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A simulated annealing heuristic for the truck and trailer routing problem with time windows

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ABSTRACT

European firms have been using a combination of trucks and trailers in the delivery/collection of food products for years. Thus, some previous studies had been devoted to improving the efficiency of the resulting truck and trailer routing problem (TTRP). Since time window constraints are present in many real-world routing applications, in this study, we introduce the truck and trailer routing problem with time windows (TTRPTW) to bring the TTRP model closer to the reality. A simulated annealing (SA) heuristic is proposed for solving the TTRPTW. Two computational experiments are conducted to test the performance of the proposed SA heuristic. The results indicate that the proposed SA heuristic is capable of consistently producing quality solutions to the TTRPTW within a reasonable time.

1. Introduction

For many years, European companies have been using trucks and trailers to deliver/collect food products, such as dairy products and compound animal feeds. However, there are only a few studies devoted to improving the routing efficiency of trucks and trailers. Chao (2002) is the first to formally define this problem as the truck and trailer routing problem (TTRP). This paper extends the TTRP by considering the time window constraints that arise in many practical situations. The resulting problem is called the truck and trailer routing problem with time windows (TTRPTW) which can be regarded as a variant of the vehicle routing problem (VRP) or the vehicle routing problem with time windows (VRPTW).

The VRP is one of the most studied combinatorial optimization problems due to its complexity in nature and extensive applications in practice (Bodin, Golden, Assad, & Ball, 1983; Dantzig & Ramser, 1959; Gillett & Miller, 1974; Golden & Assad, 1988; Laporte, 1992; Laporte, Gendreau, Potvin, & Semet, 2000; Laporte & Nobert, 1987; Toth & Vigo, 2002; Van Breedam, 1995). In the typical VRP, a set of customers with known demands is to be serviced by a fleet of homogeneous vehicles with known capacity. The goal is to design least-cost vehicle routes starting from and terminating at a central depot to satisfy individual customer's demand without violating the vehicle capacity constraints. A side constraint is that each customer must be serviced exactly once by exactly one vehicle. In TTRP, the use of a combination of vehicles and trailers in servicing customers is considered. Customers are serviced either by a single truck or a truck pulling a trailer (called a complete vehicle), due to practical constraints, including government regulations, limited maneuvering space at customer site, road conditions, etc. These constraints exist in many practical situations.

Gerdessen (1996) described two real-world TTRP applications. The first one occurred in the dairy products distribution of the Dutch dairy industry. Since many customers were located in cities with heavy traffic and limited parking spaces, maneuvering a complete vehicle was very difficult. Thus, the trailer was often temporarily parked at some place while the truck continued to deliver products to customers. Another application arose in the delivery of compound animal feed to farmers. Because there were some narrow roads and/or small bridges on the delivery routes, different types of vehicle were used to make the deliveries. One commonly used vehicle type, called double bottoms, consisted of a truck and a trailer. The trailer might be parked at a parking place while the truck serviced some farmers on the delivery route.

Semet and Taillard (1993) gave another application related to the TTRP. It occurred in a major food chain store in Switzerland where 45 company-owned chain stores were serviced by a fleet of 21 trucks and 7 trailers. Delivery scheduling for a combination of trucks and trailers was of great interest.

Hoff (2006) considered another real-world applications occurred at a Norwegian dairy company. The company used a fleet of heterogeneous trucks with tanks to collect raw milk from farmers. A single truck or a truck pulling a trailer with an additional tank might be used to service farmers. Because most Norwegian farms were small and inaccessible for vehicles carrying a trailer,

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the trailer needed to be left at a parking place so that the truck could collect milk from farmers along a sub-tour. When the truck returned to the place where the trailer was parked, it could transfer the milk from the truck tank to the trailer tank and then went on another sub-tour from there. It could also hook up the trailer and move to a new parking place, fill the milk over, and then start a new sub-tour from the new parking place. The problem could be regarded as a special case of the TTRP.

In many practical VRP applications, additional operational requirements and restrictions, such as time windows, may exist. Some customers may only be serviced during a customer specific time period. The resulting problem is known as the vehicle routing problem with time windows. Similarly, time window constraints exist in the applications of TTRP and the resulting problem is called the truck and trailer routing problem with time windows.

The TTRP is computationally more difficult to solve in comparison with VRP because VRP can be considered as a special case of TTRP. Similarly, TTRPTW can also be reduced to VRPTW. Since VRPTW itself is a very difficult combinatorial optimization problem and instances of practical size are usually tackled by heuristic approaches (Abramson, 1991; Bent & Van Hentenryck, 2004; Berger & Barkaoui, 2004; Braysy, 2003; Braysy, Dullaert, & Gendreau, 2004; Braysy & Gendreau, 2005a, 2005b; Braysy, Hasle, & Dullaert, 2004; Cordeau, Laporte, & Mercier, 2001; Cordone & Calvo, 2001; Homberger & Gehring, 2005; Ioannou, Kritikos, & Prastacos, 2001; Le Bouthillier and Crainic, 2005; Mester & Braysy, 2005; Russell & Chiang, 2005; Ting & Huang, 2005), it is natural to develop heuristic approaches for the TTRPTW. In light of Lin et al. (2009, 2010) promising demonstration that simulated annealing (SA) heuristic is an efficient and effective approach for solving the TTRP, this study develops an SA heuristic for the TTRPTW. Since there are currently no benchmark instances for TTRPTW in the literature, at least to the authors' knowledge, we converted 12 Solomon's VRPTW benchmark problems (Solomon, 1987) and six Homberger's extended Solomon's VRPTW instances (Homberger, 2009) into 54 TTRPTW benchmark problems to test the performance of the proposed SA heuristic. Computational results indicate that the proposed SA heuristic is competitive with prior approaches on some VRPTW instances, and is capable of consistently producing quality TTRPTW solutions.

