2. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.

a. List the sample space.

b. List the elements that make up the following events: (1) A = the sum of the two values is at least 5, (2) B = the value of the first die is higher than the value of the second, (3) C = the first value is 4.

c. List the elements of the following events: (1) $A \cap C$, (2) $B \cup C$, (3) $A \cap (B \cup C)$.

(a) $\Omega =$

$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$

(b) (1) $A =$

$(1,4),(1,5),(1,6),$
$(2,3),(2,4),(2,5),(2,6),$
$(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$

(2) $B =$

$(2,1),$
$(3,1),(3,2),$
$(4,1),(4,2),(4,3),$
$(5,1),(5,2),(5,3),(5,4),$
$(6,1),(6,2),(6,3),(6,4),(6,5)$

(3) $C = (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

(c) (1) $A = (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

(2) $B =$

$(2,1),$
$(3,1),(3,2),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),$
$(6,1),(6,2),(6,3),(6,4),(6,5)$

(3) $C = A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap B = (3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),$
$(5,4),(6,1),(6,2),(6,3),(6,4),(6,5)$

$A \cap C = (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

$C = (3,2),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(6,1),(6,2),(6,3),(6,4),$
$(6,5)$

11. The first three digits of a university telephone exchange are 452. If all the sequences of the remaining four digits are equally likely, what is the probability that a randomly selected university phone number contains seven distinct digits?

$452-\underline{7}\,\underline{6}\,\underline{5}\,\underline{4}$

Ways to pick 4 numbers: $\binom{7}{4}$

Arrangements for 4 numbers: $4!$

$$P(7 \text{ distinct}) = \frac{4! \times \binom{7}{4}}{10^4}$$

$$= \frac{24 \cdot 35}{10000} = \frac{840}{10000} = 0.084$$

17. In acceptance sampling, a purchaser samples 4 items from a lot of 100 and rejects the lot if 1 or more are defective. Graph the probability that the lot is accepted as a function of the percentage of defective items in the lot.

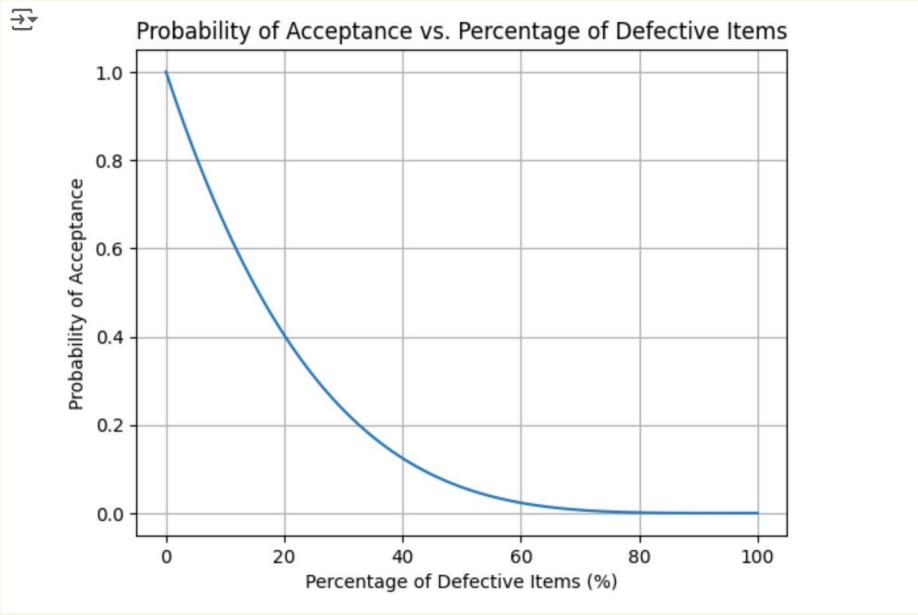
$p \Rightarrow$ percentage of defective items

$\binom{100}{4} \Rightarrow$ number of ways to choose 4 items

$100p \Rightarrow$ number of defective items

$100 - 100p \Rightarrow$ number of non-defective items.

$$P(\text{accept}) = \frac{\binom{100-100p}{4}}{\binom{100}{4}}$$



24. If n balls are distributed randomly into k urns, what is the probability that the last urn contains j balls?

total ways for n balls into k : k^n

ways to pick j : $\binom{n}{j}$

remaining balls : $n-j$

remaining urns : $k-1$

$$P(\text{last urn contains } j \text{ balls}) = \frac{\binom{n}{j}(k-1)^{n-j}}{k^n}$$

30. A group of 60 second graders is to be randomly assigned to two classes of 30 each. (The random assignment is ordered by the school district to ensure against any bias.) Five of the second graders, Marcelle, Sarah, Michelle, Katy, and Camerin, are close friends. What is the probability that they will all be in the same class? What is the probability that exactly four of them will be? What is the probability that Marcelle will be in one class and her friends in the other?

A. Probability all will be in same class.

$P(\text{Class A Marcelle}) = \frac{30}{60}$ $P(\text{Class A Sarah}) = \frac{29}{59}$

$P(\text{Class A Michelle}) = \frac{28}{58}$ $P(\text{Class A Katy}) = \frac{27}{57}$

$P(\text{Class A Camerin}) = \frac{26}{56}$

$P(\text{all Class A or B}) = 2 \cdot \left(\frac{30}{60} \cdot \frac{29}{59} \cdot \frac{28}{58} \cdot \frac{27}{57} \cdot \frac{26}{56}\right) = 0.052$

B. Probability exactly 4 in one class.

$P(\text{Class A Marcelle}) = \frac{30}{60}$ $P(\text{Class A Sarah}) = \frac{29}{59}$

$P(\text{Class A Michelle}) = \frac{28}{58}$ $P(\text{Class A Katy}) = \frac{27}{57}$

$P(\text{Class A Camerin}) = \frac{30}{56}$

$P(\text{4 in one class}) = 2 \cdot \binom{5}{4} \left(\frac{30}{60} \cdot \frac{29}{59} \cdot \frac{28}{58} \cdot \frac{27}{57} \cdot \frac{30}{56}\right) = 0.301$

C. Probability Marcelle in one class.

$P(\text{Class A Marcelle}) = \frac{30}{60}$ $P(\text{Class A Sarah}) = \frac{30}{59}$

$P(\text{Class A Michelle}) = \frac{29}{58}$ $P(\text{Class A Katy}) = \frac{28}{57}$

$P(\text{Class A Camerin}) = \frac{27}{56}$

$P(\text{Marcelle by herself}) = 2 \cdot \frac{30}{60} \cdot \frac{30}{59} \cdot \frac{29}{58} \cdot \frac{28}{57} \cdot \frac{27}{56} = 0.0602$

33. An elevator containing five people can stop at any of seven floors. What is the probability that no two people get off at the same floor? Assume that the occupants act independently and that all floors are equally likely for each occupant.

Probability that person P_i gets off on floor i: 7^5

Ways to choose 5 floors: $\binom{7}{5}$

Rearrangements for 5 people: $5!$

$$P(\text{no two people same floor}) = \frac{5!\binom{7}{5}}{7^5}$$

$$= 120 \cdot \frac{\binom{7}{5}}{7^5}$$

$$= \boxed{0.1499}$$