CSCI-2100 Data Structures Lab
Profiling Dijkstra's Algorithm

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I. Introduction

Dijkstra's algorithm addresses the problem of the minimum distance from a source vertex, v, to every other vertex in the graph.

Dijkstra's algorithm requires a subroutine which returns the index of the univisted vertex which is closest to the source vertex. For this analysis, we will refer to this subroutine as `minVertex`.

The pseudocode is outlined as follows:

```
D := array of best known distances from v
     to every other vertex
while (there exist unvisited verticies):
    call minVertex, returns a vertex v.
    mark v as visited.
    for each neighbor, w, of v:
        if D[w] > D[v] + weight(v, w):
            D[w] = D[v] + weight(v, w)
```

A trivial implementation of `minVertex` can be achieved by linearly searching through the array of best known distances, and returning the index of the unvisited vertex with the smallest value. The time complexity of this implementation is described by:

$$\Theta(|V|^2 + |E|)$$

where the $|E|$ term arises from the fact that we visit each edge once.

A. Comparison

From this we can see that if $|E| \in |V|$, then the upper bound for the run time is $O(|V|^2)$.

The use of a binary-heap in `minVertex` yields a lower time complexity on average relative to the linear implementation

This can be broken down into two components. The $|V| \log |E|$ term comes from the fact that for every vertex, we must call the `minVertex` function. The $|E| \log |E|$ term is the cost of adding an element to the heap for every edge (worst case).

The total runtime is

$$\Theta((|V| + |E|) \cdot \log |E|)$$

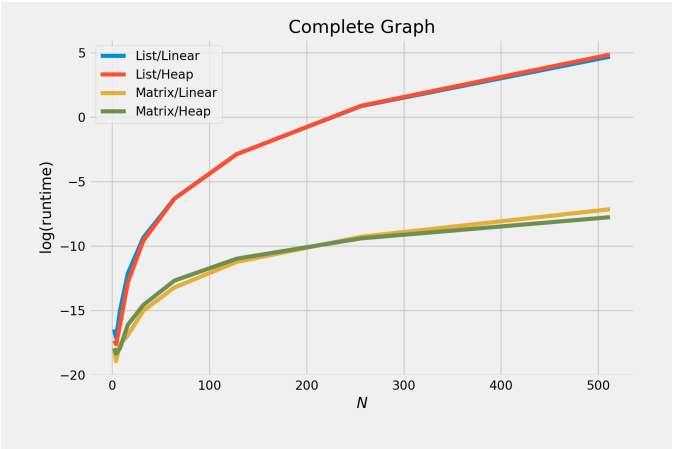
To understand the asymptotic performance of this algorithm, we need to consider a few cases. If $|E|$ is bounded above by $|V|$, (ie. $|E| \in O(|V|)$), then the linear implementation is $\Theta(|V|^2)$, and the heap implementation is $O(|V| \log |V|)$.

II. Results

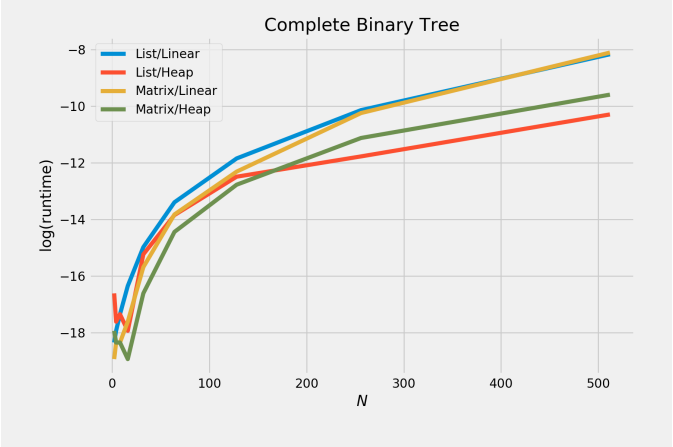
An analysis of each implmentation was tested for three cases, namely:

Fig. 1: Runtime for Dijkstra's Algorithm

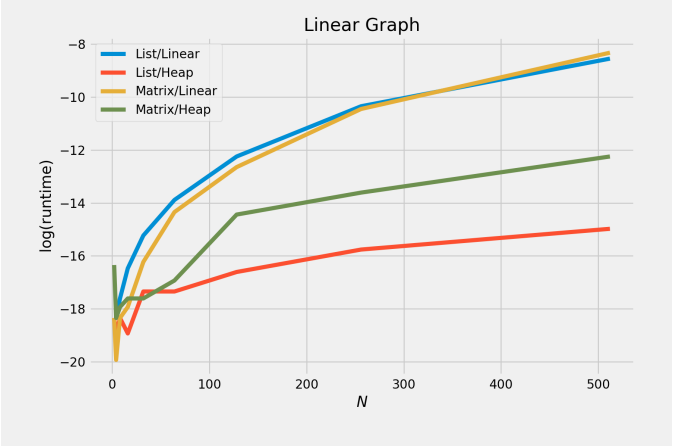
(a)



(b)



(c)



III. Conclusion

If E is sufficiently smaller compared to V (as in E « V / logV), then using heap becomes more efficient.

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TABLE I: Runtimes

	DENSITY	LINEAR	HEAP
Adjacency List	Linear Graph	0.0	0.0
	Complete Binary Tree	0.0	0.0
	Complete Graph	0.0	0.0
Adjacency Matrix		0.0	0.0
		0.0	0.0
		0.0	0.0

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REFERENCES

[1] C. A. Shaffer, *A Practical Introduction to Data Structures and Algorithm Analysis*. Citeseer, 1997.