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T3 ________	Problem Chosen	F3 ________
T4 ________	**A**	F4 ________

2014 Mathematical Contest in Modeling (MCM) Summary Sheet

A Study on Freeway Driving Rules

Summary

Most countries in the world adopt the right-hand traffic while the others (mostly old British colonies) adopt the left-hand traffic. It is believed that the performance of transportation on multi-lane freeways would be different if the driving rules are different. We build a cellular automaton model to analyze how the performance is influenced. Our model is based on the well-known NaSch model and we make some refinements and optimization (the addition of the slow-to-start rule in the BJH model, different value of the braking probability, etc.). We observe stop-and-go waves and phantom jams in the spatial-temporal diagrams. In order to study the keep-right-except-to-pass rule, we propose a new rule called the free driving rule, with which drivers need not keep right and are free to overtake, as a control. Experiment results show that the keep-right-except-to-pass rule considers more about safety, while the free driving rule can present a higher flowrate.

Contents

I. INTRODUCTION

One of the most important rules of freeway transportation in countries worldwide is the *Right- and Left-hand Traffic*. As is implied by the name, some countries drive on the right while others on the left. Statistics suggest that today about 35% of the world population drives on the left, and the countries that do are mostly old British colonies [Anonymous, 2013].

The cellular automaton (CA) model for traffic flow simulation proposed by Nagel and Schreckenberg [1992] (also known as NaSch model) is a simple and yet effective way to study the traffic flow on single-lane roads in realistic life. Benjamin et al. [1996] introduce a slow-to-start rule which simulates a possible delay before a car pulls away from being stationary, which makes a realistic generalization of the basic NaSch model. (It is known as the BJH model.) The new model proposed by Clarridge and Salomaa [2010] includes a slow-to-start rule that exhibits more realistic microscopic driver behavior than the BJH model. Rawat et al. [2011] further make refinements on the model. Xiang et al. [2013] diversify the probability of braking according to different driving states, which promotes vehicle flowrate. On this basis, our model further lets vehicles of different types have different maximum acceleration and the probability of slow-to-start change according to some criteria.

In order to avoid vehicle crashes and to increase vehicle flow, some countries require drivers on multi-lane freeways to always stay in the right-most lane unless they are going to overtake another vehicle. After passing the vehicle, they should return to the original lane. This rule is referred to as the *keep-right-except-to-pass rule*. Since most countries adopt the right-hand traffic, we are going to first analyze how this rule can influence vehicle flowrate, safety, or some other important factors embodied in freeway transportation.

In addition to the keep-right-except-to-pass rule, we institute a new rule of switching between four lanes according to the work by WU et al. [2005]. Experiment results show that the new rule promotes the flowrate of traffic transportation, and the keep-right-except-to-pass rule is safer.

The remaining part of the paper is organized as follows: Section II gives our assumptions on the problem and also includes all the variables used in our model; Section III presents the model in detail; Section IV includes model testing and sensitivity analysis (and some other analysis); Section V discusses the strengths and weaknesses of our model; Section VI includes some further discussion and the conclusion.

II. PRELIMINARY

I. Assumptions

We make following assumptions on the problem:

- We only test and analyze four-lane freeways, since freeways with more than four lanes are not common (though our model can be applied to freeways with more than four lanes);
- There is only a fixed number of vehicle types on the freeway, and vehicles of the same type have the same attributes (length, maximum acceleration and maximum velocity);
- We ignore the influence of freeway facilities (deceleration strips, caution signs, toll-gates, etc.) and road shape;
- We ignore the impacts of natural factors on freeway transportation, e.g., atrocious weather can reduce both vehicle flowrate and safety.

II. Variables

All the variables used in our model are listed in Table 1.

III. MODEL

The basic model is the well-known NaSch model, which is given pictorially in Figure 1. The model is defined on a one-dimensional array of L cells, and each cell may be occupied by a vehicle or empty [Nagel and Schreckenberg, 1992].

In NaSch model, the system is updated by four steps:

i) **Acceleration**: $v_i^{t+1/3} = \min\{v_i^t + 1, v_{\max}\}$.

