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Theory and Methodology

Vehicle routing problem with trailers

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Abstract

In this research the vehicle routing problem is extended to the vehicle routing problem with trailers. The optimal deployment of a vehicle fleet of truck–trailer combinations is investigated. Truck–trailer combinations may encounter manoeuvring problems at certain customer sites. Therefore the opportunity is introduced to leave the trailer at a parking-place and visit some 'difficult' customers with the easy manoeuvrable truck only. Construction and improvement heuristics are presented.

Keywords: Routing; Vehicle routing problem; Heuristics; Trailers

1. Introduction

In logistics management one may distinguish material management and physical distribution management. The term material management comprises the activities that are concerned with the management of the flow of goods up to the end of the production/assembly process. Physical distribution management deals with the management of the flow of goods from the end of the production process to the final customers, possibly via distribution centres.

The cost of physical distribution often form a considerable part of the total cost of a product and of its final price (Van Goor, 1980). Therefore control of these cost is worthwhile. A major aspect of physical distribution is the transport of goods from production sites to warehouses and from warehouses to customers. Routing is concerned with the design of routes along delivery points (e.g. customers or warehouses) in such a way that the total cost are as low as possible while certain conditions are satisfied. Routing problems also occur outside the area of physical distribution, e.g. the collection of mail from pillar boxes.

This paper discusses a routing problem: the vehicle routing problem with trailers.

The vehicle routing problem (VRP) is concerned with the delivery of goods to customers. These goods have to be delivered from a depot. The delivery is carried out by a fleet of vehicles. The VRP can be formulated as follows: Find a set of routes for the vehicles that minimizes the total cost of delivery to all customers, subject to restrictions on the loads of the vehicles. These total cost can contain fixed and variable cost.

The problem can be extended in various ways, e.g. by adding:
- time windows,
- combined pick-up and delivery,
- restrictions on the duration of the separate vehicle trips.

The variant that will be discussed here is the Vehicle Routing Problem with Trailers (VRPT). In the VRPT vehicles are considered that consist of a

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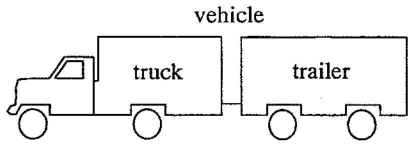

Fig. 1. A vehicle consists of a truck and a trailer.

truck and a trailer (see Fig. 1). Both parts can carry goods. The combination of truck and trailer will be called a 'vehicle'.

The use of a vehicle may cause problems when serving customers that are located in the centre of a city or customers near which little space is available for manoeuvring the vehicle. Time and trouble could be saved if these customers were served by the truck only, instead of the complete vehicle. An additional advantage of driving without the trailer is that the truck uses less fuel than the complete vehicle and drives faster.

Because of the advantages of serving some customers without a trailer it could be worthwhile to consider the following form of the daily task of a vehicle (see Fig. 2):

- Leave the depot with the truck and the trailer.
- Serve (with the truck plus the trailer) some customers that can 'easily' be reached with the complete vehicle.
- Leave the trailer at a parking-place.
- Serve (with only the truck) some customers that cause problems when visited with the complete vehicle.
- Return to the parking-place and pick up the trailer.
- Serve customers that can 'easily' be reached with the complete vehicle.
- Return to the depot.

This approach causes an additional capacity restriction: capacity needed for transporting goods for

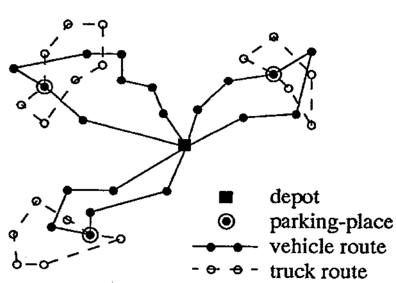

Fig. 2. Solution to the VRPT.

customers that will be visited by the truck only should not exceed the capacity of the truck.

Fig. 2 shows a situation in which each vehicle parks its trailer exactly once. One can also imagine that trailers are left at parking-places several times, or that some are not left at a parking-place at all.

An interesting application area for the VRPT is the distribution of dairy products. The Dutch dairy industry uses truck–trailer combinations (among other kinds of vehicles) for the distribution of final products. The products have to be distributed in various regions. Many customers are located in crowded cities. This implies that serving them with a truck–trailer combination may demand much more time than serving them with the truck. Therefore the trailer is often parked while the truck load is being distributed.

At the moment there are developments in the Dutch dairy industry that may lead to an increase of scale. Consequently the average distance between a distribution centre and its market increases. Therefore it will become more and more favourable to use large units, like the truck–trailer combinations, for the distribution activities.

Another application area for the VRPT can be found in the distribution of compound animal feed. Compound animal feed has to be distributed among farmers. Many farmers can be reached by narrow roads and small bridges only. Therefore they can only be served by relatively small vehicles. For the distribution of compound animal feed various types of vehicles are used, including so-called 'double bottoms', vehicles that consist of a truck and a trailer. The trailer is left behind at a parking-place during part of the route.

