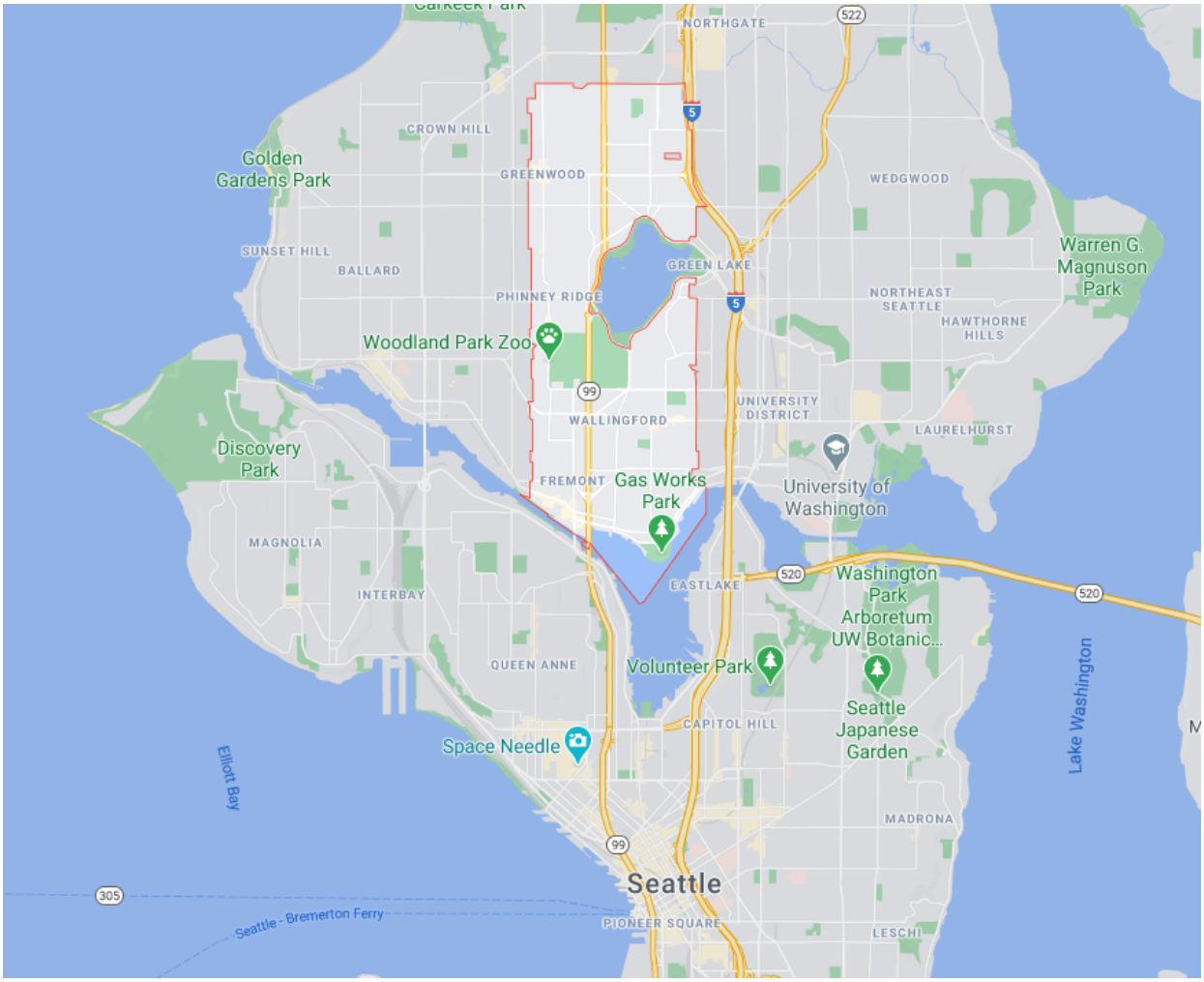
In [1]:
```
import copy
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.style as style
import matplotlib.gridspec as gridspec
import seaborn as sns
from mpl_toolkits import mplot3d
from scipy import stats
%matplotlib inline
```

In [95]:
```
df = pd.read_csv('./data_mod_1.csv')
```

§0 Exploring State Zip WA 98103

The 148 samples in WA 98103 is a comfortable amount of work with, to verify our machine learning algorithm results with hand calculations. The algorithm can be extended to the rest of the 4550 samples in the original dataframe, that comprises the entire Seattle region.

