Truss Analytics Algorithms and Integration in Arkouda

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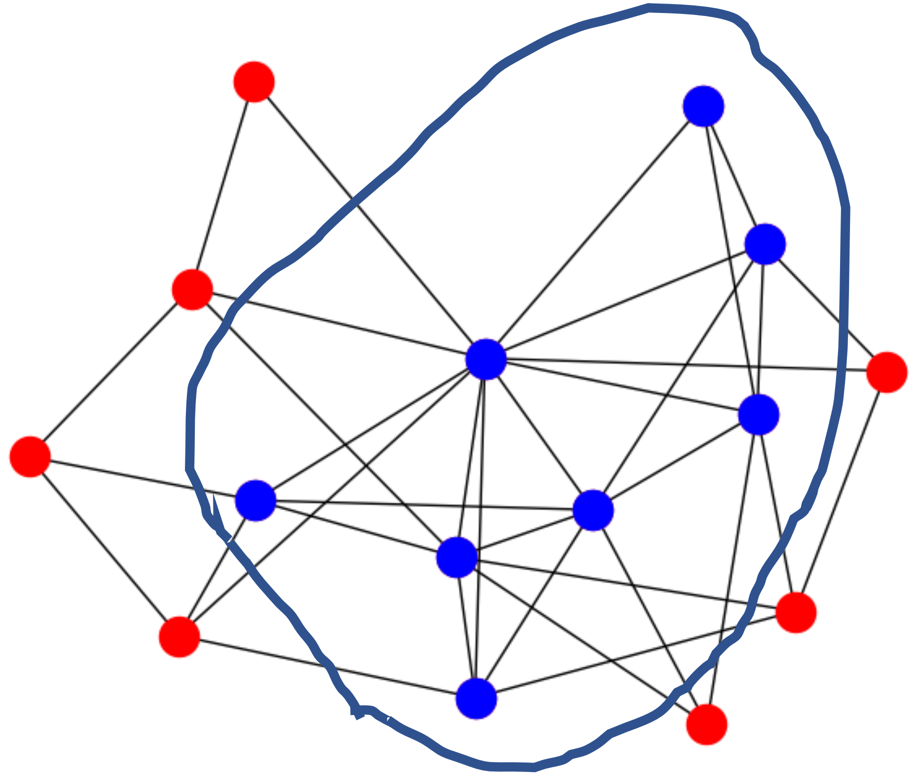
What is a Truss

- K-Truss
  - a cohesive subgraph to explore **close relationships** in a graph
    - every edge must be part of k−2 triangles in the subgraph
  - value K
    - the degree of closeness and the size of the subgraph
    - Larger k means a larger group with a closer connection
  - Widely used
    - polynomial time, a tractable problem (no NP-complete problem) in theory



K=4 truss
Connected by the blue vertices

Why Truss Analytics

- Answer three questions about a group of elements
  - Who are in the **group** that meets k requirement?
    - K-Truss problem
  - Who are in the **maximal group** that all members have the closest connection?
    - Max K-Truss problem
  - What are all the **different groups** that meet the different degrees of relationships?
    - Truss Decomposition problem
- A typical community detection method
  - Explore insights from a large graph

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Algorithm Core and Contributions

- Algorithm Core
  - Triangle Counting
- Contributions
  - A parallel triangle counting core with O(logn) instead of O(n) time
  - Productive and High-Performance Tool for truss analytics
    - Use Chaple to implement the algorithm and Use Python as the Interface (Arkouda)