2. Model

Modelling the actual situation is very difficult. As a starting point for research a simplified version of reality has been chosen. The model uses the following assumptions.

- The customers and the depot are vertices in a euclidian network.
- The time a truck or a vehicle needs to cover a certain distance is proportional to that distance.
- There are no time windows.
- Each customer site can be used as a parking-place.

- Every customer is served by exactly one vehicle (or one truck).
- Each trailer is parked exactly once.
- All customers have unit demand.
- All vehicles possess identical capacities.
- The amount of inconvenience caused by visiting a certain customer with a vehicle is denoted by the so-called *manoeuvring time*. The manoeuvring time is the amount of additional time that is needed when a customer is visited by a vehicle compared to the time that is needed when it is visited by a truck. 'Difficult' customers have high manoeuvring times.

There are three important goals: (i) minimize travelling time (distance divided by speed), (ii) minimize manoeuvring time, (iii) minimize fuel cost. The goals (i) and (iii) are compatible. Each of them may conflict with the second goal.

The objective function that is used is the sum of (a) the time that the vehicles need to cover the routes to their customers (distance divided by speed), (b) the time that the trucks need to cover the routes to their customers (distance divided by speed) and (c) the total manoeuvring time. An exact formulation of the VRPT as a mixed integer linear programming problem can be found in Gerdessen (1993).

The VRP is a special case of the VRPT. The VRP belongs to the class of NP-hard problems (Golden and Assad, 1988), so the VRPT is an NP-hard problem too. This means that it is highly unlikely that an algorithm will exist that finds an optimal solution to the VRPT in polynomial time.

Heuristics have been developed to find 'good' solutions to the VRPT in reasonable time. These heuristics can be divided into two groups: construction heuristics and improvement heuristics. The construction heuristics try to find a 'good' feasible solution, the improvement heuristics try to improve this feasible solution. The construction heuristics will be described in Section 3 and the improvement heuristics in Section 4.

3. Construction heuristics

3.1. Heuristic I

Heuristic I constructs a solution to the VRPT in a very straightforward way. Initially the possibility of leaving trailers behind at parking-places is ignored. The resulting standard VRP is solved heuristically. Next, each separate route of the resulting VRP-solution is divided into a truck route and a vehicle route, and a parking-place is chosen. The latter problem is called a Travelling Salesman Problem with a Trailer (a TSPT).

Heuristic I consists of the following steps:

I.1. Ignore the possibility of leaving trailers behind and solve the resulting standard VRP.

I.2. Solve a TSPT for the vertex set of each separate route.

As to step I.1:

The standard VRP is solved heuristically using a seed heuristic. This heuristic consists of the following steps:

S1. Calculate the minimum number of vehicles, call it p:

$$p = \left\lceil \frac{\text{total demand}}{\text{vehicle capacity}} \right\rceil .$$

S2. Select p vertices that are spread evenly over the area in the following way. First select the vertex that is farthest away from the depot. Then repeat the following steps $(p-1)$ times: (i) for each unselected vertex calculate the distance to the nearest selected vertex, (ii) select the vertex with the maximal distance. Each selected vertex is assigned to one vehicle. This vertex is called the seed of the route of that vehicle. After this step every vehicle route contains two vertices: its seed and the depot.

S3. For each of the unassigned vertices i and the seeds p calculate the saving $s(i, p)$ that can be achieved when i is served by the vehicle that is serving p:

$$s(i, p) = d_{p0} + d_{i0} - d_{ip}$$

where d_{ij} denotes the distance from vertex i to vertex j and 0 denotes the depot.

S4. Find the i and p that result in the maximum saving: find $\max_{i,p}\{s(i, p)\} = s(i_{\max}, p_{\max})$. Insert $i_{\max}$ into the route along $p_{\max}$.
For each unassigned vertex i that can (with respect to capacity) be inserted into the route along $p_{\max}$ recalculate $s(i, p_{\max})$ using a cheapest insertion rule. Repeat step S4 until no more assignments can be made.

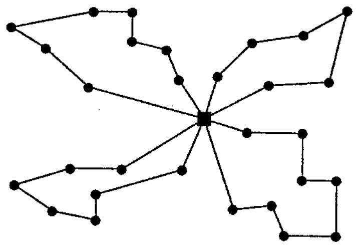

Fig. 3. Solution to the standard VRP.

S5. Improve the solution with the improvement heuristics that are described in Sections 4.1, 4.3, 4.4 and 4.5.
The solution now has a shape as shown in Fig. 3.

As to step I.2:

The input for the TSPT heuristic is a set of vertices that are visited by one vehicle after step I.1. The TSPT heuristic consists of the following steps:

TSPT1. The depot is assigned to the vehicle route. So the vehicle route now contains one vertex.