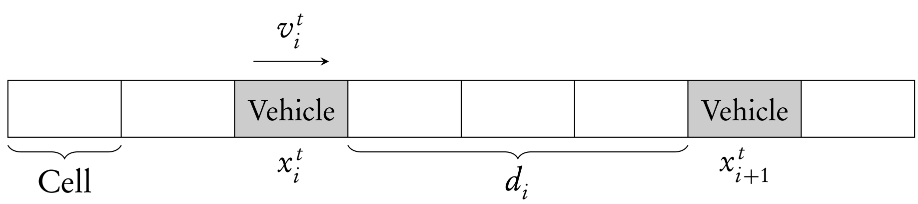


Figure 1: The NaSch Model

Table 1: Variable List

Variable	Meaning
L	Length of the freeway considered
N	Number of vehicles on the freeway
ρ	Vehicle density (vehicles per cell)
v_i^t	Velocity of the ith vehicle at time t
x_i^t	Position of the ith vehicle at time t (head end)
l_i	Length of the ith vehicle
d_i	Distance between the ith vehicle and the vehicle ahead of it on the same lane
$d_{i,\text{left}}$	Distance between the ith vehicle and the vehicle ahead of it on the left side
$d^*_{i,\text{left}}$	Distance between the ith vehicle and the vehicle behind it on the left side
$d_{i,\text{right}}$	Distance between the ith vehicle and the vehicle ahead of it on the right side
$d^*_{i,\text{right}}$	Distance between the ith vehicle and the vehicle behind it on the right side
s_{behind}	Safe distance kept with the vehicle behind
$v_{\max}$	Maximum velocity of the vehicle
$v_{\min}$	Minimum velocity of the vehicle
a	Acceleration of the vehicle
p_{brake}	Probability of braking
p_{change}	Probability of the driver changing lane when allowed
$p_{\text{s-to-s}}$	Probability of slow-to-start

ii) **Deceleration**: $v_i^{t+2/3} = \min\{v_i^{t+1/3}, d_i\}$.

iii) **Randomization**: $v_i^{t+1} = \max\{v_i^{t+2/3} - 1, 0\}$ with probability p_{brake}.

iv) **Vehicle Motion**: $x_i^{t+1} = x_i^t + v_i^{t+1}$.

According to the work by Rawat et al. [2011], we improve the basic NaSch model to a more realistic one. The size of the cells in the original model is defined to be 7.5 meters, and each vehicle can only occupy one cell. The new model is modified to have cells with size of 0.5 meters and one vehicle can take multiple cells. Thus, we assume that there are two types of vehicles—the light vehicles (e.g., cars) will take 12 cells (6 meters long), with $v_{\max} = 60$ cells, and the heavy vehicles (e.g., trucks) will take 20 cells (10 meters long), with $v_{\max} = 40$ cells. The accelerations also differ. First, the acceleration of one vehicle changes with its current velocity. Second, different types of vehicle have different maximum acceleration.

The probability of braking also changes with different driving states [Xiang et al., 2013]. In the original NaSch model, the probability p_{brake} in step iii) won't change according to the results of the former two steps. In realistic life, this is often not the case. Thus, the new model will have different values of p_{brake}.

We include the slow-to-start rule [Benjamin et al., 1996; Clarridge and Salomaa, 2010] in the new model. This rule captures the little delay when the driver is going to drive away from a blocked state (when the vehicle has reached the head of a jam queue). The delay can be caused by the driver's loss of attention, or the slow starting of the engine.

Now we are ready to present our model mathematically. The lane-changing rule will be discussed soon afterwards.

I. Our Model

In our model, the system is updated by six steps:

i) **Slow-to-Start**: If $v_i^t = 0$ and $x_{i+1}^{t-1} - x_i^{t-1} - l_{i+1} = 0$, then $v_i^{t+1} = 0$ with probability $p_{\text{s-t-s}}$. The vehicle just stays where it is, and the following steps can be skipped.

ii) **Acceleration**: $v_i^{t+1/3} = \min\{v_i^t + a, v_{\max}\}$. If the vehicle is light, then

$$a = \begin{cases} 4, & v_i^t \le 22, \\ 3, & 22 < v_i^t \le 32, \\ 2, & v_i^t > 32. \end{cases} \tag{1}$$

If the vehicle is heavy, then

$$a = \begin{cases} 3, & v_i^t \leq 12, \\ 2, & 12 < v_i^t \leq 22, \\ 1, & v_i^t > 22. \end{cases} \tag{2}$$

iii) **Deceleration**: $v_i^{t+2/3} = \min\{v_i^{t+1/3}, d_i\}$.

