25·14·2. Digital Transducers. These transducers convert the input physical phenomenon into an electrical output which is in the form of pulses.

Fig. 25·7 shows a digital linear displacement transducer. This transducer uses digital code marks to identify the position of a movable piece by a binary system of notation. The position is given out as a train of digital pulses.

At the binary system uses only two symbols 0 and 1 (Binary systems are explained in Appendix A) it can easily be represented by opaque and transparent areas on a glass scale. Another alternative is to use conducting and non-conducting areas on a metal scale. In Fig. 25·7, a scale is constructed to show the linear position of a movable objects. This digital transducer uses 5 digits. The complete binary number denoting the position is obtained by scanning the pattern across the scale at a stationary index mark. Glass scales can be read optically by means of a light source, an optical system and photocells. Metal scales are scanned by brushes making electrical contact with individual tracks.

Fig. 25·7. Digital displacement transducer using 5-digit scale for digital indication of linear position of a movable object.

25·15. Electrical Phenomena used in Transducers

The transducers may be classified into different categories depending upon the principle employed by their transduction elements to convert the physical phenomenon (input) into output electrical signals. The different electrical phenomena employed in transduction elements of transducers are listed below. These phenomena may be combined with appropriate primary sensing elements (detectors) to produce a variety of transducers.

These phenomena are :

1. Resistive. 2. Inductive. 3. Capacitive. 4. Electromagnetic. 5. Piezoelectric. 6. Ionization. 7. Photoelectric or Photo-emissive. 8. Photoconductive or Photoresistive. 9. Photovoltaic. 10. Potentiometric. 11. Thermo-electric or Thermo-voltaic. 12. Electrokinetic.

Table 25·2 on page 756-757 shows a classification of transducers according to electrical principles involved. Although it is impossible to classify the sensors, the devices listed in Table 25·2 do represent a wide cross-section of commercially available transducers in instrumentation engineering.

25·16. Resistive Transducers

It is generally seen that methods which involve the measurement of change in resistance are preferred to those employing other variables. This is because both alternating as well as direct currents and voltages are suitable for resistance measurements.

The resistance of a metal conductor is expressed by a simple equation that involves a few physical quantities. The relationship is $R=\rho L/A$.

where R=resistance ; Ω, L=length of conductor ; m,

A=cross-sectional area of conductor ; m^2, and ρ=resistivity of conductor material ; Ω—m.

Any method of varying one of the quantities involved in the above relationship can be the design basis of an **electrical resistive transducer**. There are a number of ways in which resistance can be changed by a physical phenomenon. The translational and rotational potentiometers which work on the basis of change in the value of resistance with change in length of the conductor can be used for measurement of translational or rotary displacements. **Strain gauges** work on the principle that the resistance of a conductor or a semi-conductor changes when strained. This property can be used for measurement of displacement, force and pressure. The resistivity of materials changes with change of temperature thus causing a change of resistance. The property may be used for measurement of **temperature**. Thus electrical resistance transducers have a wide field of application.

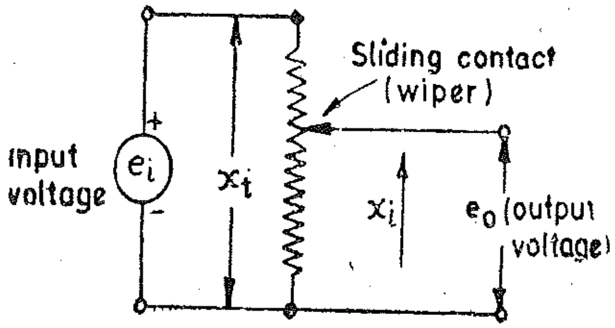
25·17. Potentiometers

Basically, a resistive potentiometer, or simply a **pot,** (A potentiometer used for the purposes of voltage division is called a **pot**) consists of a resistance element provided with a sliding contact. This sliding contact is called a **wiper.** The motion of sliding contact may be translatory or rotational. Some pots use the combination of the two motions, *i.e.* translational as well as rotational. These potentiometers have their resistive element in the form of helix and thus, are called **helipots.**

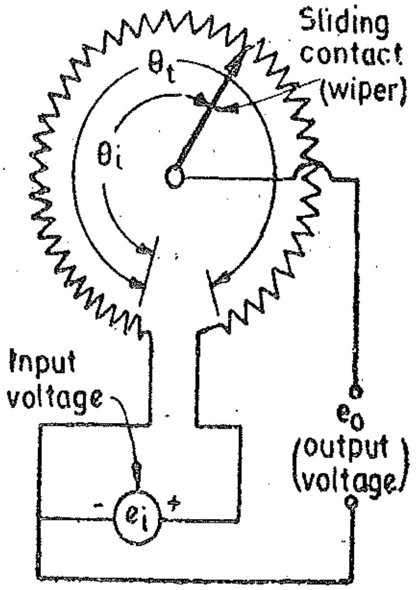
The translational resistive elements are straight devices and have a stroke of about 2 mm to 0·5 m. The rotational resistive devices are circular in shape and are used for measurement of angular displacement. They have a range of 10° to 60 full turns. The helical resistive elements are multiturn rotational devices which can be used for measurement of either translational or rotary motion. The resistance element of a potentiometer may be excited either with a d.c. or an a.c. voltage source. The **pot,** is thus, a **passive transducer** since it requires an external power source for its operation.