TSPT2. For each of the vertices i calculate the time v_i it would take to serve i with the vehicle, assuming that no other vertices are served on the same trip:

$$v_i = \frac{2 \times \text{distance from the depot to } i}{\text{speed of the vehicle}} + \text{manoeuvring time at } i.$$

The vertex with the largest v_i is assigned to the truck route, so the truck route now contains one vertex.

TSPT3. For each of the unassigned vertices i the increase Δv_i in the duration of the current vehicle route caused by insertion of that vertex i via a cheapest insertion rule (Lawler, 1985) is calculated:

$$\Delta v_i = \frac{\text{length of cheapest insertion of } i \text{ in } v}{\text{speed of the vehicle}} + \text{manoeuvring time at } i.$$

Similarly, the increase Δt_i in the duration of the current truck route caused by insertion of that vertex i into the truck route via a cheapest insertion rule is calculated:

$$\Delta t_i = \frac{\text{length of cheapest insertion of } i \text{ in } t}{\text{speed of the truck}}.$$

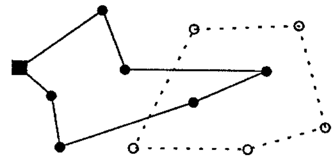

Fig. 4. Routes for both sets.

The vertex with maximum value of $|\Delta v_i - \Delta t_i|$ is inserted into the appropriate route.

Step 3 is repeated until the truck route or the vehicle route is filled to capacity. The remaining unassigned vertices are inserted into the other route. The solution now has the shape as shown in Fig. 4.

TSPT4. For each vertex that is served by the vehicle it is calculated how much the cheapest insertion into the truck route would cost (in time units). The vertex with the lowest cost is chosen as the parking-place for the trailer and inserted into the truck route, see Fig. 5.

Several variants of Steps 2 and 3 have been investigated. Numerical tests have shown that they did not lead to better results than the version given above, so they shall not be described here.

Improvement heuristics are applied to the solution of Heuristic I (See section 4).

An advantage of Heuristic I is its simplicity. It is a straightforward extension of the standard VRP. A disadvantage is the possibility that in Step I.1 some routes are made that contain a lot of vertices that cause large manoeuvring times (so-called 'difficult' customers). In such a case it will not be possible to visit all the 'difficult' customers by the truck, so some of them will have to be visited by the vehicle. This can imply large total manoeuvring times. Therefore we may need a heuristic that probably is less likely to put too many 'difficult' vertices into one route. Heuristic II has been designed for this purpose.

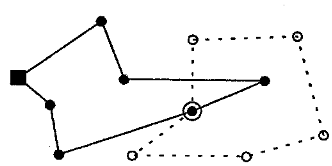

Fig. 5. Solution to the TSPT.

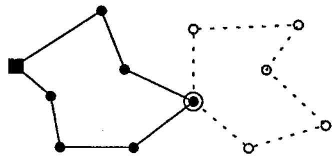

Fig. 6. Shape of the solution that minimizes total distance.

3.2. Heuristic II

Heuristic II tries to partition the set of customers into a truck set and a vehicle set before the actual routing phase starts. The heuristic assigns the vertices to routes one by one.

Heuristic II consists of three steps:

II.1. Construct routes for the trucks.

II.2. Choose a parking-place for each trailer and insert it into the corresponding truck route.

II.3. Use the parking-places as seeds for the construction of vehicle routes.

As to II.1:

The objective function consists of two parts: (i) total driving time (distance divided by speed), (ii) total manoeuvring time.

The amount of driving time in the final solution can be minimized by constructing a solution in which the total distance is minimized. In such a solution each route will have roughly the shape that is shown in Fig. 6: the vertices that are close to the depot are served by the vehicle, and the vertices that are far from the depot are served by the truck. So it is favourable to serve vertices that are far from the depot with trucks.

The amount of manoeuvring time in the final solution is minimal when the solution contains as few 'difficult' vertices in the vehicle routes as possible. So for minimizing the total manoeuvring time it is favourable to visit those vertices with trucks that cause large manoeuvring times.

It seems to be advantageous to visit with a truck those vertices that (i) are far from the depot, (ii) would cause large manoeuvring times for vehicles. Heuristic II aims at assigning vertices with these properties to trucks using a weighted combination a_i of the manoeuvring time of the vertex and its distance to the depot (weighing coefficient α):

$$a_i = \alpha \times \text{manoeuvring time at } i + \text{distance from depot to } i.$$

At first the vertices with the highest values of a_i are assigned to the trucks until 70% of the truck capacity is used. Then the rest of the truck capacity is filled with vertices that have a relatively high value of a_i and that are relatively close to the vertices that are already assigned to the truck routes.

After step 1 the truck routes are constructed (see Fig. 7).

As to II.2:

Every route needs a parking-place for the trailer: the first and the last vertex of a truck route is the parking-place. For a given truck route a 'good' parking-place is a parking-place that is close to that route. So for each combination of a truck route t and an unassigned vertex i we calculate the cost (in time units) to divert route t along vertex i. Subsequently the cheapest way of adding exactly one vertex to each route is searched. These vertices will be the parking-places (see Fig. 8).