iv) **Braking Probability Determination**:

$$p_{\text{brake}} = \begin{cases} p_1, & v_i^{t+2/3} > v_i^t, \\ p_2, & v_i^{t+2/3} < v_i^t, \\ p_3, & v_i^{t+2/3} = v_i^t. \end{cases} \tag{3}$$

The relation of the three randomization probabilities is $p_2 > p_3 > p_1$.

v) **Randomization**: $v_i^{t+1} = \max\{v_i^{t+2/3} - 1, 0\}$ with probability p_{brake}.

vi) **Vehicle Motion**: $x_i^{t+1} = x_i^t + v_i^{t+1}$.

II. The Lane-changing Rule

Figure 2 shows the model with four lanes. Vehicle i changes its driving lane if the following three criteria are fulfilled [Li et al., 2006]: (Assume the vehicle is going to the left side.)

- $d^*_{i,\text{left}} > s_{\text{behind}}$
- $d_{i,\text{left}} > d_i$
- $d_i < \min\{v_i^t + a, v_{\max}\}$

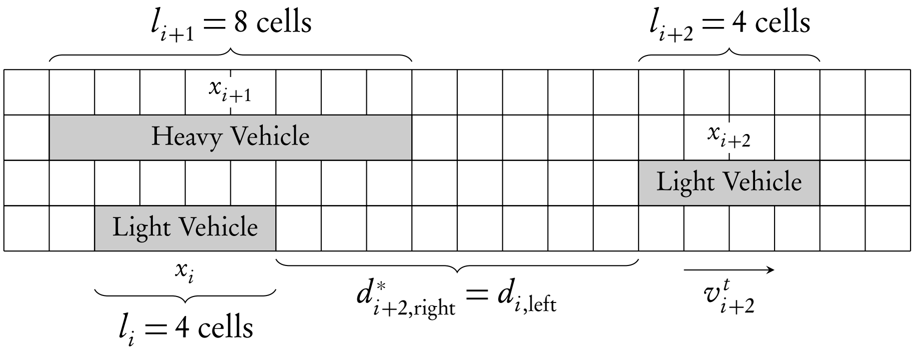


Figure 2: The Model with Four Lanes (Schematic View)

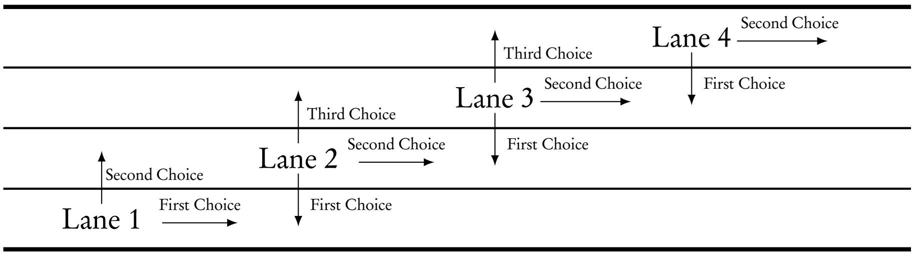


Figure 3: The Keep-Right-Except-to-Pass Rule

III. The Keep-Right-Except-to-Pass Rule

The keep-right-except-to-pass rule requires drivers to always stay in the right-most lane unless they want to overtake the vehicle ahead of him on the same lane. Figure 3 shows how this rule works on four-lane freeways. When a vehicle is in one lane, the driver's first choice is to go to the right side. If he can't go to the right side, or he has already been in the right-most lane, he should keep driving in the current lane. If this state still can't be satisfied (he wants to overtake the vehicle ahead of him), he should go to the left side.

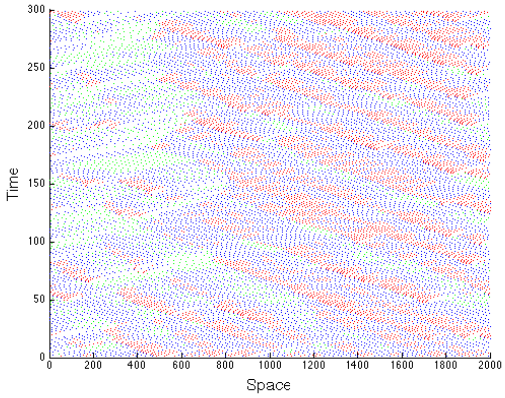
IV. The Free Driving Rule

The free driving rule doesn't require drivers to stay in the left-most lane or in the right-most lane. When the driver wants to overtake another vehicle, he can choose to go to the left side or the right side, with certain probabilities.

