Diverse Online Feature Selection

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Abstract—Online feature selection has been an active research area in recent years. We propose a novel diverse online feature selection method based on Determinantal Point Processes (DPP). Our model aims to provide diverse features which can be composed in either a supervised or unsupervised framework. The framework aims to promote diversity based on the kernel produced on a feature level, through at most three stages: feature sampling, local criteria and global criteria for feature selection. In the feature sampling, we sample incoming stream of features using conditional DPP. The local criteria is used to assess and select streamed features (i.e. only when they arrive), we use unsupervised scale invariant methods to remove redundant features and optionally supervised methods to introduce label information to assess relevant features. Lastly, the global criteria uses regularization methods to select a global optimal subset of features. This three stage procedure continues until there are no more features arriving or some predefined stopping condition is met. We demonstrate based on experiments conducted on that this approach yields better compactness, is comparable and in some instances outperforms other state-of-the-art online feature selection methods.

1 Introduction

Every day more and more domains are increasing the breadth and depth of their data every year. It becomes critical to find ways to create compact and interpretable representations of our data [1]. In this paper we focus on the problem of diverse online feature selection, where diversity is defined in terms of the features themselves, and online enabled means that the feature streams may arrived in mini-batch format or stream-wise fashion.

In this paper we will consider the online feature selection problem, where features flow into the model dynamically, this can be in groups or one by one. As the features arrive, a feature selection process is performed. This formulation differs from the typical online learning problem, where the feature space is assumed to remain constant while new instances are shown to the model and the weights subsequently updated [2].

Existing techniques generally do not consider diversity and instead rely on other measures, whether it be through use of a regularizer, statistical tests or correlation measures for feature selection. To this end, we propose an online feature selection approach called *Diverse Online Feature Selection* (DOFS). Our framework is composed of three stages: feature sampling, local criteria and global criteria for feature selection. In the feature sampling, we sample incoming stream of features using conditional DPP. In the local criteria, this is used to assess and select features only when they arrive, we use unsupervised scale invariant methods to remove redundant features and optionally supervised methods to introduce label information to assess relevant features. Lastly we use global criteria which uses regularization methods to select a globally optimal subset of features. This three stage procedure continues until there are no more features arriving or some predefined stopping condition is met.

This work makes the following contributions.

- We propose using conditional DPP as a means for selecting diverse features from stream of features. In order to do so, we provide a new and novel truncated DPP sampling algorithm.
- To evaluate a stream of features, we introduce an unsupervised, scale invariant criteria to remove redundant features and supervised approach to address the shortcomings of using only DPP for sampling the feature stream.
- our proposed *Diverse Online Feature Selection* (DOFS) achieves the strong classification results whether working in supervised or unsupervised framework

The paper is organized into the following sections. Section 2 we will lay the preliminary foundations and review related approaches to the online feature selection problem. In section 3, we will introduce our framework for Diverse Online Feature Selection (DOFS). In section 4 we will provide experimental results to demonstrate the effectiveness of DOFS. We will conclude this work in section 5.

2 Preliminaries and Related Work

In this section we first give a review of offline feature selection and the state-of-the-art online feature selection counterparts. Representative methods reviewed are Grafting, Alpha-investing, Online Streaming Feature Selection (OSFS), Online Group Feature Selection (OGFS). Afterwards, we will provide a review of determinantal point processes and the feature sampling problem.

2.1 Feature Selection

Traditionally, feature selection has been performed in an offline setting. The feature selection problem can be framed as follows. We are given a matrix $X = [x_1, \dots, x_n] \in \mathbb{R}^{d \times n}$ which has n instances and d-dimension feature space $F = [f_1, f_2, \dots, f_d] \in \mathbb{R}^d$. The goal of feature selection is to selection a subset of the feature space such that $U \in \mathbb{R}^l$ where l is the number of desired features, where in most cases $l < d$ [3]. Offline feature selection is a widely studied topic with

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many different reviews [1]. Rather than provide a comprehensive review, we will instead focus on several selected techniques and their online feature selection counterparts. We will cover feature selection from two perspectives as a filter and wrapper method. From the filter approach, we will consider batch approaches using statistical significance and spectral feature selection as well as their online variants being Online Streaming Feature Selection and Online Group Feature Selection respectively. We will also consider the wrapper methods in the batch setting using regularization and information criterion approaches, as well as the online variants being grafting and alpha-investing respectively.

