer type. For task type 3, 4, 7 or 8, the truck does not need to visit a TEP before servicing the task since the correct trailer has been brought to the place in advanced. In contrast, trucks that serve sub-job type 3, 4, 7 or 8 must not have any trailers. In this case, if a

Table 1
The task type and its description

Task type	Task description	Source	Destination	Trailer type
1	Sub-trip of import job	Port	Warehouse	20
2	Sub-trip of import job	Port	Warehouse	40
3	Sub-trip of import job	Warehouse	Depot	20
4	Sub-trip of import job	Warehouse	Depot	40
5	Sub-trip of export job	Depot	Warehouse	20
6	Sub-trip of export job	Depot	Warehouse	40
7	Sub-trip of export job	Warehouse	Port	20
8	Sub-trip of export job	Warehouse	Port	40
9	Empty container movement	Port	Depot	20
10	Empty container movement	Depot	Port/Depot	20
11	Empty container movement	Port	Depot	40
12	Empty container movement	Depot	Port/Depot	40

trailer is attached to the truck, it must be returned to a trailer exchange point before servicing the task. For example, a truck that serves sub-job type 7 leaves the destination (port) of a previous task with a trailer. If the same truck is to serve another task type 3, 4, 7 or 8, it must travel to a TEP to drop the trailer obtained previously. In brief, a truck is required to visit a trailer exchange point under the following conditions:

- It needs a trailer for task type 1, 2, 5, 6, 9, 10, 11 or 12 and it does not have a trailer.
- It needs a trailer for task type 1, 2, 5, 6, 9, 10, 11 or 12 and it has an incorrect trailer type.
- It has a trailer but it has to service sub-job type 3, 4, 7 or 8, e.g., the truck needs to travel to a TEP for dropping the trailer before servicing the task.

Obviously the availability of trailers at TEPs should be updated frequently since the number of trailers changes with the pick-up and return activities, e.g., a trailer that is returned earlier in a day will be available for pick-up later in the same day. To model these activities, the approach of time segmentation for trailer resources is used as follows:

- Working hours per day: 12 hours \times 60 minutes = 720 minutes.
- Time per segment: 10 minutes.
- Number of time slots available: $\frac{720}{10}$ slots = 72 slots.

Hence the number of trailers available for pick-up in a particular time slot is equal to the number of trailers in previous time slot, added by the trailers returned in previous time slot and deducted the trailers picked up in previous time slot. In this approach, different trailer types are managed and updated in separate lists. For example, a TEP has 3 trailers (with type 20) and the following events occur in the current time slot: one trailer (type 20) is returned and two trailers (type 20) are picked up. In this case, the trailer exchange point should have two trailers (type 20) available for pick up in the next time slot.

2.2. Mathematical model

2.2.1. Decision variables

$X_{ikm} \in \{0, 1\}$, where $i = \{1, \dots, I\}$, $k = \{1, \dots, K\}$, $m = \{1, \dots, M\}$. If task i is assigned to truck k as the m th task, $X_{ikm} = 1$, otherwise $X_{ikm} = 0$;

$X_{i0} \in \{0, 1\}$, $i \in \{1, \dots, I\}$. If task i is sub-contracted to companies, $X_{i0} = 1$, otherwise $X_{i0} = 0$.

2.2.2. Parameters

I	number of tasks;
K	maximum number of trucks;
M	maximum number of jobs that can be handled by one truck in a planning horizon;
J	number of trailer exchange points;
y	task type, i.e., $y \in \{1, \dots, 12\}$;
$I(y)$	the set of task with type y ;
$\cup_y I(y)$	all tasks = $\{1, \dots, I\}$;
TW	time segment for trailer resources = 10;
MTW	maximum number of time slots = 72.

Symbol

$\lceil x \rceil$	the smallest integer larger or equal to x ;
$\lfloor x \rfloor$	the largest integer smaller or equal to x .

Distance of tasks' location

D_{hji}	distance from destination of previous task h to trailer point j followed by source of task i ;
D_{hi}	distance from destination of previous task h to source of task i ;
D_i	distance from source of task i to destination of task i .

Task handling time

H_{i1}	handling time at source of task i ;
H_{i2}	handling time at destination of task i .

Task time window

R_{i0}	start-time at the source of task i ;
R_{i1}	end-time at the source of task i ;
R_{i2}	start-time at the destination of task i ;
R_{i3}	end-time at the destination of task i ;
A_{k0}	start available time for truck k ;
A_{kf}	end available time for truck k .

Cost

P_i	routing cost of task i for internal fleet operation;
S_i	routing cost of task i for outsourced.

Number of trailers at trailer exchange point

TP_{40j}	initial number of trailer type 40 at point j ;
TP_{20j}	initial number of trailer type 20 at point j .

2.2.3. Minimization objectives

The routing solutions should minimize both the criteria of routing cost and the number of trucks concurrently as follows:

$$\text{Routing cost} = \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M X_{ik_m} P_i + \sum_{i=1}^I X_{i0} S_i;$$

$$\text{Number of trucks} = \sum_{k=1}^K \left\lceil \frac{\sum_{i=1}^I \sum_{m=1}^M X_{ik_m}}{I} \right\rceil;$$

subject to the following requirements and constraints:

Task and trailer types requirements

$$\text{pickup}_{20,k,m} = \sum_{y=1,2,5,6,4,8,11,12} \sum_{y'=1,5,9,10} \sum_{j \in I(y)} \sum_{i \in I(y')} (X_{ik_m})(X_{jk_{m-1}}) \quad \text{for } m > 1;$$

$$\text{pickup}_{20,k,m} = \sum_{y=1,5,9,10} \sum_{i \in I(y)} X_{ik_m} \quad \text{for } m = 1;$$

$$\text{pickup}_{40,k,m} = \sum_{y=1,2,3,5,6,7,9,10} \sum_{y'=2,6,11,12} \sum_{j \in I(y)} \sum_{i \in I(y')} (X_{ik_m})(X_{jk_{m-1}}) \quad \text{for } m > 1;$$

$$\text{pickup}_{40,k,m} = \sum_{y=2,6,11,12} \sum_{i \in I(y)} X_{ik_m} \quad \text{for } m = 1;$$

$$\text{return}_{20,k,m} = \sum_{y=3,7,9,10} \sum_{y'=2,6,11,12,3,4,7,8} \sum_{j \in I(y)} \sum_{i \in I(y')} (X_{ik_m})(X_{jk_{m-1}});$$

$$\text{return}_{40,k,m} = \sum_{j \in I(y)} \sum_{i \in I(y)} (X_{ik_m})(X_{jk_{m-1}}) \quad \text{for } y = 4, 8, 11, 12; y' = 1, 5, 9, 10, 3, 4, 7, 8;$$

$$\text{visit}_{k,m} \in \{0, 1\};$$

$$\text{pickup}_{20,k,m} + \text{pickup}_{40,k,m} + \text{return}_{20,k,m} + \text{return}_{40,k,m} \geq \text{visit}_{k,m};$$

$$\text{pickup}_{20,k,m} + \text{pickup}_{40,k,m} + \text{return}_{20,k,m} + \text{return}_{40,k,m} \leq 2\text{visit}_{k,m}.$$

Single assignment

A task is only assigned to one truck k (as the m th task) or outsourced to other companies,

$$\sum_{k=1}^K \sum_{m=1}^M X_{ik_m} + X_{i0} = 1 \quad \text{for } i \in \{1, \dots, I\}.$$

Jobs must be assigned sequentially

For $k \in \{1, \dots, K\}$, $m \in \{1, \dots, M-1\}$, $\sum_{i=1}^I X_{ik_{(m+1)}} \leq \sum_{i=1}^I X_{ik_{(m)}}$.