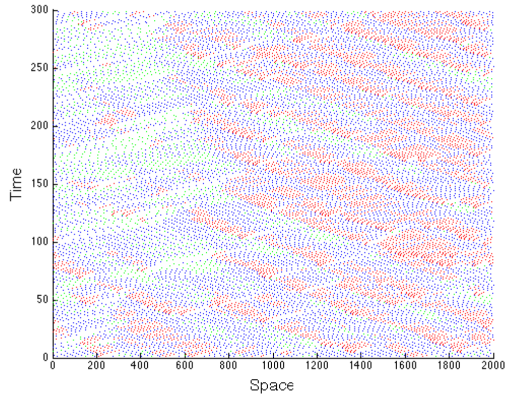
IV. Testing and Analysis

I. Model Testing

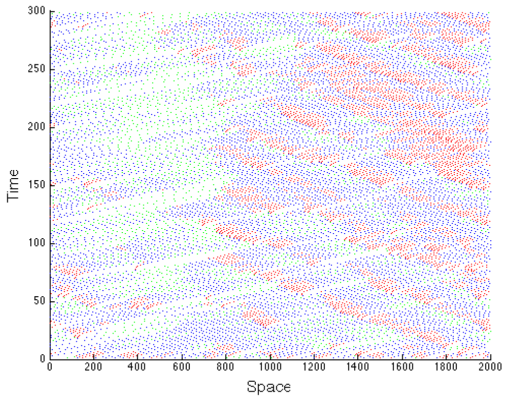
During the simulation, we define the four-lane freeway to be a loop. Initially, we randomly put vehicles in the four lanes. The spatial-temporal diagrams of the four lanes using the keep-right-except-to-pass rule are shown in Figure 4. Those using the free driving rule are shown in Figure 5. Vehicles with large velocity are marked green, medium velocity blue, and small velocity red. From the diagrams, we can observe stop-and-go waves (the red strips) and phantom jams, which are common in realistic traffic. In the simulation using the keep-right-except-to-pass rule, the right-most lane (lane 1) are more congested than the left lanes, which conforms to the rule itself. The free driving rule doesn't give much difference between the lanes.