The remainder of this paper is organized as follows. Section 2 describes the TTRPTW and surveys related literature. Section 3 details the main features of the proposed SA heuristic. Section 4 gives descriptions of the test problem generation scheme, the parameter settings and results of the computational study. Section 5 summarizes this study.

2. Problem definition and literature review

The TTRPTW model discussed in this paper is an extension of the TTRP model proposed by Chao (Chao, 2002). The problem can be formally defined on an undirected graph $G = (V, A)$, where $V = \{0, 1, 2, \ldots, n\}$ is the set of vertices and $A = \{(i, j): i, j \in V\}$ is the set of edges. Vertex 0 represents the central depot, while the remaining vertices in $V/\{0\}$ correspond to customers. Each vertex i is associated with a non-negative demand d_i, a service time window (ET_i, LT_i), a service time ST_i, and a customer type CT_i, where ET_i and LT_i denote the earliest time and latest time that the service to customer i can start, respectively. ST_i denotes the time required to service customer i. $CT_i = 1$ indicates that customer i is a truck customer (TC) that can only be serviced by a single truck, while $CT_i = 0$ means customer i is a vehicle customer (VC) that can be serviced by either a single truck or a complete vehicle. Each edge (i, j) is associated with a non-negative cost c_{ij}, which can be interpreted as the travel time required on the edge or simply the travel distance of the edge.

The number of trucks and trailers used in the routes is not determined *a priori*. All trucks have identical capacity Q_k and all trailers have identical capacity Q_r. If a trailer is assigned to a truck, it has to stay with the truck while the truck is on the main tour. The goal of the TTRPTW is to find a set of least-cost vehicle routes that originates and terminates at the central depot such that each customer is serviced exactly once within their specific time windows and the total demand on each vehicle route does not exceed the total capacity of the vehicles used for that route.

There are three types of routes in a TTRPTW solution: (1) a pure truck route (PTR) traveled by a single truck; (2) a pure vehicle route (PVR) without any sub-tour traveled by a complete vehicle; and (3) a complete vehicle route (CVR) consisting of a main tour traveled by a complete vehicle, and at least one sub-tour traveled by the truck alone. A sub-tour starts and ends at the same VC site or the depot on the main tour. This parking place is called the root of the sub-tour. After left the trailer at the root, the truck proceeds to service customers on the sub-tour. When all customers on the sub-tour are serviced, the truck returns to the root, picks up its trailer and continues to service remaining customers on the same route.

To our best knowledge, there are no articles devoted to TTRPTW. In fact, there are only a few TTRP related articles in the literature despite of its practical importance. Semet and Taillard (1993) discussed a practical VRP that allows limited use of trailers under accessibility restrictions. The problem differs from the TTRP in that a VC cannot be serviced in a sub-tour. In addition, other constraints such as time windows and vehicle-dependent variable costs are included in the problem. The authors developed a clustering-based construction heuristic and a tabu search (TS) heuristic for the problem.

Semet (1995) considered the "partial accessibility constrained VRP" which is similar to the TTRP. The problem is different from TTRP because of the following assumptions: (1) two sub-tours cannot have the same root; (2) all trucks must be used, number of trailers used must be determined *a priori*; and (3) the central depot cannot be visited in the middle of a vehicle route. Semet proposed a heuristic based on Fisher and Jaikumar (1981) generalized assignment method for the VRP for the problem.

Gerdessen (1996) studied the vehicle routing problem with trailers (VRPT) which is very similar to the TTRP. The problem differs from the TTRP because: (1) all customers have unit demand; (2) a maneuvering cost instead of customer type is assigned to each customer; (3) each trailer is parked exactly once; and (4) speeds of truck and complete vehicle are different. The authors developed four different construction heuristics for the problem.

Bodin and Levy (2000) studied a postal delivery problem which is also similar to the TTRP. In this problem, the postmen correspond to the trucks and their postal cars correspond to the trailers. Recently, Scheuerer (2004) discussed two extensions of the TTRP: the multiple depots TTRP and the periodic TTRP. Drexl (2006, 2007b) studied a more general TTRP called the "vehicle routing problem with trailer and transshipments". In this problem, the assignment of trailers to trucks is not fixed as in the TTRP. Drexl (2007a, 2007b) also studied a TTRP related problem features more practical considerations.

Since TTRP is a hard combinatorial problem, heuristic approaches based on tabu search and simulated annealing had been proposed for solving the TTRP. Chao (2002) and Scheuerer (2006) solved the TTRP by a 2-phase approach. Construction heuristics were used in the first phase to obtain an initial TTRP solution, which was then improved by a tabu search algorithm in the second phase. Recently, Lin, Yu, and Chou (2009) applied an SA based heuristic to the truck and trailer routing problem (TTRP) and obtained results that are competitive with those obtained by Scheuerer (2006). The only exact approach for the TTRP is due to Drexl

(2007a). The author developed a branch-and-price algorithm for the TTRP. However, the algorithm had only been tested on relatively small instances of the TTRP.