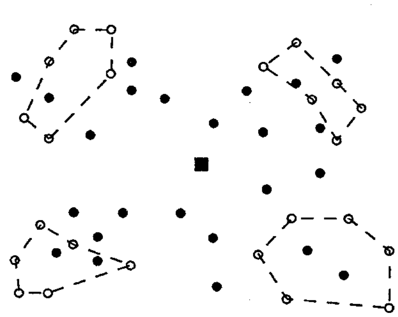

Fig. 7. A set of truck routes.

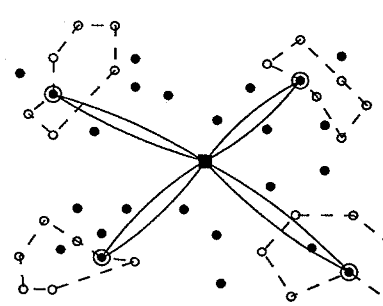

Fig. 8. All vehicle routes contain two vertices: the parking-place and the depot.

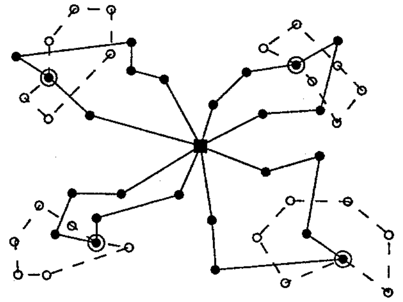

Fig. 9. The parking-places are used as seeds for the vehicle routes.

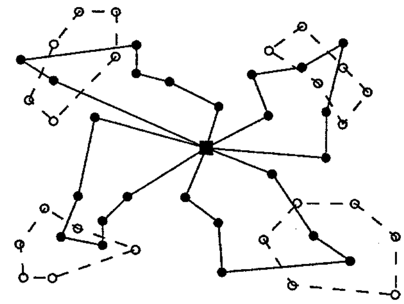

Fig. 10. Situation after Step III.2.

As to II.3:

All vehicle routes now contain exactly two vertices: the depot and the parking-place that belongs to that route. The parking-places are used as seeds for the construction of vehicle routes with the seed heuristic that is described in Step I.1 of Heuristic I (see Section 3.1). After this step all vehicle and truck routes are known (see Fig. 9).

In the description of Heuristic II only the construction elements are mentioned. After Steps II.1 and II.3 improvement heuristics are carried out. These will be described in Section 4. Yet it is necessary to mention the improvement heuristics here because applying them reveals a drawback of Heuristic II: the parking-places are chosen in a very early stage. When applying an improvement heuristic to the set of vehicle routes (after Step II.3) vertices can be moved from one vehicle route to another. Special attention is needed when the parking-place is involved: the implemented improvement heuristics can not move parking-places from one route to another. This restricts the number of possible improvements. The choice of the parking-places is permanent, and one can not remove an ill-chosen parking-place. Heuristic III is designed to avoid this disadvantage.

3.3. Heuristic III

Heuristic III has a start that is similar to Heuristic II. The difference is that in Heuristic III separate truck routes and trailer routes are constructed and improved before the parking-places are chosen.

Heuristic III consists of three steps:

III.1 Construct routes for the trucks.

III.2. Construct routes for the traillers.

III.3. Construct a feasible solution by connecting each truck route to a vehicle route using parking-places.

As to III.1:

Step III.1 is identical to Step II.1 of Heuristic II.

As to III.2:

Every vehicle route needs a seed. These seeds are chosen in the same way in which the parking-places are chosen in Step II.2. These seeds are used for constructing the vehicle routes in the same way as in Step I.1 of Heuristic I. After this step the solution has the shape as shown in Fig. 10.

After Step III.2 improvement heuristics are used that will be described in Section 4. Parking-places do not exist, so there are many possibilities for exchanging vertices between routes (see Fig. 11).

As to III.3

In this step all separate truck routes and vehicle routes have to be combined into a feasible solution

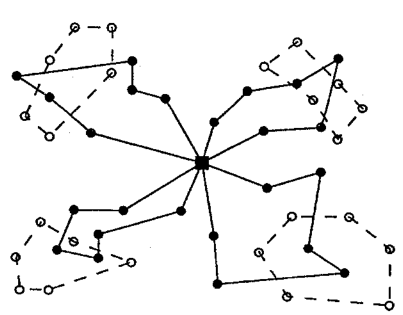

Fig. 11. Improved situation after Step III.2.