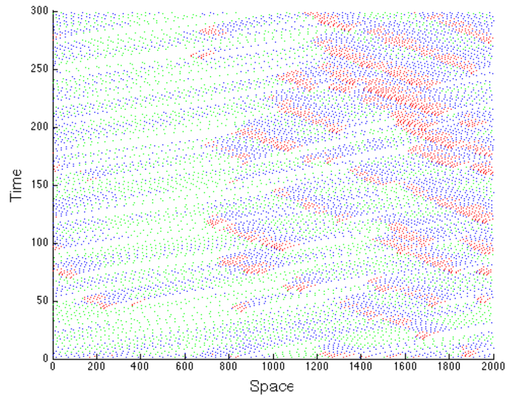
(a) Lane 1



(b) Lane 2



(c) Lane 3



(d) Lane 4

Figure 4: Spatial-temporal Diagrams of the Keep-Right-Except-to-Pass Rule

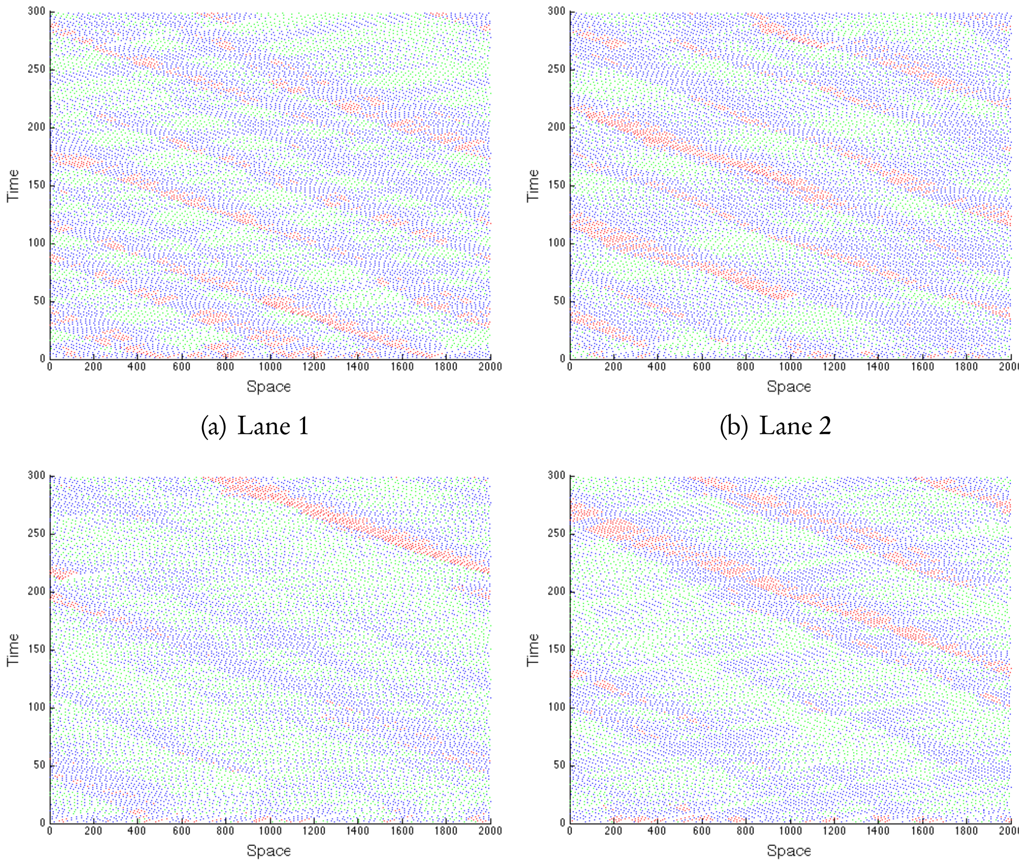
II. Sensitive Analysis

We investigate the performance of the two driving rules (the keep-right-except-to-pass rule and the free driving rule) using the model, by changing the configurations. The arguments are listed below.

- Vehicle density (in vehicles per cell);
- Maximum velocity (in cells per second);
- Ratio of light vehicles (range from 0 to 1).

The performance of the driving rules is determined by the following factors:

- Vehicle flowrate (in vehicles per minute);



(a) Lane 1 (b) Lane 2

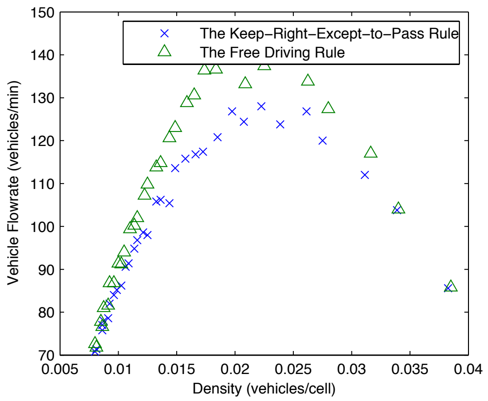
(c) Lane 3 (d) Lane 4

Figure 5: Spatial-temporal Diagrams of the Free Driving Rule

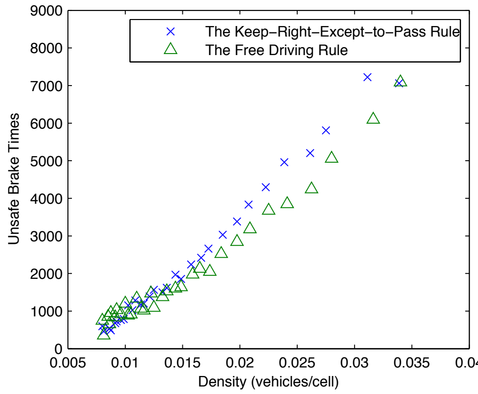
- Unsafe brake times;
- Lane-changing times.

We define deceleration greater than 8 cells to be unsafe brake. The two factors later (unsafe brake times and lane-changing times) determine the safety of the driving rules.