The resistance elements of potentiometers are wire wound but some other commonly used resistive materials are cermet, hot moulded carbon, carbon film and thin metal films.

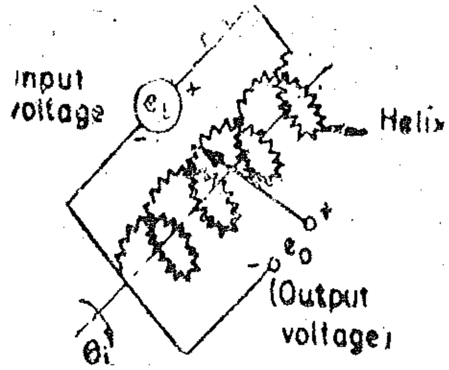
Fig. 25·8 shows that the schematic diagrams of a translational, single turn rotational and multiturn rotational helix potentiometers.



(*a*) Translational



(*b*) Rotational



(*c*) Helipot (rotational)

Fig. 25·8. Resistive potentiometers.

Let us confine our discussion to d.c. excited potentiometers Consider a translational potentiometer as shown in Fig. 25·8 (*a*).

Let e_i=input valtage ; V, R_p=total resistance of potentiometer ; Ω,

x_t=total length of a translational pot ; m,

x_i=displacement of the slider from its 0 position ; m,

and e_0=output voltage ; V.

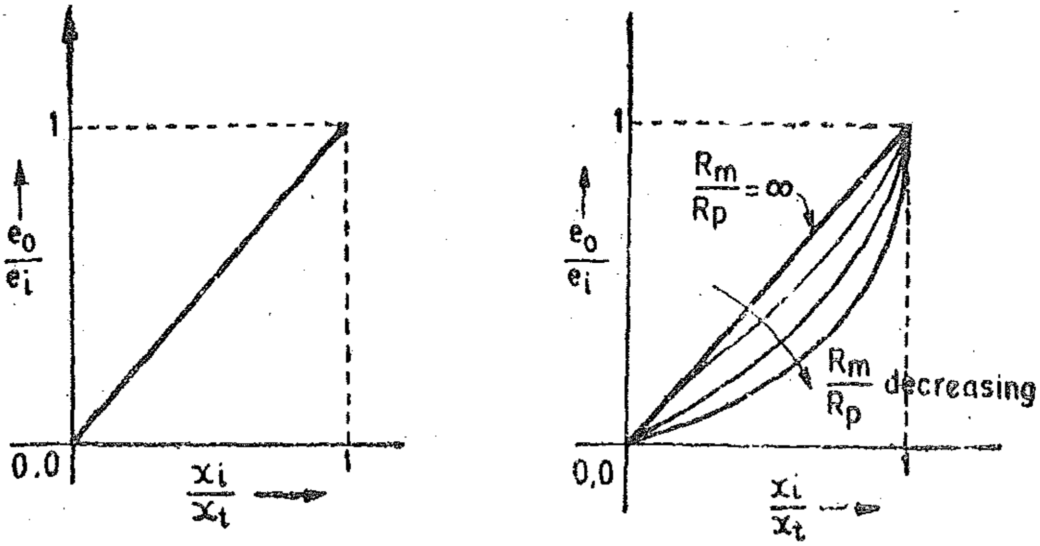
If the distribution of the resistance with respect to translational movement is linear, the esistance per unit length is R_p/x_t.

The output voltage under ideal conditions is :

$$e_0=\left(\frac{\text{resistance at the output terminals}}{\text{resistance at the input terminals}}\right)\times\text{input voltage}$$

$$=\left[\frac{(R_p/x_t\times x_i)}{R_p}\right]\times e_i=\frac{x_i}{x_t}\,e_i \qquad \text{...(25.5)}$$

Thus under ideal circumstances, the output voltage varies linearly with displacement as shown in Fig. 25.9 (*a*).



(*a*) Unloaded pot (*b*) Loaded pot

Fig. 25.9. Characteristics of potentiometers.

$$\text{Sensitivity}\quad S=\frac{\text{output}}{\text{input}}=\frac{e_0}{x_i}=\frac{e_i}{x_t} \qquad \text{...(25.6)}$$

Thus under ideal conditions the sensitivity is constant and the output is faithfully reproduced and has a linear relationship with input. The same is true of rotational motion.

Let θ_i=input angular displacement in degrees, and θ_t=total travel of the wiper in degrees.

$\therefore$ Output voltage $e_0=e_i/\theta_i/\theta_t$ ...(25.7)

This is true of single turn potentiometers only.

The circuits shown in Fig. 25.8 are called **potentiometer dividers** since they produce an output voltage which is a fraction of the input voltage. Thus the input voltage is "divided". The potential divider is a device for dividing the potential in a ratio determined by the position of the sliding contact.