For completeness, the third approach for feature selection is the embedded method which perform feature selection in the process of training as they are specific to models. Approaches here could include decision tress such as CART, which have built-in mechanism to perform feature selection [1]. To the best of our knowledge, there are not any embedded methods present from an online feature selection perspective.

2.1.1 Correlation Criteria and OSFS

The first approach uses the filter method, which evaluates features by certain criterion and select features by ranking their evaluation values or by some chosen threshold.

One common approach is to consider the correlation related criteria [1] such as mutual information, maximum margin, or independence criterion. Of particular interest is conditional independence criterion which is constructed through consideration of relevance and redundancy of features in terms of condition independence [4]. In this setting the process of labelling a feature to be relevant or redundant is performed using statistical tests based on conditional independence.

Online Streaming Feature Selection (OSFS) uses this framework of relevance and redundancy to determine whether incoming features are added. When a feature arrives, OSFS first analysis correlation with the label and determines whether the feature is relevant [5]. Once a feature is successfully chosen, then OSFS performs redundancy test to determine if both previous and current features are redundant and can be removed. In this setting the redundancy is a key component of OSFS approach.

Spectral Feature Selection and OGFS

A similar approach which also uses statistical tests and falls under the filter method for feature selection is the use of the spectral feature selection. In spectral feature selection a graph is constructed. From this graph where the ith vertex corresponds to $\mathbf{x_i} \in X$, with an edge between all vertex pairs. In this graph construct its adjacency matrix W, and degree matrix D. The adjacency matrix is constructed differently depending on the supervised or unsupervised context. In the spectral analysis setting the adjancy matrix can be the similarity metric of choice [6], [7]. For example in the unsupervised context this is can be the RBF kernel function [6], [7], or a weighted sum of correlation metric and rank coefficient metric [8]. Once the appropriate metric is chosen then, a feature ranking function is used for filtering the features. This function can change depending on context, and can be constructed. The choice of this function can be used to determine the statistical significance of each individual feature using trace ratio criterion approach [9].

To extend Spectral feature selection to the online setting, the Online Group Feature Selection (OGFS) has been proposed which considers incoming *groups* of features and applying spectral feature selection on a group-wise level. This is used to determine the relevancy over the particular group of features which has been shown to extend into the online setting.

2.1.2 Regularization and Grafting

The wrapper method, which uses the machine learning algorithm of interest as a black box to score subsets of features.

Regularization is typically labelled as a wrapper method in the feature selection framework, meaning that it uses a model algorithm to jointly build a model and select features. This is typically employed through both minimizing empirical error and a penalty. In the context of regularization, the goal is to encourage sparsity on the feature subset. Regularizer penalties are typically framed as [10]

$$\Omega_p(\theta) = \lambda \sum_{i=1}^{m} \alpha_{\mathbf{i}} |\theta_{\mathbf{i}}|^p$$

where a choice of $p = 1$ is typically chosen to promote sparsity, commonly referred to as the Lasso penalty.

To alter this framework to an online setting, the grafting algorithm is used. Grafting is performed on any model which can be subjected to Lasso regularizer. The idea behind grafting is to determine whether the addition of a new feature would cause the incoming feature or alternatively, any existing feature to have a non-zero weight. With a chosen parameter λ, the regularizer penalty is then $\lambda|w_j|$. Thus gradient descent will accept a new incoming feature w_j if:

$$\left| \frac{\partial \bar{\mathcal{L}}}{\partial w_j} \right| > \lambda$$

where $\bar{\mathcal{L}}$ is the mean loss. In other words, if the reduction in $\bar{\mathcal{L}}$ outweighs the regularizer penalty $\lambda|w_j|$, then the new incoming feature w_j will be chosen. If this test is not passed, then the feature is discarded. As Grafting makes no assumption on the underlying model, it can be used in both linear and non-linear models.