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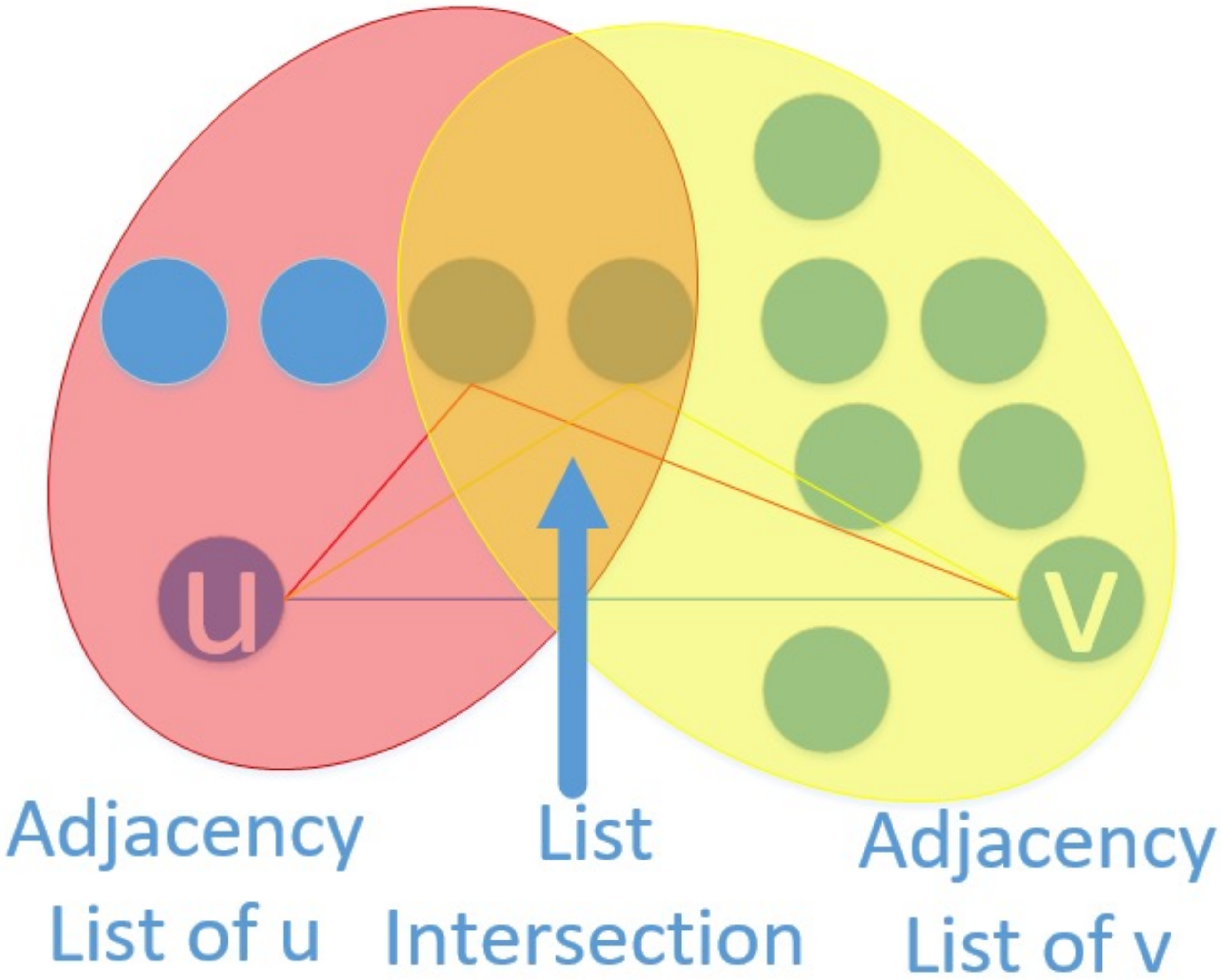
Core Algorithm Description

- Existing Methods
  - List Intersection
    - Binary Search (O(nlog(n)), two adjacency lists
    - Path Merge (O(n)), two adjacency lists
- Proposed Method
  - Minimum Search (three adjacency lists)
    - Time complexity O(logn)
      - degree(u)<degree(v), w in adjacency of u
      - If degree(w)<degree(v)
        - Search v in the adjacency of w
      - If degree(w)>=degree(v)
        - Search w in the adjacency of v
    - degree(u) parallel threads
    - Significant time savings for real-world graphs



Building Parallel k-Truss Algorithms

Algorithm 1: Naive K-Truss Parallel Algorithm

$NaiveKTruss(G, k)$
/* $G = \langle E, V \rangle$ is the input graph with edge set E and vertex set V. k is the given K-Truss value.
$EdgeDel[\,] = -1$ // initialize all edges as not deleted
while *there is any edge can be deleted* **do**
    $sup[\,] = 0$ // initialize the triangle counting array
    forall (*undeleted edge* $e = \langle u, v \rangle \in E$) && ($e$ *is local*) **do**
        calculate $sup(e, G)$ using list intersection or minimum search method
        $sup[e] = sup(e, G)$
    end
    forall ($e = \langle u, v \rangle \in E$) && ($e$ *is local*) **do**
        if ($EdgeDel[e] == -1$) && ($sup[e] < k - 2$) **then**
            $EdgeDel[e] = k - 1$
        end
    end
end
return $EdgeDel$