Eqns 25.6 and 25.7 are based upon the assumption that the distribution of resistance with respect to linear or angular displacement are uniform and the resistance of the voltage measuring device (*i.e.* output device) is infinite. However, in practice, the output terminals of the pot are connected to a device whose impedance is finite. Thus when an electrical instrument, which forms a load for the pot, and is connected across

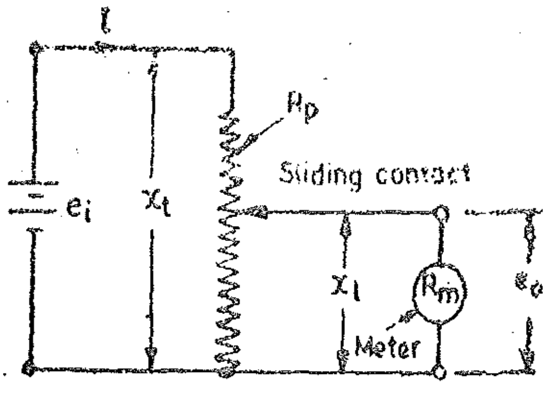


Fig 25.10. Loaded potentiometer.

the output terminals. The indicated voltage is less than that given by Eqn. 25·6. The error, which is referred to as a loading error is caused by the input resistance of the output device.

Let us consider the case of a translational potentiometer as shown in Fig. 25·10. Let the resistance of a meter or a recorder monitoring the output be R_m

As explained earlier if the resistance across the output terminals is infinite, we get a linear relationship between the output and the input voltage.

$$e_0=(x_i/x_t)e_i=Ke_i \qquad \text{...(25·8)}$$

However, under actual conditions the resistance, R_m, is not infinite. This causes a non-linear relationship between the output and input voltages.

25·17·1. Loading Effect. The resistance of the parallel combination of load resistance and the portion of the resistance of the potentiometer is :

$$\frac{(x_i/x_t)\,R_pR_m}{(x_i/x_t)\,R_p+R_m}=\frac{K\,R_pR_m}{K\,R_p+R_m} \qquad \text{...(25·9)}$$

The total resistance seen by the source is :

$$R=R_p\,(1-K)+\frac{K\,R_pR_m}{K\,R_p+R_m}=\frac{KR_p^2(1-K)+R_pR_m}{K\,R_p+R_m} \qquad \text{...(25·10)}$$

$$\therefore \text{ Current } \quad i=\frac{e_i}{R}=\frac{e_i\,(K\,R_p+R_m)}{K\,R_p^2\,(1-K)+R_pR_m} \qquad \text{...(25·11)}$$

The output voltage under loaded conditions is :

$$e_0=e_i-iR_m\,(1-K)=\frac{e_i\,K}{1+(R_p/R_m)K-(R_p/R_m)K^2} \qquad \text{...(25·12)}$$

$$=\frac{e_i\,K}{K(1-K)(R_p/R_m)+1} \qquad \text{...(25·13)}$$

The ratio of output to input voltage under loaded conditions is :

$$\frac{e_0}{e_i}=\frac{K}{K(1-K)(R_p/R_m)+1} \qquad \text{...(25·14)}$$

The Eqn. 25·14 shows that there exists a non-linear relationship between output voltage e_0, and input voltage e_i. In case $R_m=\infty$, $\frac{e_0}{e_i}=K$.

It is evident from Eqn, 25·14, that as the ratio of R_m/R_p decreases, the non-linearity goes on increasing. This is shown in Fig. 25·9 (*b*). Thus, in order to keep linearity the value of R_m/R_p should be as large as possible. However, when we have to measure the output voltage with a given meter, the resistance of the potentiometer, R_p, should be as small as possible.

$\therefore$ Error = output voltage under load − output voltage under no load

$$=\frac{e_i\,K}{[K(1-K)(R_p/R_m)+1]}-e_iK=e_i\left[\frac{K^2\,(K-1)}{K(1-K)+(R_m/R_p)}\right] \qquad \text{...(25·15)}$$

Based upon full-scale output, this relationship may be written as :

Percentage error

$$\%\epsilon=\left[\frac{K^2(K-1)}{K(1-K)+(R_m/R_p)}\right]\times 100 \qquad \text{...(25·16)}$$

Except for the two end points where $K=0$ *i.e.* $x_i=0$ and $K=1$ where $x_i=x_t$ the error is always negative. Fig. 25·11 shows a plot of the variation in error with the slider position for different ratios of the load (meter) resistance to the potentiometer resistance.