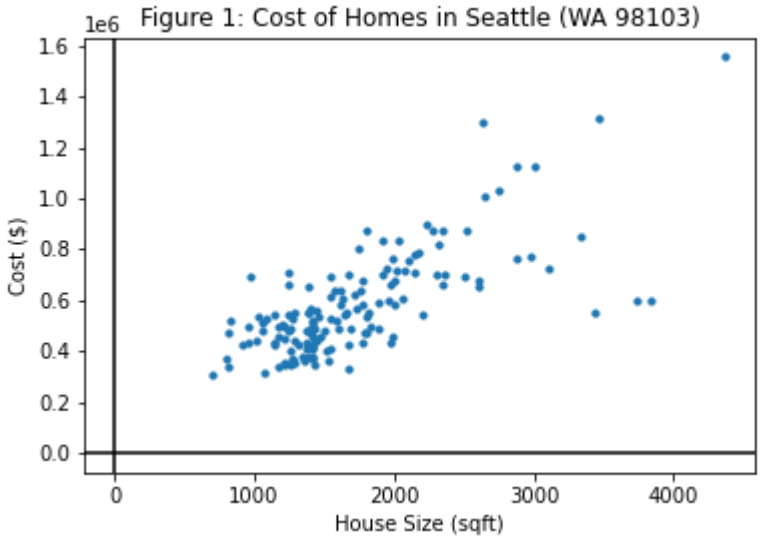


In [118…
```
df_4 = df.loc[ (df['statezip'] == 'WA 98103'), ['bedrooms', 'bathrooms', 'sqft_living',

X_ = df_4.sqft_living  # sqft is the most telling of price
X = X_.to_numpy()  # we want to convert from series to a numpy array
ones = np.ones(len(X))  # adding a ones column in front
X_with_ones_col = np.column_stack((ones, X))  # Data Preprocessing
```

```
y_ = df_4.price  # target
y = y_.to_numpy()
```

§1.0 Applying Machine Learning Models

In [805…
```
fig4, ax4 = plt.subplots()

ax4.scatter(X, y, s=10)  # plot looks fine
ax4.axhline(0, color='black')
ax4.axvline(0, color='black')
ax4.set_title("Figure 1: Cost of Homes in Seattle (WA 98103)")
ax4.set_xlabel("House Size (sqft)")
ax4.set_ylabel("Cost ($)")
```



Out[805…
```
Text(0, 0.5, 'Cost ($)')
```

§1.1 Linear Regression with the sklearn Library

In [21]:
```
from sklearn import datasets
from sklearn import linear_model
from sklearn.model_selection import train_test_split
```

In [108…
```
model = l_reg.fit(X_with_ones_col, y)
```

In [109…
```
l_reg.coef_
```

Out[109…
```
array([  0.        , 229.49521863])
```

In [80]:
```
l_reg.intercept_
```

Out[80]:
```
184435.24482113082
```

From the sklearn, the weights for w_0 and w_1 are 184435.2 and 229.5 respectively.

§1.2 An Analytic Approach

If we wanted to do this at a lower level, there are two ways to solve this. After determining the weights from with the linear regression machine learning model provided by the sklearn package, we will find the weights 'by hand' as well. There are many ways to do this. In this project, we will be employing the following two approaches to find the weights, similar to Lab 2.

Its very convenient to be able to calculate the weights with the following approach. In fact this is the best way to solve this problem for this situation (excluding sklearn)

As mentioned above, since there's only one feature to consider (sqft), we can solve for the weights with an analytic approach. Essentially, this means using basic linear algebra techniques, to solve for our weight, weights being our slope and intercept in this single-featured example.

The advantages of an analytic approach:

- no need for gradient descent
- no need to perform thousands of iterations
- no need for learning rate guesswork
- no need for feature scaling

In fact, the entire solution can be solved with an analytic approach. If the code is vectorized, an analytic approach would work up to even 10,000 features.

