Expert Systems with Applications 37 (2010) 899–903

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Short Communication

A note on the truck and trailer routing problem

Shih-Wei Lin[a], Vincent F. Yu[b,*], Shuo-Yan Chou[b]

[a] Department of Information Management, Chang Gung University, No. 259, Wen-Hwa 1st Road, Kwei-Shan, Tao-Yuan 333, Taiwan
[b] Department of Industrial Management, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Road, Taipei 106, Taiwan

ARTICLE INFO

Keywords:
Vehicle routing problem
Truck and trailer routing problem
Simulated annealing

ABSTRACT

This study considers the relaxed truck and trailer routing problem (RTTRP), a relaxation of the truck and trailer routing problem (TTRP). TTRP is a variant of the well studied vehicle routing problem (VRP). In TTRP, a fleet of trucks and trailers are used to service a set of customers with known demands. Some customers may be serviced by a truck pulling a trailer, while the others may only be serviced by a single truck. This is the main difference between TTRP and VRP. The number of available trucks and available trailers is limited in the original TTRP but there are no fixed costs associated with the use of trucks or trailers. Therefore, it is reasonable to relax this fleet size constraint to see if it is possible to further reduce the total routing cost (distance). In addition, the resulting RTTRP can also be used to determine a better fleet mix. We developed a simulated annealing heuristic for solving RTTRP and tested it on 21 existing TTRP benchmark problems and 36 newly generated TTRP instances. Computational results indicate that the solutions for RTTRP are generally better than the best solutions in the literature for TTRP. The proposed SA heuristic is able to find better solutions to 18 of the 21 existing benchmark TTRP instances. The solutions for the remaining three problems are tied with the best so far solutions in the literature. For the 36 newly generated problems, the average percentage improvement of RTTRP solutions over TTRP solutions is about 5%. Considering the ever rising crude oil price, even small reduction in the route length is significant.

1. Introduction

In this paper we consider a relaxation of the truck and trailer routing problem (TTRP), called the relaxed truck and trailer routing problem (RTTRP). TTRP is a variant of the well known vehicle routing problem (VRP) which is one of the most studied combinatorial optimization problems in the last few decades (Christofides, Mingozzi, & Toth, 1979; Dantzig & Ramser, 1959; Fisher, 1995; Gillett & Miller, 1974; Laporte, 1992; Laporte, Gendreau, Potvin, & Semet, 2000; Laporte & Nobert, 1987; Laporte & Semet, 2002). In the standard VRP, a set of customers is serviced by a fleet of homogeneous vehicles based at a central depot. The demand of each customer and vehicle capacity are known in advance. The goal is to design vehicle routes originating from and terminating at the central depot to fulfill each customer's demand so that the total cost (or route length) is minimized. The total demand on each route should not exceed the vehicle capacity and each customer can only be serviced once by exactly one vehicle. For a more in depth treatment of VRP, readers are referred to Bodin, Golden, Assad, and Ball (1983), Christofides et al. (1979), Fisher (1995), Golden and Assad (1988) and Toth and Vigo (2002).

TTRP was first proposed by Chao (2002) and subsequently studied by Scheuerer (2006) and Lin, Yu, and Chou (2009). In TTRP, the use of trailers (a commonly neglected feature in the VRP) is considered. Some customers can be serviced by a complete vehicle (i.e., a truck pulling a trailer), while other customers can only be serviced by a single truck due to some practical limitations, such as government regulations, limited maneuvering space at customer site, road conditions. Such limitations exist in many real-world settings (Gerdessen, 1996; Hoff, 2006; Semet & Taillard, 1993). Those customers who can only be serviced by a single truck are referred to as tuck customers (TCs); the other customers that can be serviced by either a single truck or a complete vehicle are called vehicle customers (VCs).

TTRP can be formally defined as follows. Let $G = (V,A)$ be an undirected graph, where $V = \{0,1,2,\ldots,n\}$ is the set of vertices and $A = \{(i,j)\colon i,j \in V\}$ is the set of edges. Vertex 0 corresponds to the central depot, while the remaining vertices in $V\backslash\{0\}$ represent customers. Each vertex i in $V\backslash\{0\}$ is associated with a known demand $d_i \geqslant 0$ and a customer type $t_i \in \{0,1\}$, where $t_i = 1$ indicates that customer i is a TC; $t_i = 0$ denotes that customer i is a VC. Each edge (i,j) is associated with a non-negative cost c_{ij} that represents the travel time required on the edge or simply the travel distance between node i and node j.