3. Simulated annealing heuristic for the TTRPTW

Simulated annealing is a local search-based heuristic that is capable of escaping from being trapped into a local optimum by accepting, in small probability, worse solutions during its iterations. It has been applied successfully to a wide variety of highly complicated combinatorial optimization problems (Abramson, 1991; Chwif, Barretto, & Moscato, 1998; Jayaraman & Ross, 2003; Lim, Rodrigues, & Zhang, 2006; Lin, Chou, & Chen, 2007; McKendall, Shan, & Kuppusamy, 2006; Van Breedam, 1995; Yu, Lin, & Chou, 2010; Yu, Lin, Lee, & Ting, 2010) as well as various real-world problems (Candalino, Kobza, & Jacobson, 2004; Kim & Moon, 2003; Lee, Cao, & Meng, 2007). The SA usually starts with a randomly generated initial solution. At each iteration, the algorithm selects a new solution from the neighborhood of the current solution. If the objective function value of the new solution is better than that of the current solution, the new solution becomes the current solution from which the search continues. A new solution with a worse objective function value may also be accepted as the current solution with a small probability determined by the Boltzmann function, $\exp(-\Delta/KT)$, where K is a predetermined constant and T is the current temperature. The essential idea is not to restrict the search moves only to better solutions. By accepting a worse solution, the procedure may escape from a local optimum.

3.1. Solution representation

A TTRPTW solution is represented by a string of numbers consisting of a permutation of n customers denoted by the set $\{1,2,\ldots,n\}$ and N_{dummy} zeros (artificial depot or the root of a sub-tour), followed by the service vehicle types of individual VCs. The zeros are used to separate routes or terminate a sub-tour, in addition to vehicle capacity constraints. The parameter N_{dummy} is calculated by $\lfloor \sum_i d_i/Q_k \rfloor$, where $\lfloor \cdot \rfloor$ denotes the largest integer smaller than or equal to the enclosed number. The jth non-zero number in the first $n + N_{dummy}$ positions denotes the jth customer to be serviced.

The service vehicle type of a VC is either 0 or 1. The VC's service vehicle type is set to be 1 if it is serviced by a complete vehicle; otherwise, its service vehicle type is 0. Since a TC must be serviced by a truck alone, its service vehicle type is not included in the solution.

In the following, we describe the solution representation in more details. The first non-zero element in a solution indicates the first customer to be serviced in the first route. If the first customer on a route is to be serviced by a single truck, the route is set to be a PTR. Other customers are added to the route one by one from left to right to represent the sequence in which they are serviced, provided that both the capacity constraint of the vehicle in use and the time window constraint of each customer on the route are not violated. The capacity of the service vehicle may be $(Q_k + Q_r)$ or Q_k. A zero in the solution representation indicates that the vehicle will either return to the root of a sub-tour or the depot.

The time window constraint stipulates that service to a customer must start within the customer's service time window. If the vehicle arrives at the customer site earlier than the customer's service time window, the vehicle must wait until the service time window starts. If adding the next customer to the current route violates the customer's time window constraint, i.e. arrival time at the customer site is later than the latest start time of the customer's service time window, the current route ends here and a new route will be initiated.

Whenever a route is terminated and there are still customers need to be serviced, a new route will be generated starting with the next customer in the solution representation. It can be verified that this solution representation always gives a feasible TTRPTW solution without violating the capacity constraint of vehicles in use and the time window constraint of customers.

Table 1
A TTRPTW instance with 25 customers.

No.	*X*	*Y*	*W*	*ET*	*LT*	*ST*	*CT*
0	35.0	54.0	–	–	–	–	–
1	45.0	68.0	10.0	912.0	967.0	90.0	1
2	45.0	70.0	30.0	825.0	870.0	90.0	1
3	42.0	66.0	10.0	65.0	146.0	90.0	1
4	42.0	68.0	10.0	727.0	782.0	90.0	1
5	42.0	65.0	10.0	15.0	67.0	90.0	1
6	40.0	69.0	20.0	621.0	702.0	90.0	0
7	40.0	66.0	20.0	170.0	225.0	90.0	1
8	38.0	68.0	20.0	255.0	324.0	90.0	0
9	38.0	70.0	10.0	534.0	605.0	90.0	0
10	35.0	66.0	10.0	357.0	410.0	90.0	0
11	35.0	69.0	10.0	448.0	505.0	90.0	0
12	30.0	70.0	20.0	652.0	721.0	90.0	0
13	27.0	60.0	30.0	30.0	92.0	90.0	0
14	27.0	70.0	10.0	567.0	620.0	90.0	0
15	25.0	65.0	40.0	384.0	429.0	90.0	0
16	25.0	70.0	40.0	475.0	528.0	90.0	1
17	23.0	62.0	20.0	99.0	148.0	90.0	0
18	20.0	60.0	20.0	179.0	254.0	90.0	1
19	20.0	65.0	10.0	278.0	345.0	90.0	1
20	30.0	50.0	10.0	10.0	73.0	90.0	0
21	30.0	52.0	20.0	914.0	965.0	90.0	0
22	28.0	52.0	20.0	812.0	883.0	90.0	0
23	28.0	55.0	10.0	732.0	777.0	90.0	0
24	25.0	50.0	10.0	65.0	144.0	90.0	0
25	25.0	52.0	40.0	169.0	224.0	90.0	0

3.2. Illustration of solution representation

Table 1 gives a TTRPTW instance with 25 customers. Each customer's location (X, Y), demand (W), earliest time (ET) and latest time (LT) of service time window, service time required (ST), and customer type (CT) are listed in the table. A sample solution to this instance is shown in Fig. 1, in which four dummy zeros are introduced. Customers 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 20, 21, 22, 23, 24 and 25 are vehicle customers (shown in boldface) and their service vehicle types are as shown in the solution representation. Shaded customers are to be serviced by a single truck. Among them, customer 6, 8, 9, 10, 11, 12 and 14 are VCs that will be serviced by trucks since their service vehicle type is 1 in the solution representation. The remaining nine truck customers must be serviced by trucks. A visual illustration of this solution representation is given in Fig. 2.

