INVERSE KINEMATICS SOLVING

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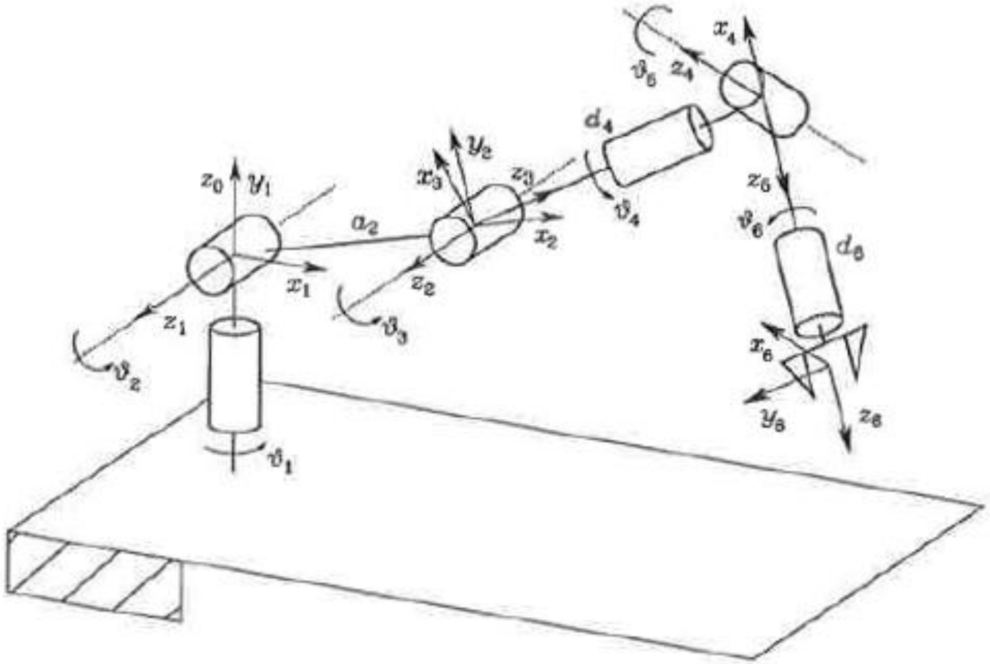
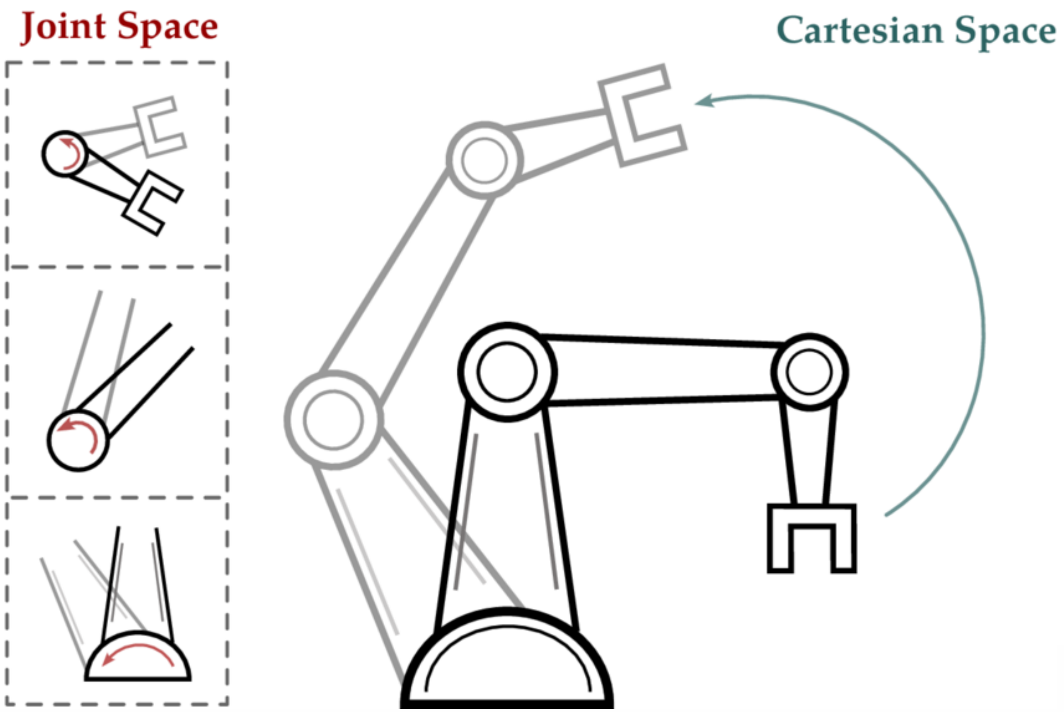
KINEMATIC IN ROBOTICS