There are m_k available trucks and m_r available trailers ($m_k \geqslant m_r$) but the number of trucks and trailers that are actually used in the

* Corresponding author. Tel.: +886 2 2730 3276; fax: +886 2 2737 6344.
E-mail addresses: vincent@mail.ntust.edu.tw, vince.yu@gmail.com (V.F. Yu).

0957-4174/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2009.06.077

vehicle routes are not determined *a priori*. Therefore, it is not uncommon that some trucks or trailers are not used at all in a TTRP solution. The capacity of each truck and each trailer is Q_k and Q_t, respectively. Once a trailer is assigned to a truck, it may not be assigned to another truck. The goal of the TTRP is to find a set of least cost vehicle routes that originate from and terminate at the central depot so that each customer is serviced exactly once and the cumulated demand on each route does not exceed the total capacity of the vehicle(s) used for that route.

There are three types of routes in a TTRP solutions: (1) a pure truck route (PTR) that is traveled by a single truck; (2) a pure vehicle route (PVR) that is traveled by a complete vehicle without any sub-tour; and (3) a complete vehicle route (CVR) which consists of a main tour traveled by a complete vehicle, and one or more sub-tours traveled by the truck alone. A sub-tour starts and ends at the depot or the same vehicle customer site on the main tour, i.e. the trailer is dropped-off at the depot, or a VC site while the truck proceeds to service customers on the sub-tour. The trailer drop-off point is called the root of the sub-tour. After all customers on the sub-tour are serviced, the truck must return to the root where its trailer is parked, pick up the trailer and move on to service remaining customers on the same route.

Note that in TTRP, there are no fix costs associated with the vehicles although there are limitations on the number of available trucks and available trailers. Thus, it is possible to construct better vehicle routes by utilizing more vehicles or allowing vehicles to take on multiple trips. Further, if the reduction in costs resulting from such relaxation is significant, it may be worthwhile to acquire or lease extra vehicles provided that the acquisition/lease costs can be justified. In view of this, we relax the fleet size constraint in TTRP and call the resulting problem the relaxed truck and trailer routing problem. This is the only difference between the TTRP and the RTTRP.

TTRP is more difficult to solve than VRP because VRP is a special case of the TTRP. Since the VRP itself is a hard combinatorial optimization problem and is usually solved by heuristics, Chao (2002), Scheuerer (2006), Lin et al. (2009) all applied heuristic approaches to solve TTRP. Both Chao and Scheuerer solved the TTRP by a two-phase approach. They first construct an initial solution with some heuristics. The initial solution is then improved with a tabu search (TS) algorithm. Lin et al. (2009) developed a simulated annealing (SA) based heuristic for the TTRP. Computational results indicate that their SA heuristic performs slightly better than other TS based heuristics in solving the TTRP. Thus, we developed an SA heuristic for the RTTRP and compared the results with the solutions to TTRP obtained from TS based heuristics and SA based heuristics in the literature whenever applicable.

2. Simulated annealing heuristic for the RTTRP

Simulated annealing has been applied successfully to a wide variety of highly complicated combinatorial optimization problems (Chwif, Barretto, & Moscato, 1998; Jayaraman & Ross, 2003; Lim, Rodrigues, & Zhang, 2006; McKendall, Shan, & Kuppusamy, 2006), including TTRP (Lin et al., 2009). In the following subsections, we discuss the proposed SA heuristic for RTTRP in detail, including solution representation, generation of the initial solution, calculation of the objective function value, neighborhood structure, parameters used in the computational study, and finally the SA procedure for RTTRP.