In this example, the first route starts by servicing customer 20, and then servicing customers 24, 25, 23, 22, and 21, followed by a zero representing the depot. Thus, the vehicle will return to the depot after finishing servicing customer 21. The customers in the first route are all VCs. All of them are serviced by a complete vehicle

20	**24**	**25**	**23**	**22**	**21**	0	0	5	3	7	**8**	**13**	**17**	18	19	**15**	16	**14**	**12**	0	**10**	**11**	**9**	**6**	4	2	1	0
Route 1								Route 2				Route 3									Route 4							

Service vehicle types of VCs for customer															
1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0
6	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**17**	**20**	**21**	**22**	**23**	**24**	**25**

Fig. 1. An example of solution representation.

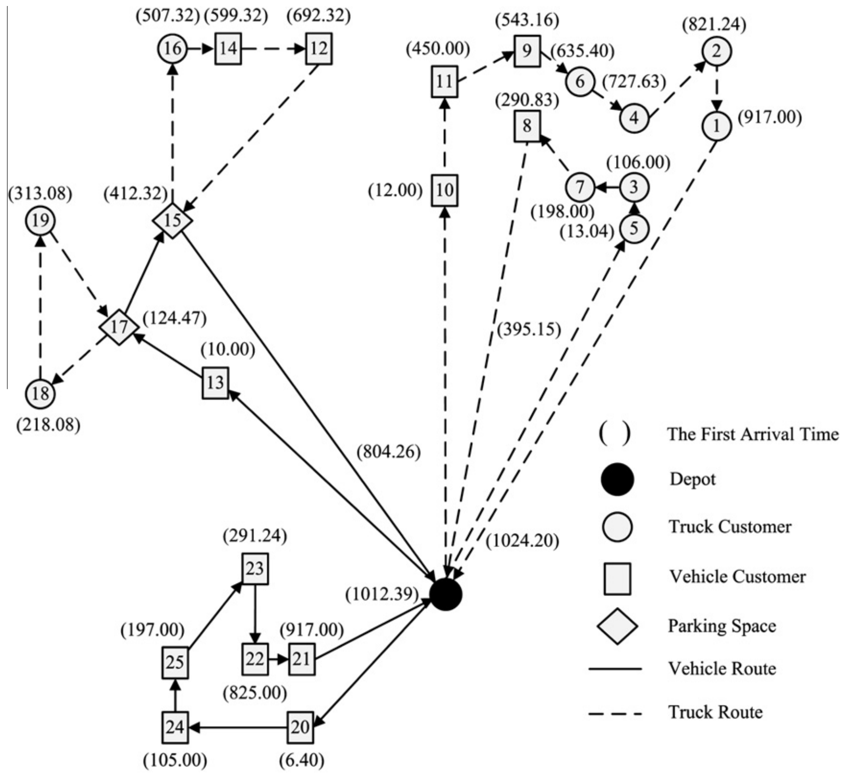

Fig. 2. A visual illustration of the example solution given in Fig. 1.

because their service vehicle types are all zeros in the solution representation. Thus, this route is a pure vehicle route.

Note that there are two consecutive dummy zeros at the end of this route. In this case, the second zero can be disregarded since there are no customers between the zeros.

The first customer to be serviced in the second route is customer 5, followed by customers 3, 7, and 8. Because customer 5 is a truck customer, this route is set to be a pure truck route. Note that in this route, customer 8 is a VC whose corresponding service vehicle type is 1, thus it is serviced by a truck. Since the next customer 13 is a VC that that will be serviced by a complete vehicle (service type of customer 13 is 0) and the current route is a PTR, after finishing servicing customer 8, this truck will return to the depot and the route is terminated.

The third route starts with VC 13. Since VC 13's service vehicle type is 0, it will be serviced by a complete vehicle. The vehicle goes on to service VC 17. Since the next customer 18 is a truck customer, the trailer has to be parked at customer 17 so that the truck can go on to service customers 18 and 19. The next customer 15 is a vehicle customer, whose service vehicle type is 0, indicating that it is serviced by a complete vehicle. Thus, the truck needs to return to customer 17 to pick up its trailer before continuing onto the route to service customer 15. The customer to be serviced after customer 15 is customer 16. Since customer 16 is a truck customer, the trailer has to be parked at customer 15, while the truck continues to service customers 16, 14 and 12 (both customer 14 and customer 12 are VCs whose service vehicle type is 1, thus they are serviced by a truck) sequentially. After finishing servicing customer 12, the next customer to be serviced is zero. Since the vehicle is currently on a sub-tour, the zero indicates that the next customer is the root (trailer parking place) of the current sub-tour. Therefore, the vehicle will return to customer 15. The next customer to be served is customer 10. Since adding customer 10 to the current route will violate the time window constraint of customer 10, the vehicle will return to the depot, and the route is terminated.