Time sequence for each task

For $k \in \{1, \dots, K\}$, $m \in \{1, \dots, M-1\}$, $T_{k_{(m+1)}(0)} = T_{k_{(m)}(2)}$;

For $k \in \{1, \dots, K\}$, $m \in \{1, \dots, M\}$.

$$T_{k_m(1)} \geq T_{k_m(0)} + \sum_{i=1}^I X_{ik_{(m)}} \left\{ H_{i1} + \sum_{h=1}^I X_{hk_{m-1}} [\text{visit}_{k,m} D_{hji} + (1 - \text{visit}_{k,m}) D_{hi}] \right\};$$

$$T_{k_m(2)} \geq T_{k_m(1)} + \sum_{i=1}^I X_{ik_m}(D_i + H_{i2}).$$

Time window constraints

For $k \in \{1, \dots, K\}$, $m \in \{1, \dots, M-1\}$, $A_{k0} \leq T_{k_m(0)} \leq A_{kf} - (T_{k_m(2)} - T_{k_m(0)})$;

$$R_{i0} \leq \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(T_{k_m} - H_{i1}) + X_{i0}R_{i0} \leq R_{i1}.$$

For every particular $i \in \{1, \dots, I\}$,

$$R_{i2} \leq \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(T_{k_m(2)} - H_{i2}) + X_{i0}R_{i2} \leq R_{i3}.$$

Trailer constraints

$X_{ik_m}(t) \in \{0, 1\}$, where $X_{ik_m}(t) = 1$ when the event falls into time window t ,

$$X_{ik_m}(t) = 1 - \left\lfloor \frac{\left\lfloor \left\lfloor \frac{T_{k_m(1)}}{MTW} \right\rfloor - t \right\rfloor}{MTW} \right\rfloor.$$

The number of trailer type 20 at time slot $t = 0$, i.e., $B_{20j}(0) = TP_{20j}$.

For every $t = 0$ to 71, and every j , the number of trailer type 20 available for next time slot, $t + 1$, is

$$B_{20j}(t+1) = B_{20j}(t) + \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(t) \cdot \text{return}20_{k,m} - \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(t) \cdot \text{pickup}20_{k,m};$$

where $B_{20j}(t) \geq 0$.

The number of trailer type 40 at time slot $t = 0$, i.e., $B_{40j}(0) = TP_{40j}$.

For every $t = 0$ to 71, and every j , the number of trailer type 40 available for next time slot, $t + 1$, is

$$B_{40j}(t+1) = B_{40j}(t) + \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(t) \cdot \text{return}40_{k,m} - \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M X_{ik_m}(t) \cdot \text{pickup}40_{k,m};$$

where $B_{40j}(t) \geq 0$.

2.3. Test cases generation

The TTVRP models various factors affecting the routing performance, particularly on the importance of trailer resources such as the trailers allocation in multiple trailer exchange sites and the location of trailer exchange points. In order to examine these factors thoroughly, a number of test cases with different combination of variables are generated according to the following criteria:

- Number of tasks.
- Total number of trailers.
- Number of trailers and allocation.
- Number of trailer exchange points (with trailer resources assigned initially).

The test cases are generated based on the scenario of one-day activity for a logistic company. The jobs schedule starts from 8:00 am to 8:00 pm (12 hours a day). All the tasks must be finished within a day and

the details of every task are generated. The service map for the problem contains one port, three depots and five trailer exchange points. The five TEPs are named as TEP1, TEP2, TEP3, TEP4 and TEP5, which are located at disperse places and may have different initial number of trailers. The problem also defines the location of 80 customer sites spreading across the area randomly. The service map for the problem is a 120×120 grid and the locations of customers are given as a pair of (x, y) coordinates. The distance (traveling time) among any two points is calculated as $0.5 \times (\text{triangular distance})$, where the value of 0.5 is merely a scaling factor such that a truck can serve around three tasks per day in average. The timing constraint is also specified in the test cases, e.g., the handling time at the source and destination (i.e., port, depot, and customer warehouses) requires 10 minutes, which must be included in calculating the time needed for a complete job handling. The time windows for the source and destination of each job are generated according to the type of jobs. The availability of trailer resources is quantified into 10-minutes slots. The return of a trailer is only visible to others after the current time slot, where the retrieval of a trailer gives immediate effect to the current count of trailers. The cost for each task type is based on the way tasks are accomplished, i.e., by self-fleet service or outsourced to external companies. There is no hard rule to specify whether the cost for internal fleet is cheaper than outsource fleet and vice versa, i.e., the cost merely depends on the type of jobs to be served.

There are a total of 28 test cases generated in this study, which differs in terms of the number of task orders, the number of trailers, allocation of trailers, and the number of trailer exchange points. However, information about customer warehouses and other important locations like port and depots remains unchanged. Table 2 lists the test cases for NORM (Normal) category, where the trailers are allocated “equally” to TEPs. As shown in Table 2, the test cases in this category are divided into four groups with different number of tasks in the range of 100–132, and all TEPs can contribute to the supply of any demands for trailers. As shown in Table 3, the eight test cases for TEPC (Trailer Exchange Point Case) category contain a constant of 132 tasks, but are assigned with extreme trailer allocation strategies. In some cases, only one TEP is allocated with trailers, while the available number of trailers remains constant at 30 for all test cases in this category. As shown in Table 4, the LTTC (Less Trailer Test Case) category comprises of eight test cases with an equal number of trailers. In this category, the available number of trailers is set as 10, e.g., the trailer resources for both TEPC and LTTC test cases share the same distribution ratio but are assigned with different quantity of trailers.

Table 2
Test cases for the category of NORM

Group	Test case ^a	Job number	Trailers at each TEP	TEPs allocated with trailers	Distribution
100	test_100_1_2	100	1 or 2	5	Uniform
	test_100_2_3	100	2 or 3	5	Uniform
	test_100_3_4	100	3 or 4	5	Uniform
112	test_112_1_2	112	1 or 2	5	Uniform
	test_112_2_3	112	2 or 3	5	Uniform
	test_112_3_4	112	3 or 4	5	Uniform
120	test_120_1_2	120	1 or 2	5	Uniform
	test_120_2_3	120	2 or 3	5	Uniform
	test_120_3_4	120	3 or 4	5	Uniform
132	test_132_1_2	132	1 or 2	5	Uniform
	test_132_2_3	132	2 or 3	5	Uniform
	test_132_3_4	132	3 or 4	5	Uniform

^a The last digit denotes the number of trailers allocated for each TEP.

Table 3
Test cases for the category of TEPC

Test case	Job number	Number of trailers at TEPs	TEPs allocated with trailers	Distribution ^a
test_132_tep5	132	30	5	Uniform
test_132_tep1a	132	30	1	TEP1
test_132_tep1b	132	30	1	TEP2
test_132_tep1c	132	30	1	TEP3
test_132_tep1d	132	30	1	TEP4
test_132_tep1e	132	30	1	TEP5
test_132_tep3a	132	30	3	Distributed among TEP1, TEP3 and TEP5
test_132_tep3b	132	30	3	Distributed among TEP1, TEP2 and TEP4

^a Fixed number of trailers and different distribution of TEPs.

Table 4
Test cases for the category of LTTC

Test case	Job number	Number of trailers at TEPs	TEPs allocated with trailers	Distribution ^a
test_132_ltt5	132	10	5	Uniform
test_132_ltt1a	132	10	1	TEP1
test_132_ltt1b	132	10	1	TEP2
test_132_ltt1c	132	10	1	TEP3
test_132_ltt1d	132	10	1	TEP4
test_132_ltt1e	132	10	1	TEP5
test_132_ltt3a	132	10	3	Distributed among TEP1, TEP3 and TEP5
test_132_ltt3b	132	10	3	Distributed among TEP1, TEP2 and TEP4

^a Less trailers and different distribution of TEPs.