2.1. Solution representation and initial solution

An RTTRP solution is represented by a string of numbers consisting of a permutation of n customers denoted by the set $\{1,2,\ldots,n\}$, N_{dummy} zeros (artificial depot or the root of a sub-tour), and the service vehicle types of individual VCs. The N_{dummy} zeros serve the purpose of separating routes or terminating sub-tours. The parameter N_{dummy} is defined by $\left\lfloor \sum_i \frac{d_i}{Q_k} \right\rfloor$, where $\lfloor \cdot \rfloor$ denotes the largest integer which is smaller than or equal to the enclosed number. The ith non-zero number in the first $n + N_{dummy}$ positions indicates the ith customer to be serviced.

The service vehicle type of a VC is either 0 or 1. If the VC is serviced by a truck alone, its service vehicle type is set to be 1. Otherwise, it is serviced by a complete vehicle, and its service vehicle type is 0.

The first number in the solution representation indicates the first customer to be serviced in the first route. Other customers are added to the route one at a time from left to right to represent the order in which they are serviced, provided that the capacity of the vehicle currently in use is not violated. Note that the capacity of the vehicle in use varies depending on the type of the route and the portion of the route under consideration. The capacity of the vehicle in use is $(Q_k + Q_t)$ if the route is a PVR or the vehicle is on the main tour of a CVR; or Q_t if the vehicle is on a PTR or on a sub-tour of a CVR. When encountering zero in the solution representation, the vehicle will either return to the root of current sub-tour or the depot. More specifically, if the vehicle is currently on a sub-tour of a CVR, it will return to the root of the sub-tour where the trailer was dropped-off and the sub-tour is terminated. Otherwise, it is currently on a PTR, on a PVR, or on a main tour of a CVR. In either case, the vehicle will return to the depot and the route is terminated.

Whenever a route is terminated and there are still customers that have not been serviced, a new route will be generated with the next customer in the solution representation being the first customer of the route. It can be verified that this solution representation always gives a feasible RTTRP solution.

The initial solution is generated at random. It includes a random sequence of the customers and the dummy zeros, and randomly generated service vehicle types of individual VCs.

2.2. Neighborhood structure

We adopted a standard SA procedure with a random neighborhood structure that features several move types, namely insertion, swap, and change of service vehicle type, to solve the RTTRP. Let $\mathcal{N}(X)$ denote the neighborhood of the current solution X. In each iteration, a new solution Y is selected from $\mathcal{N}(X)$ to be the next solution, either by insertion, swap, or change of service vehicle type of VCs. In the following, we discuss the new solution generations process in detail.

The insertion move is carried out by randomly selecting the ith customer in X and inserting it into the position immediately before another randomly selected jth customer of X. The swap move randomly selects two customers in X, and then switches their positions. The change of service vehicle type of VCs is carried out by randomly selecting a VC from X, and then changing its service vehicle type from 0 to 1 or from 1 to 0. In other words, if the selected VC was serviced by a complete vehicle before the move, it will be serviced by a single truck after the move, and vice versa. We set the probability of choosing the swap move, insertion move, or the change of service vehicle type of VCs move to be 0.2, 0.2, and 0.1, respectively.

To increase the chance of obtaining a better solution, in addition to the aforementioned random moves, we also include in our algorithm the best-of-N-trials moves, in which the best solution among the N trial solutions is chosen as the next solution, where N is a predetermined number of trails. For swap move and insertion move, this number is set to be N_{trial}, obtained by $\lfloor (n + N_{dummy})/3 \rfloor$. For the change of service vehicle type of VCs, each VC's service vehicle type is changed one at a time, thus the number of trials is the same as the number of VCs.

We set the probabilities of performing the best-of-N-trials moves to be 0.2 and 0.2 for swap move and insertion move, respectively. The probability of performing the best-of-N-trials moves for the change of service vehicle type of VCs is set to be 0.1. Note that the probabilities of performing swap, insertion, and change of service vehicle type of VCs on one or two randomly selected customers total to 0.5, and the probability of performing the best-of-N-trials moves also adds up to 0.5. Thus, the total probability of performing these neighborhood moves is 1.

2.3. Parameters and the SA procedure

The SA procedure starts by setting the current temperature T to be T_0, the initial temperature, and generating an initial solution X at random. Initially, the current best solution X_{best} and the best objective function value obtained so far are set to be X and obj(X), respectively.