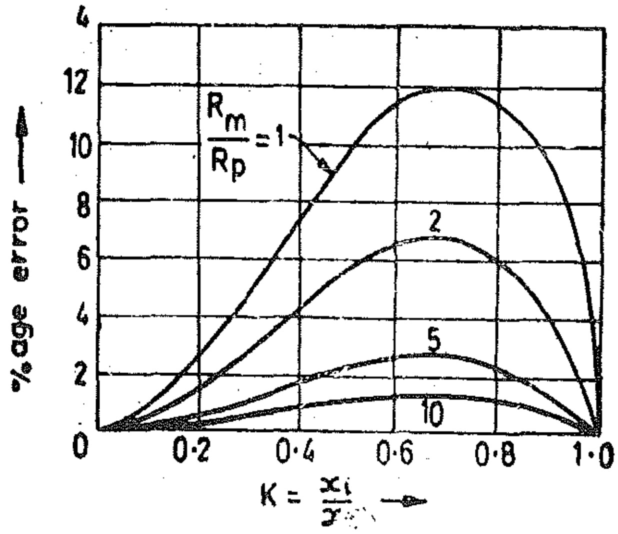


Fig. 25·11. Variation of error due to loading effect of a potentiometer

The error as indicated in Fig. 25·11 is actually negative. Examining Fig. 25·11, the maximum error is about 12 per cent of full scale if $R_m/R_p=1$. This error drops down to about 1·5 per cent when $R_m/R_p=10$. For values of $R_m/R_p>10$ the position of maximum error occurs in the vicinity of $x_i/x_t=0{\cdot}67$.

Maximum percentage error $\epsilon_{max} = 15 \times (R_p/R_m)$ ...(25·17)

25·17·2. Power Rating of Potentiometers. The potentiometers are designed with a definite power rating which is related directly to their heat dissipating capacity. The manufacturer normally designs a series of potentiometers of single turn with a diameter of 50 mm with a wide range of ohmic values ranging from 100 Ω to 10 kΩ in steps of 100 Ω. These potentiometers are essentially of the same size and of the same mechanical configuration. They have the same heat transfer capabilities. Their rating is typically 5 W at an ambient temperature of 21°C. This limits their input excitation voltage. Since power $P = e_i^2/R$, the maximum input excitation voltage that can be used is :

$$(e_i)_{max} = \sqrt{PR_p} \text{ volt} \qquad ...(25·18)$$

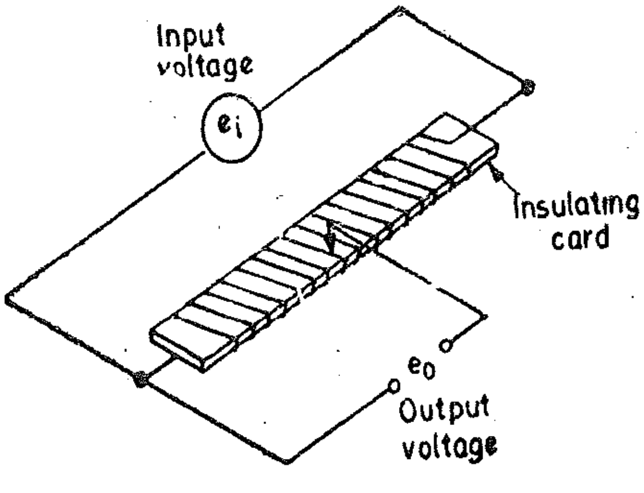
25·17·3. Linearity and Sensitivity. It has been explained earlier that in order to achieve a good linearity, the resistance of potentiometer R_p, should be as low as possible when using a particular meter.

In order to get a high **sensitivity** the output voltage e_0 should be high which in turn requires a high input voltage, e_i. Due to limitations of power dissipation as is clear from Eqn. 25·18, the input voltage is limited by the resistance of the potentiometer. In order to keep the power dissipation at a low level, the input voltage should be small and resistance of the potentiometer should be high. Thus for a high **sensitivity**, the input voltage should be large and this calls for a high value of resistance R_p, On the other hand if we consider the **linearity**, the resistance of potentiometer R_p, should be as low as possible. The resistance of the potentiometer, R_p, cannot be made low because if we do so the power dissipation goes up with the result that we have to make the input voltage small to keep down the power dissipation to the acceptable level. This results in lower **sensitivity**

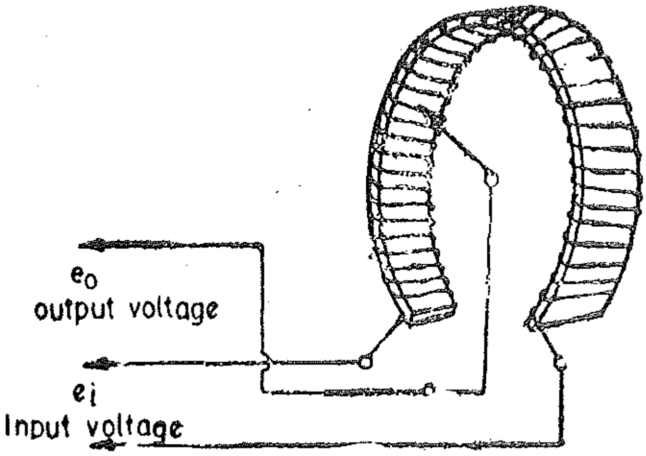
Thus **linearity** and **sensitivity** are two conflicting requirements. If R_p is made small, the **linearity** improves, but a low value of R_p requires a lower input voltage e_i in order to keep down the power dissipation and a low value of e_i results in a lower value of output voltage e_0 resulting in lower **sensitivity**. Thus the choice of potentiometer resistance, R_p, has to be made considering both the **linearity** and **sensitivity** and a compromise between the two conflicting requirements has to be struck.