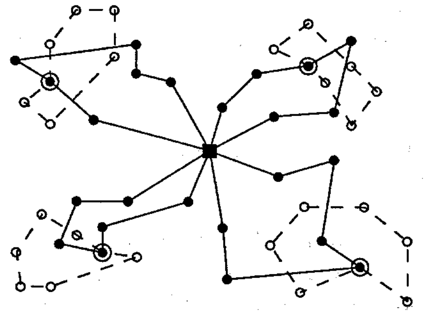

Fig. 12. Truck routes and trailer routes are combined into a connected solution.

to the VRPT. This is done in the following way. For each possible combination of a vehicle route v and a truck route t find the vertex p_{vt} in v that can be inserted into t at minimal cost (that is: minimal among the vertices of v). Vertex p_{vt} will be the parking-place in case vehicle route v and truck route t are combined. Call the cost of inserting p_{vt} into t the cost c_{vt} of combining vehicle route v and truck route t into one connected route. Find the least-cost assignment of truck routes to trailer routes in which each vehicle route and each truck route are used exactly once. The result of this is a connected solution to the VRPT (see Fig. 12).

3.4. An alternative way of applying Heuristic I: Heuristic Ib

The version of Heuristic I that is described in Section 3.1 has the same disadvantage as Heuristic II: the parking-places are chosen in a very early stage, which limits the possibilities for improvements afterwards. However, Heuristic I can also be applied in a way that is similar to Heuristic III: step TSPT4 can be omitted, so that the improvement heuristics can be applied to routes without parking-places, which increases the number of possible improvements. The resulting truck routes and trailer routes can be combined into a feasible solution in the way that is described in Step III.3 (see Section 3.3).

In the sequel the original version of Heuristic I will be referred to as Heuristic Ia. The version in which the parking-places are chosen after the application of the improvement heuristics will be referred to as Heuristic Ib.

4. Improvement heuristics

In the description of the improvement heuristics the concept *chain of vertices* is used. A chain of vertices is a set of vertices that are successively visited in a certain route. A chain can not contain more vertices than the route to which it belongs.

In the following the word 'similar' refers to the conveyance that covers a certain route: two truck routes are similar, and two vehicle routes are similar. A truck route and a vehicle route are dissimilar.

The used improvement heuristics can be divided into two categories:

(i) improvement heuristics that try to improve the assignment of vertices to different routes,
(ii) improvement heuristics that try to improve the sequence in which the vertices in a certain route are visited.

Category (i) contains the following heuristics:

(a) exchange of chains of vertices between similar routes,
(b) exchange of chains of vertices between dissimilar routes,
(c) a 2-opt exchange between vehicle routes,
(d) a 3-opt exchange between vehicle routes.

Category (ii) contains the following heuristics:

(e) improvement of a single route (a truck route or a vehicle route),
(f) exchange of vertices between the truck part and the vehicle part within a route,
(g) application of a TSPT heuristic to the vertices of a route.

All improvement heuristics verify that capacity constraints are not violated.

4.1. Exchange of chains of vertices between similar routes

This heuristic tries to improve the solution by exchanging a chain of 1, 2 or 3 vertices of a route for a chain of 1, 2 or 3 vertices of a similar route. In the situation in Fig. 13 the exchange of the chain B–C for the chain F–G is profitable if

$$d_{AB} + d_{CD} + d_{EF} + d_{GH} > d_{AF} + d_{GD} + d_{EB} + d_{CH}.$$

All profitable exchanges that do not include a parking-place are carried out.

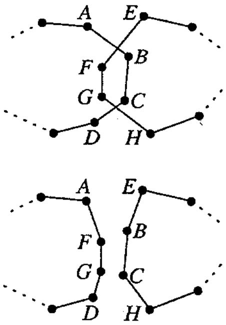

Fig. 13. Exchange between similar routes.

4.2. Exchange of chains of vertices between dissimilar routes

This heuristic tries to improve the solution by exchanging a chain of 1, 2 or 3 vertices of a truck route for a chain of 1, 2 or 3 vertices of a vehicle route.

The exchange in Fig. 14 is profitable if

$$\frac{d_{AB}+d_{BC}+d_{CD}}{s_v}+\frac{d_{EF}+d_{FG}+d_{GH}}{s_t}+m_B+m_C > \frac{d_{AF}+d_{FG}+d_{GD}}{s_v}+\frac{d_{EB}+d_{BC}+d_{CH}}{s_t}+m_F+m_G,$$

where s_v is the speed of a vehicle, s_t is the speed of a truck, and m_i is the manoeuvring time at i.

All profitable exchanges that do not include a parking-place are carried out.

4.3. A 2-opt exchange between vehicle routes

In a 2-opt exchange between routes the goal is to improve the current solution by breaking two edges belonging to two different routes, and adding two

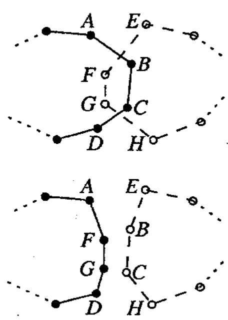

Fig. 14. Exchange between dissimilar routes.