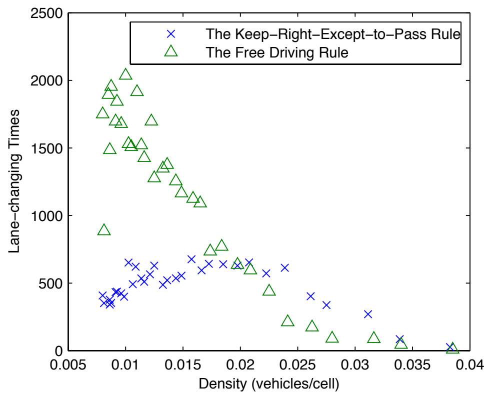
Experiment results are shown in Figure 6–8. In Figure 6, when the vehicle density changes, the flowrates first increase, reach its maximum, and then decrease. The free driving rule has greater flowrate than the keep-right-except-to-pass rule, since it allows drivers to overtake whenever they want to. For the same reason, the free driving rule makes much more lane changes when the density is low, which we consider unsafe. The unsafe brake times increase when the density gets large, and the two rules almost have the same number of brake. To this end,



(a) Traffic flow (in vehicles per minute) *vs.* vehicle density (in vehicles per cell).



(b) Unsafe brake times *vs.* vehicle density (in vehicles per cell).



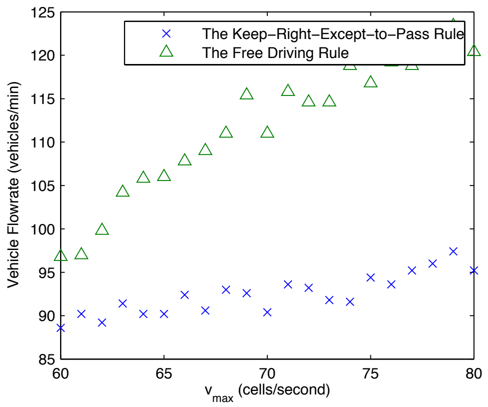
(c) Lane-changing times *vs.* vehicle density (in vehicles per cell).

Figure 6: How traffic flow, unsafe brake times and lane-changing times change with vehicle density.

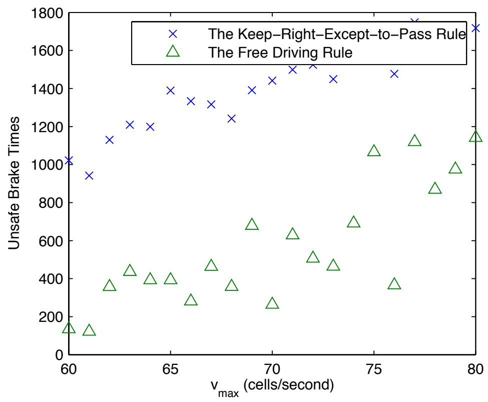
we may consider the keep-right-except-to-pass rule as a safe one, while the free driving rule an efficient one.

In Figure 7, when the maximum velocity changes, vehicle flowrate, unsafe brake times and lane-changing times all demonstrate a tend of increasing. In Figure 7(a), the free driving rule has greater flowrate because it allows both side lane-changing. In Figure 7(c) shows the randomness of lane-changing in the free driving rule.

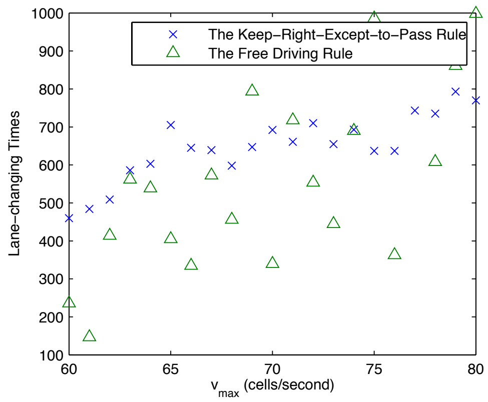
In Figure 8, when the ratio of light vehicles changes, flowrates also increase, since the ratio of light vehicles becoming larger means the average velocity being increased.



(a) Traffic flow (in vehicles per minute) *vs.* maximum velocity (in cells per second).



(b) Unsafe brake times *vs.* maximum velocity (in cells per second).



(c) Lane-changing times *vs.* maximum velocity (in cells per second).

Figure 7: How traffic flow, unsafe brake times and lane-changing times change with maximum velocity.