Typical values of sensitivity are of the order of 200 mV/degree for a rotational potentiometer and 200 mV/mm for a translational potentiometer. The short stroke devices have generally a high values of sensitivity.

25·17·4. Construction of Potentiometers. The resolution of the potentiometers influences the construction of their resistance elements. Normally, the resistive element is a single wire of conducting material which gives a continuous stepless variation of resistance as the wiper travels over it. Such



(a) Linear potentiometer.



(b) Circular potentiometer.

Fig. 25·12. Wire wound potentiometers.

potentiometers are available but their length (in the case of translational potentiometers) and diameter (in the case of rotational potentiometers) restrict their use on account of space considerations.

The resolution of the potentiometers is dependent upon the construction of the resistive element and in order to get high values of resistance in small space **wire wound** potentiometers are used extensively. The resistance wire is wound on a mandrel or a card for translational displacement as shown in Fig. 25 12 (*a*). For the measurement of rotational motion these mandrels or cards are formed into a circle or a helix. This is shown in Fig. 25 12 (*b*). If wire wound type of construction is adopted, the variation of resistance is not a linear continuous change but is in small steps as the sliding contact (wiper) moves from one turn to another. This is shown in Fig. 25·13.

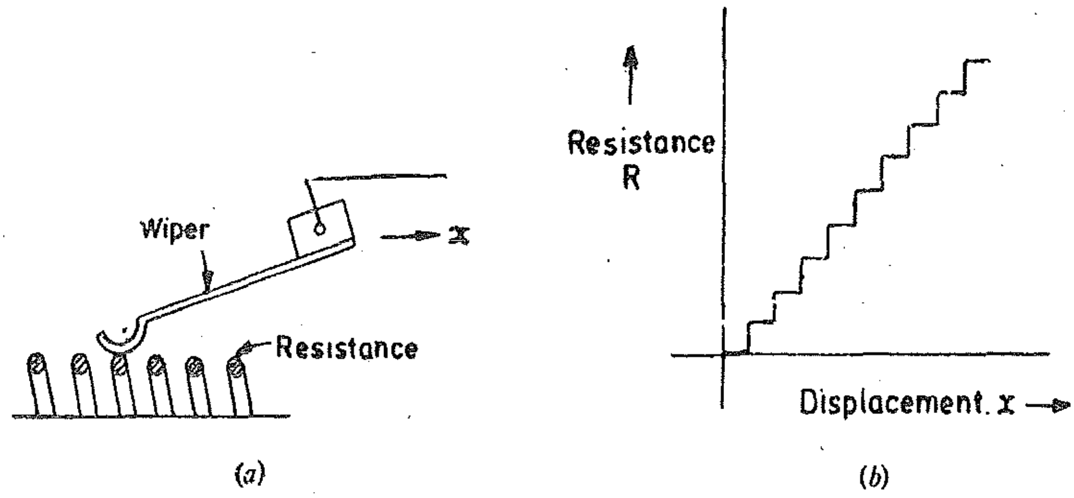


Fig. 25·13. Translational potentiometer and its characteristics.

Since the variation in resistance is in steps the resolution is limited. For instance, a translational potentiometer has about 500 turns of a resistance wire on a card of 25 mm in length and for this device the resolution is limited to $25/500 = 0{\cdot}05$ mm $= 50$ μm.

The actual practical limit is 20 and 40 turns per mm. Thus for translational devices the resolution is limited to 25—50 μm.

For rotational devices, the best angular resolution$=\dfrac{(50-100)\times10^{-3}}{D}$ degrees ...(25·19)

where D=diameter of the potentiometer ; m.

In order to get higher resolution, thin wires which have a high resistance have to be put close to each other and they can be closely wound on account of their small diameter. Thus the resolution and total resistance are interdependent.

In case a fine resolution and high resistance are required a carbon film or a conductive-plastic resistance elements are used. Carbon film resistive elements have a resolution of 12·5 nm.

25·17·5. Helipots. The resolution can be increased by using multi-turn potentiometers. These are called helipots. The resistance element is in the form of a helix and the wiper travels along a "lead screw". The number of turns is still limited to 200 to 400 per cm but an increase in resolution can be obtained by using a gearing arrangement between the shaft whose motion is to be measured and the potentiometer shaft. As an example, one rotation of the measured shaft can cause 10 rotations of the potentiometer shaft. This increases the resolution of the measured shaft motion by 10 times. Multiturn potentiometers are available to about 60 turns.

Similar, magnifying or amplifying techniques can be used for translational motion as well.

25·17·6. Types of Potentiometers and their Characteristics. There are five types of potentiometers available :

1. Wire Wound. These use nickel chromium, nickel copper, or some other precious resistance elements. Wire wound potentiometers can carry relatively large currents at high temperatures. Their temperature co-efficient is usually small, is of the order $20 \times 10^{-6}\ \Omega/\Omega\text{-}^\circ C$ or less and also they are relatively inexpensive. Their resolution is about 0.05 mm and is limited by the number of turns. Multiturn potentiometers using 3 to 10 turn units are used when the potentiometer is required to have close settings.