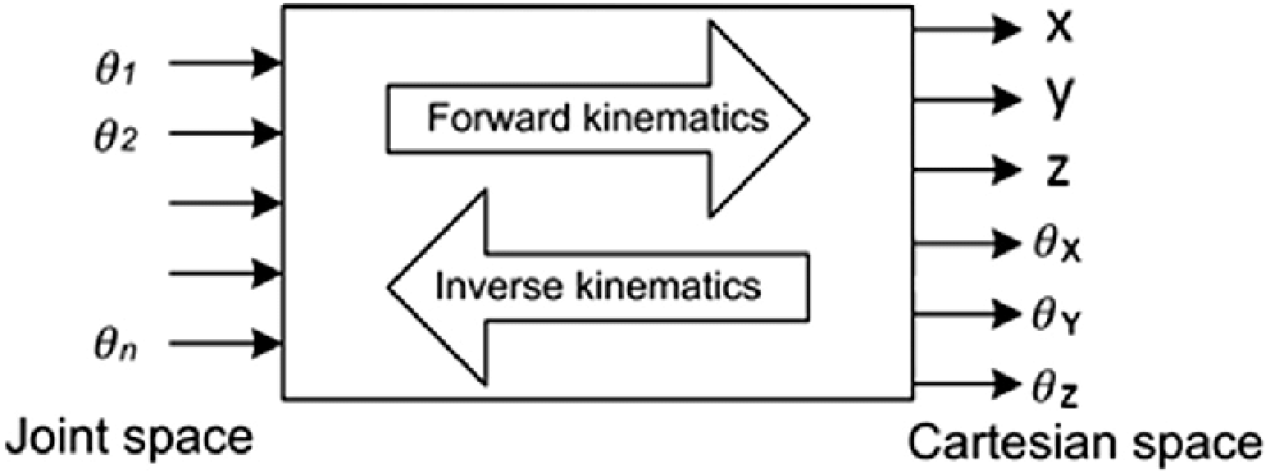
Property of motion of a rigid body.



