System model

$$x_{k+1} = f_k(x_k, u_k, w_k) \qquad k = 0, 1, \ldots, N-1$$

$x_k \in S$, $u_k \in C(x_k)$, $w_k \in D_k$. D_k is countable.

$P_k(w_k)$ is the probability that the disturbance at time k takes on the value, $w_k \in D_k$.

The disturbances at each time point are <u>independent</u> but may <u>not</u> be <u>identically distributed</u>.
The disturbances at each time point have <u>finite expectation</u>.

$\Pi := \{ (\mu_0, \ldots, \mu_{N-1}),\ \mu_i : S \times L \to C \}$ is the set of admissible control policies.

L is a finite subset of $(0,1)$, the set of confidence levels.

$c : S \to \mathbb{R}$ is a bounded stage cost.

$f_k : S \times C \times D_k \to S$ is bounded (Lipshitz continuous?).

<u>Claim</u>: Lemma 22 of Pflug & Pichler 2015 implies the following CVaR-decomposition for our system model,

$$\mathrm{CVaR}_\alpha\left[\sum_{i=k+1}^{N} c(x_i) \,\middle|\, x_k, \Pi_k \right] = \max_{R \in \mathcal{B}(\alpha, P_k)} \mathbb{E}\left[R \cdot \mathrm{CVaR}_{\alpha R}\left[\sum_{i=k+1}^{N} c(x_i) \,\middle|\, x_{k+1}, \Pi_k \right] \,\middle|\, x_k, \Pi_k \right]$$

time-dependent
Is this okay?

$$= \max_{R \in \mathcal{B}(\alpha, P_k)} \sum_{w_k \in D_k} R(w_k) \cdot \mathrm{CVaR}_{\alpha R(w_k)}\left[\sum_{i=k+1}^{N} c(x_i) \,\middle|\, x_{k+1} = f_k(x_k, \mu_k(x_k, \alpha), w_k), \Pi_k \right] \cdot P_k(w_k),$$

where $\Pi_k := (\mu_k, \ldots, \mu_{N-1})$ is a given policy for the sub-problem starting at time k, $\mu_k : S \times L \to C$ for each k,

and $\mathcal{B}(\alpha, P_k) := \left\{ R : D_k \to [0, \tfrac{1}{\alpha}],\ \sum_{w_k \in D_k} R(w_k) \cdot P_k(w_k) = 1 \right\}$.

What else needs to be specified about $\mathcal{B}(\alpha, P_k)$?
In Pflug & Pichler, R is measurable with respect to some sub-σ-algebra. How do we specify that here?