The fourth route starts by servicing customer 10, and then sequentially servicing customer 11, 9, 6, 4, 2 and 1. A zero appears immediately after customer 1 in the solution representation so the vehicle will go back to the depot after finishing servicing customer 1. Since this route starts with a VC that is served by a truck (service vehicle type is 1), this route is set to be a PTR.

The solution representation determines the customers on each route and the service vehicle type of each VC. Thus, it is easy to calculate the objective function value (total travel distance), denoted by obj(X), of a given solution X.

3.3. Neighborhood

We use a standard SA procedure with a random neighborhood structure that features various types of moves, including insertion, swap, and change of service vehicle type of VCs, to solve the TTRPTW. We define the set $N(X)$ to be the set of solutions neighboring a solution X. At each iteration, a new feasible solution Y is generated from $N(X)$ by one of these three types of move described as follows.

The swap move randomly selects the ith and the jth customers of X and then exchanges their positions. The insertion move is carried out by randomly selecting the ith customer of X and then inserting it into the position immediately before another randomly selected jth customer of X. The move of changing service vehicle type of VCs is performed by randomly selecting a VC from X and then changing its service vehicle type from 1 to 0 or from 0 to 1. In other words, if the chosen VC was serviced by a truck before the move, it will be serviced by a complete vehicle after the move, and vice versa. Following Lin et al. (2009), the probabilities of performing the three moves: swap, insertion, and change of service vehicle type of VCs are set to be 0.2, 0.2, and 0.1, respectively. In order to increase the chance of obtaining a better solution, besides randomly choosing one or two customers to undergo one of the three moves, the strategy of best-of-N-trials is also performed, where N is a predetermined number of trials; that is, the best solution among the N trials is chosen as the solution selected from the neighborhood of X.

For swap and insertion moves, this number is set to be N_{trial}, calculated by $\lfloor (n + N_{dummy})/3 \rfloor$, where $\lfloor \cdot \rfloor$ denotes the largest integer which is smaller than or equal to the enclosed number. For the move of changing service vehicle type of VCs, each VC's service vehicle type is changed one at a time, thus the number of trials equals the number of VCs.

The probabilities of performing the best-of-N-trials strategy are 0.2, 0.2, and 0.1 for swap, insertion, and change of service vehicle type of VCs moves, respectively.

3.4. Parameters used

The proposed SA heuristic requires six parameters I_{iter}, T_0, T_F, K, $N_{non\text{-}improving}$ and α. I_{iter} denotes the number of iterations the search proceeds at a particular temperature. T_0 represents the initial temperature and T_F represents the final temperature below which the SA procedure is terminated. K is the Boltzmann constant used in the probability function to determine whether a worse solution is accepted or not. $N_{non\text{-}improving}$ is the maximum allowable number of temperature reductions during which the best objective function value is not improved. Finally, α is a coefficient used to control the speed of the cooling schedule.

3.5. The SA procedure

In the beginning, the current temperature T is set to be T_0 and an initial solution X is randomly generated. The solution consists of a random sequence of all customers and the dummy zeros, followed by a 0–1 string representing the randomly assigned service vehicle types of VCs. The current best solution X_{best} and the best objective function value obtained so far, denoted by F_{best}, are set to be X and obj(X), respectively.

At each iteration, a new solution Y is generated from the neighborhood of the current solution X, $N(X)$, and its objective function value is evaluated. Let Δ = obj(Y) − obj(X). If Δ is less than or equal to zero (i.e., Y is better than X), X is replaced with Y. Otherwise, the probability of replacing X with Y is $\exp(-\Delta/KT)$. X_{best} and F_{best} record the current best solution and the best objective function value obtained so far, as the algorithm progresses.

The current temperature T is decreased after I_{iter} iterations after the previous temperature decrease, according to the formula $T = \alpha T$, $0 < \alpha < 1$. After every three temperature reductions, a local search procedure that sequentially performs 2-opt, swap, insertion, and change of service vehicle types is used to improve the current best solution.

The algorithm is terminated when the current temperature T is lower than T_F or the current best solution X_{best} has not improved for $N_{non\text{-}improving}$ consecutive temperature decreases. Following the termination of SA procedure, the (near) optimal routing plan can be derived from X_{best}. A flowchart depicting the proposed SA heuristic is given in Fig. 3.

4. Computational study

The proposed SA heuristic was implemented in C language and tested on a PC with an Intel Pentium 4 1.5 GHz processor and 512 MB memory. To the best of our knowledge, currently there are no existing TTRPTW benchmark problems. Therefore, new TTRPTW benchmark problems are generated for the computational study.

Two experiments are performed to verify the performance of the proposed heuristic. In the first experiment, several well-known VRPTW benchmark problems were selected and solved by the proposed SA heuristic. Results are compared with best known solutions in the literature. In the second experiment, more VRPTW problems are converted into TTRPTW test problems and then solved by the proposed SA approach.