3. A hybrid multi-objective evolutionary algorithm

As described in Section 1, the TTVRP should be best solved via multi-objective optimization, e.g., it involves optimizing routes for multiple trucks to meet all constraints and to minimize the conflicting costs of routing distance and number of trucks concurrently. This section presents a hybrid multi-objective evolutionary algorithm designed for solving the TTVRP problem. Section 3.1 gives a brief review on the applications of multi-objective evolutionary algorithms for domain-specific combinatorial problems. The program flowchart of the HMOEA is illustrated in Section 3.2. The remaining sections present various features of HMOEA, including the variable-length chromosome representation in Section 3.3, specialized genetic operators in Section 3.4, Pareto fitness ranking in Section 3.5, and fitness sharing in Section 3.6. Following the concept of hybridizing local optimizers with multi-objective evolutionary algorithms for better local exploitation (Tan et al., 2001c), Section 3.7 describes the local heuristics that are incorporated in HMOEA.

3.1. Multi-objective evolutionary algorithms in combinatorial applications

Evolutionary algorithms (Bäck, 1996) are global search optimization techniques based upon the mechanics of natural selection and reproduction, which have been found to be very effective in solving complex multi-objective optimization problems (Burke and Newall, 1999; Jaskiewicz, 2003; Bagchi, 1999; Deb,

2001; Cvetkovic and Parmee, 2002; Knowles and Corne, 2000; Fonseca and Fleming, 1993; Refanidis and Vlahavas, 2003; Kursawe, 1991). Without the need of linearly combining multiple attributes into a composite scalar objective function, evolutionary algorithms incorporate the concept of Pareto's optimality to evolve a family of solutions at multiple points along the trade-off surface. Among some famous multi-objective evolutionary algorithms, NSGA, NSGA II, PAES, SPGA and MOMGA-II are widely applied in solving problems with continuous domain space. These algorithms can be found in several surveys such as: Coello Coello (1999), Coello Coello et al. (2002), Fonseca (1995), Van Veldhuizen and Lamont (2000) and Zitzler and Thiele (1999).

Although multi-objective evolutionary algorithms have been applied to solve a number of domain-specific combinatorial problems, such as flowshop and jobshop scheduling, nurse scheduling, and timetabling, these algorithms are often designed with specialized genetic representation or operators for specific applications, which are hard to be used directly for solving the TTVRP addressed in this paper. For example, Murata and Ishibuchi (1996) presented two hybrid genetic algorithms (GAs) to solve a flowshop scheduling problem that is characterized by unidirectional flow of work with a variety of jobs being processed sequentially in a one-pass manner. Jaszkievicz (2001) proposed the algorithm of Pareto simulated annealing (PSA) to solve a multi-objective nurse scheduling problem. Chen et al. (1996) proposed a GA-based approach to tackle continuous flowshop problem in which intermediate storage is required for partially finished jobs. Dorndorf and Pesch (1995) proposed two different implementations of GA using priority-rule-based and machine-based representations to solve a jobshop scheduling problem (JSP). The JSP concerns the processing of several machines with mutable sequence of operations, i.e., the flow of work may not be unidirectional as encountered in the flowshop problem. Ben et al. (1998) later devised a specific representation with two partitions in a chromosome to deal with the priority of events (in permutation) and to encode the list of possible time slots for events, respectively. Jozefowicz et al. (2002) solved a multi-objective capacitated vehicle routing problem using a parallel genetic algorithm with hybrid Tabu search. Paquete and Fonseca (2001) proposed an algorithm with modified mutation operator (and without recombination) to solve a multi-objective examination timetabling problem.

3.2. Program flowchart of HMOEA

Unlike many parametric optimization problems, the TTVRP does not have a clear neighborhood structure, i.e., it is difficult to trace or predict good solutions for the TTVRP since feasible solutions may not be located at the neighborhood of current candidate solutions. To design an evolutionary algorithm that is capable of solving such an ordered-based multi-modal and multi-objective optimization problem, a few features like variable-length chromosome representation, specialized genetic operators, Pareto ranking, fitness sharing, and local search heuristics are incorporated in the HMOEA. The program flowchart of EA is shown in Fig. 1. As can be seen, the simulation begins by reading the information of all tasks. An initial population is then built such that each individual must at least be a feasible candidate solution, i.e., every individual and route in the initial population must be feasible. The initialization process is started by inserting tasks into an empty route one-by-one in a random order, where any task violating the constraints is deleted from the current route. The route is then accepted as part of the solutions and a new empty route is added to serve the deleted and remaining tasks. This process continues until all tasks are routed and a feasible initial population is built as depicted in Fig. 2.

Once an initial population is formed, all individuals in the population will be evaluated based upon the cost functions and will be ranked according to the Pareto ranking scheme. A simple fitness sharing approach (Fonseca and Fleming, 1998) is applied to distribute the population along the Pareto front uniformly. The tournament selection scheme (Tan et al., 2001c) with a tournament size of 2 is then performed, where individuals in the population are randomly grouped into pairs and those individuals with a lower rank in partial order will be selected for reproduction. A simple elitism mechanism (Tan et al.,

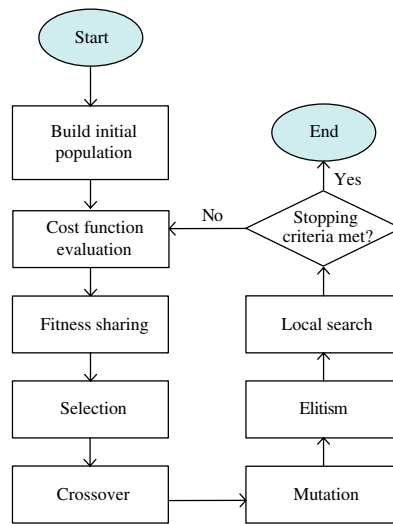


Fig. 1. The program flowchart of EA.

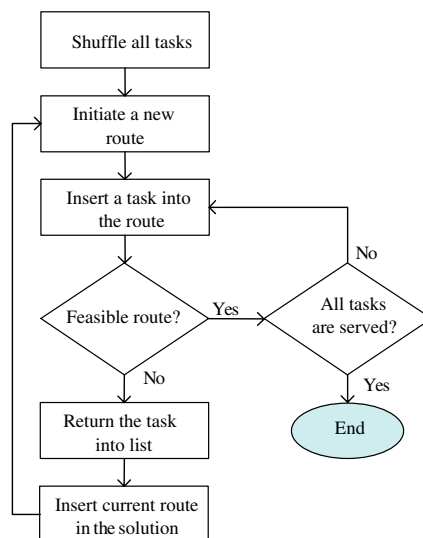


Fig. 2. The procedure of building a feasible initial population in HMOEA.

2001c) is employed in the HMOEA to achieve a faster convergence and better routing solutions. The specialized genetic operators in HMOEA consist of route-exchange crossover and multi-mode mutation. To improve the local exploitation and internal routing of individuals, simple heuristics are performed at each generation of the HMOEA. It should be noted that the feasibility of all new individuals reproduced after the process of specialized genetic operations and local heuristics is retained without the need of any repairing mechanism. The evolution process repeats until a predefined number of generations are reached or no significant performance improvement is observed over the last G generations, where G is a parameter to be set in experiments.

3.3. Variable-length chromosome representation

The chromosome in an evolutionary algorithm is often represented as a fixed-structure bit string and the bits position in a chromosome are usually assumed to be independent and context insensitive. However, such a representation is not suitable for the order-oriented combinatorial TTVRP problem, for which the sequence among customers is essential. In HMOEA, a variable-length chromosome representation is adopted, where each chromosome encodes a complete routing plan including the number of routes and tasks served by the trucks, e.g., a route is a sequence of tasks to be served by a truck. In every route there must be at least one task assignment, and any task that is not assigned to a route is considered for outsourcing (all the outsourced tasks are contained in a list). The number of trailers must be up-to-date and a routing plan must include supplementary information of trailers availability in every trailer exchange points. As shown in Fig. 3, a chromosome may consist of several routes and each route or gene is not a constant but a sequence of tasks to be served. Such a variable-length representation is efficient and allows the number of trucks to be manipulated and minimized directly for the multi-objective optimization in TTVRP.