In each iteration, the next solution Y is generated from $\mathcal{N}(X)$ and its objective function value, obj(Y), is evaluated. Let Δ = obj(Y) − obj(X), i.e. the improvement in objective function values of Y over X. The probability of replacing X with Y is 1 when $\Delta \leqslant 0$; otherwise, it is determined by $\exp(-\Delta/KT)$, where K is the Boltzmann constant. In our implementation, this is accomplished by first generating a random number $r \in [0,1]$ and then replacing X with Y if $r < \exp(-\Delta/KT)$.

After running I_{iter} iterations at the current temperature T, we decrease the current temperature by setting $T \leftarrow \alpha T$, where $0 < \alpha < 1$. After every three temperature reductions, a local search procedure that performs 2-opt, swap, insertion, and change of service vehicle types sequentially is applied to improve the current best solution.

The algorithm terminates when the current temperature T is lower than T_F, the predetermined final temperature, or when the current best solution X_{best} has not been improved for $N_{non\text{-}improving}$ consecutive temperature reductions. A (near) optimal routing plan for the RTTRP can easily be derived from X_{best} after the SA is terminated.

3. Computational study

We coded the proposed SA heuristic for RTTRP in C++ and compiled it with Microsoft Visual C++ 6.0. We then applied the program to Chao's 21 TTRP benchmark problems (Chao, 2002) on a Pentium IV 1.5 GHz PC with 1 GB RAM running Microsoft Windows XP operating system.

Except for K, the parameter values for the computational study are adopted from Lin et al. (2009). That is, α = 0.965, I_{iter} = 150,000, T_0 = 100, T_F = 1, and $N_{non\text{-}improving}$ = 30. The value of K is set to be 1/6 in this study. The parameter P for penalty cost used in Lin et al. (2009) is not needed since the fleet size constraint is dropped in RTTRP.

To evaluate the improvement in solution quality of RTTRP over that of TTRP, we compared the solution to RTTRP with the best TTRP solutions reported in the literature (Lin et al., 2009; Scheuerer, 2006). TTRP results obtained by Scheuerer (2006) and Lin et al. (2009), and the RTTRP results by the proposed SA heuristic are presented in Table 1. It can been seen that in 18 out of the 21 benchmark TTRP instances, RTTRP solution is better than the best TTRP solution obtained by Scheuerer (2006) and Lin et al. (2009). For

Table 1
A comparison of RTTRP solutions obtained by the proposed SA heuristic with TTRP solutions reported in the literature for Chao's problem set (Chao, 2002).