2.1.3 Information Criterion and Alpha-investing

Another approach to feature selection in the wrapping sense is the usage of penalized likelihoods. In the context of single pass feature selection techniques, penalized likelihoods are preferred [11]. This set of approaches can be framed as:

$$-2\log(\text{likelihood}) + F$$

where the parameter F indicates how a criterion is to penalize model complexity directly.

The alpha-investing algorithm [11], makes use the information in order to determine whether a new incoming stream of features is considered to be relevant or not. It makes use of the *change* in log-likelihood and is equivalent to a t-statistic, which means a feature is added to the model if its p-value is greater than some α. Alpha-investing works through adaptively controlling the threshold for adding features. This works through increasing the wealth α when a feature is chosen to reduce the change of incorrect inclusion of features. Similarly when a feature is assessed wealth is "spent", which reduces the threshold, in order to avoid adding additional spurious features.

In contrast to the previous work, we will tackle feature selection through the use of feature sampling through determinantal point processes.

2.2 Determinantal Point Process

We begin by reviewing determinantal point processes (DPPs) and conditional DPP.

A point process $\mathcal{P}$ on a discrete set $\mathcal{Y} = \{1, 2, \dots, N\}$ is a probability measure over all $2^{\mathcal{Y}}$ subsets. $\mathcal{P}$ is a determinantal point process (DPP) if $\boldsymbol{Y}$ range over finite subsets of $\mathcal{Y}$, we have for every $A \subseteq \mathcal{Y}$

$$P(\mathcal{A} \subseteq \boldsymbol{Y}) = \det(\mathbf{K}_{\mathcal{A}})$$

where $K \in \mathbb{R}^{M \times M}$ is a positive semidefinite kernel matrix, where all eigenvalues of K are less than or equal to 1. An alternative construction of DPP is defined by L-ensembles where L_{ij} is a measurement of similarity between elements i and j, then DPP assigns higher probability to subsets that are diverse. The relationship between K and L has been shown to be [12]

$$K = (L + I)^{-1} L$$

Where I is the identity matrix. Then the choice of a specific subset Y is shown to be [12]

$$\mathcal{P}_L(\boldsymbol{Y} = Y) = \frac{\det(L_Y)}{\det(L + I)}$$

2.2.1 Conditional Determinantal Point Process

In our situtation, often we would like to sample future unchosen/unseen points with the additional constraints based on the currently chosen features. Suppose that we have input X and set $\mathcal{Y}(X)$ of iterms dervied from the input. Then conditional DPP is defined to be $\mathcal{P}(\boldsymbol{Y} = Y|X)$ which is a conditional probability that assigns a probability to every possible subset $Y \subseteq \mathcal{Y}(X)$. Then the model will take form

$$\mathcal{P}(\boldsymbol{Y} = Y|X) \propto \det(L_Y(X))$$

DPP have demonstrated its use in discovering diverse sample points which has found use in applications such as computer vision and document summarisation [12] [13]. In this context we will consider sampling feature vectors.

Algorithm 1 Conditional Feature Sampling using DPP

```
Input: Best candidate feature set: $X \in \mathbb{R}^{d \times n}$ , new set of
    features $G$, reconstruction error $\alpha$
Output: Sample of features from $G$
1: Construct similarity matrix $L$ based on $X \cup G$.
2: Sample features from $G$ conditioning on $X$ using conditional DPP
```

Assuming that the similarity matrix and eigenvalues decomposition L is provided, DPP sampling has been shown to have complexity $O(k^3)$ [14] though Markov Chain DPP sampling (under certain conditions) is linear in time with respect to the size of data [15].