3.4. Specialized genetic operators

Since standard genetic operators may generate individuals with infeasible routing solutions for TTVRP, specialized genetic operators of route-exchange crossover and multi-mode mutation are incorporated in HMOEA as described in the following sub-sections.

3.4.1. Route-exchange crossover

Classical one-point crossover may produce infeasible routing sequence for combinatorial problems because of the duplication and omission of vertices after reproduction. Goldberg and Lingle (1985) proposed a PMX crossover operator suitable for combinatorial optimization problems. The operator cuts out a section of the chromosome and puts it in the offspring. It maps the remaining bits to the same absolute position or the corresponding bit in the mate's absolute position to avoid any redundancy. Whitley et al. (1989) proposed a genetic edge recombination operator to solve a TSP problem. For each node, an edge-list

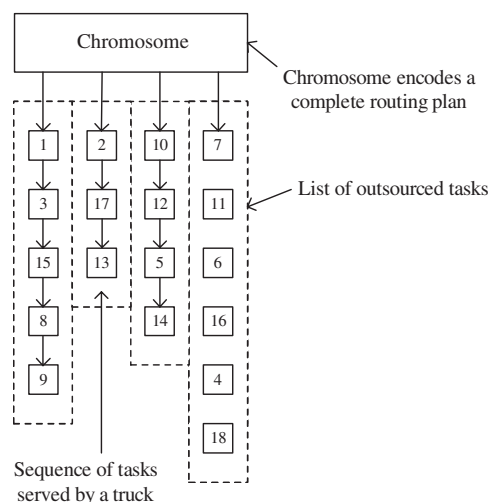


Fig. 3. The data structure of chromosome representation in HMOEA.

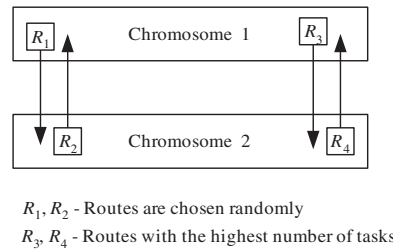


Fig. 4. The route-exchange crossover in HMOEA.

containing all nodes is created. The crossover parents share the edge-lists where several manipulations on edge-list are repeated until all edge-lists are processed. Oliver et al. (1987) proposed a permutation-based crossover operator for TSP problems, e.g., the cycle crossover that focuses on preserving the order of sequence from parents to children. Potvin and Bengio (1996) implemented two crossover operators for bit string representation when solving a vehicle routing problem. Ishibashi et al. (2000) proposed a two-point ordered crossover that randomly selects two crossover points from the parents and decides which segment should be inherited to the offspring.

As shown in Fig. 4, a simple route-exchange crossover is adopted in HMOEA, which allows good sequence of routes or genes in a chromosome to be shared with other chromosomes in the population. The operation starts by grouping chromosomes into pairs randomly and the crossover is performed according to a predefined crossover rate (PC). The operation consists of two independent steps: (1) two random routes (one from each chromosome) are selected and swapped between the two chromosomes; (2) the route with the highest number of tasks from each chromosome is swapped. To ensure the feasibility of chromosomes after crossover, each task can only appear once in a chromosome, i.e., any task in a chromosome that is also found in the newly inserted route will be deleted during the insertion of new routes. Such a crossover operation will not cause any violation in terms of time windows in TTVRP. Deleting a task from a route will only incur certain waiting time before the next task is served, and thus will not result in any conflicts for the time windows. Besides, any task that violates the trailer resources constraint will be assigned for outsourcing and hence all the reproduced chromosomes will remain as feasible routing solutions.

3.4.2. Multi-mode mutation

Gendreau et al. (1999b) proposed a RAR mutation operator, which extracts a node and inserts it at a random point of the routing sequence in order to retain the feasibility of solutions. Ishibashi et al. (2000) extends the approach to a shift mutation operator, which extracts a segment or a number of nodes (instead of a node) and inserts it at a new random point to generate the offspring. During the crossover operation by HMOEA, routes' sequence is exchanged in a whole chunk and no direct manipulation is made to the internal ordering of the nodes for TTVRP. The sequence in a route is modified only when any redundant nodes in the chromosome are deleted. A multi-mode mutation is adopted in HMOEA, which serves to complement the crossover by optimizing the local route information of a chromosome. The mutation is expected to trigger changes of tasks sequence within a chromosome and the mutation rate is considerably small since it could be destructive to the chromosome structure and information of routes. A random number is generated to choose between two possible operations in the mutation. The first operation picks two routes in a chromosome randomly and concatenates the first route to the second route before deleting the first route from the chromosome. In the second operation, the sequence containing all the outsourced tasks is evaluated as a new route. The approach also checks feasibility on the route in order to delete any tasks that cause violation to any of the constraints, and those deleted tasks will be considered as outsourced tasks.

3.5. Pareto fitness ranking

The TTVRP is a multi-objective optimization problem where a number of objectives such as the number of trucks and the cost of routing need to be minimized concurrently, subject to a number of constraints like time windows and availability of trailers. A solution in multi-objective optimization is Pareto-optimal if, in shifting from a point to another point in the Pareto-optimal set, any improvement in one of the objective functions from its current value causes at least one of the other objective functions to deteriorate from its current value (Fonseca, 1995). The role of HMOEA for multi-objective optimization in TTVRP is thus to discover such a set of Pareto-optimal solutions concurrently, for which the decision-maker could select an optimal solution depending on the current situation, as desired.

The Pareto fitness ranking scheme (Fonseca, 1995; Tan et al., 2001c) for evolutionary multi-objective optimization is adopted here to assign the relative strength of individuals in a population. The ranking approach assigns the same smallest rank for all non-dominated individuals, while the dominated individuals are inversely ranked according to how many individuals in the population dominating them based on the following criteria:

- A smaller number of trucks but an equal cost of routing.
- A smaller routing cost but an equal number of trucks.
- A smaller routing cost and a smaller number of trucks.

Therefore the rank of an individual p in a population is given by $(1 + q)$, where q is the number of individuals that dominating the individual p based on the above criteria.

3.6. Fitness sharing

A simple fitness sharing (Fonseca and Fleming, 1998) is incorporated in HMOEA to prevent genetic drift, which is a phenomenon where a finite population tends to settle on a single optimum even if many other local optima exist. The fitness sharing models the competitions among individual for finite resource available in a niche. When the number of individuals in its neighborhood increases, the fitness of an individual is degraded as a result of the competition. The sharing approach measures the niching distance in the objective domain to achieve diversity of solutions on the trade-off curve. The niche radius, σ , is a parameter that defines the size of neighborhood where all individuals within this distance would contribute towards the sharing function. The distance between individuals is normalized to the maximum range of objective space, which is dynamically computed at each generation. Let $\text{dist}(x, y)$ be the normalized distance between individual x and individual y , the sharing function sh can be defined as follows:

$$sh(\text{dist}(x, y)) = \begin{cases} (1 - \text{dist}(x, y)/\sigma)^2 & \text{if } \text{dist}(x, y) < \sigma; \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The sharing value of an individual will be increased by other individuals that are found located within the niche radius and the sharing value is higher when the distance between the individuals is shorter. With the help of sharing function, the niche count nc is defined as

$$nc(x) = \sum_{y \in \text{individuals}} sh(\text{dist}(x, y)). \quad (2)$$

During the tournament selection, individuals with a lower rank in partial order will be selected for reproduction, where the partial order ranking between two individuals depends on both their Pareto rank and niche counts. Rigorously, the partial order \geq_p for two individuals i and j is defined as

$$i \geq_p j \quad \text{if} \quad [\text{rank}(i) > \text{rank}(j)] \quad \text{or} \quad [\text{rank}(i) = \text{rank}(j) \text{ and } nc(i) > nc(j)].$$

3.7. Local search exploitation

The role of local search is vital to encourage better convergence and to discover any missing trade-off regions in evolutionary multi-objective optimization (Jaszkiewicz, 1998; Tan et al., 2001c). The local search can contribute to intensification of the optimization results, which is usually regarded as a complement to evolutionary operators that mainly focused on global exploration (Ishibuchi et al., 2003). In HMOEA, the local search starts by scanning through all routes in a chromosome, where any routes that contain a smaller number of tasks than a threshold are identified. These identified routes are grouped into pairs randomly and all tasks in each pair are combined to form a new route that is sorted in ascending order by the earliest service time. After the merging, feasibility check is performed such that any infeasible tasks are deleted from the route and moved to the outsourced list.