ID	Scheuerer (λ = 15,000)			Lin et al. (K = 1/3)			SA heuristic for RTTRP (K = 1/6)					
	Min $c(s^*)$[a]	Avg $c(s^*)$[b]	T[c]	Min $c(s^*)$[d]	Avg $c(s^*)$[e]	T[f]	Min $c(s^*)$[g]	Avg $c(s^*)$[h]	T[i]	Trucks used[j]	Trailers used[k]	Improvement in Avg $c(s^*)$ over Lin et al.[l] (%)
1	566.80	567.98	9.51	566.82	568.86	6.80	**557.11**	559.59	6.77	5	4	1.71
2	615.66	619.35	9.60	612.75	617.48	6.67	**608.22**	610.38	6.66	5	4	0.74
3	620.78	629.59	11.24	**618.04**	620.50	5.59	**618.04**	618.5	5.35	5	3	0.00
4	801.60	809.13	18.49	808.84	817.71	16.32	**784.73**	790.76	15.53	8	7	2.98
5	**839.62**	858.98	15.16	**839.62**	858.95	14.42	**839.62**	845.89	14.73	9	5	0.00
6	936.01	949.89	18.62	934.11	942.60	13.65	**930.64**	935.76	12.58	9	5	0.37
7	830.48	832.91	33.60	830.48	838.50	24.96	**810.38**	815.09	24.63	6	6	2.42
8	878.87	881.26	25.66	875.76	882.70	24.03	**873.80**	882.55	24.29	8	3	0.22
9	942.31	955.95	30.47	912.64	921.97	21.75	**911.49**	918.2	21.27	9	2	0.13
10	1039.23	1052.65	60.94	1053.90	1074.38	63.61	**1018.62**	1027.76	63.55	10	9	3.35
11	1098.84	1107.47	56.17	1093.57	1108.88	60.33	**1076.88**	1088.84	60.10	10	9	1.53
12	1175.23	1184.58	63.71	1155.44	1166.59	51.70	**1154.13**	1166.27	51.22	12	6	0.11
13	1288.46	1296.33	165.41	1320.21	1340.98	119.56	**1263.62**	1284.81	119.51	13	13	4.29
14	1371.42	1384.13	132.06	1351.54	1367.91	113.75	**1336.03**	1354.1	112.66	15	13	1.15
15	1459.55	1488.71	154.10	1436.78	1454.91	93.87	**1422.22**	1454.59	92.27	17	10	1.01
16	1002.49	1003.00	43.14	1004.47	1007.26	41.46	**975.65**	978.22	40.64	6	5	2.87
17	1042.35	1042.79	33.73	1026.88	1035.23	38.81	**1006.79**	1008.97	37.55	6	5	1.96
18	1129.16	1141.94	31.78	1099.09	1110.13	31.34	**1097.56**	1103.61	31.09	7	5	0.14
19	813.50	813.98	28.84	814.07	823.01	29.58	**797.19**	801.51	29.18	8	7	2.07
20	848.93	852.89	24.57	855.14	859.06	28.47	**847.21**	849.85	29.00	9	6	0.93
21	**909.06**	914.04	26.84	**909.06**	915.38	24.03	**909.06**	914.33	23.94	10	5	0.00
Avg.	962.40	970.84	47.32	958.06	968.24	39.56	944.71	952.84	38.18	-	-	1.33

[a] Best TTRP solutions from 10 runs by Scheuerer (λ = 15,000).
[b] Average TTRP solutions from 10 runs by Scheuerer (λ = 15,000).
[c] Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for Scheuerer (λ = 15,000).
[d] Best TTRP solutions from 10 runs by Lin et al. (K = 1/3).
[e] Average TTRP solutions from 10 runs by Lin et al. (K = 1/3).
[f] Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for Lin et al. (K = 1/3).
[g] Best RTTRP solutions from 10 runs by the SA heuristic (K = 1/6).
[h] Average RTTRP solutions from 10 runs by the SA heuristic (K = 1/6).
[i] Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for the SA heuristic (K = 1/6).
[j] Number of Truck used in the best solution that attains Min $c(s^*)$.
[k] Number of Trailer used in the best solution attains Min $c(s^*)$.
[l] Calculated by (Min $c(s^*)^{d}$ − Min $c(s^*)^{g}$)/Min $c(s^*)^{d}$.

Table 2
A comparison of RTTRP solutions obtained by the proposed SA heuristic with TTRP solutions obtained by the SA heuristic of Lin et al. (2009).

