Early work in chance constrained optimal control developed conditions under which the problem is convex [1,2], which was followed by approximation methods such as a quadratic approximation [3], a conditional value-at-risk (CVaR) approximation [4] which were then extended [5] by solving a series of convex approximations that iteratively tightens the convex bounding function. The problem was also studied within a Model Predictive Control framework [6] using an ellipsoidal set bounding approach, which was then extended to non-convex sets [7]. These ideas were extended by using risk allocation [8] to distribute the risk of violating each chance constraint while still guaranteeing the specified level of safety. The chance constrained optimization problem was also treated with a scenario approach which draws samples for the uncertain parameters and requires the constraints to be satisfied for each sample. In this approach, the number of samples required needs to be determined in order for the original chance constraints to hold with a large probability [9,10]. Another sampling approach is to draw samples from the uncertainty distributions and use binary variables to count the number of constraint violations [11]. This transforms the original stochastic control problem into a deterministic mixed integer program.

Despite this progress, there remain roadblocks in using these algorithms for the applications in this proposal which involve real time robotic motion planning under uncertainty. These include the lack of computational efficiency for use in online control, over-conservativeness of the existing methods for feedback control design in this framework which uses a conservative bounding approach and assumes a fixed risk for the constraints, and the need to consider both uncertain constraint parameters and variables, as well as future constraint parameter sensing.

In our work [12-16], the feedback controller was designed to shape the uncertainty of the system to facilitate the satisfaction of the stochastic constraints. Our work developed a new hybrid approach that uses a combination of sampling and analytic functions to represent the uncertainty. This approach results in a convex optimization program, guaranteeing the optimal solution and reducing the complexity over other methods. The formulation can incorporate future measurements of the uncertain environment to increase the performance of the system. The proposed stochastic control methods can be solved in real-time to plan trajectories for the exploratory system, using both quadrotor unmanned aerial vehicles navigating in a three-dimensional cluttered, uncertain environment. The solution method enables the quadrotors to explore the environment to gather more information, allowing the system to successfully complete its objectives.

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