4. Computational results

The HMOEA was programmed in C++ based on a Pentium III 933 MHz processor with 256MB RAM under the Microsoft Windows 2000 operating system. From the empirical results of preliminary experiments, we found that HMOEA performed equally well with small changes of parameter values. As the general rules of thumb, the crossover rate is relatively larger than mutation rate. The choice is reasonable as high mutation rate tends to destroy the good chromosomes and preventing the preservation of good parents. Table 5 shows the parameter settings chosen after some preliminary experiments. These settings should not be regarded as an optimal set of parameter values, but rather a generalized one for which the HMOEA performs fairly well over the test problems.

This section contains the computational results and analysis of optimization performances for all problem instances. Section 4.1 studies the performance of convergence trace and Pareto-optimality for multi-objective optimization using the 12 test cases in normal category (the different categories were presented in Section 2.3). In the same section, several other performance metrics such as the utilization rate, the progress ratio and a simple scenario of using the results of the routing plan are included. Section 4.2 analyzes the optimization problem when different trailer allocation scenarios happen based on the test cases of TEPC and LTTC (each of the TEPC and LTTC categories contains eight test cases). In Section 4.3, the optimization performance of HMOEA is compared with two other multi-objective evolutionary algorithms based upon various performance measures.

4.1. Multi-objective optimization performance

4.1.1. Convergence trace

Convergence trace is an important performance indicator to show the effectiveness of an optimization algorithm. The two objectives in TTVRP are the number of trucks and the routing cost as defined in

Table 5
Parameter settings for the simulations

Parameter	Value
Crossover rate	0.8
Mutation rate	0.3
Population size	800
Generation size	1000 or no improvement over the last five generations
Niche radius	0.04
G generations	5

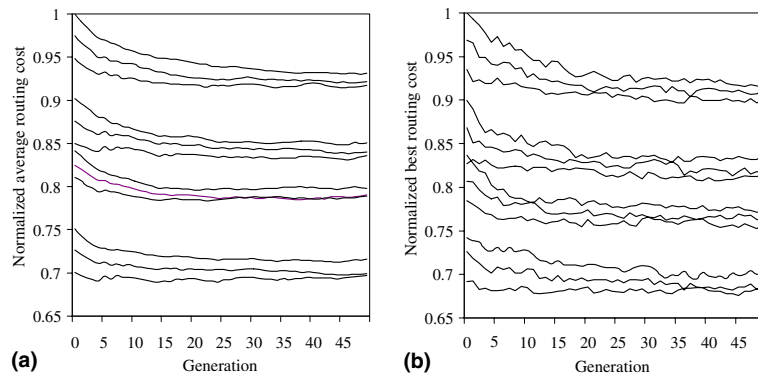


Fig. 5. Convergence trace of the normalized average (a) and best (b) routing costs.

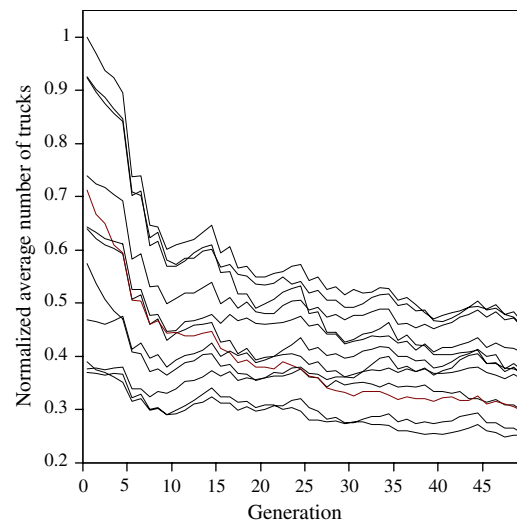


Fig. 6. Convergence trace of the normalized average number of trucks.

Section 2. Fig. 5 shows the normalized average and best routing costs at each generation for the 12 test cases in normal category, where each line represents the convergence trace for each of the test cases. As can be seen, the routing costs decline nicely as the evolution proceeds. The same observation can be found in Fig. 6, where the normalized average number of trucks at each generation is plotted. The rapid reduction of the number of trucks in Fig. 6 is expected as the initial population in HMOEA was generated randomly.

4.1.2. Pareto front

In solving a vehicle routing problem, the logistic manager is often interested in not only getting the minimum routing cost, but also the smallest number of trucks required to service the plan. In order to reduce the routing cost, more number of trucks is often required and vice versa, i.e., the two criteria are non-commensurable and often competing with each other. Fig. 7 shows the evolution progress of Pareto front for all the 12 test cases in normal category. In the simulation, the largest available vehicle number is limited to 35, which is more than sufficient to cater the number of tasks in each test case. The various Pareto fronts

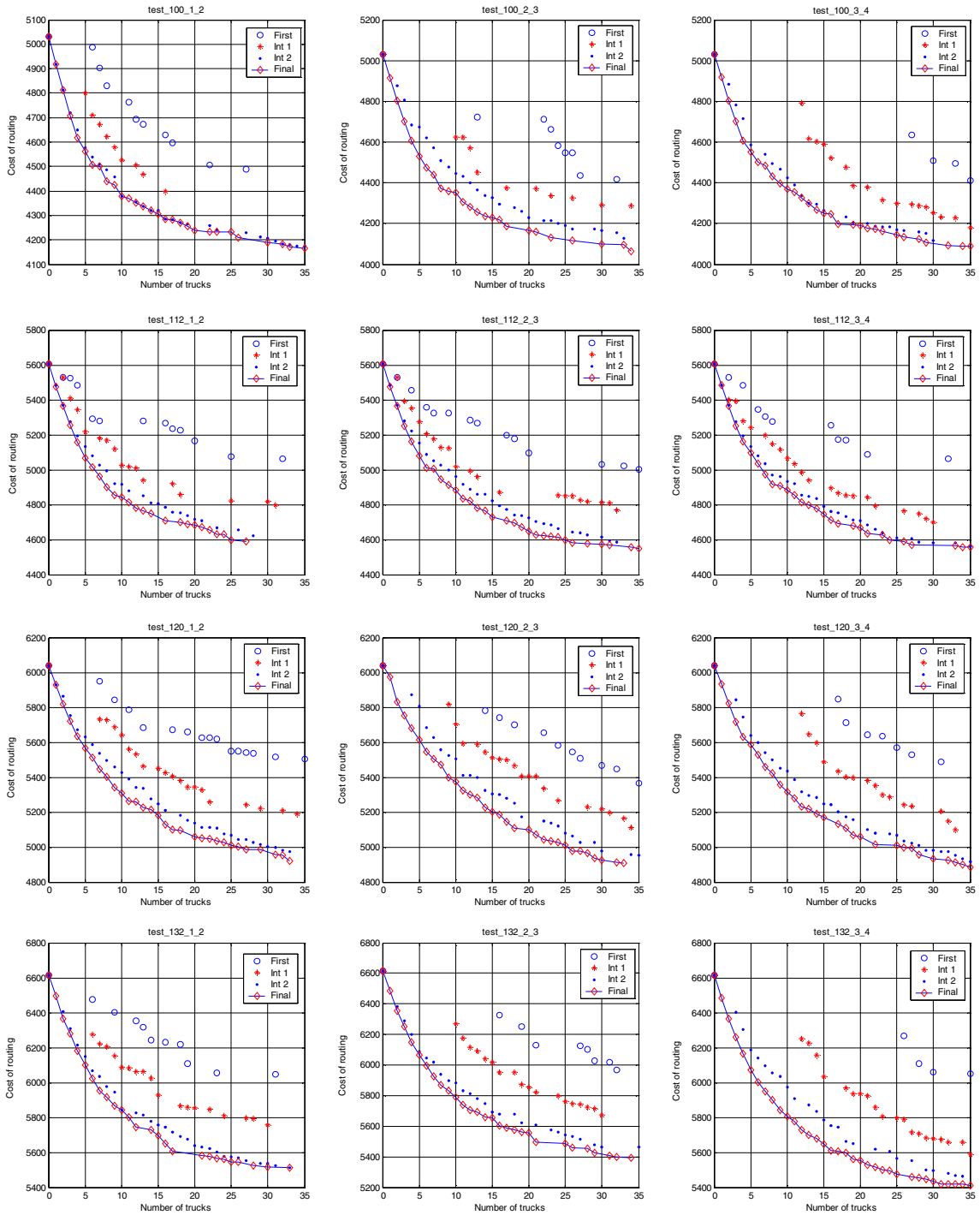


Fig. 7. The evolution progress of Pareto front for the 12 test cases in normal category.