The interwinding capacitance between turns and between winding and arm, housing etc. limits the use of wire wound potentiometers to low frequencies. The response is limited to about 5 Hz.

2. Cermet. Cermet uses precious metal particles fused into ceramic base. These fused metal particles act as resistance elements. The advantages of using Cermet are large power ratings at high temperatures, low cost and moderate temperature coefficients of the order 100×10^{-6} to $200 \times 10^{-6}\ \Omega/\Omega\text{-}^\circ C$. Cermet is very useful for a.c. applications.

3. Hot Moulded Carbon. The resistance element is fabricated by moulding together a mixture of carbon and a thermosetting plastic binder. Hot moulded carbon units are useful for a.c. applications.

4. Carbon Film. A thin film of carbon deposited on a non-conductive base forms the resistance element. The advantage of carbon film potentiometers is their low cost. Temperature coefficients are upto $1000 \times 10^{-6}\ /^\circ C$.

5. Thin Metal Film. A very thin, vapour deposited layer of metal on glass or ceramic base is used as a resistance element. The advantages of this potentiometer are its excellent resistance to changes in environments and use on a.c. The cost is also moderate.

Advantages and Disadvantages. Resistance potentiometers have the following major advantages :

(*i*) They are inexpensive.

(*ii*) They are simple to operate and are very useful for applications where the requirements are not particularly severe.

(*iii*) They are very useful for measurement of large amplitudes of displacement.

(*iv*) Their electrical efficiency is very high and they provide sufficient output to permit control operations without further amplification.

(*v*) It should be understood that while the frequency response of wire wound potentiometers is limited, the other types of potentiometers are free from this problem.

(*vi*) In wire wound potentiometers the resolution is limited while in Cermet and metal film potentiometers, the resolution is infinite.

The disadvantages are :

(*i*) The chief disadvantage of using a linear potentiometer is that they require a large force to move their sliding contacts (wipers).

(*ii*) The other problems with sliding contacts are that they can be contaminated, can wear out, become misaligned and generate noise. So the life of the transducer is limited. However, recent developments have produced a roller contact wiper which, it is claimed, increases the life of the transducer by 40 times.

Example 25.3. A linear resistance potentiometer is 50 mm long and is uniformly wound with a wire having a resistance 10,000 Ω. Under normal conditions, the slider is at the centre of the potentiometer. Find the linear displacement when the resistance of the potentiometer as measured by a Wheatstone bridge for two cases is :

(*i*) 3850 Ω, and (*ii*) 7560 Ω.

Are the two displacements in the same direction ?

If it is possible to measure a minimum value of 10 Ω resistance with the above arrangement, find the resolution of the potentiometer in mm.

Solution. The resistance of potentiometer at the normal position $=10{,}000/2=5000\ \Omega$.

(*i*) Resistance of potentiometer wire per unit length$=10{,}000/50=200\ \Omega/\text{mm}$.

(*ii*) Change in resistance of potentiometer from its normal position$=5000-3850=1150\ \Omega$.

$\therefore$ Displacement$=\dfrac{1150}{200}=5{\cdot}75$ mm.

(*iii*) Change in resistance of potentiometer from its normal position$=7560-5{,}000=2{,}560\ \Omega$.

$\therefore$ Displacement$=\dfrac{2560}{200}=12{\cdot}55$ mm.

Since one of the displacements represents a decrease and the other represents an increase in resistance of the potentiometer as compared with the resistance of the potentiometer at its normal position, the two displacements are in the opposite direction.

Resolution $=$ minimum measurable value of resistance $\times \dfrac{\text{mm}}{\Omega}=10\times\dfrac{1}{200}=0{\cdot}05$ mm.

Example 25.4. A variable potential divider has a total resistance of 2 kΩ and is fed from a 10 V supply. The output is connected across a load resistance of 5 kΩ. Determine the loading error for the slider positions corresponding to $x_i/x_t=0, 0{\cdot}25, 0{\cdot}5, 0{\cdot}75$ and $1{\cdot}0$. Use the results to plot a rough graph of loading error against the ratio x_i/x_t.

Solution. From Eqn. 25.16, the percentage loading error $=\left[\dfrac{K^2(K-1)}{K(1-K)+(R_m/R_p)}\right]\times 100$

In terms of input voltage the error$=\left[\dfrac{K^2(K-1)}{K(1-K)+(R_m/R_p)}\right]e_i$

(*i*) when $K=\dfrac{x_i}{x_t}=0$, Error$=\left[\dfrac{0(0-1)}{0(1-0)+(5000/2000)}\right]\times 10=0$.

(*ii*) when $K=0{\cdot}25$, Error$=\left[\dfrac{(0{\cdot}25)^2(0{\cdot}25-1)}{0{\cdot}25(1-0{\cdot}25)+(5000/2000)}\right]\times 10=-0{\cdot}174$ V.

