HW 5 E94081018 趙鴻輝

(a)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & -\frac{1}{2} \\ 0 & 2 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$\text{rank}(A) = 3$

(b) $$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda, & 0, & 1 \\ -1, & 4-\lambda, & -1 \\ -1, & 2, & 0-\lambda \end{vmatrix} = -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = -(\lambda-1)\cdot(\lambda-2)\cdot(\lambda-3) = 0$$

$\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$

(c) $\lambda_1 = 1$:

$$A - \lambda_1 I = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 3 & -1 \\ -1 & 2 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 + x_3 = 0 \\ x_2 = 0 \end{cases} \quad x = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix} \quad V_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda_2 = 2$

$$A - \lambda_2 I = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 2 & -1 \\ -1 & 2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 1 \\ -1 & 2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 - 2x_2 = 0 \\ x_3 = 0 \end{cases} \quad x = \begin{bmatrix} 2x_2 \\ x_2 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_3 = 3$

$$A - \lambda_3 I = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 2 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 - x_3 = 0 \\ x_2 - 2x_3 = 0 \end{cases}$$

$$x = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$