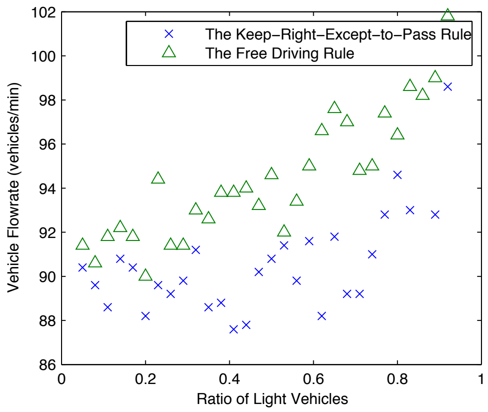
V. Strengths and Weaknesses

The **strengths** of our model are

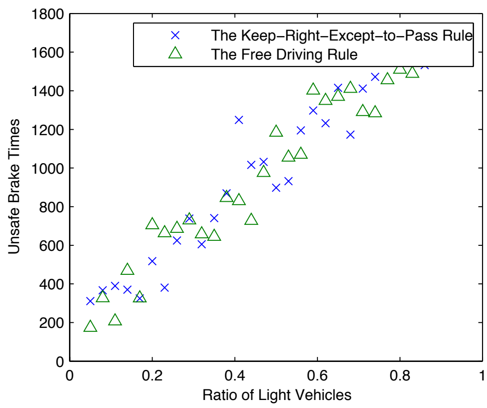
i) Different types of vehicles have different length, maximum velocity and acceleration.

ii) The model makes refinements and optimization on the original cellular automaton model (the NaSch model).

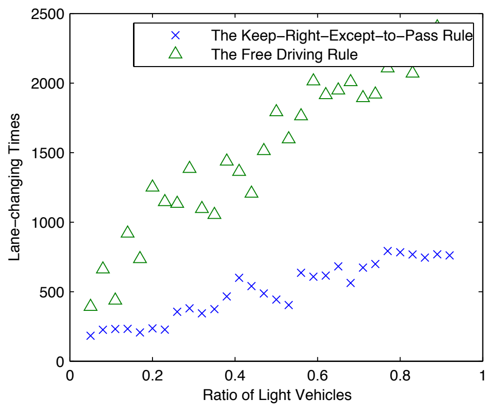
iii) The model takes the "human factors" during the drive into consideration, including the probability of changing lane if allowed, the probability of



(a) Traffic flow (in vehicles per minute) *vs.* ratio of light vehicles.



(b) Unsafe brake times *vs.* ratio of light vehicles.



(c) Lane-changing times *vs.* ratio of light vehicles.

Figure 8: How traffic flow, unsafe brake times and lane-changing times change with the ratio of light vehicles.

unconscious braking, etc.

iv) The model sets a safe distance between vehicles, to ensure that the vehicles keep a certain distance when running on the freeways, which is closer to the realistic traffic.

v) The model uses probabilities to make choices between turning left or right.

The **weaknesses** of our model are

i) The model doesn't consider the vehicles that drive onto or out of the freeway from the two sides.

ii) In order to ensure the safety of vehicles, the model doesn't consider the limitation on deceleration.

iii) Acceleration and deceleration are supposed to be done instantly, and the velocity at the end of each second is regarded the same as the average velocity in this second. Actually, the velocity at the end of a second after acceleration is often larger than the average velocity, and the velocity after deceleration is often smaller than the average velocity.

iv) The model doesn't consider the time to change lane. Lane-changing and moving forward are done in one second, so the time of lane-changing can't be determined.

VI. Further Discussion and Conclusion

I. Intelligent System

The intelligent system that fully takes control of a freeway might either be part of the road network or imbedded in the design of all vehicles using the freeway. We investigate the two situations individually, and analyze how the system will influence our model.

- The system is part of the road network, i.e., each vehicle can be detected and thus communicates with the system. Besides, each vehicle can have an idea of what the other vehicles are going to act. Also, it have to receive the command from the central system. A vehicle may take consideration of every possibility and make the best choice.
- The system is imbedded in each vehicle. It can evaluate the distance between another vehicle and itself. The vehicle will no longer need human to manipulate the velocity and direction. Therefore, all the rules in our model that have "human factors" (with probabilities) should be modified.

II. Conclusion

Though the keep-right-except-to-pass rule doesn't present a high flowrate, it is justifiable on safety grounds. Some experiment results have successfully shown this conclusion. However, because some factors that appear important, such as freeway facilities, mingled vehicles, etc., are not taken into consideration, our model still need improvement.

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