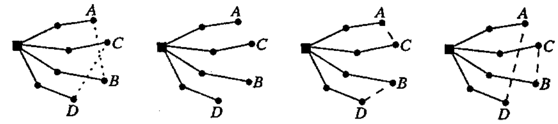

Fig. 15. There are two ways of reconnecting the four chains into a feasible new solution.

new edges to restore feasibility. There are two ways of adding new edges to a solution in which two edges have been broken (see Fig. 15). The exchange of the old edges (A, B) and (C, D) for two new edges is profitable if

$$d_{AB}+d_{CD} > d_{AC}+d_{BD}$$

or

$$d_{AB}+d_{CD} > d_{AD}+d_{BC}.$$

All profitable exchanges that result in two routes with one parking-place each are carried out. Note that the 2-opt heuristic can only be applied to sets of routes that have a common vertex (in this case: the depot). Therefore it can only be applied to vehicle routes and not to truck routes.

4.4. A 3-opt exchange between vehicle routes

In a 3-opt heuristic three edges, belonging to three different routes, are broken. There are eight ways of

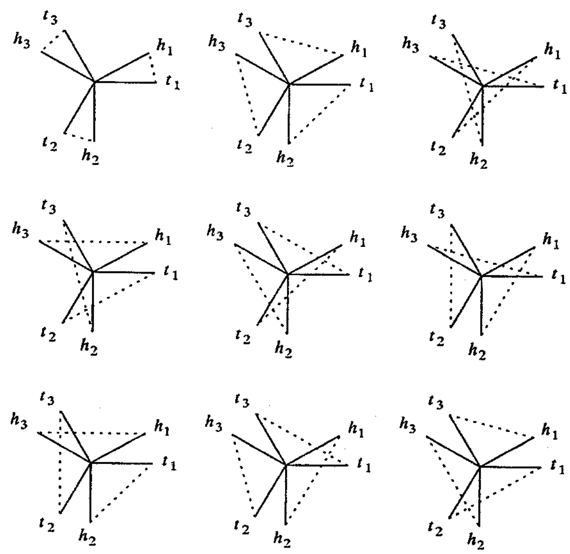

Fig. 16. The 8 loose ends can be reconnected in 8 different new ways. The figure in the left-upperhand corner represents the original situation.

adding three new edges to join the heads and tails of the three old routes into three new routes (see Fig. 16). The lowest cost set of new edges is searched. The exchange is carried out if this set has lower cost than the set of broken edges.

Testing this heuristic revealed that applying the k-opt heuristic (Lin and Kernighan, 1973) to the newly formed routes often led to a significant further improvement. Therefore cases in which the best set of new edges leads to a slight increase of the total distance are subjected to an additional test: the resulting new routes are improved by the k-opt heuristic. Subsequently the total length of the three new, improved routes is compared with the total length of the original routes. The exchange is carried out if the total length is decreased.

Note that the 3-opt heuristic can only be applied to sets of routes that have one common vertex (in this situation: the depot). Therefore it can only be applied to vehicle routes.

4.5. Improvement of a single route

The separate truck routes and vehicle routes are TSPs. Therefore two well known improvement heuristics, 2-opt (Lawler et al., 1985) and k-opt (Lin and Kernighan, 1973), are applied to improve these separate routes.

4.6. Exchange of vertices between the truck part and the vehicle part within a certain route

This heuristic tries to exchange chains of 1, 2 or 3 vertices between the truck part and the vehicle part of a certain route. At first it is tried to add truck vertices to the vehicle route. This never causes violation of the capacity constraints. If 'space' is created in the truck route, then we try to add vehicle vertices to the truck route (see Fig. 17).

Moving a vertex from the truck route to the vehicle route (or vice versa) causes three changes in the objective function:

(i) a change in the duration of the truck route (distance divided by speed of the truck),
(ii) a change in the duration of the vehicle route (distance divided by speed of the vehicle),
(iii) a change in the total manoeuvring time of the vehicle route.

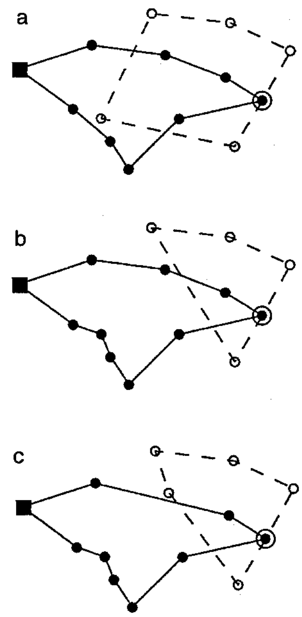

Fig. 17. (a) Original truck route and vehicle route. (b) One truck vertex is moved to the vehicle route. (c) One vehicle vertex is moved to the truck route.

The overall effect has to be calculated. Profitable moves are carried out.

4.7. Application of a TSPT heuristic on the vertices of a route

This heuristic is applied to the vertices of a truck route and a vehicle route that are connected by a parking-place. The heuristic ignores the current arrangement of the vertices, and considers the set of vertices as if they are not assigned to a route yet. The heuristic searches a good solution to the TSPT that is formed by the vertices. When the solution that is found has lower cost than the current solution the current solution is replaced by the new one. Otherwise nothing is changed.