Calculate # Triangles in parallel

Remove Edges that cannot Support k-2 Triangles

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Building Optimized Parallel K-Truss Algorithms

- Calculate Triangles in Parallel
- Remove edges that cannot support k-2 triangles using minimum search method
- LOOP (search affected edges)
  - Use minimum search method to identify affected edges and update triangle supports
  - Remove edges that cannot support k-2 triangles

Algorithm 2: Optimized K-Truss Parallel Algorithm

```
OptKTruss(G,k)
/* G = ⟨E,V⟩ is the input graph with edges set E and vertices set V. k is the given K-Truss value.
EdgeDel[] = −1 // initialize all edges as undeleted
sup[] = 0 // initialize the support array of each edge
SetDel = ϕ;SetAff = ϕ
forall (edge e ∈ E) && (e is local) do
    sup[e] = sup(e, G) using minimum search method
end
forall (e ∈ E) && (e is local) do
    if (EdgeDel[e] == −1) && (sup[e] < k − 2) then
        EdgeDel[e] = 1 − k
        Add e into SetDel
    end
end
while (SetDel is not empty) do
    forall (e1 ∈ SetDel) && (e1 is local) do
        using minimum search method to find e2 and e3 that can form a triangle with e1
        reduce the support of e2 and e3 if they are undeleted edges
        add the affected edges into SetAff if their supports are less than k − 2
    end
    forall (e ∈ SetDel) && (e is local) do
        if (EdgeDel[e] == 1 − k) then
            EdgeDel[e] = k − 1
        end
    end
    SetDel.clear()
    SetDel <=> SetAff // switch the values of the two sets.
end
return EdgeDel
```

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Building Parallel Max K-Truss Algorithms

- Estimate the upper bound value of kUp
- Modified Binary Search in [3, kUp]
  - The k-truss is not empty but (k+1) truss is
    - If kLow is not empty then check kUp
    - If kUp is empty then check kMid
      - While kMid is empty then update kMid and check again
      - If kMid is not empty then update kLow and search recursively

Algorithm 3: Max K-Truss Parallel Algorithm

$MaxKTruss(G)$
/* $G = \langle E, V \rangle$ is the input graph with edges set E and vertices set V.
Let $k_{low} = 3$ and set k_{up} based on the proposed analysis method
return $DownWardSearch(G, k_{low}, k_{up})$
$function\ DownWardSearch(G, k_{low}, k_{up})$
$EdgeDel = kTruss(G, k_{low})$
if *(All edges have been deleted)* **then**
    return $(k_{low} - 1, EdgeDel)$
end
else
    $EdgeDel = kTruss(G, k_{up})$
    if *(there are undeleted edges in $EdgeDel$)* **then**
        return $(k_{up}, EdgeDel)$
    end
    else
        $k_{mid} = (k_{low} + k_{up})/2$
        $EdgeDel = kTruss(G, k_{mid})$
        while *(All edges have been deleted in $EdgeDel$)* **do**
            $k_{up} = k_{mid} - 1$
            $k_{mid} = (k_{low} + k_{up})/2$
            $EdgeDel = kTruss(G, k_{mid})$
        end
        if $(k_{mid} == k_{up} - 1)$ **then**
            return $(k_{mid}, EdgeDel)$
        end
        else
            $k_{low} = k_{mid} + 1$
            return $DownwardSearch(G, k_{low}, k_{up})$
        end
    end
end

Data Structure Design and Selection

- High-level Set/DistBag and Low-level Arrays
  - Easy<->Performance
  - List intersection using set.contains operation is expensive
- Atomic Variables
  - The number of triangles updated by multiple removed edges
  - The total number of removed edges updated by multiple removed edges
- ForAll/CoForAll
  - Implicit synchronization among different parallel threads