obtained at the initial generation (First), two intermediate generations (Int 1 and Int 2) and the final generation (Final) are plotted in Fig. 7 with different markers. As can be seen, there is only a small number of non-dominated solutions appeared at the initial generations, which are also congested at a small portion of the solution space. However, as the evolution proceeds, the diversity of the population increases significantly and the non-dominated solutions gradually evolve towards the final trade-off curve. A dashed line connecting all the final non-dominated solutions is drawn for each test case in Fig. 7, which clearly shows the final trade-off or routing plan obtained by the HMOEA. It should be noted that the Pareto front includes the plan with zero truck number that sub-contracts all tasks to external company, although such a policy is apparently not practical to adopt because it is against the will of the logistic management.

4.1.3. Routing plan

The average best routing cost for each truck number of the 12 test cases in normal category is plotted in Fig. 8, which shows an obvious trade-off between the two objectives of routing cost and truck number in TTVRP. This trade-off curve is useful for the decision-maker to derive an appropriate routing schedule according to the current situation. The information about the number of tasks to be serviced and the number of trailers available at each trailer exchange point is often available. Based on the information, if the number of trucks available in a company is fixed, the logistic manager can estimate the required routing cost from the trade-off curve in Fig. 8. In contrast, if the manager is given a specified budget or routing cost, he or she can then determine the minimum number of internal trucks to be allocated so that the spending can be kept below the budget. For example, if the routing cost is to be kept below 5100, then the company must allocate at least 10 trucks for serving the task orders. However, if only 15 trucks are allocated by the company, then the incurred routing cost would be around 4900–5000, including the cost payment for outsourced companies.

Fig. 9 shows the average progress ratio at each generation for the 12 test cases in normal category, which is a useful convergence measures for the Pareto front in multi-objective optimization. The progress ratio at any generation is defined as the domination of one population to another (Tan et al., 2001c),

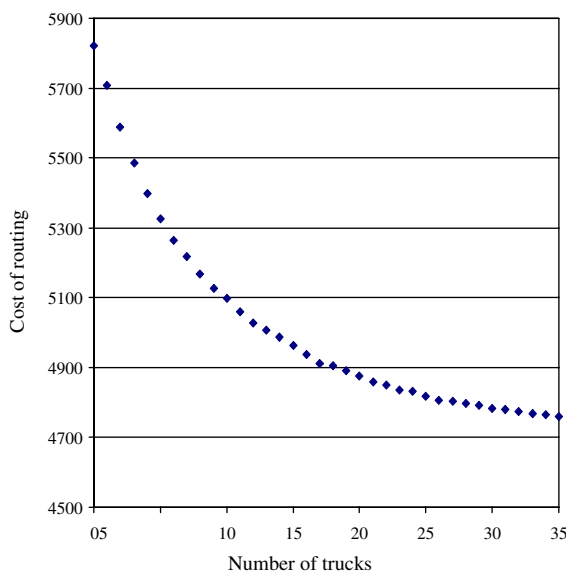


Fig. 8. The trade-off graph between cost of routing and number of trucks.

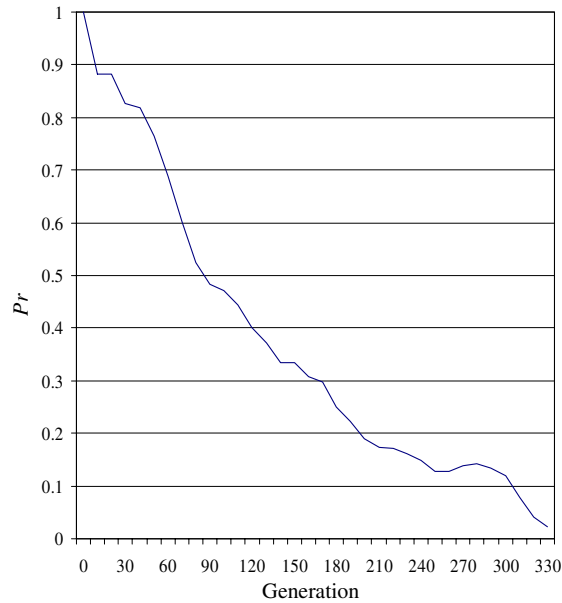


Fig. 9. The average pr at each generation for the 12 test cases in normal category.

$$pr^{(n)} = \frac{nondom_indiv^{(n)} \text{ dominating } nondom_indiv^{(n-1)}}{nondom_indiv^{(n)}}. \quad (3)$$

As shown in Fig. 9, the average pr starts from a value close to one indicating the high probability of improvement to the solutions at the initial stage. As the evolution continues, the pr decreases to a small value which means that the evolution is nearly converged since the possibility of finding new improved non-dominating solution is low.

4.1.4. Utilization of trucks

Besides the trade-off curve, one interesting aspect to be investigated in TTVRP is the utilization of trucks, which is defined as the average number of tasks completed by a truck. A higher utilization means fewer trucks with smaller associated fixed cost are needed to perform the tasks. Fig. 10 shows the average utilization of all test cases in the normal category based on the non-dominated solutions at the initial (Initial) and final (Final) generation, respectively. Clearly, the utilization performance of TTVRP after the optimization by HMOEA has been improved consistently for different number of trucks.

Fig. 11 shows that the utilization of trucks for all the test cases in normal category increases as the number of trailers increases, which is plotted by taking the average utilization of every individuals in the final population served by the internal fleets. Besides the number of trucks employed, utilization performance in TTVRP is also correlated to the trailer allocation, e.g., the utilization is higher as the number of trailers increases, since abundant trailer resources help to reduce unnecessary traveling to farther TEPs as well as to eliminate infeasibility caused by the trailer constraints in TTVRP. Obviously, such information is useful for the management to achieve a high utilization performance in TTVRP before arriving at the final routing plan.

4.2. Computational results for TEPC and LTTC

The test cases in TEPC and LTTC categories are designed to examine situations such as excessive or inadequate trailer resources. The following sub-sections study the extreme trailers allocation policy: Section

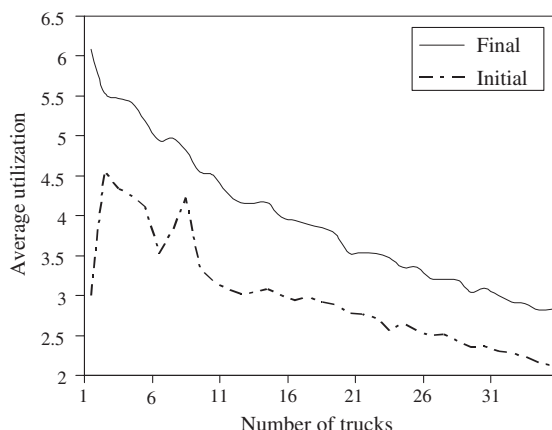


Fig. 10. The average utilization of all test cases in the normal category.