ID	Characteristic of test problem							With fleet size constraint ($K = 1/3$, $P = 200$)					Without fleet size constraint ($K = 1/6$)					Improvement rate (%)	
	Source	VC	TC	Avail. trucks	Avail. trailers	Q_k	Q_t	Avg $c(s^*)$	Time (s) in Min	Min $c(s^*)$	Trucks used	Trailers used	Avg $c(s^*)$	Time (s) in Min	Min $c(s^*)$	Trucks used	Trailers used	Avg (s)	Min (s)
1	tai75a	57	18	13	7	750	750	1855.24	20.06	1833.20	12	7	1670.23	17.63	1656.62	11	10	−9.97	−9.63
2	tai75a	38	37	13	7	750	750	1911.36	18.73	1898.60	12	7	1747.46	18.65	1746.25	11	10	−8.58	−8.02
3	tai75a	19	56	13	7	750	750	1996.86	16.79	1991.03	12	7	1923.62	17.63	1918.60	11	10	−3.67	−3.64
4	tai75b	57	18	12	6	850	850	1462.89	19.94	1455.47	12	6	1377.36	14.15	1375.08	11	8	−5.85	−5.52
5	tai75b	38	37	12	6	850	850	1495.07	17.59	1491.30	12	6	1437.08	18.65	1435.68	12	8	−3.88	−3.73
6	tai75b	19	56	12	6	850	850	1656.76	15.33	1643.83	12	6	1618.39	14.47	1614.21	12	7	−2.32	−1.80
7	tai75c	57	18	11	6	600	600	1462.62	18.78	1457.44	11	6	1343.18	17.10	1338.10	10	9	−8.17	−8.19
8	tai75c	38	37	11	6	600	600	1495.75	17.49	1486.18	11	6	1428.24	17.10	1423.89	11	7	−4.51	−4.19
9	tai75c	19	56	11	6	600	600	1552.15	15.69	1539.14	11	6	1506.93	12.88	1506.50	11	7	−2.91	−2.12
10	tai75d	57	18	12	6	850	850	1541.25	19.14	1540.11	12	6	1417.65	16.42	1412.72	9	9	−8.02	−8.27
11	tai75d	38	37	12	6	850	850	1584.35	18.99	1579.16	12	6	1513.74	16.42	1506.95	10	8	−4.46	−4.57
12	tai75d	19	56	12	6	850	850	1748.56	16.16	1735.56	12	6	1739.62	14.90	1733.86	12	6	−0.51	−0.10
13	tai100a	75	25	14	7	750	750	2258.13	33.51	2233.42	14	7	2081.04	31.01	2047.43	11	10	−7.84	−8.33
14	tai100a	50	50	14	7	750	750	2341.37	32.10	2324.23	14	7	2251.00	31.01	2223.02	11	10	−3.86	−4.35
15	tai100a	25	75	14	7	750	750	2503.30	27.50	2489.78	14	7	2476.08	25.06	2470.66	14	8	−1.09	−0.77
16	tai100b	75	25	14	7	950	950	2276.45	33.11	2217.58	14	7	1983.52	28.83	1957.00	11	10	−12.87	−11.75
17	tai100b	50	50	14	7	950	950	2398.71	28.22	2320.43	14	7	2138.68	28.83	2114.81	11	10	−10.84	−8.86
18	tai100b	25	75	14	7	950	950	2531.27	27.38	2508.92	14	7	2331.88	24.74	2329.98	12	10	−7.88	−7.13
19	tai100c	75	25	14	7	1050	1050	1463.76	32.59	1454.65	14	7	1394.56	32.41	1383.14	12	10	−4.73	−4.92
20	tai100c	50	50	14	7	1050	1050	1497.65	30.40	1492.64	14	7	1439.79	32.41	1436.41	12	9	−3.86	−3.77
21	tai100c	25	75	14	7	1050	1050	1770.67	25.81	1755.33	14	7	1761.74	24.97	1755.33	14	7	−0.50	0.00
22	tai100d	75	25	15	8	650	650	1807.50	31.77	1790.48	14	8	1683.52	29.28	1662.28	12	11	−6.86	−7.16
23	tai100d	50	50	15	8	650	650	1880.66	31.72	1872.48	15	8	1810.37	29.28	1799.13	13	10	−3.74	−3.92
24	tai100d	25	75	15	8	650	650	2008.42	26.81	1988.54	15	8	1990.56	24.38	1978.66	14	9	−0.89	−0.50
25	tai150a	113	37	20	10	800	800	3443.60	74.29	3410.24	18	10	3134.90	70.91	3122.79	14	14	−8.96	−8.43
26	tai150a	150	75	20	10	800	800	3564.45	70.41	3514.94	18	10	3293.80	70.91	3286.83	15	14	−7.59	−6.49
27	tai150a	100	112	20	10	800	800	3713.70	60.05	3694.24	19	10	3526.10	59.00	3501.39	16	13	−5.05	−5.22
28	tai150b	113	37	18	9	1000	1000	2880.74	72.27	2848.54	18	9	2698.36	68.73	2680.50	14	12	−6.33	−5.90
29	tai150b	150	75	18	9	1000	1000	3027.07	70.12	2982.58	18	9	2835.00	68.73	2803.86	15	12	−6.34	−5.99
30	tai150b	100	112	18	9	1000	1000	3135.74	57.96	3075.35	18	9	3040.46	57.14	2999.32	16	12	−3.04	−2.47
31	tai150c	113	37	19	10	1050	1050	2452.15	72.07	2429.72	18	10	2379.57	69.79	2360.47	16	14	−2.96	−2.85
32	tai150c	150	75	19	10	1050	1050	2561.24	65.93	2511.30	18	10	2470.30	69.79	2448.50	16	12	−3.55	−2.50
33	tai150c	100	112	19	10	1050	1050	2934.67	56.98	2856.72	19	10	2895.43	57.19	2844.30	18	11	−1.34	−0.43
34	tai150d	113	37	19	10	950	950	3026.12	73.60	2997.12	18	10	2802.23	68.68	2783.68	14	14	−7.40	−7.12
35	tai150d	150	75	19	10	950	950	3102.56	68.38	3066.94	18	10	2948.06	68.68	2901.66	14	14	−4.98	−5.39
36	tai150d	100	112	19	10	950	950	3267.00	57.54	3244.30	18	10	3226.23	55.34	3180.48	16	13	−1.25	−1.97
Avg.								2266.94	38.20	2242.54			2147.69	36.75	2131.67			−5.18	−4.89

