Problem 1: A small market orders copies of a certain magazine for its magazine rack each week. Let X denote the demand for the magazine, with probability mass function (pmf)

$$P(X=1)=\tfrac{1}{15},\quad P(X=2)=\tfrac{2}{15},\quad P(X=3)=\tfrac{3}{15},\quad P(X=4)=\tfrac{4}{15},\quad P(X=5)=\tfrac{3}{15},\quad P(X=6)=\tfrac{2}{15}$$

Suppose the store owner actually pays \$2.00 for each copy of the magazine and the price to customers is \$4.00. If magazines left at the end of the week have no salvage value, is it better to order three or four copies of the magazine? [Hint: For both three and four copies ordered, express net revenue as a function of demand X, and then compute the expected revenue.]

Expected net revenue: $E[R(n)]$, n = # 3 or 4

3 copies → $n=3$

$X=1 \to 4-6=-2$
$X=2 \to 2(4)-6=2$
$X=3 \to 3(4)-6=6$
$X>3 \to 3(4)-6=6$

4 copies $n=4$

$X=1 \to 4-8=-4$
$X=2 \to 2(4)-8=0$
$X=3 \to 3(4)-8=4$
$X=4 \to 4(4)-8=8$
$X>4 \to 4(4)-8=8$

$E[R(3)]=\frac{1}{15}\times(-2)+\frac{2}{15}\times 2+\frac{3}{15}\times 6+\frac{4}{15}\times 6+\frac{3}{15}\times 6+\frac{2}{15}\times 6$
$=4.933$

$E[R(4)]=\frac{1}{15}\times(-4)+\frac{2}{15}\times 0+\frac{3}{15}\times 4+\frac{4}{15}\times 8+\frac{3}{15}\times 8+\frac{2}{15}\times 8$
$=5.333$

It is better for the store to order 4 copies. $5.33 > 4.933$

Problem 2: Let X be the damage incurred (in \$) in a certain type of accident during a given year. Possible X values are 0, 1000, 5000, and 10000, with probabilities .8, .1, .08, and .02, respectively. A particular company offers a \$500 deductible policy. If the company wishes its expected profit to be \$100, what premium amount should it charge?

$P(X=0)=0.8$, $P(X=1000)=0.1$,

$P(X=5000)=0.08$, $P(X=10000)=0.02$

if $X=0$, payout $=0$

if $X=1000$, payout $=500$

if $X=5000$, payout $=4500$

if $X=10{,}000$, payout $=95000$

$$E[\text{Payout}]=P(X=0)\cdot 0+P(X=1000)\cdot 500+P(X=5000)\cdot 4500+P(X=10000)\cdot 9500$$

$$=0.8\times 0+0.1\times 500+0.08\times 4500+0.02\times 9500$$

$$=0+50+360+190=600$$

Profit = Premium − Expected Payout

100 = Premium − 600

Premium = 700

The premium should be \$700

Problem 3: A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite. The geologist instructs a laboratory assistant to randomly select 15 of the speci- mens for analysis.

(a) What is the probability that all specimens of one of the two types of rock are selected for analysis?
(b) What is the probability that the number of granite specimens selected for analysis is within 1 standard deviation of its mean value?

A Either 10 basalic and 5 granite or 10 granite and 5 basaltic.

$$P(10B \text{ and } 5G) = \frac{\binom{10}{10}\times\binom{10}{5}}{\binom{20}{15}} = \frac{252}{15504} = 0.01625$$

Same

$$P(10G \text{ and } 5B) = 0.01625$$

$$P(\text{all one type}) = 0.01625 + 0.01625 = \boxed{0.0325}$$

B HyperGeometric$(n=15, M=10, N=20)$

$$\mu = n\cdot\frac{M}{N} \quad \sigma = \sqrt{n\cdot\frac{M}{N}\left(1-\frac{M}{N}\right)\left(\frac{N-n}{N-1}\right)} \quad \mu = 15\times\frac{10}{20} = 15\times\frac{1}{2} = 7.5$$

$$\sigma = \sqrt{15\times\frac{10}{20}\times\left(1-\frac{10}{20}\right)\left(\frac{20-15}{20-1}\right)} = \sqrt{15\times\frac{1}{2}\times\frac{1}{2}\times\frac{5}{19}} = 0.9934$$

$$\text{Range} = 7.5 \pm 0.9934 \rightarrow (6.5067, 8.4933)$$

$$P(\text{Granite falls within 1 std of } \mu) = P(X=7) + P(X=8)$$

$$P(X=7) = \frac{\binom{10}{7}\cdot\binom{10}{8}}{\binom{20}{15}} = 0.3483 \quad P(X=8) = \frac{\binom{10}{8}\binom{10}{7}}{\binom{20}{15}} = 0.3483$$

$$P(7 \le X \le 8) = 0.3483 + 0.3483 = \boxed{0.6966}$$

Problem 4: Organisms are present in ballast water discharged from a ship according to a Poisson process with a concentra- tion of 10 organisms per cubic meter.

(a) What is the probability that one cubic meter of discharge contains at least 8 organisms?
(b) What is the probability that the number of organisms in 1.5 cubic meter of discharge exceeds its mean value by more than one standard deviation?
(c) For what amount of discharge would the probability of containing at least 1 organism be .999 ?

A $P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $\lambda = 10$

$P(\text{atleast } 8) = 1 - P(X \le 7) \rightarrow P(X \le 7) = P(X=0) + \ldots + P(X=7)$

$P(X \le 7) = \sum_{x=0}^{7} \frac{10^x e^{-10}}{x!} = 0.22022$

$P(X \ge 8) = 1 - P(X \le 7) = 1 - 0.22022 = \boxed{0.77978}$

B $\lambda_{1.5} = 1.5 \times 10 = 15$ $\sigma = \sqrt{\lambda_{1.5}} = \sqrt{15}$

$P(X > \lambda_{1.5} + \sigma) = P(X > 15 + \sqrt{15}) = P(X > 15 + 3.87)$

$= P(X > 18.87) \rightarrow$ needs to be integer $\rightarrow P(X \ge 19)$

$P(X \ge 19) = 1 - P(X \le 18)$ where $\lambda = 15$

$= 1 - 0.81948 = \boxed{0.18052}$

C $P(X \ge 1) = 1 - P(X=0) = 0.999$

$P(X=0) = \frac{10^0 e^{-10}}{0!} = e^{-10v}$

$e^{-10v} = 1 - 0.999$

$e^{-10v} = 0.001$

$-10v = \ln(0.001)$

$-10v = -6.907$ $\rightarrow V = \frac{6.907}{10} = \boxed{0.6907}$

Problem 5: Automobiles arrive at a vehicle equipment inspection station according to a Poisson process with rate of 10 per hour. Suppose that with probability 0.5 an arriving vehicle will have no equipment violations. What is the probability that ten "no-violation" cars arrive during the next hour?

$X \sim \text{Poisson}(\lambda = 10)$

$\lambda_{\text{no violation}} = \lambda \times 0.5 = 10 \times 0.5 = 5$

# of no violation cars arriving per hour $\lambda = 5$

$$P(X=10) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{5^{10} e^{-5}}{10!}$$

$$= \frac{9765625 \times e^{-5}}{3628800}$$

$$= \boxed{0.01813}$$

87. Under what conditions is the sum of independent negative binomial random variables also negative binomial?

The sum of independent negative binomial random variables $X_1, X_2, \dots, X_n$ is also negative binomial if they have the same probability of success p.

So it will be $NB(r_1 + r_2 + \dots + r_n, P)$ where the total number of successes is the sum of the individual success parameters $r_1, r_2, \dots, r_n$