As the above algorithm is inherently unsupervised (i.e. makes no assumption on the response vector). This sampling approach could easily be suitable for both supervised and unsupervised problems. Furthermore, we propose two different approaches for removing redundant features; first approach in an unsupervised, scale-invariant manner and second in a supervised way, leveraging the label information to improve the consistency of the features chosen.

2.3 Local Criterion

Feature sampling alone is insufficient to provide suitable subset of features without redundancy. Although DPP seeks to promote diversity within its features it may not necessarily remove all redundant features. Depending on choice of kernels, kernels may not necessarily be scale invariant and almost never consistent with respect to response. In order to address both of these concerns, we turn turn to other criteria to promote further compactness and reduce redundancy in the feature selection framework; irrespective of the type of kernel chosen.

2.3.1 Unsupervised Criterion

In order to address the *scale-invariant* aspect, we turn towards non-parametric pair-wise tests to remove redundant features, such as the Wilcoxon signed-rank test [16]. In our scenario, any two pairs of features can be viewed as a pair of measurements.

If N is the sample size, and the pairwise measurements are x_i, y_i for the ith measurement for feature x and y respectively, then the test statistic is calculated through first ranking the pairs by smallest to largest absolute difference, $|x_i - y_i|$. Each pair is then given a rank, in this scenario we will define R_i to be the rank of the ith ranked pair. Then the statistic is calculated as

$$W = \sum_{i=1}^{N} (\text{sign}(x_i - y_i) R_i)$$

where W converges to approximately normal distribution, with z-score is given by

$$z = W / \left(\sqrt{\frac{N(N+1)(2N+1)}{6}} \right)$$

Here we propose Wilcoxon signed-rank test to remove any incoming features which are redundant compared with the present features.

As the Wilcoxon signed-rank test requires sorting along a vector of size d, and all other computation are simple arithmetic, then a single test will have complexity $O(d \log(d))$, as this test is repeated k times under the proposed Wilcoxon criterion, then it has complexity $O(kd \log(d))$.

Although redundancy would already be minimised due to the nature of DPP sampling, Wilcoxon signed-rank test will provide an approach to removing redundant features

Algorithm 2 Wilcoxon Criterion

```
Input: Best candidate feature set: $X \in \mathbb{R}^{d \times k}$, proposed single new feature $f$, significance level $\alpha$
Output: Boolean, if feature $f$ is discarded or kept
for each feature $x$ in $X$ do
    $p \leftarrow$ Wilcoxon signed-rank test.
    If $p > \alpha$. Then discard $f$ and terminate, otherwise continue
end for
Keep feature $f$
```

which will help augment the existing approach through addressing *scale-invariant* aspect which would have been missed. In addition to using this criteria to detect and remove redundant features in a scale invariant way, it is also worthwhile to incorporate information relating to our label in order to select features that are consistent with our label.

2.3.2 Supervised Criterion

Another approach is to make use of information embedded in our label vector Y. Under this situation it would help address *consistency* aspect which DPP alone would fail to account for.

Our criteria is based class separability critera in conjunction with trace ratio criterion [17] and criterions devised by Wang et al. (2015). We will define the selected feature set to be U, S_w to be the within class scatter matrix and S_b to be the between class scatter matrix. There are several ways for class separability to be defined:

First it can be defined using the mean and variance measures of the class [18]:

$$S_w(U) = \sum_{j=1}^{c} \pi_j \sigma_j$$

$$S_b(U) = \sum_{j=1}^{c} (\mu_j - M_o)(\mu_j - M_o)^T$$

$$M_o(U) = \sum_{j=1}^{c} \pi_j \mu_j$$

Where π_j is the priori probability that a pattern belongs to class y_j, U is the current candidate feature vector, μ_j is the sample mean vector of class y_j, M_o is the sample mean vector for the enture data point, σ_j is the sample covariance matrix of class y_j.