5. Computational results

5.1. General remark

During the numerical experiments it soon became clear that the objective function for the VRPT contains a great number of local optima. Slight changes in the execution of the heuristics could result in solutions with objective function values that differed substantially, and these changes could lead to both positive and negative results: positive for one data set and negative for another. This makes it very hard to judge the quality of the heuristics. Yet the results of experiments allow some tentative conclusions about the performance of the heuristics.

5.2. Design of the test problems

For the experiments three classes of test problems have been generated. In all test problems the x- and y-coordinates of the vertices were integers, each randomly chosen from the interval (1, 400) with uniform distribution. The depot was always situated in (200, 200). The three classes differed in the way in which manoeuvring times were assigned:

- In the test problems of class U each vertex has been assigned an integer manoeuvring time from the interval (1, 100) with uniform distribution.
- In the test problems of class W the problem area was divided into 25 squares, see Fig. 18. The vertices in the dark squares were assigned an integer manoeuvring time from the interval (70, 100) with uniform distribution. The manoeuvring times of the vertices in the light squares were drawn from (0, 30) with uniform distribution.
- In the test problems of class S the problem area was also divided into 25 squares. Now the vertices in the dark squares were assigned a manoeuvring time of 100, and the vertices in the light squares were assigned a manoeuvring time of 0.

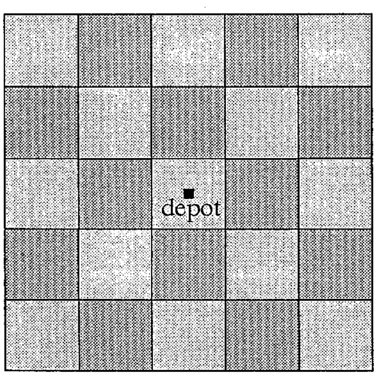

Fig. 18. The problem area was divided into 25 squares.

Table 1
Average objective function values for problem class U (in time units)

Class U	Ia	Ib	II	III
50/5/5	4977	4881	5126	5042
100/5/5	8675	8662	8797	8627
100/10/10	7075	6922	7014	6988
200/10/10	12121	11989	11920	11780
200/5/15	14689	14630	14506	14476

In short: the class U was designed to investigate the performances of the heuristics in situations with *u*niformly distributed manoeuvring times; the classes W and S were designed to investigate the performances in situations with *w*eakly and *s*trongly clustered manoeuvring times respectively.

For each of the 3 classes 5 situations were considered:

- problem size = 50 vertices, capacity of a truck = 5, capacity of a trailer = 5,
- problem size = 100 vertices, capacity of a truck = 5, capacity of a trailer = 5,
- problem size = 100 vertices, capacity of a truck = 10, capacity of a trailer = 10,
- problem size = 200 vertices, capacity of a truck = 10, capacity of a trailer = 10,
- problem size = 200 vertices, capacity of a truck = 5, capacity of a trailer = 15.

In the sequel these situations will be referred to as the problem instances 50/5/5, 100/5/5, etc. Each of these 15 test problems has been generated 10 times with different random seeds. This resulted in a total of $3 \times 5 \times 10 = 150$ instances.

5.3. Numerical results

Tables 1 to 3 show the results of the heuristics in aggregated form. The entry "4977" in column Ia and row 50/5/5 of Table 1 is the average over the 10 different 50/5/5 instances in problem class U of the objective function value produced by Heuristic Ia (in time units). The other entries have been calculated analogously. For the listed results of Heuristic II and III a value of $\alpha = 6$ has been used (see Sections 3.2 and 3.3). All results relate to the objective function values that were produced by the construction heuristics combined with the improvement heuristics.

For all problem classes Heuristic Ib generally performed better than Heuristic Ia and Heuristic III generally performed better than Heuristic II. This is due to the greater flexibility that is caused by choosing the parking-places after the application of improvement heuristics in Heuristics Ib and III.

In class U Heuristics Ia and Ib generally produced the best solutions to the instances with 50 vertices. Heuristics Ib and III performed almost equally for the instances with 100 vertices. Heuristic III performed slightly better than Heuristic Ib for the 200/10/10 and 200/5/15 instances.

A possible explanation is that the relative amount of manoeuvring time in the final solution tends to increase with increasing problem size. Heuristics Ia and Ib produce a starting solution without taking into account the manoeuvring times of the vertices; they consider the distance aspect only. This appears to be advantageous in the small problems, because the driving times are relatively important in the final solution of these small problems. Heuristic III considers the manoeuvring times of the vertices from the first step. As manoeuvring times are relatively important in the final solution to the 200/10/10 instances Heuristic III can perform better than Heuristics Ia and Ib for these problem instances. In the 200/5/15 instances the trucks can serve relatively few customers, so that the amount of manoeuvring time in the final solution is smaller than in the 200/10/10 instances. This reduces the advantage of Heuristic III.