4.1. Test problems

In the first experiment, six of Solomon's (1987) VRPTW benchmark problems with 100 customers are selected as the test problems. These benchmark instances vary in vehicle capacity, travel time of vehicles, spatial distribution of customers, time window density and width, and hence are classified into three categories: R-type (uniformly distributed customers), C-type (clustered customers), and RC-type (a mix of R and C types). Each of these three categories contains two sets of problems. Problem sets R1, C1 and RC1 have narrow scheduling horizon, while problem sets R2, C2 and RC2 have large scheduling horizon. Problems with narrow scheduling horizon have vehicles with small capacities and short route times. Thus, a vehicle can only service a few customers. On the other hand, large scheduling horizon problems use vehicles with large capacities and long travel times, so more customers can be serviced by the same vehicle.

We select the first problem with 100 customers from each data-set (C101, C201, R101, R201, RC101, and RC201) to perform the first experiment, so a total of six Solomon's VRPTW benchmark problems are used in this experiment. Results are compared to existing approaches for the VRPTW to verify the performance of the proposed SA heuristic.

For the second experiment, in addition to the six problems selected for the first experiment, the first problem with 50 customers from each Solomon's data set and the first problem with 200 customers from each data set of Homberger's extended Solomon's instances (Homberger, 2009) are also included in the experiment. Each of the 18 selected VRPTW benchmark problems is converted into three TTRPTW benchmark problems in the following manner similar to what Chao (2002) used to generate TTRP instances. For each customer i in a Solomon's problem, the distance between customer i and its nearest neighboring customer is calculated and denoted by A_i. In the first TTRPTW problem, 25% of the customers with the smallest A_i values are specified as truck customers. This percentage is increased to 50% and 75% in the second and third TTRPTW problems, respectively. See Chao (2002) and Lin et al. (2009) for a more detailed discussion on the test problems generating scheme.

4.2. Computational results

Parameter settings may have great influence on the computational results. Thus, a set of pilot experiments with the following combinations of parameters was conducted to identify the best combination of parameters for the proposed SA procedure:

$$\alpha = 0.965, 0.975,$$
$$I_{iter} = 30000, 50000, 70000, 90000, 120000, 150000, 200000,$$
$$K = 1, 1/2, \ldots, 1/9.$$

From the results of the pilot study, we observed that setting $\alpha = 0.965$, $I_{iter} = 150000$, and $K = 1/4$ gave the best results. Therefore, these parameter values were used for further computational studies. Other parameters used in the experiment are: $T_0 = 100$, $T_F = 1$, and $N_{non\text{-}improving} = 30$. Since $T_0\alpha^{130} = 100 \times 0.965^{130} < 1 = T_F$, the current temperature will fall below the final temperature no more than 130 temperature reductions. Not that the algorithm may terminate early if X_{best} is not improved in 30 successive temperature reductions.

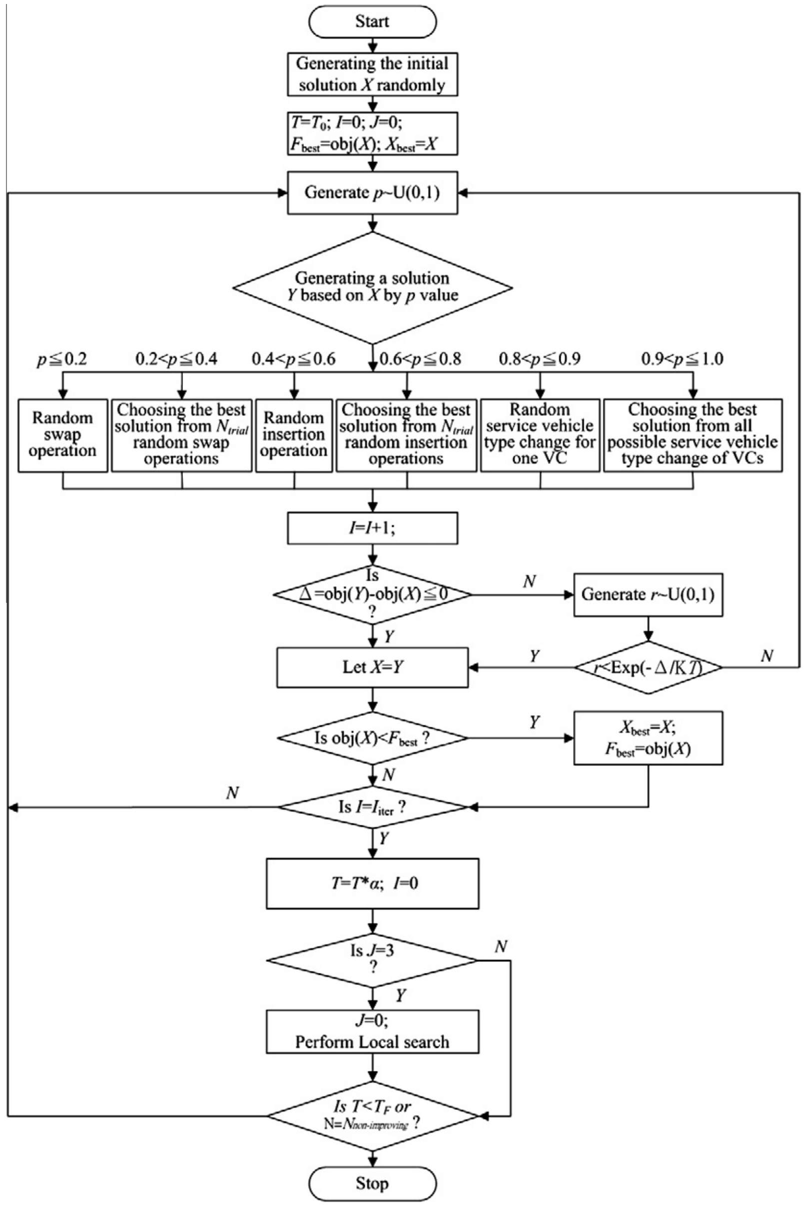

Fig. 3. A flowchart for the proposed SA heuristic.