each the remaining three problems, the objective function value of the RTTRP solution is the same as that of the best TTRP solution. On average, the objective function value is improved by 1.33% when the fleet size constraint is dropped. Note that we only compared the average results of RTTRP with the TTRP results of Lin et al. (2009) since their solutions are slightly better than Scheuerer's results (2006) on average.

To gain more insights into the difference of solution quality between RTTRP solutions and TTRP solutions, we conducted more computational experiments on a second set of 36 newly generated TTRP instances. These instances are converted from 12 basic VRP problems given by Rochat and Taillard (1995) in a manner similar to what Chao used to generate TTRP benchmark problems (Chao, 2002). For each customer i in an original problem, the distance between i and its nearest neighbor customer is calculated and denoted by A_i. Each problem is then converted into three TTRP problems. In the first problem, 25% of the customers with the smallest A_i values are specified as truck customers. This percentage was increased to 50% and 75% in the second and third problem respectively.

The SA parameters used in the computational study for the second set of TTRP problems remain the same except for the penalty cost P, which is increased to 200. The RTTRP solutions obtained by the proposed SA heuristic are compared with the TTRP solutions obtained by the SA heuristic of Lin et al. (2009). Each problem is run 10 times. The best and average solutions from 10 runs, as well as the required computational time for each problem are presented in Table 2. It can be seen that in 35 out of the 36 new TTRP instances, the minimum RTTRP solutions from 10 runs are better that of TTRP solutions (tied on other one). Moreover, the average RTTRP solutions from 10 runs are better than that of TTRP solutions for all instances. The percentage of improvement of average and minimum RTTRP solutions over those of TTRP solutions ranges from 0.5% to 12.87% and 0% to 11.75% respectively. On average the improvement rate over TTRP solutions is about 5% for average and minimum RTTRP solutions.

It is interesting to note that for most test problems, the number of trucks used in RTTRP solutions is less than or equal to that used in TTRP solutions, while the number of trailers used in RTTRP solutions is greater than or equal to that used in TTRP solutions. In other words, the trailer/truck ratio is closer to 1 in RTTRP solutions which is more consistent with the common practice in less than truck load industry. Since the acquisition costs and maintenance costs for trailers are generally less than those of trucks, it may prove worthwhile to use more trailers and fewer trucks in the fleet mix. In addition, using fewer trucks also implies that fewer drivers are needed to service all customers which could result in more savings in addition to routing costs.

4. Conclusions

In this paper, we proposed an SA heuristic for the RTTRP, a relaxation of TTRP without the constraint on fleet size. The RTTRP solutions obtained by the SA and the best TTRP solutions reported in prior studies are compared on the 21 benchmark TTRP instances. The RTTRP solution is better than the best TTRP solution in 18 of the 21 problems. The solution quality of the RTTRP solutions and the best TTRP solutions are the same for the remaining three problems. Further, the average RTTRP results are slightly better than the TTRP results of Lin et al. (2009), which is by far the best solutions to the TTRP. Further experiments are conducted on 36 new TTRP instances generated by this research and obtained similar results. The percentage of improvement of RTTRP solutions over TTRP solutions are about 5%. Consider the rising crude oil price and the low margin of trucking companies, it may be worthwhile for trucking companies to consider leasing or acquiring additional vehicles, especially trailers as they are inexpensive compared to trucks, provided that the resulting savings is greater than the leasing or acquisition costs of the additional vehicles.