Anthropomorphic arm with spherical wrist

OPERATIONAL VS JOINT SPACE





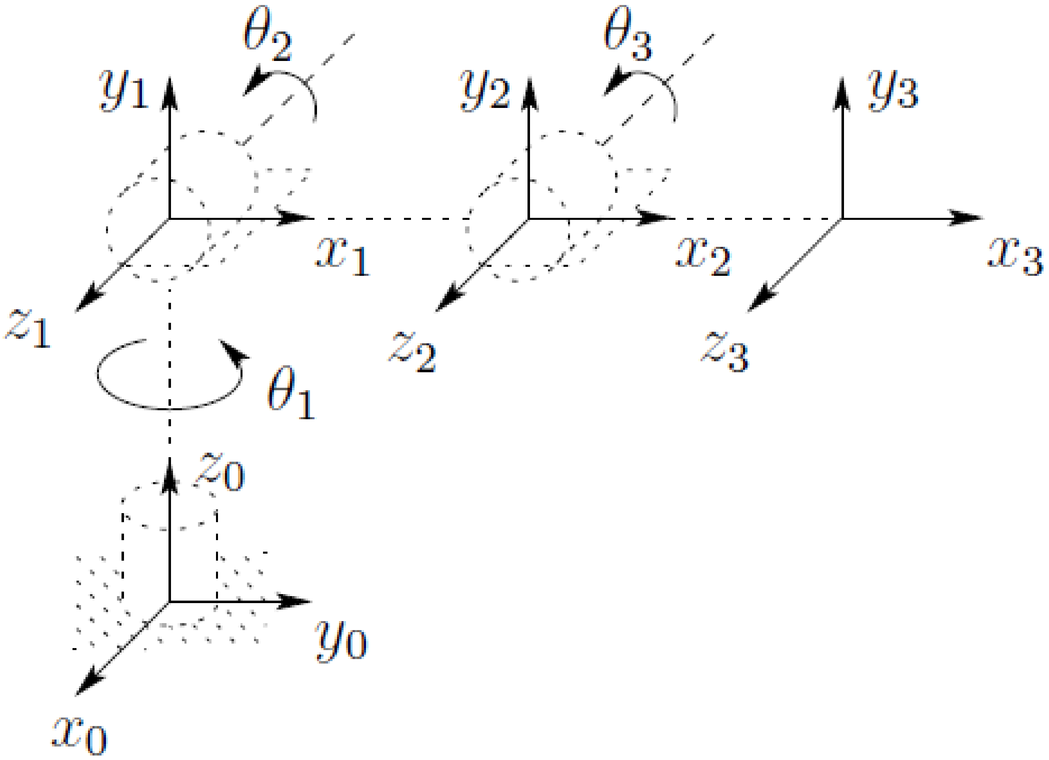
KINEMATIC CHAIN

$$q_i = \begin{cases} \theta_i & \text{if joint } i \text{ is revolute} \\ d_i & \text{if joint } i \text{ is prismatic} \end{cases}$$

$$A_i = A_i(q_i)$$

Transformation matrix:

$$H = T_n^0 = A_1(q_1) \cdots A_n(q_n)$$



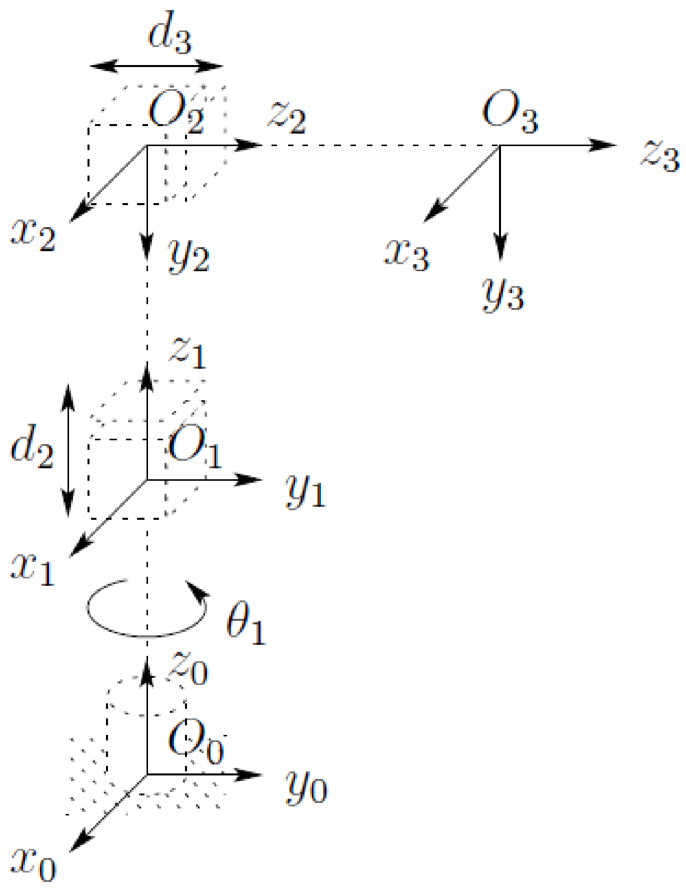
DH CONVENTION

Denavit-Hartenberg: provides a **systematic** procedure to perform forward kinematics

Each homogeneous transformation is represented by a product of four basic transformations:

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}Rot_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