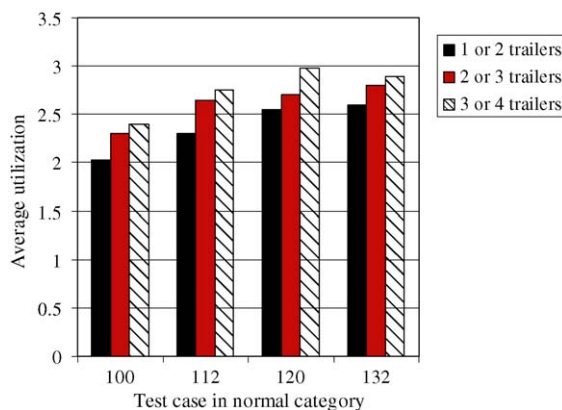


Fig. 11. The average utilization of all individuals in the final population.

4.2.1 compares the optimization results of TEPC and LTTC while Section 4.2.2 investigates the effects of the number and location of trailer exchange points in TTVRP.

4.2.1. Scenario of extreme trailer allocation

The resource of trailers is one of the key elements in TTVRP. For the normal test category, since the variation of trailer number at TEPs is small and the tasks that require trailers are only a proportion of the total tasks, the effect of trailer number to routing cost is insignificant as discussed in Section 4.1. In this sub-section, the scenario of excessive and limited trailer resources is compared based on the test cases in TEPC (with 30 trailers) and LTTC (with 10 trailers) categories. Fig. 12 shows the box plot of routing costs for the final non-dominated solutions in different test cases of TEPC and LTTC categories. Each box plot represents the distribution of a sample set where a vertical line within the box encodes the median, while the right and left ends of the box are the upper and lower quartiles. Dashed appendages illustrate the spread and shape of distribution, and dots represent the outside values. In the figure, 132_tep1 and 132_ltt1 represents the combined result for the test cases with only one TEP for TEPC and LTTC, respectively. As can

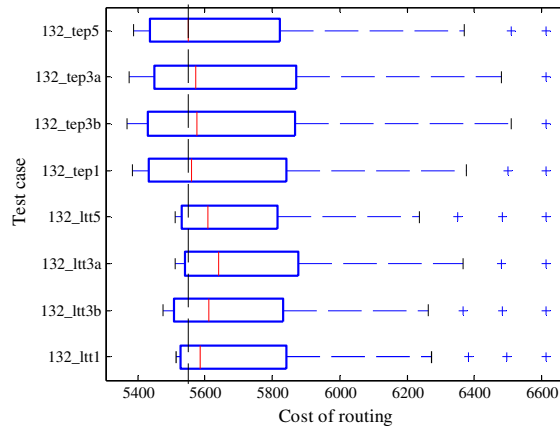


Fig. 12. The performance comparison of abundant TEPC with limited trailers in LTTC.

be seen, the mean routing costs for test cases in TEPC are consistently lower than the cases in LTTC. When the number of trailers is abundant as in TEPC, a feasible solution can be found more easily as compared to LTTC where resource of trailers is limited and the search for better solutions is restricted by the lack of trailers. The results show that the trailers and their distribution greatly affect the final routing performance. It is thus important to have enough trailers allocation at the initial of planning horizon, and a good routing policy should favor the choice that brings more trailers back to TEPs at the end of each planning horizon.

4.2.2. The number and location of TEPs

This sub-section compares the routing performance among the different test cases within each category of TEPC and LTTC. Figs. 13 and 14 show the box plots of routing costs for the final non-dominated solutions in different test cases of TEPC and LTTC, respectively. In Fig. 13, the mean value of test_132_tep5 is extended vertically and chosen as a reference since this test case has its trailer resources distributed uniformly to all the TEPs. It can be seen that the range of routing costs for the various test cases is rather closed to test_132_tep5. In addition, there is only minor difference in terms of the mean routing cost, except for the case of test_132_tep1e where the trailers are allocated to only one TEP. Hence the location of TEP is

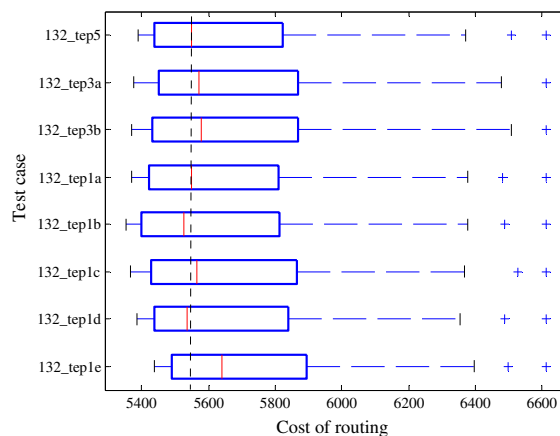


Fig. 13. The performance comparison of different test cases in TEPC category.

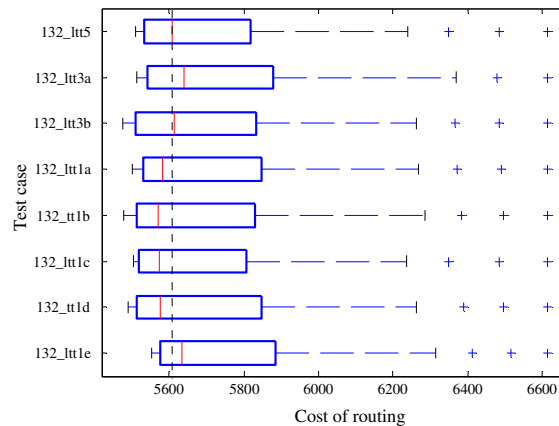


Fig. 14. The performance comparison of different test cases in LTTC category.

not strategic for TTVRP. Similarly, the mean routing cost of test_132_ltt_1e is also inferior as compared to other test cases in the LTTC category as shown in Fig. 14. The results suggest that the final destinations of trailers should be properly planned and allocated at suitable TEPs that support the routing for the next planning horizon.

4.3. Comparison results

In this section, the performance of HMOEA is compared with two variants of evolutionary algorithms, i.e., MOEA with standard genetic operators as well as MOEA without hybridization of local search. The comparison allows the effectiveness of the various features in HMOEA, such as specialized genetic operators and local search heuristics, to be examined. The multi-objective evolutionary algorithm with standard generic operators (STD_MOEA) includes the commonly known cycle crossover and RAR mutation. The cycle crossover is a general crossover operator that preserves the order of sequence in the parent partially and was applied to solve the traveling salesman problems by Oliver et al. (1987). The remove and reinsert (RAR) mutation operator removes a task from the sequence and reinsert it at a random position (Gendreau et al., 1999b). The multi-objective evolutionary algorithm without hybridization of local search (NH_MOEA) employs the specialized genetic operators in HMOEA but excludes the local search heuristic. The experimental setups and parameter settings of STD_MOEA and NH_MOEA are similar to the settings of HMOEA in Table 5.

4.3.1. Average routing cost

To compare the quality of solutions produced by the algorithms, the average routing cost (ARC) of the non-dominated solutions in the final population is calculated for various test cases with different number of tasks as shown in Fig. 15. In the figure, the average value of ARC is plotted for each group of the test cases with equal number of tasks in the normal category. As can be seen, the STD_MOEA that employs standard genetic operators incurs the highest ARC since its operators are not tailored made for the TTVRP problem. According to the no free lunch theorem (Wolpert and Macready, 1996), any optimization methods should be tailored to the problem domain for best performance. The results in Fig. 15 also illustrate that the HMOEA outperforms NH_MOEA and STD_MOEA consistently, which produces the lowest routing cost for all test cases. The average routing cost of the non-dominated solutions in the final population for test

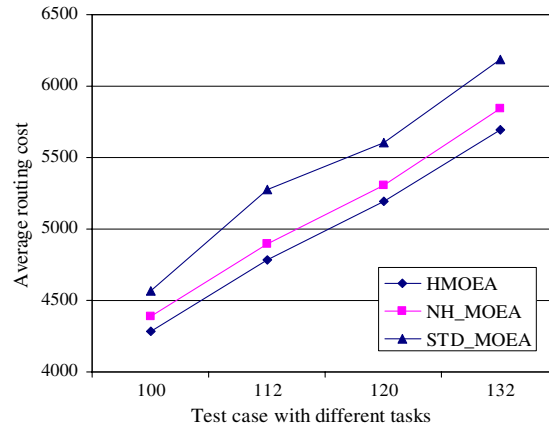


Fig. 15. The average routing cost for the normal category.