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Experimental Setup

- Data sets
  - 9 real-world graphs
  - 7 synthetic graphs
- Comparison algorithms
  - Naïve List Intersection (set)
  - Naïve Min Search (set)
  - Optimized Min Search (array)
  - Path Merge (not given in the paper)

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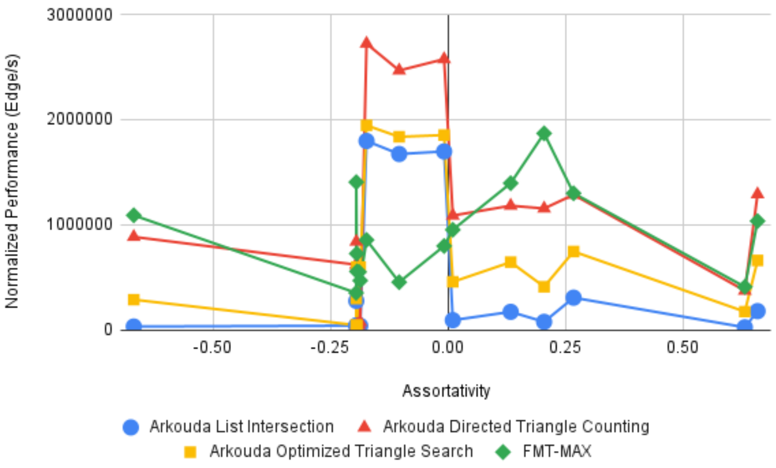
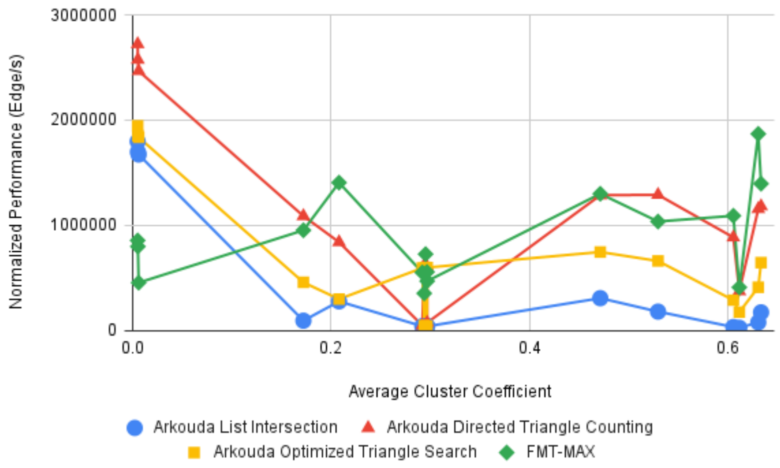
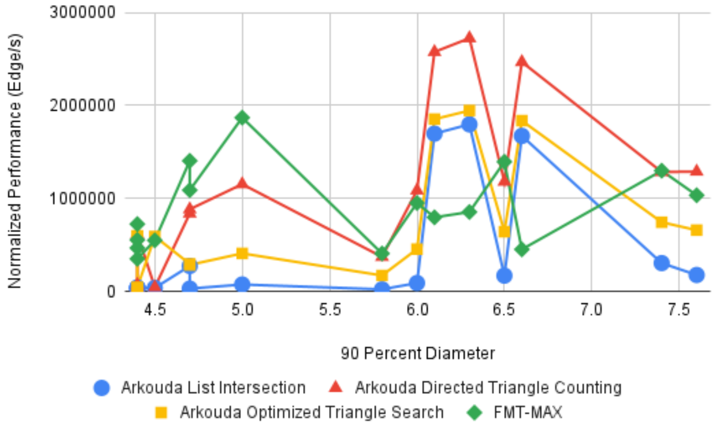
Performance Comparison