Similarly it can be constructed through the use of any kernel to define measure of similarity [19]:

$$S_w(U) = \frac{1}{c} \sum_{j=1}^{c} \frac{1}{N_j^2} \left(\sum_{k=1}^{N_j} \sum_{l=1}^{N_j} ||x_k^{(j)} - x_l^{(j)}||^2 \right)$$

$$S_b(U) = \frac{2}{c(c-1)} \sum_{i=1}^{c} \sum_{j=1, j \neq i}^{c} \frac{1}{N_i N_j} \left(\sum_{k=1}^{N_j} \sum_{l=1}^{N_j} ||x_k^{(j)} - x_l^{(j)}||^2 \right)$$

Where c represents the total number of classes for the supervised classification problem.

Furthermore class separability can also be defined using the label information directly [3]:

$$S_w(U) = \begin{cases} \frac{1}{n} - \frac{1}{n_c} & y_i = y_j = l \\ \frac{1}{n} & \text{otherwise} \end{cases}$$

$$S_b(U) = \begin{cases} \frac{1}{n_c} & y_i = y_j = l \\ 0 & \text{otherwise} \end{cases}$$

Where n_c represents the number of instances in class c.

Using any of the between and within class separation criteria defined above, we can use use these to determine whether a feature is informative or not. The feature level criterion we will define based on a single feature f:

$$s(f) = \frac{S_b(f)}{S_w(f)}$$

We can extend this to yield a score for a subset of features based on a subset of features U, where the goal would be to maximise the following criterion:

$$F(U) = \frac{\text{tr}(S_b(U))}{\text{tr}(S_w(U))}$$

Both of these criterion can be used to select a stream of features.

Supervised Criterion 1 *Given U to be the previously selected subset, x_i denoting the newly arrived feature. Then feature x_i will be selected if*

$$F(U \cup f) - F(U) > \epsilon$$

where ϵ is a small positive parameter.

Supervised Criterion 2 *Given U to be the previously selected subset, f denoting the newly arrived feature. Then feature f will be selected if it is a significant feature with discriminative power*

The significance of the feature can be evaluated by t-test

$$t(f, U) = \frac{\hat{\mu} - s(f)}{\hat{\sigma}/\sqrt{|U|}}$$

Where $\hat{\mu}, \hat{\sigma}$ are the sample mean and standard deviation of scores of all features in U. If the t-value reaches the chosen significance level (in experiments conducted here, chosen to be 0.05) then the feature is assumed to be significant.

Algorithm 3 Supervised Criterion

```
Input: Incoming set of features: $U \in \mathbb{R}^{d \times k}$, significance level $\alpha$
Output: A set $G$, representing the set of selectioned features
    Initialize: $G = \{\}$
Sort $U$ according to function $s$
for each feature $f$ in $U$ do
    If $F(G \cup f) - F(G) > \epsilon$ then $G = G \cup f$
    If $t(f, G) > \alpha$ then $G = G \cup f$
end for
Return $G$
```

As both of these criterion are in linear time [3], then the remaining complexity comes from the construction of the class separability critera. The class separability critera has different time complexity depending on the choice of criterion. In the class separation criterion from Mitra et all (2002), it relies on the construction of a covariance matrix, with all other operations being simple arithmetic operations. As the complexity of covariance matrix calculation is $O(k^2d)$, this suggests that the criteria is of complexity $O(ck^2d)$, as the covariance is needed to be computed for each class, and dominates this criterion. Similarly for the class separation which uses the kernel, the time complexity is $O(c^2N^2)$. However if we use class separation criterion which uses the label information directly, then it would be in linear time as well [3].

In our supervised criterion, we will accept features if they pass either **supervised criterion 1** or **supervised criterion 2**. It can also be used in conjunction with unsupervised criterion to result in providing additional representative features.

After the various criterions which are selected is run, we can proceed with global criterion to remove redundant features both assessed from the streaming process and previously accepted features.