DH FRAMES - EXAMPLE



$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

* variable

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IK PROBLEM

Input: End-effector position and orientation

Output: Robot's joint angles

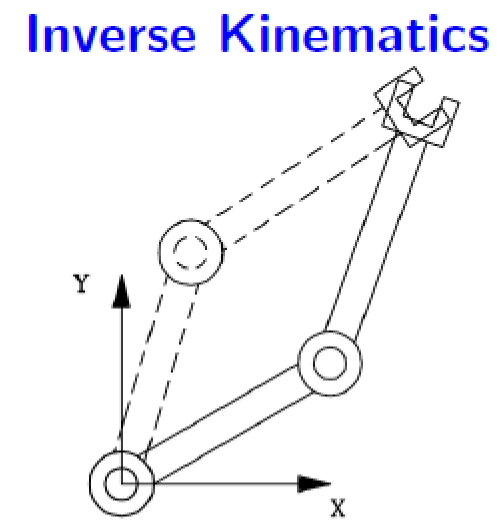
Given the 4x4 homogeneous transformation:

$$H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} \in SE(3)$$

Where:

$$T_n^0(q_1, \ldots, q_n) = A_1(q_1)\, A_2(q_2) \ldots A_n(q_n) = H$$

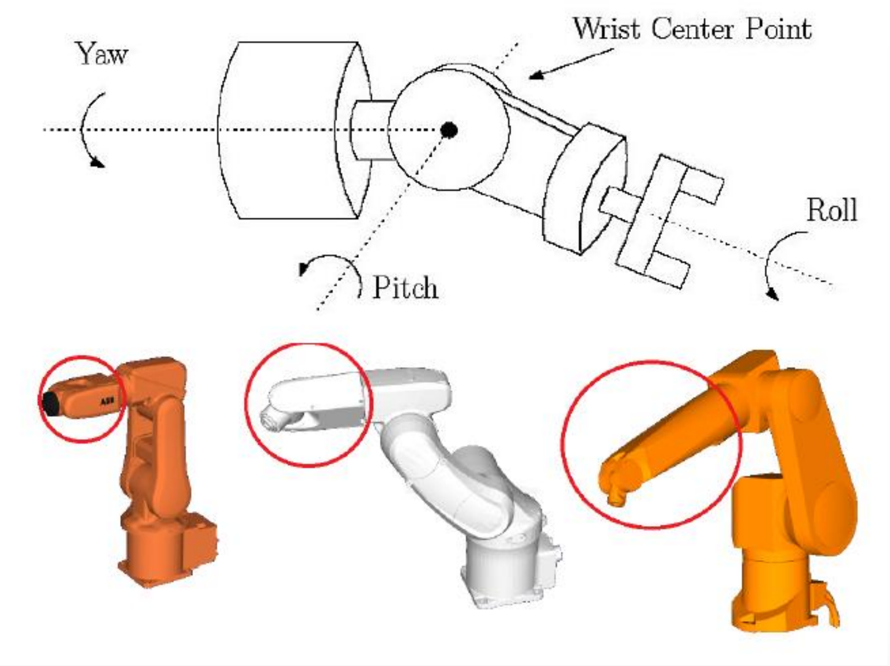
12 equations to solve!!