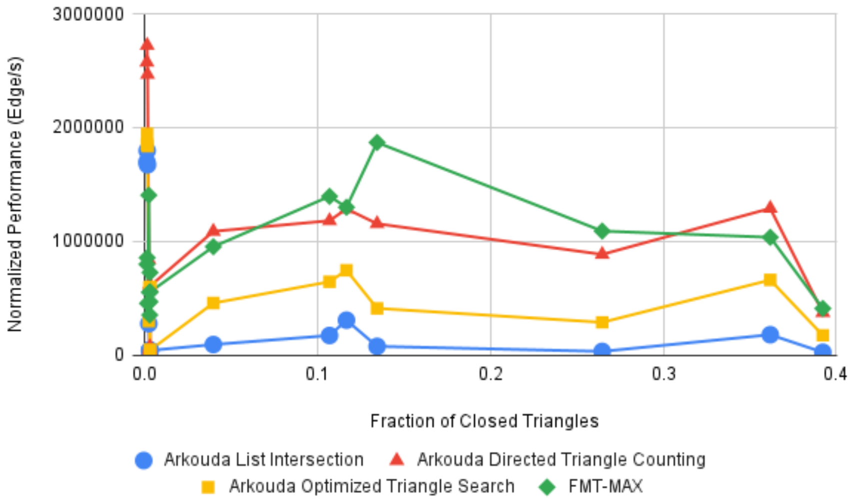
Graph	LI Naive K-Truss	MS Naive K-Truss	MS Opt K-Truss	MS Max K Truss	MS Truss Decomposition	Speedup 1	Speedup 2
amazon0601	1008.58	509.29	60.61	93.22	66.22	2.0	16.6
as-caida20071105	16.70	2.98	1.00	1.73	0.88	5.6	16.7
ca-AstroPh	113.28	56.11	9.64	11.16	5.17	2.0	11.7
ca-CondMat	23.52	11.58	2.11	2.58	2.21	2.0	11.2
ca-GrQc	2.49	1.24	0.29	0.35	0.36	2.0	8.6
ca-HepPh	29.33	14.69	3.07	3.22	3.45	2.0	9.6
ca-HepTh	3.88	1.93	0.50	0.61	0.61	2.0	7.7
com-Youtube	4885.27	302.37	55.72	71.89	61.94	16.2	87.7
delaunay_n10	2.04	1.05	0.08	0.09	0.08	1.9	25.5
delaunay_n11	5.50	2.81	0.16	0.18	0.16	2.0	34.2
delaunay_n12	14.00	7.15	0.32	0.36	0.31	2.0	44.3
delaunay_n13	36.69	18.74	0.62	0.70	0.61	2.0	58.9
delaunay_n14	98.61	50.46	1.23	1.46	1.22	2.0	79.9
delaunay_n15	266.96	136.52	2.49	2.93	2.45	2.0	107.3
delaunay_n16	735.75	378.16	4.91	5.83	4.87	1.9	149.8

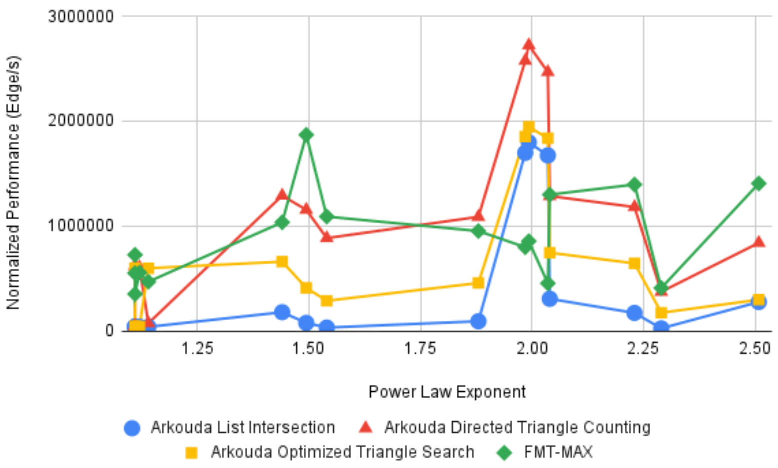
Preliminary Graph topology and Normalized Performance Analysis











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Conclusion

- Algorithm
  - The novel parallel **minimum search**-based method can really improve the performance Truss Analytics Algorithms compared with the widely used existing list intersection methods.
- Language
  - Chapel's high-level data structure (set/distbag…), atomic variables and parallel structure forall/coforall are very helpful to implement the parallel truss algorithms **productively**
- Data Science Environment
  - Arkouda can help **Python users** to employ the provided large-scale truss analytics with high performance

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Thank You!

Q&A