In problem class W all heuristics performed almost equally in the instances with 50 vertices. Heuristics Ib and III also performed almost equally in the instances with 100 vertices. Heuristic III performed better than Heuristic Ib in the 200/10/10 instances and slightly better in the 200/5/15 instances. The explanation for these results is probably the same as for the problems in class U.

In problem class S Heuristic III generally performed better than Ib. This is due to the fact that placing a customer from the dark area (with a manoeuvring time of 100) in a vehicle route has a large impact on the value of the objective function. The construction steps of Heuristic II and III avoid the assignment of these customers to vehicle routes, whereas Heuristics Ia and Ib do not have this possibility. Heuristics Ia and Ib can only get rid of unfavourable assignments via the improvement heuristics.

Table 2
Average objective function values for problem class W (in time units)

Class W	Ia	Ib	II	III
50/5/5	4833	4841	4877	4852
100/5/5	8364	8197	8323	8182
100/10/10	6658	6543	6553	6494
200/10/10	11230	10945	10883	10568
200/5/15	13748	13468	13403	13265

5.4. Performance of the improvement heuristics

It is very hard to measure the performance of the individual improvement heuristics. They are all integrated in the construction heuristics and their individual contributions to the quality of the final solutions is hard to quantify. Therefore only a few notes on their performance shall be given.

Most improvement heuristics manage to improve the solutions on which they are applied. Only two of them seem to be less effective: 2-opt exchange between vehicle routes (see Section 4.3) and exchange of vertices between the truck part and the vehicle part of a route (see Section 4.6).

The improvement heuristic that exchanges chains of vertices between dissimilar routes tends to balance the manoeuvring times and the driving times: solutions with a relatively high amount of manoeuvring time usually contain less manoeuvring time after the execution of the heuristic, and vice versa.

It turns out that the improvement heuristics do improve the solutions considerably indeed. They of-

Table 3
Average objective function values for problem class S (in time units)

Class S	Ia	Ib	II	III
50/5/5	4710	4713	4710	4641
100/5/5	7875	7754	7670	7496
100/10/10	6278	6228	5937	5935
200/10/10	9964	9711	9641	9315
200/5/15	12897	12675	12638	12465

Table 4
Running times (in seconds)

	Ia	Ib	II	III
50/5/5	2	2	<1	1
100/5/5	7	8	3	4
100/10/10	8	9	4	4
200/10/10	35	40	13	18
200/15/5	35	39	13	22

ten manage to turn bad starting solutions into reasonably good final solutions.

5.5. Running times

The running times were measured on a 486DX2-33 MHz personal computer. It was found that the running times did only depend on the problem size and not on the problem class. The running times are displayed in Table 4. Each entry represents the amount of running time that the full heuristic needed (all improvement procedures included), averaged over all test problems.

The table shows that the running times of Heuristics Ia and Ib are considerably longer than those of Heuristics II and III. This is due to the quality of the starting solutions. The starting solutions that are produced by Heuristics Ia and Ib need more improvement than those produced by Heuristics II and III.

6. Discussion

The goal of this research was to investigate the vehicle routing problem with trailers. As this is an unknown field of research it was desirable to conduct a model study and investigate the problem with as little additional complications as possible. Therefore several assumptions have been made. The two most important assumptions are: (a) all customers have unit demand, (b) each trailer is parked exactly once.

Assumption (a) simplifies the problem, because one can calculate the minimum number of vehicles beforehand. It is not hard to find a feasible solution with a number of vehicles that is more than or equal to the minimum number. This is not the case in a situation with unequal demand. It can be hard to calculate the minimum number of vehicles and it can be hard to find a feasible solution in which this minimum number (or a few more) vehicles are used. So the variant with unequal demand is harder than the variant with equal demand. However, the problem of determining the optimal number of vehicles is hard in both variants.

Assumption (b) states that each trailer is parked exactly once. In practice it could be useful if the number of times that a trailer is parked would be determined by the model. In that case the cost of the act of unhitching and hitching the trailer should be inserted into the objective function, e.g. in time units. Since in this research the number of parking-places was fixed to one, it was not necessary to calculate these cost. Developing a model with a free number of parking-places could be one of the next steps in the research of the VRPT.

In relation to the parking-places one additional remark can be made: it is assumed that each customer site can be used as a parking-place. We do realise that for the sake of security not every site may be equally eligible. Restriction of possible parking-places to a subset of the customers sites should not cause many difficulties. The distributors of compound animal feed for that matter have so far not encountered any problems with respect to security.

7. Concluding remark

In this research a tool has been developed to cope with a relatively new extension of the VRP: the VRP with trailers. The tool is not finished yet, many practical aspects have to be incorporated. In fact, real operational systems are always customer specific. Yet the problem has proven to be an interesting research topic with many possibilities for future research.

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