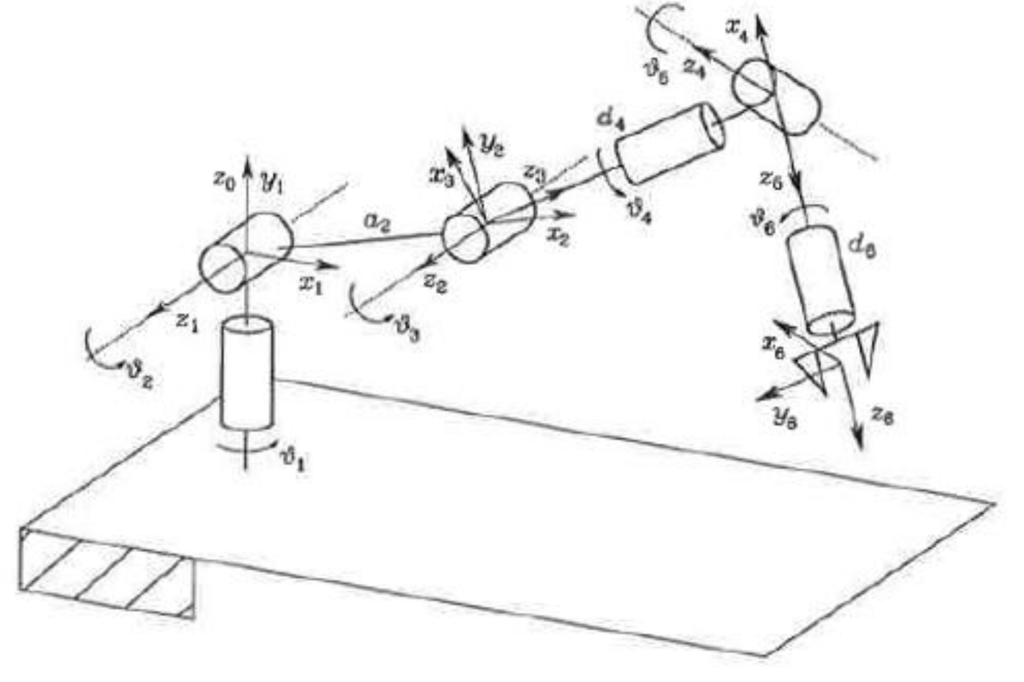


KINEMATIC DECOUPLING

Wrist: joints between the arm and end-effector

Spherical wrist: axes of the three last joints intersect at one point.





Anthropomorphic arm with spherical wrist

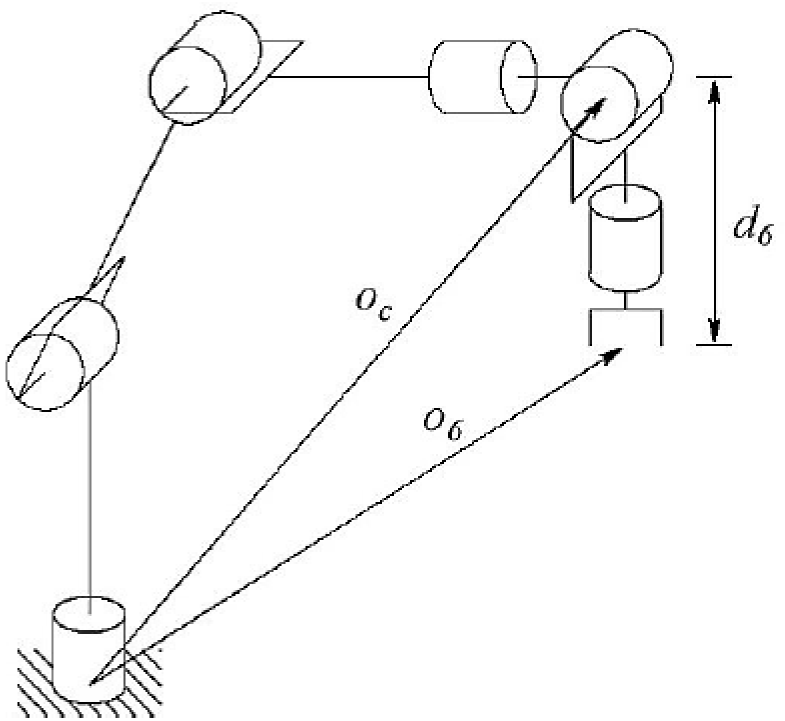
KINEMATIC DECOUPLING – INVERSE POSITION

- **Spherical wrist ->** position of wrist center point Oc **independent on the end-effector orientation.**