2.4 Global Criterion

Similar to approaches used by Grafting [10], we also use regulariser to remove redundant features after the conditional sampling step is complete. This approach was also used in OGFS algorithm under "inter-group selection" criteria which used the Lasso regulariser specifically [3]. In this setting we will consider elasticnet implementation as an alternative to using lasso to promote sparsity. The regularizer penalty is framed as

$$\Omega_p(\theta) = \lambda \sum_{i=1}^{m} \alpha_{\mathbf{i}} |\theta_{\mathbf{i}}|^p$$

Where Elasticnet penalty is specifically $\alpha_1\Omega_1 + \alpha_2\Omega_2$ which is elasticnet, typically chosen where $\alpha_1, \alpha_2 > 0$ and $\alpha_1 + \alpha_2 = 1$

Similar to the approach taken by Lasso methods, elasticnet can be used to select features by having some tolerate $\lambda \geq 0$ in mind [20]. Without loss of generality, assume that the coefficient of a predictor for a particular feature f, is β_f, then we will remove a feature if:

$$|\beta_f| < \lambda$$

Using this, we can now form our global criterion.

Algorithm 4 Global Criterion

```
Input: Incoming set of features: $U \in \mathbb{R}^{d \times n}$, tolerance level $\lambda$
Output: A set $G$, representing the set of selectioned features
  Initialize: $G = \{\}$
  Fit a model with elasticnet regularizer
  for each feature $f$ in $U$ do
      If $|\beta_f| \geq \lambda$ then $G = G \cup f$
  end for
  Return $G$
```

3 Framework for Diverse Online Feature Selection

The framework for online feature selection is as follows. First, assume the current best candidate subset model matrix $G = [x_1, \ldots, x_n] \in \mathbb{R}^{d \times n}$, where d is the number of selected features and n is the number of instances. Let the incoming matrix $\mathbf{G}'$ be $\mathbf{G}' = [x'_1, \ldots, x'_n] \in \mathbb{R}^{(d+m) \times k}$, where m is the number of newly available features. Without loss of generality we can assume that $\mathbf{G}' \in \mathbb{R}^{(d+m) \times n}$, that is the incoming feature stream have the same number of instances as the best subset model matrix. Then the difference between the new batch and best subset is that the new incoming stream of data contains additional features.

Then the online feature selection problem at each iteration selects the best subset of features of size m', where $0 \leq m' \leq m$.

$$\begin{bmatrix} \text{G} \\ \end{bmatrix}_{d \times n} \longrightarrow \begin{bmatrix} \text{G}' \\ \\ \\ \end{bmatrix}_{(d+m) \times n} \longrightarrow \begin{bmatrix} \text{G}'' \\ \end{bmatrix}_{(d+m') \times n}$$

If the initial best subset was size d and there were an additional m features available to be selected, the online feature selection algorithm will then select $d + m'$ features.

3.1 Diverse Online Feature Selection

Algorithm 5 Diverse Online Feature Selection

```
Input: Best feature candidate feature set $X$, Feature stream $F$, label vector $Y$
Output: A set $G$, representing the set of selectioned features
  while features are arriving do
      $G \leftarrow$ generate new group of features
      Sample features from $G$ using DPP to get sampled subset $G'$ conditional on $X$
      for $f$ in $G'$ do
          Local Criterion: Evaluate feature $f$ using unsupervised and/or supervised criterions to determine relevancy
          Global Criterion: Perform redundacy check based on regulariser
      end for
  end while
```

As the complexity of the various parts have been touched on in the previous sections, we can put them all together to get the overall complexity of DOFS. If a single iteration has the best candidate feature set to be $G \in \mathbb{R}^{d \times n}$, with a stream of new data of size $F \in \mathbb{R}^{(d+m) \times n}$. Then the complexity of DPP sampling will be, $O((d+m)^3)$ where m represents the number of features available to be selected from the feature stream after DPP sampling. The unsupervised criterion will then have complexity at most $O((d+m)\log(d+m))$ and supervised criterion will have complexity at most $O(cn^2(d+m))$ or as little as being linear in time.

