CSCI-2100 Data Structures Lab
Profiling Dijkstra's Algorithm

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I. INTRODUCTION

Dijkstra's algorithm addresses the problem of finding the shortest path from a

A. Algorithm Complexity

$|V| :=$ the number of vertices in the graph,
$|E| :=$ the number of edges in G

For the linear implmentation of `minVertex`, the run time (for the worst case) is described by:

$$\Theta(|V|^2 + |E|)$$

where the $|E|$ term arises from the fact that we visit each edge once.

B. Comparison

From this we can see that if $|E| \in |V|$, then the upper bound for the run time is $O(|V|^2)$.

The use of a binary-heap in `minVertex` yields a lower time complexity on average relative to the linear implementation

This can be broken down into two components. The $|V|\log|E|$ term comes from the fact that for every vertex, we must call the `minVertex` function. The $|E|\log|E|$ term is the cost of adding an element to the heap for every edge (worst case).

The total runtime is

$$\Theta((|V| + |E|) \cdot \log|E|)$$

To understand the asymptotic performance of this algorithm, we need to consider a few cases. If $|E|$ is bounded above by $|V|$, (ie. $|E| \in O(|V|)$), then the linear implementation is $\Theta(|V|^2)$, and the heap implementation is $O(|V|\log|V|)$.
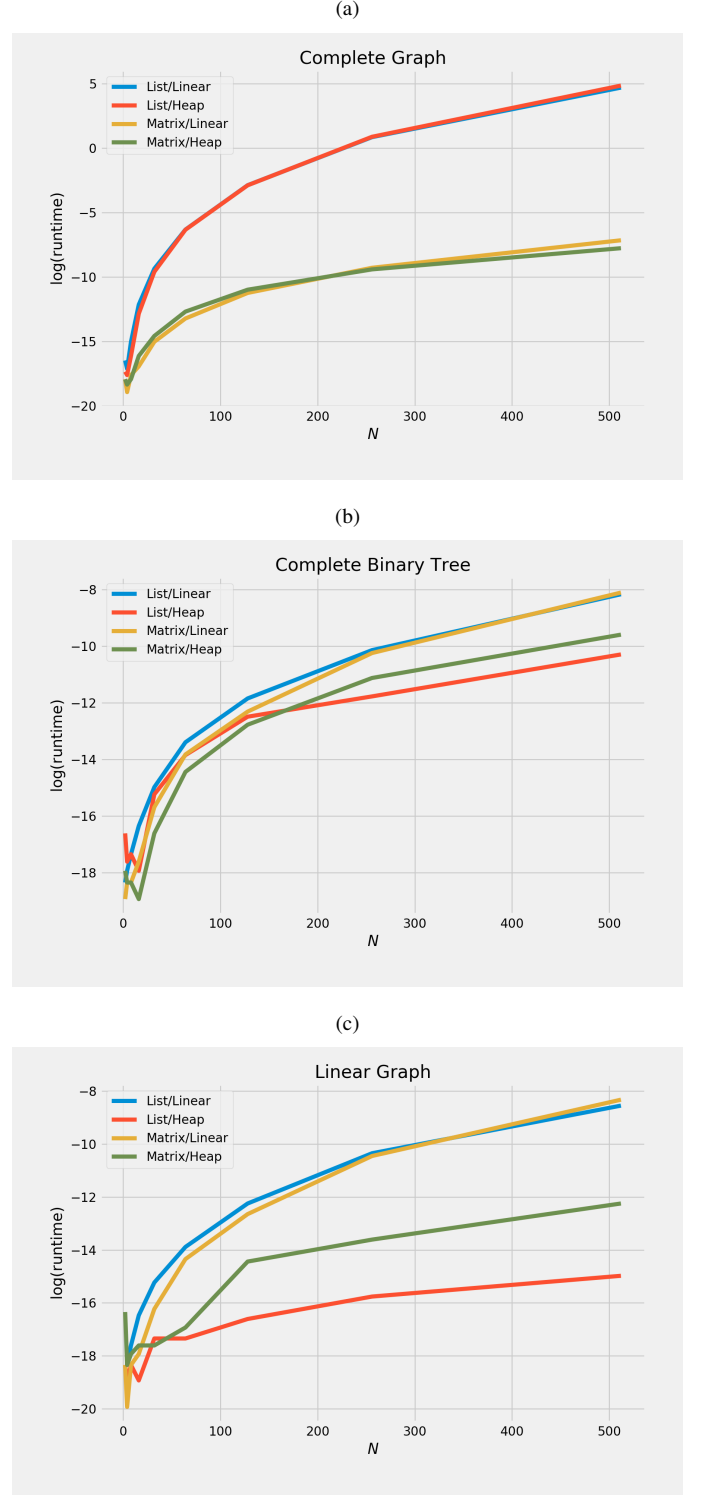
II. RESULTS

An analysis of each implmentation was tested for three cases, namely:

$$\begin{cases} |E| = |V| - 1 \\ |E| = |V| \\ |E| = |V|(|V| - 1)/2 \end{cases}$$

TABLE I: Runtimes

	DENSITY	LINEAR	HEAP
	$|E| \in |V|/2$	0.0	0.0
Adjacency List	$|E| \in |V|$	0.0	0.0
	$|E| \in |V|^2$	0.0	0.0
	$|E| \in |V|/2$	0.0	0.0
Adjacency Matrix	$|E| \in |V|$	0.0	0.0
	$|E| \in |V|^2$	0.0	0.0

Fig. 1: Runtime for Dijkstra's Algorithm

(a)

(b)

(c)

III. CONCLUSION

If E is sufficiently smaller compared to V (as in E « V / logV), then using heap becomes more efficient.

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REFERENCES

[1] C. A. Shaffer, *A Practical Introduction to Data Structures and Algorithm Analysis*. Citeseer, 1997.