In [86]:
```
def normal_equation(X, y):
    return np.linalg.inv(X.T @ X) @ X.T @ y
```

In [556…
```
normal_equation(X_with_ones_col, y)
```

Out[556… `array([184435.24482113,    229.49521863])`

From our analytic approach, the weights for w_0 and w_1 are 184435.2 and 229.5 respectively. These values are exactly the same as the values from sklearn.

§1.3 Developing a Cost Function

Approach 2 is more involved, for example, one has to adjust the number of iterations, guess the learning rate and more (more discussed below). Since more things can go wrong with gradient descent than the analytic solution, the results from gradient descent+cost function is compared to the analytic solution.

We explore how to solve for the weights with gradient descent on a cost function because in some situation, gradient descent works where the normal equation cannot be applied, like for example in logistic regression, knn.

§1.3.1 Scaling Down

For plotting purposes, it would be ideal to have weights close in value so that the cost function looks decent. I'm going to scale the X by 1000 and the scale the y by 100,000.

The cost function can be contrived with the original values, and the weights correctly come out as w_0=184435.2 and w_1= 229.5, but the cost function does not look good.

Note that, I could scale down with standard feature scaling techniques as well (subtracting the mean and divide by the standard deviation from each value) but I've opted not to because I want to get a scaled down value by tens (ie 184435 -> 1.84) of what we got from sklearn and from the analytic solution, so that all three approaches are comparable.

In [112…
```
X_scaled = X/1000
X_scaled_with_ones_col = np.column_stack((np.ones(len(X_scaled)), X_scaled))
y_scaled = y/100000
```

If we use the normal equation, we get the following weights

In [116…
```
normal_equation(X_scaled_with_ones_col, y_scaled)
```

Out[116… `array([1.84435245, 2.29495219])`

Note that, the weight values changed from

w_0=184435.2 and w_1= 229.5 to **w_0=1.84 and w_1= 2.29**

are much closer in value now, it will be much easier to visualize in a plot.

If the cost function and gradient descent are done correctly below, it will yield w_0=1.84 and w_1= 2.29. w1=2.29.

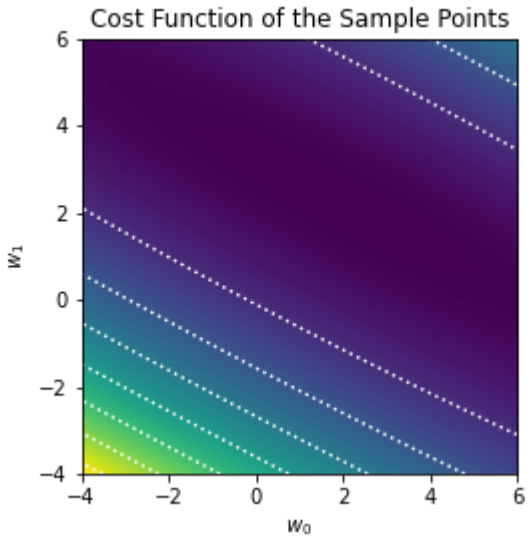
§1.4 Plotting the Cost Function

The cost function, tests all pairs of weights $w_0$and w_1 and plot them in a specified range.

§1.4.1 Contour Plot of the Cost Function

In [804…
```
def plot(g):
    g2 = g*g
    steps = np.linspace(-4, 6, g)
    w0, w1 = np.meshgrid(steps, steps)
    w0_ = w0.ravel()
    w1_ = w1.ravel()
    w = np.column_stack([w0_, w1_])

    def calculateCostArray(X, y, i):
        return computeCost(X, y, w[i])

    def computeCost(X, y, w):
        t = X @ w - y
        t_squared = t * t   # vectorized. element wise squaring
        cost = 1/(2*len(X)) * sum(t_squared)
        return cost

    costs = np.zeros(g2)
    for i in range(g2):
        costs[i] = calculateCostArray(X_scaled_with_ones_col, y_scaled, i)

    costs = costs.reshape(g, g, -1)
    costs = costs.squeeze()

    fig3, ax3 = plt.subplots()
```

```
    ax3.imshow(costs, origin='lower', extent=(w0.min(), w0.max(), w1.min(), w1.max()))
    ax3.set_title("Cost Function of all the Samples in Figure 1")
    ax3.set_xlabel('$w_0$')
    ax3.set_ylabel('$w_1$')
    ax3.contour(w0, w1, costs, colors='white', linestyles=':')

plot(100)
```



this looks good. Upon visual inspection the cost function, I can see that w_0=1.84 and w_1= 2.29 does seem to lie in the darkest part of the cost function. The cost function will be a bowl like shape. Since there's only one feature, I'm aware that there will be only one global minimum. I know this because when I did my hand calculations with just three samples, I've noticed that, the cost function ends up as a 2nd degree polynomial, and that always has a shape of a bowl. Adding more samples to our cost function will still only produce a second degree polynomial, hence one global minimum.