Data Set	#instances	#dim.
Ionosphere	351	34
Spambase	4601	57
Spectf	267	44
Wdbc	567	30
Colon	62	2000
Leukemia	72	7129
Lung Cancer	181	12533
Prostate	102	12600

Overall the worse case complexity will be $O((d+m)^3) + O(cn^2(d+m)) = O(\max((d+m)^3, cn^2(d+m))$. Where n represents the number of incoming instances used to update our feature selection, and m is the number of new available features. If we use the class separation criteria which has linear time complexity, then the overall complexity will reduce to DPP sampling, i.e. $O((d+m)^3)$.

4 Experiments

Various experiments were conducted to validate the efficiency of our proposed method. We used several benchmark datasets, several other state-of-the-art online feature selection methods are used for comparison including Grafting, OSFS, and OGFS. The classification accuracy, log-loss and compactness (the number of selected features) are used to measure performances of the algorithms in our experiments.

We divide this section into three sub-sections, including introduction to our data sets, the experimental setting and the experimental comparisons.

4.1 Benchmark Data Sets

The benchmark datasets are from UCI Machine Learning Repository, and the Micro Array datasets. The information of these datasets are described in the table below.

There are four datasets from UCI repository (Ionosphere, Spambase, Spectf, Wdbc), and four datasets from microarray dataset (colon, leukemia, lung cancer, prostate).

4.2 Experimental Settings

In our experiments, Grafting and OGFS used elasticnet setup with $\lambda = 0.15$ for the regularizer penalty and inter-group selection parameters respectively. For OSFS, OGFS, DOFS the threshold parameter α is set to 0.05.

To simulate online group feature selection, a similar setup by Wang et al. was followed. The group structures of the feature space was simulated by dividng the feature space as a global feature stream by streaming features in groups of size m. In our experiements we set $m \in [5, 10]$ as suggested by Wang et al.. Models were compared using using existing Matlab implementations such as the LOFS library [21], whilst the DOFS implementation was completed in Python using scikit-learn library. The DOFS models include the unsupervised variant (without consideration of class separability), and supervised variant using the criteria which used the label information directly.

4.3 Experimental Results

Comparison of DOFS variants

If we consider the three variants of DOFS, the usefulness of both the supervised and unsupervised algorithms are clearly warranted as if we consider accuracy to be metric of interest, supervised/unsupervised variant has better accuracy in 4 of 8 models. However, in the situations which supervised variant underperforms, the difference with the unsupervised variant is much lower. In the results above, it is clear that the unsupervised variant promotes greater compactness over the supervised variant. This can be thought of as the algorithm allowing more "chances" for a feature to be accepted and passed through the model. This is further highlighted by the difference when there is no redundancy check placed as in the variant which only uses DPP. In this setting there is a distinct possibility that extrenous set of features is selected despite the use of conditional DPP, which comes at a cost of performance, as can be observed in all the Micro Array datasets, where the number of features selected is at least 10 times, and in some cases 100 times larger than the other two variants provided.

Data Set	Alpha-investing		OSFS	
	#dim	accu.	#dim	accu.
Ionosphere	10	87.18	8	79.93
Spambase	45	77.18	54	60.99
Spectf	7	79.40	5	79.09
Wdbc	21	71.53	10	62.74
Colon	4	79.76	4	85.48
Leukemia	16	66.67	5	91.83
Lung cancer	69	86.67	7	83.43
Prostate	25	97.09	5	91.84

Data Set	Grafting		OGFS	
	#dim	accu.	#dim	accu.
Ionosphere	32	91.76	26	88.26
Spambase	50	92.28	24	91.07
Spectf	37	80.36	5	71.27
Wdbc	24	94.82	18	96.07
Colon	26	84.26	102	90.47
Leukemia	13	94.53	63	100
Lung cancer	19	96.53	33	99.44
Prostate	17	95.53	96	98.00

Overall from the results above comparing against either the supervised or unsupervised DOFS algorithm, we can see that DOFS generally has superior performance compared with Alpha-investing and OSFS algorithms, whilst it seems to be competitive with Grafting and OGFS. In generally there is a trade-off between compactness and performance; where it would perform better than alpha-investing and OSFS algorithm whilst being less compact, and competitive with Grafting and OGFS whilst having better compactness. What is interesting is that DOFS algorithm demonstrates inferior performance against all methods when using the Prostate dataset.