(*iii*) when $K=0{\cdot}5$, Error$=\left[\dfrac{(0{\cdot}5)^2(0{\cdot}5-1)}{0{\cdot}5(1-0{\cdot}5)+(5000/2000)}\right]\times 10=-0{\cdot}454$ V.

(*iv*) when $K=0{\cdot}75$, Error$=\left[\dfrac{(0{\cdot}75)^2(0{\cdot}75-1)}{0{\cdot}75(1-0{\cdot}75)+(5000/2000)}\right]\times 10=-0{\cdot}524$ V.

(*v*) when $K=1$, Error$=\left[\dfrac{(1)^2(1-1)}{1(1-1)+(5000/2000)}\right]\times 10=0$ V.

A rough graph of error versus x_i/x_t is shown in Fig. 25.14 (The errors are negative).

Example 25.5 The output of a potentiometer is to be read by a recorder of 10 kΩ input resistance. Non-linearity must be held to 1 per cent. A family of potentiometers having a thermal rating of 5 W and resistances ranging from 100 Ω to 10 kΩ in steps of 100 Ω is available. Choose from the family the potentiometer that has the greatest possible sensitivity and also meets other requirements. Find the maximum excitation voltage permissible with this potentiometer. What is the sensitivity if the potentimeter is a single turn (360°) unit ?

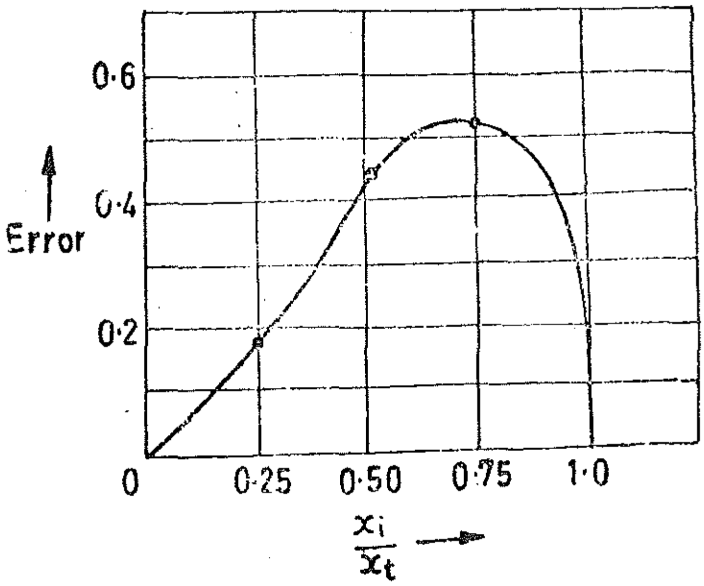


Fig. 25.14

Solution :

From Eqn. 25.17, the maximum possible percentage linearity$=15\ R_p/R_m$.

Therefore, the maximum resistance of potentiometer, with 1 per cent linearity and a recorder with a resistance $R_m=10{,}000\ \Omega$ across it is :

$$R_p=\frac{1\times R_m}{15}=\frac{1\times 10{,}000}{15}=666{\cdot}7\ \Omega.$$

Thus we are left with choice of potentiometers having a resistance of : 100 Ω, 200 Ω, 300 Ω, 400 Ω, 500 Ω and 600 Ω. The potentiometer with the highest resistance gives the highest sensitivity. Therefore, the 600 Ω potentiometer is selected.

With a power dissipation of 5 W, the maximum allowable excitation voltage is :

$$(e_i)_{max}=\sqrt{P\,R_p}=\sqrt{5\times 600}=57{\cdot}4\ \text{V}.$$

The sensitivity of the potentiometer when it is a single turn unit :

$$K=\frac{e_i}{360}=\frac{57{\cdot}4}{360}=0{\cdot}16\ \text{V/degree.}$$

Example 25·6. A helipot is provided with 40 turns/mm. The gearing arrangement is such that the motion of the main shaft by one revolution causes 5 revolutions of the potentiometer shaft. Calculate the resolution of the potentiometer.

Solution :

The resolution of the potentiometer without the gearing arrangement 1/400 mm = 25 μm.

With gearing arrangement which causes 5 revolutions of potentiometer shaft with one rotation of main shaft, the resolution $=\frac{25}{5}=5$ μm.

25·18. Strain Gauges

If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change. Also there is a change in the value of resistivity of the conductor when it is strained and this property is called **piezo resistive effect**. Therefore, resistance strain gauges are also known as **piezoresistive gauges**. the strain gauges are used for measurement of strain and associated stress in experimental stress analysis. Secondly, many other detectors and transducers, notably the load cells, torque meters, diaphragm type pressure gauges, temperature sensors, accelerometers and flow meters, employ strain gauges as secondary transducers.

25·18·1. Theory of Strain Gauges. The change in the value of resistance by straining the gauge may be partly explained by the normal dimensional behaviour of elastic material. If a strip of elastic material is subjected to tension, or in other words positively strained, its longitudinal dimension will increase while there will be a reduction in the lateral dimension So when a gauge is subjected to a positive strain, its length increases while its area of cross-section decreases. Since the resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section, the resistance of the gauge increases with positive strain. The change in the value resistance of strained conductor is more than what can be accounted for an increase in resistance due to dimensional changes.