The darkest region within this cost function represents the lowest point of our cost function. We want to let the algorithm know, that has to pay a cost, for the the pair of weights (w0, w1) it picks. Every pair of weights has an associated cost. An algorithm will pick the lowest point on the cost function as this corresponds to the lowest cost.

We have a cost function representation as a contour and a surface plot. Now we need to apply gradient descent to our cost function and hopefully we get w_0=1.84 and w_1= 2.29

§1.5 Gradient Descent

As can be seen in our function definition, we guess the weights as w_0=0 and w_1= 0.

With gradient descent, our objective is to minimize the weights w_0 and w_1 such that the cost is the lowest. Essentially, the algorithm is paying a 'cost' for the parameters it picks, so it 'learns' by picking the weights with the lowest cost.

Basically you are instructing the algorthm, 'Find me the values of w_0 and w_1 that gives me the minimum of a cost function.'

Recap that from the normal equation, what we're supposed to be getting

w_0=1.84 and w_1=2.29

To get started with gradient descent, we define a modified version from Lab 2

In [89]:
```
def gradient_descent(X, y, learn_rate=0.00004, num_steps= 10000):
    w = np.array([0, 0]) # so instead of guessing x, we're guessing w
    for i in range(num_steps):
        grad = (X.T @ X) @ w - X.T @ y  # so.. this is a very useful equation
        w = w - learn_rate*grad
    return w
```

I've noticed that, 0.0004 learning rate is quite low and 10000 iterations is quite high. But these are the values end up yielding the best results.

In [819…
```
w_from_gradient_descent = gradient_descent(X_scaled_with_ones_col, y_scaled)
w_from_gradient_descent
```

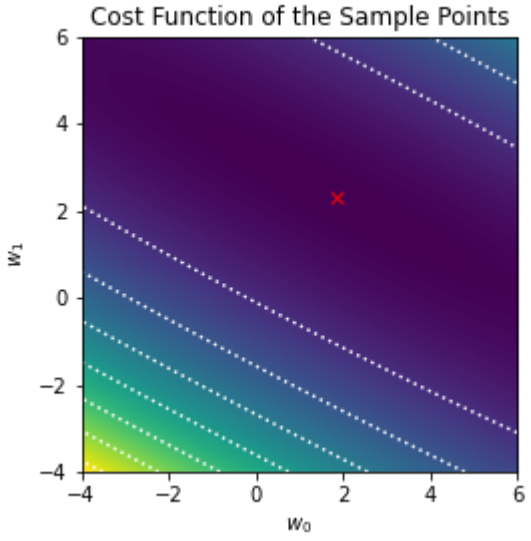
Out[819…
```
array([1.84270283, 2.29581986])
```

With gradient descent, we come up with values w_0=1.84 and w_1= 2.29, the same answer as our above approaches and the answer we were expecting.

§1.6 Gradient Descent on our Cost Function

We can plot the weights on our contour plot we've created in §1.3.1

In [820…
```
plot(100)
plt.plot(*w_from_gradient_descent, 'xr')
```



Out[820…
```
[<matplotlib.lines.Line2D at 0x1dfc9176cd0>]
```

§2.0 Conclusion

Up to now, we have only considered a single feature, the size of the house (sqft). We are limited to only a single feature if we want to be able to plot. If we had more than one feature, we cannot visualize the cost function and gradient descent actually descending. When more features are added, $\mathbb{R}$ moves into higher dimensions and we lose the visual aspect of it all because we cannot

plot the cost function anymore. This is important because a visualizing this greatly helps understand the mechanics of gradient descent for other machine learning techniques.

The plan is to see if our weights make sense. the gradient descent to work and include all the features, such as bathrooms, condition of the house etc.