- Orientation of end-effector **depends on the last three joints.**

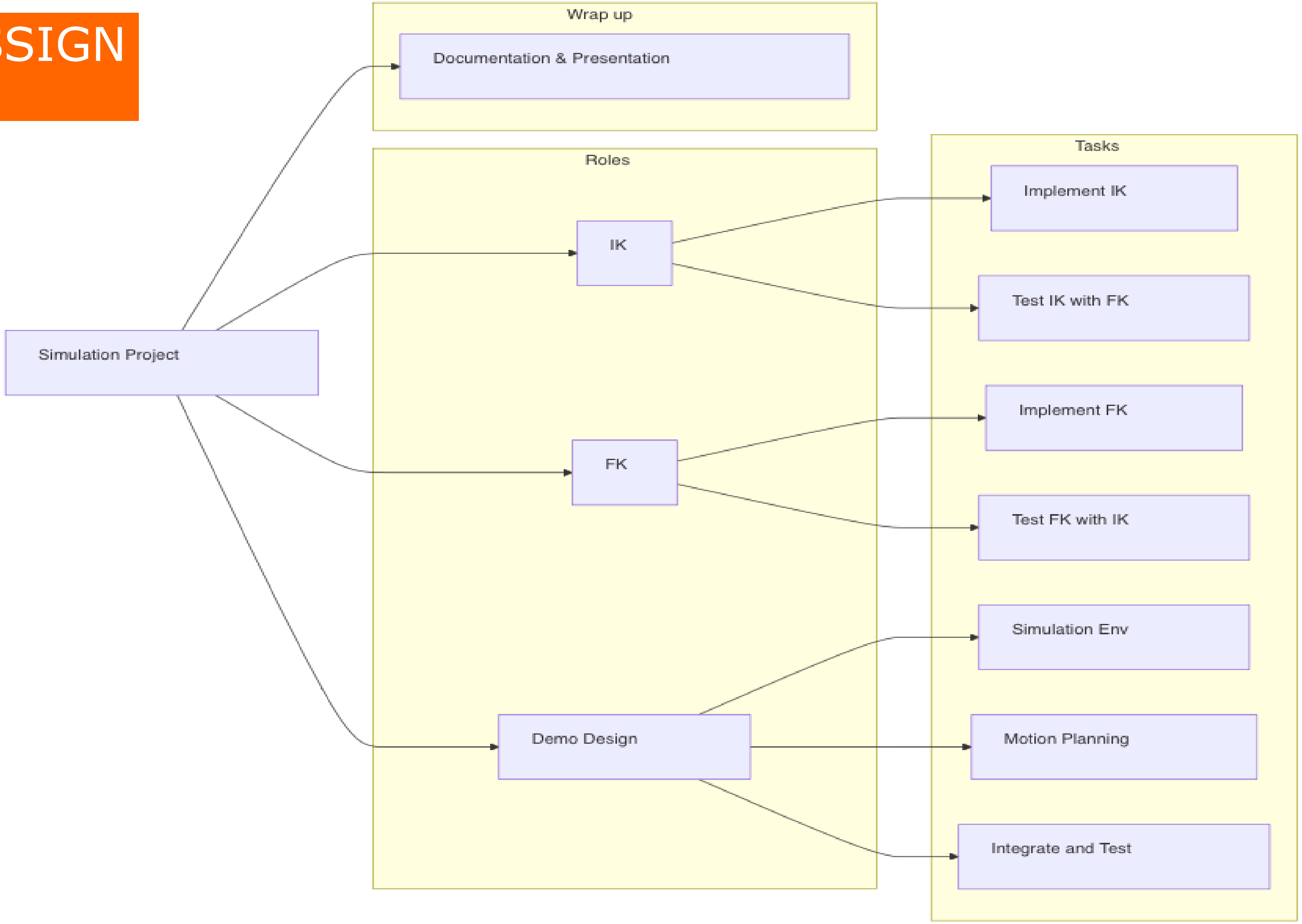
Kinematic decoupling: solving the IK of a 6 dof manipulator splits in two problems:

  - **Inverse position kinematics**
  - **Inverse orientation kinematics**



Elbow Manipulator with Spherical Wrist

HOW TO ASSIGN TASKS?



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WHATS NEXT?

1) Check and run the FK example using DH

2) Derive the DH table for the MARA

3) Perform Kinematic decoupling for the mara

→Robot Modeling and Control: Chapter 3.3.2

THANK YOU
ANY QUESTION?

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