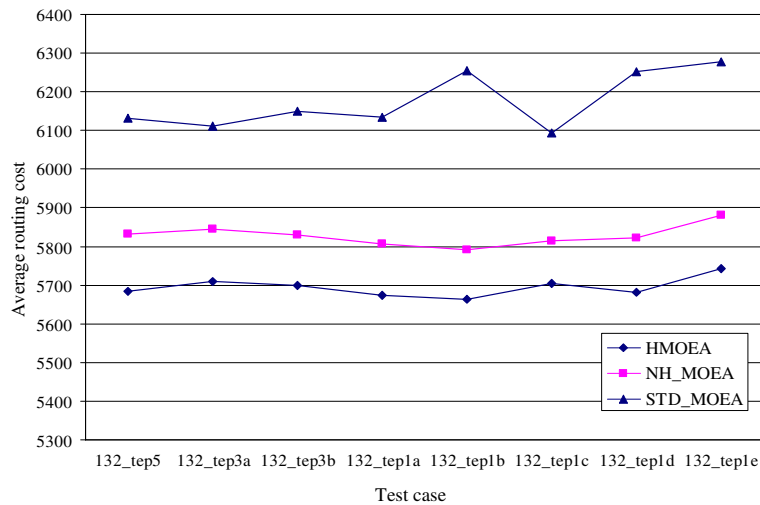


Fig. 16. The average routing cost for the TEPC category.

cases in the category of TEPC and LTTC is shown in Figs. 16 and 17, respectively, where a similar outstanding optimization performance for HMOEA is observed.

4.3.2. Ratio of non-dominated individuals

In multi-objective optimization, it is often desired to find as many as possible useful candidate solutions that are non-dominated in a given population, which could be measured by the ratio of non-dominated individuals (RNI) as proposed by Tan et al. (2001c). For a given population X , the RNI in percentage is formulated as

$$\text{RNI}(X)\% = \frac{\text{nondom_indiv}}{N} \times 100\%, \quad (4)$$

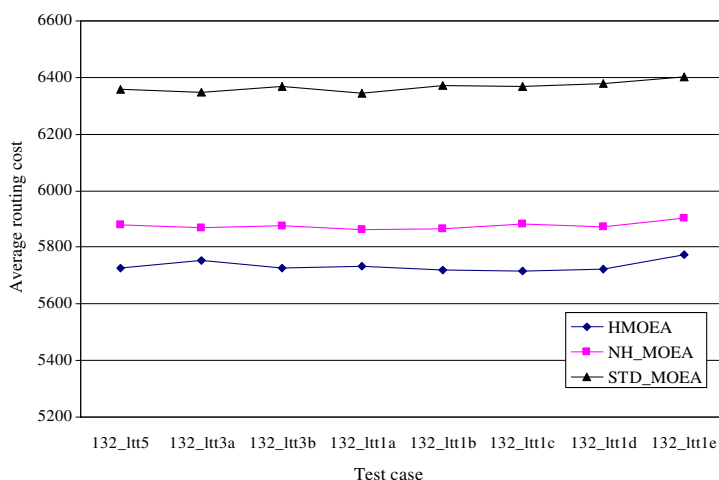


Fig. 17. The average routing cost for the LTTC category.

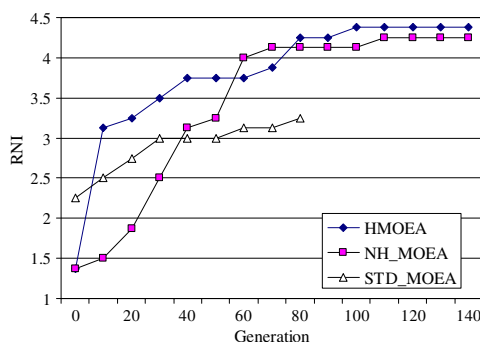


Fig. 18. The RNI of various algorithms for test case 132_3_4.

where *nondom_indiv* is the number of non-dominated individuals in population *X*, and *N* is the size of the population *X*. Without loss of generality, Fig. 18 shows the RNI for the three algorithms based on a randomly selected test case 132_3_4. As can be seen, the RNI value of STD_MOEA is the lowest among the three algorithms. The evolution in STD-MOEA stopped at around 90 generations as no improvement was observed for five generations continuously. The results also show that the search performance of HMOEA for non-dominated solutions is slightly better than NH_MOEA. Besides, the HMOEA also has the best average RNI of 1.89 as compared to the value of 1.71 and 0.44 for NH_MOEA and STD_MOEA, respectively.

4.3.3. Simulation time

Besides the multi-objective optimization performance, the computational time for different algorithms is studied in this sub-section. The three algorithms adopt the same stopping criteria in the simulation, i.e., the evolution stops after 1000 generations or when no improvement is found for the last five generations. Fig. 19 shows the normalized simulation time for the three algorithms based on three randomly selected test cases from each category, e.g., test_132_3_4, test_132_tep5 and test_132_ltt5. As can be seen, the

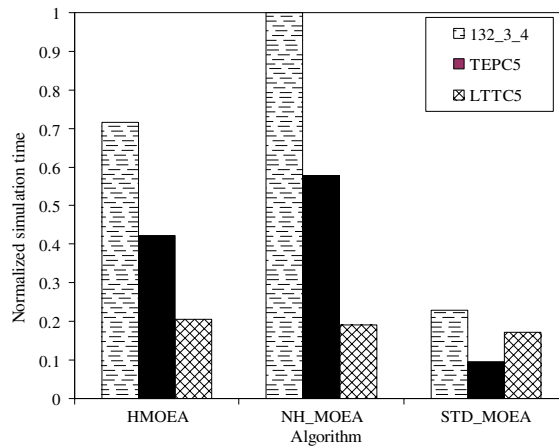


Fig. 19. The normalized simulation time for various algorithms.

STD_MOEA requires the shortest time to converge or halt the evolution, although the optimization results obtained by the STD_MOEA are much inferior as compared to NH_MOEA and HMOEA. It is believed that the population in STD_MOEA has converged prematurely to local Pareto front. The results also show that the computation time required by HMOEA is better than NH_MOEA for the normal and TEPC categories (which have abundant trailer resources where more feasible solutions exist) and is comparable to NH_MOEA for the LTTC category (which has less trailer resources with a smaller set of feasible solutions).

5. Conclusions

A transportation problem for moving empty or laden containers for a logistic company has been considered and a mathematical model for the truck and trailer vehicle routing problem (TTVRP) has been constructed in this paper. The objective of the routing problem is to minimize the routing distance and the number of trucks required, subject to a number of constraints such as time windows and availability of trailers. To solve such a multi-objective and multi-modal combinatorial optimization problem, a hybrid multi-objective evolutionary algorithm (HMOEA) featured with specialized genetic operators, variable-length representation and local search heuristic has been applied to find the Pareto optimal routing solutions for the TTVRP. Detailed studies have been performed to extract important decision-making information from the multi-objective optimization results. Besides, the relationships among different variables, such as the number of trucks and trailers, the trailer exchange points, and the utilization of trucks in the routing solutions, have been examined and analyzed. The computational results have shown that HMOEA is effective in solving multi-objective combinatorial optimization problems, such as finding useful trade-off solutions for the TTVRP routing problem. Comparisons to two other general evolutionary algorithms also show that the proposed approach are better in terms of the average routing cost and the ratio of non-dominated individuals.

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