References

Bodin, L., Golden, B. L., Assad, A., & Ball, M. O. (1983). Routing and scheduling of vehicles and crews: The state of the art. *Computers and Operations Research, 10*, 62–212.

Chao, I. M. (2002). A tabu search method for the truck and trailer routing problem. *Computers and Operations Research, 29*(1), 33–51.

Christofides, N., Mingozzi, A., & Toth, P. (1979). The vehicle routing problem. In N. Christofides, A. Mingozzi, P. Toth, & C. Sandi (Eds.), *Combinatorial optimization* (pp. 315–338). Chichester, UK: Wiley.

Chwif, L., Barretto, M. R. P., & Moscato, L. A. (1998). A solution to the facility layout problem using simulated annealing. *Computers in Industry, 36*, 125–132.

Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science, 6*, 80–91.

Fisher, M. L. (1995). Vehicle routing. In M. O. Ball, T. L. Magnanti, C. L. Monma, & G. L. Nemhauser (Eds.), *Network routing* (pp. 1–33). Amsterdam: North-Holland.

Gerdessen, J. C. (1996). Vehicle routing problem with trailers. *European Journal of Operational Research, 93*(1), 135–147.

Gillett, B., & Miller, L. (1974). A heuristic algorithm for the vehicle dispatch problem. *Operations Research, 22*(2), 340–349.

Golden, B. L., & Assad, A. (1988). *Vehicle routing: Methods and studies*. Amsterdam: North-Holland.

Hoff, A. (2006). *Heuristics for rich vehicle routing problems*. Ph.D. thesis, Molde University College, Norway.

Jayaraman, V., & Ross, A. (2003). A simulated annealing methodology to distribution network design and management. *European Journal of Operational Research, 144*, 629–645.

Laporte, G. (1992). The vehicle routing problem: An overview of exact and approximate algorithms. *European Journal of Operational Research, 59*, 345–358.

Laporte, G., Gendreau, M., Potvin, J. Y., & Semet, F. (2000). Classical and modern heuristics for the vehicle routing problem. *International Transactions in Operational Research, 7*(4–5), 285–300.

Laporte, G., & Nobert, Y. (1987). Exact algorithms for the vehicle routing problem. *Surveys in Combinatorial Optimization, 31*, 147–184.

Laporte, G., & Semet, F. (2002). Classical heuristics for the capacitated VRP. *The Vehicle Routing Problem*, 109–128.

Lim, A., Rodrigues, B., & Zhang, X. (2006). A simulated annealing and hill-climbing algorithm for the traveling tournament problem. *European Journal of Operational Research, 174*, 1459–1478.

Lin, S.-W., Yu, V. F., & Chou, S.-Y. (2009). Solving the truck and trailer routing problem based on a simulated annealing heuristic. *Computers and Operations Research, 36*(5), 1683–1692.

McKendall, A. R., Jr., Shan, J., & Kuppusamy, S. (2006). Simulated annealing heuristics for the dynamic facility layout problem. *Computers and Operations Research, 33*, 2431–2444.

Rochat, Y., & Taillard, É. D. (1995). Probabilistic diversification and intensification in local search for vehicle routing. *Journal of Heuristics, 1*(1), 147–167.

Scheuerer, S. (2006). A tabu search heuristic for the truck and trailer routing problem. *Computers and Operations Research, 33*, 894–909.

Semet, F., & Taillard, E. (1993). Solving real-life vehicle routing problems efficiently using tabu search. *Annals of Operations Research, 41*, 469–488.

Toth, P., & Vigo, D. (2002). *The vehicle routing problem*. Philadelphia, PA: SIAM Monographs on Discrete Mathematics and Applications.