The first experiment is performed on the six Solomon's VRPTW problems with 100 customers. Two different tests are conducted to evaluate the performance of the proposed approach. In the first test, all customers are set to be truck customers and the capacities of the trucks are the same as those in the original VRPTW problems. Thus, these TTRPTW problems are essentially the same as the original VRPTW problems. In the second test, all customers are set to be vehicle customers. The original vehicle capacity is equally distributed to a truck and a trailer, that is, truck capacity and trailer capacity is one half of the original vehicle capacity, respectively. Note that the second test is different from the first test in that VCs can also be serviced by a single truck.

For each problem, the best solution from 5 runs of the proposed SA heuristic with different random seeds is reported. The results are compared with the best known solutions obtained by heuristics in the literature (Solomon, 2005), as shown in Table 2. For the first test, the solutions to C101 and C201 obtained by the SA heuristic are the same as the best known solution, and the solutions to R101, R201, RC101, and RC201 are better than the best known solutions. For the second test, the solutions from the SA heuristic are either the same as or better than the best known solutions for five out of the six test problems. The only exception is R101, but the difference between the SA solution and the best know solution is insignificant.

Note that these better results are due to the fact that TTRPTW only concerns with minimizing the total travel distance of vehicles, while in VRPTW, minimizing the number of vehicles used is also considered. This is evident from the solutions for R201 and RC201, in which about twice as many trucks are used compared to the best VRPTW solutions. Nevertheless, at least, the result of the first experiment indicates the proposed SA approach is effective on solving the VRPTW type problems, when minimizing the number of vehicles used is not as important as minimizing the total route distance.

In the second experiment, the proposed approach is executed 10 times on each of the 54 newly generated TTRPTW benchmark problems. The results for benchmark problems with 50, 100, and 200 customers are reported in Tables 3–5, respectively.

As can be seen from Tables 3–5, overall the differences between the best solutions from 10 runs (Min $c(s^*)$), the average solutions from 10 runs (Avg $c(s^*)$), and the best known solutions obtained

Table 2
Results of the first experiment.

Problem	BKS	Test 1		Test 2		
	NV/TD	Truck capacity	NK/TD	Truck capacity	Trailer capacity	NK/NR/TD
C101	10/828.94	200	10/828.94	100	100	10/10/828.94
C201	3/591.56	700	3/591.56	350	350	3/3/591.56
R101	19/1645.79	200	20/1642.88	100	100	20/20/1648.53
R201	4/1252.37	1000	8/1148.17	500	500	9/9/1149.84
RC101	14/1696.94	200	15/1638.92	100	100	17/17/1666.39
RC201	4/1406.91	1000	9/1269.01	500	500	10/10/1273.27

BKS, best known solutions (Solomon, 2005).
NV, number of vehicle used.
NK, number of truck used.
NR, number of trailer used.
TD, total distance.

Table 3
Results for TTRPTW with 50 customers ($K = 1/4$).

ID	Original problem	VC	TC	Truck Capacity	Trailer Capacity	Min c(s*)[a]	Avg c(s*)[b]	T[c]	c(s**)[d]
1	C101	38	12	100	100	415.67	415.67	10.75	415.67
2	C101	25	25	100	100	491.07	497.83	8.21	424.11
3	C101	37	13	100	100	500.59	504.77	11.42	424.11
4	C201	38	12	350	350	403.95	412.01	12.15	403.95
5	C201	25	25	350	350	404.67	404.67	8.28	404.67
6	C201	37	13	350	350	404.67	404.67	10.22	404.67
7	R101	38	12	100	100	1046.70	1046.70	14.00	1046.70
8	R101	25	25	100	100	1046.70	1046.70	11.91	1046.70
9	R101	37	13	100	100	1046.70	1046.70	10.28	992.07
10	R201	38	12	500	500	794.34	801.41	13.59	794.34
11	R201	25	25	500	500	794.34	794.50	10.76	794.34
12	R201	37	13	500	500	794.34	794.34	9.17	794.34
13	RC101	38	12	100	100	973.34	974.71	13.21	960.79
14	RC101	25	25	100	100	986.23	986.78	12.55	986.23
15	RC101	37	13	100	100	992.07	993.43	11.20	992.07
16	RC201	38	12	500	500	686.31	686.31	9.13	686.31
17	RC201	25	25	500	500	686.31	686.31	7.93	686.31
18	RC201	37	13	500	500	686.31	686.39	9.30	686.31

[a] Best solutions from 10 runs by the proposed SA heuristic.
[b] Average solutions from 10 runs by the proposed SA heuristic.
[c] Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for the proposed SA heuristic.
[d] Best known solutions obtained by the proposed SA heuristic during parameter analysis.

Table 4
Results for TTRPTW with 100 customers ($K = 1/4$).

