46

48

52

39

70

72

46. Urn A has three red balls and two white balls, and urn B has two red balls and five white balls. A fair coin is tossed. If it lands heads up, a ball is drawn from urn A; otherwise, a ball is drawn from urn B.

a. What is the probability that a red ball is drawn?

b. If a red ball is drawn, what is the probability that the coin landed heads up?

a $P(\text{Head}) = \frac{1}{2}$, $P(\text{Tail}) = \frac{1}{2}$

$P(\text{Red } A) = \frac{3}{5}$ $P(\text{Red } B) = \frac{2}{7}$

$$P(\text{Red}) = P(\text{Red} \mid \text{Head}) \times P(\text{Head}) + P(\text{Red} \mid \text{Tail}) \times P(\text{Tail})$$

$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{7} = \frac{3}{10} + \frac{2}{14} = \frac{3}{10} + \frac{1}{7} = \frac{21}{70} + \frac{10}{70} = \boxed{\frac{31}{70}}$$

b $$P(\text{Head} \mid \text{Red}) = \frac{P(\text{Red} \mid \text{Head}) P(\text{Head})}{P(\text{Red})}$$

from a $\rightarrow$ $$\frac{\frac{3}{5} \times \frac{1}{2}}{\frac{31}{70}} = \frac{\frac{3}{10}}{\frac{31}{70}} = \frac{3}{10} \times \frac{70}{31} = \frac{210}{310} = \boxed{\frac{21}{31}}$$

48. An urn contains three red and two white balls. A ball is drawn, and then it and another ball of the same color are placed back in the urn. Finally, a second ball is drawn.

a. What is the probability that the second ball drawn is white?

b. If the second ball drawn is white, what is the probability that the first ball drawn was red?

/

(a)

$$P(\text{White} \mid \text{first Red}) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{White} \mid \text{first White}) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{Red first}) = \frac{3}{5}, \quad P(\text{White first}) = \frac{2}{5}$$

$$P(\text{White second}) = P(\text{White} \mid \text{Red first}) \times P(\text{Red First}) + P(\text{White} \mid \text{White first}) \times P(\text{White first})$$

$$= \frac{1}{3} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{5} = \frac{3}{15} + \frac{2}{10} = \frac{1}{5} + \frac{1}{5} = \boxed{\frac{2}{5}}$$

(b) $P(\text{Red first} \mid \text{White Second})$?

$$= \frac{P(\text{White} \mid \text{Red first}) \times P(\text{Red first})}{P(\text{White Second})}$$

from (a) $\rightarrow$ $= \dfrac{\frac{1}{3} \times \frac{3}{5}}{\frac{2}{5}} = \dfrac{\frac{3}{15}}{\frac{2}{5}} = \dfrac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{5} \times \frac{5}{2} = \boxed{\frac{1}{2}}$

52. Suppose that 5 cards are dealt from a 52-card deck and the first one is a king. What is the probability of at least one more king?

if first card is king : 3 kings, 51 cards

Remaining non-king → 51-3=48 cards

A_i → event that non-king is drawn

$P(A_1) = \frac{48}{51}$, $P(A_2) = \frac{47}{50}$, $P(A_3) = \frac{46}{49}$, $P(A_4) = \frac{45}{48}$

$$P(A_i) = \frac{48}{51} \times \frac{47}{50} \times \frac{46}{49} \times \frac{45}{48}$$

$$= \frac{47}{51} \times \frac{46}{50} \times \frac{45}{49} = \frac{97290}{124950}$$

$$P(\text{Atleast 1 more king}) = 1 - P(A_i)$$

$$= 1 - \frac{97290}{124950}$$

$$= \boxed{0.2223}$$

39. A monkey at a typewriter types each of the 26 letters of the alphabet exactly once, the order being random.

a. What is the probability that the word *Hamlet* appears somewhere in the string of letters?

b. How many independent monkey typists would you need in order that the probability that the word appears is at least .90?

(a)

total = 26!

positions for "Hamlet" $26-6+1=21$

ways to arrange rest of letters = 20!

$$P(\text{Hamlet appears}) = \frac{21 \times 20!}{26!}$$

$$P(\text{Hamlet}) = \boxed{\frac{21!}{26!}}$$

(b)

$$P(\text{not Hamlet}) = 1 - \frac{21!}{26!}$$

$$P(n \text{ Monkey no Hamlet}) = \left(1 - \frac{21!}{26!}\right)^n$$

So,

$$1 - \left(1 - \frac{21!}{26!}\right)^n \geq 0.9$$

$\approx$ $\boxed{18175685 \text{ Monkeys}}$

70. If $A \subset B$, can A and B be independent?

if A is a subset of B, independent?

$A = \{6\}$, $B = \{2,4,6\}$ -> rolling dice

$P(A) = \frac{1}{6}$, $P(B) = \frac{1}{2}$

$P(A|B) = P(A)$?

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$ -> $\frac{1}{3} \neq \frac{1}{6}$

$P(B|A) = P(B)$?

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1 \neq \frac{1}{2}$

$P(A \cap B) = P(A) \times P(B)$?

$P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} \neq \frac{1}{6}$

No, A and B can not be independent as long as $0 < P(A) < P(B) < 1$

72. Suppose that n components are connected in series. For each unit, there is a backup unit, and the system fails if and only if both a unit and its backup fail. Assuming that all the units are independent and fail with probability p, what is the probability that the system works? For $n = 10$ and $p = .05$, compare these results with those of Example F in Section 1.6.

$P(\text{Fail}) = p$

$P(\text{Primary Fail} \cap \text{Backup Fail}) = p \times p = p^2$

$P(\text{Atleast 1 unit works}) = 1 - p^2$

$P(\text{System works}) = \underline{(1-p^2)^n}$

$n = 10, \; p = 0.05$

$P(\text{System works}) = (1 - 0.05^2)^{10} = (1 - 0.0025)^{10}$

$= (0.9975)^{10} = \boxed{0.975}$

The probability of the system works in Example F is 0.6. With a back up unit for each component with n=10 and p=0.05 is 0.975. So adding a back up for each unit increases the reliability of the system.