DOFS vs Alpha-investing

Both variants of DOFS manages to outperform alpha-

Data Set	DOFS (DPP only)		DOFS (Unsupervised)		DOFS (Supervised)	
	#dim	acc.	#dim	accu.	#dim	accu.
Ionosphere	11	87.75	9	88.12	23	86.47
Spambase	24	86.44	10	82.54	37	88.26
Spectf	10	79.40	31	79.26	29	79.57
Wdbc	8	86.29	13	86.62	13	86.34
Colon	750	90.32	47	95.70	38	94.52
Leukemia	836	63.37	5	68.41	58	100
Lung cancer	1366	94.48	7	91.08	88	98.37
Prostate	1441	78.43	42	92.02	34	86.61

investing in 6 of the 8 datasets. Excluding Prostate dataset, in the ionosphere the performance is within 2%. When comparing compactness, alpha-investing is generally more compact. Overall DOFS (Unsupervised) has roughly ˜5-7% improvement and DOFS (Supervised) ˜8-10% improvement over alpha-investing approach for online feature selection. In terms of compactness, the unsupervised variant has even better compactness for 5 of the 8 datasets chosen, demonstrating that the unsupervised variant of DOFS consistently outperforms alpha-investing both in terms of accuracy and compactness. Overall our algorithm is able to select sufficient features with discriminative power.

DOFS vs OSFS

Unsupervised and supervised variant of DOFS outperforms OSFS in 7 of the 8 datasets, with roughly ˜4-6% improvement for the unsupervised variant and ˜10% for the supervised variant. OSFS achieves greater compactness in all combination of datasets and variants of DOFS algorithm, with the exception of unsupervised DOFS and Spambase dataset. This demonstrates the trade-off in compactness of representation in this algorithm against the accuracy in performance. Overall our algorithm is able to select sufficient features with discriminative power.

DOFS vs Grafting

Across the board Grafting appears to be a superior algorithm in terms of accuracy. Unsupervised DOFS outperforms Grafting in only 1 of the 8 datasets, whilst supervised variant outperformed Grafting in 3 of the 8 datasets. On average the difference in accuracy for the supervised variant suggests that we suffer ˜1-2% loss in accuracy, demonstrating minimal loss in performance. With this in mind, in 4 of the 5 datasets where performance was worse than grafting, the supervised DOFS achieved improved compactness by ˜30%.

DOFS vs OGFS

Compared with OGFS, DOFS unsupervised variant outperforms in 2 of 8 datasets and DOFS supervised outperforms in 3 of 8. On average the difference in accuracy for the supervised variant suggests that we suffer ˜1-2% loss in accuracy on average, demonstrating minimal loss in performance. Given this trade-off, supervised variant of DOFS manages to have an improved compactness by ˜12%. This demonstrates that DOFS is a competitive algorithm retaining similar level of performance whilst promoting further compactness.

5 Conclusion

In this paper, we have presented a new algorithm called DOFS which can select diverse features both in a supervised or unsupervised environment. We have explored the limitations of using DPP for feature sampling alone, and demonstrated the necessity and value of introducing additional redundancy checks to provide a competitive performance. This framework allows us to efficient select features that arrive by groups and also one by one. We have divided online feature selection into three stages: DPP sampling, local criteria and global criteria. We have designed several criteria for selecting the optimal number of k to sample from DPP, trace rank approach for supervised learning problem, group wilcoxon signed rank test and Lasso to reduce redundancy. Experiments have demonstrated that DPP is on par or better than other state-of-the-art online feature selection methods whilst being more compact through the use of the UCI and Micro Array benchmark datasets.

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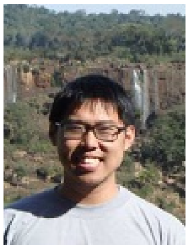

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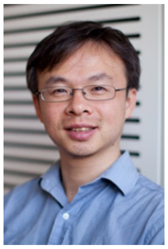

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