ID	Original problem	VC	TC	Truck capacity	Trailer capacity	Min $c(s^*)$[a]	Avg $c(s^*)$[b]	T[c]	$c(s^{**})$[d]
1	C101	75	25	100	100	971.82	971.92	52.71	971.82
2	C101	50	50	100	100	1107.81	1111.72	62.00	1106.90
3	C101	25	75	100	100	1154.80	1166.35	58.19	1154.80
4	C201	75	25	350	350	671.37	685.14	41.70	671.31
5	C201	50	50	350	350	713.06	723.68	47.72	713.06
6	C201	25	75	350	350	711.45	711.45	44.61	711.45
7	R101	75	25	100	100	1651.16	1659.96	69.89	1651.16
8	R101	50	50	100	100	1650.56	1660.73	60.13	1644.64
9	R101	25	75	100	100	1644.64	1645.68	49.29	1644.64
10	R201	75	25	500	500	1177.79	1193.16	56.52	1166.64
11	R201	50	50	500	500	1156.29	1172.90	53.65	1148.17
12	R201	25	75	500	500	1148.17	1157.80	43.78	1147.80
13	RC101	75	25	100	100	1719.74	1739.52	70.86	1717.01
14	RC101	50	50	100	100	1776.69	1793.11	62.75	1770.40
15	RC101	25	75	100	100	1786.11	1794.70	50.12	1784.06
16	RC201	75	25	500	500	1279.33	1287.16	60.95	1277.05
17	RC201	50	50	500	500	1282.10	1289.75	45.84	1273.69
18	RC201	25	75	500	500	1266.11	1276.67	38.34	1266.11

[a] Best solutions from 10 runs by the proposed SA heuristic.
[b] Average solutions from 10 runs by the proposed SA heuristic.
[c] Average times in minutes from 10 runs on a Pentium IV 1.5 GHz PC for the proposed SA heuristic.
[d] Best known solutions obtained by the proposed SA heuristic during parameter analysis.

during parameter analysis ($c(s^{**})$) are very small, indicating that the proposed SA heuristics is capable of consistently producing quality TTRPTW solutions within a reasonable amount of time, regardless of different problem characteristics. The results may be an indication that the proposed approach is applicable to solving the TTRPTW problem of interest, efficiently and effectively.

Table 5
Results for TTRPTW with 200 customers ($K = 1/4$).

ID	Original problem	VC	TC	Truck capacity	Trailer capacity	Min $c(s^*)$[a]	Avg $c(s^*)$[b]	T[c]	$c(s^{**})$[d]
1	C101	50	150	100	100	3070.34	3147.37	262.70	3070.34
2	C101	100	100	100	100	3348.21	3535.60	279.78	3348.21
3	C101	150	50	100	100	3828.60	4005.90	241.83	3828.60
4	C201	50	150	350	350	2223.86	2289.77	211.37	2222.22
5	C201	100	100	350	350	2372.56	2409.55	227.59	2298.83
6	C201	150	50	350	350	2360.23	2400.66	180.16	2351.91
7	R101	50	150	100	100	5106.72	5165.43	290.98	5094.25
8	R101	100	100	100	100	5324.81	5403.86	269.25	5324.81
9	R101	150	50	100	100	5484.90	5517.14	209.05	5459.20
10	R201	50	150	500	500	3654.23	3724.04	259.58	3654.23
11	R201	100	100	500	500	3549.33	3583.74	225.64	3527.81
12	R201	150	50	500	500	3533.16	3574.75	190.86	3516.68
13	RC101	50	150	100	100	3816.75	3914.46	291.93	3816.75
14	RC101	100	100	100	100	4209.23	4247.30	281.49	4170.70
15	RC101	150	50	100	100	4702.97	4763.81	240.13	4681.76
16	RC201	50	150	500	500	2907.93	2968.04	255.72	2854.18
17	RC201	100	100	500	500	2951.21	2995.58	232.25	2854.18
18	RC201	150	50	500	500	2858.70	2879.14	200.13	2843.15

[a] Best solutions from 10 runs by the proposed SA heuristic.
[b] Average solutions from 10 runs by the proposed SA heuristic.
[c] Average times in minutes from 10 runs on a Pentium IV1.5 GHz PC for the proposed SA heuristic.
[d] Best known solutions obtained by the proposed SA heuristic during parameter analysis.

However, further study to derive lower bounds for small TTRPTW instances may be necessary to verify this.

5. Conclusions and future research directions

The truck and trailer routing problem has rarely been studied despite of its practical implication, especially in the distribution/collection of food products. In this paper we introduced an extension of the truck and trailer routing problem, namely the truck and trailer routing problem with time windows, to bring this type of routing problem closer to the reality. Due to the complexity of the TTRPTW, heuristic approaches may be the only viable way to find good solutions within a reasonable time for realistic problems. Thus, in this study, we developed a simulated annealing based heuristic for the TTRPTW and performed extensive testing on the proposed approach. Two computational experiments are performed on a set of six Solomon's VRPTW benchmark problems, and 54 TTRPTW instances converted from Solomon's VRPTW benchmark problems, respectively. Based on the results, we believe that the proposed SA heuristic is a promising solution approach for the TTRPTW.

Future research may try to extend the TTRPTW to include more practical considerations, such as split deliveries, multiple time windows, and time dependent travel times, to move the problem even closer to real-world problems. Other efficient heuristics for VRPTW and TTRP may also be modified to solve the TTRPTW. In this regard, the benchmark problems generated in this study may serve as a test bed for future research to test the efficiency of specific algorithms for TTRPTW. Exact solution approaches and lower bounds for the TTRPTW may be developed to verify the effectiveness of various heuristics for the TTRPTW.

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