The extra change in the value of resistance is attributed to a change in the value of resistivity of a conductor when strained. This property, as described earlier, is known as **peizoresistive effect.**

Let us consider a strain gauge made of circular wire. The wire has the dimensions : length $=L$, area $=A$, diameter $=D$ before being strained. The material of the wire has a resistivity ρ.

∴ Resistance of unstrained gauge $R=\rho L/A$.

Let a tensile stress s be applied to the wire. This produces a positive strain causing the length to increase and area to decrease. Thus when the wire is strained there are changes in its dimensions. Let $\Delta L=$change in length, $\Delta A=$change in area, $\Delta D=$change in diameter and $\Delta R=$change in resistance.

In order to find how ΔR depends upon the meterial physical quantities, the expression for R is differentited with respect to stress s. Thus we get :

$$\frac{dR}{ds}=\frac{\rho}{A}\frac{\partial L}{\partial s}-\frac{\rho L}{A^2}\frac{\partial A}{\partial s}+\frac{L}{A}\frac{\partial \rho}{\partial s} \qquad \text{...(25·20)}$$

Dividing Eqn. 25·20 throughout by resistance $R=\rho L/A$, we have

$$\frac{1}{R}\frac{dR}{ds}=\frac{1}{L}\frac{\partial L}{\partial s}-\frac{1}{A}\frac{\partial A}{\partial s}+\frac{1}{\rho}\frac{\partial \rho}{\partial s} \qquad \text{...(25·21)}$$

It is evident from Eqn. 25·21, that the per unit change in resistance is due to :

(*i*) per unit change in length$=\Delta L/L$. (*ii*) per unit change in area$=\Delta A/A$.

Area $\quad A=\frac{\pi}{4}D^2 \quad \therefore \quad \frac{\partial A}{\partial s}=2.\frac{\pi}{4}D.\frac{\partial D}{ds} \qquad$ ...(25·22)

or

$$\frac{1}{A}\frac{dA}{ds}=\frac{(2\pi/4)D}{(\pi/4)D^2}\frac{\partial D}{\partial s}=\frac{2}{D}\frac{\partial D}{\partial s} \qquad \text{...(25·23)}$$

$\therefore$ Eqn. 25·21 can be written as :

$$\frac{1}{R}\frac{dR}{ds}=\frac{1}{L}\frac{\partial L}{\partial s}-\frac{2}{D}\frac{\partial D}{\partial s}+\frac{1}{\rho}\frac{\partial \rho}{\partial s} \qquad \text{...(25·24)}$$

Now, Poisson's ratio $\quad \nu=\frac{\text{lateral strain}}{\text{longitudinal strain}}=-\frac{\partial D/D}{\partial L/L} \qquad$ ...(25·25)

or $\quad \partial D/D=-\nu\times\partial L/L$

$$\therefore \quad \frac{1}{R}\frac{dR}{ds}=\frac{1}{L}\frac{\partial L}{\partial s}+\nu\frac{2}{L}\frac{\partial L}{\partial s}+\frac{1}{\rho}\frac{\partial \rho}{\partial s} \qquad \text{...(25·26)}$$

For small variations, the above relationship can be written as :

$$\frac{\Delta R}{R}=\frac{\Delta L}{L}+2\nu\frac{\Delta L}{L}+\frac{\Delta \rho}{\rho} \qquad \text{...(25·27)}$$

The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

Gauge factor $\quad G_f=\frac{\Delta R/R}{\Delta L/L} \qquad$ ...(25·28)

or $\quad \frac{\Delta R}{R}=G_f\frac{\Delta L}{L}=G_f\times\epsilon \qquad$ ...(25·29)

where $\quad \epsilon=\text{strain}=\frac{\Delta L}{L}$

The gauge factor can be written as :

$$G_f=\frac{\Delta R/R}{\Delta L/L}=1+2\nu+\frac{\Delta\rho/\rho}{\Delta L/L}=1+2\nu+\frac{\Delta\rho/\rho}{\epsilon} \qquad \text{...(25·30)}$$

The strain is usually expressed in terms of microstrain. 1 microstrain$=1$ μm/m.

If the change in the value of resistivity of a material when strained is neglected, the gauge factor is :

$$G_f=1+2\nu \qquad \text{...(25·31)}$$

Eqn. 25·31 is valid only when **Piezoresistive Effect** *i.e.* change in resistivity due to strain is almost negligible.

The Poisson's ratio for all metals is between 0 and 0·5. This gives a gauge factor of approximately, 2. The common value for Poisson's ratio for wires is 0·3. This gives a value of 1·6 for wire wound strain gauges.

Table 25·3 gives the value of gauge factors for the various materials.

TABLE 25·3

Gauge Factors

Material	*Gauge Factor*	*Material*	*Gauge Factor*
Nickel	−12·1	Platinum	+4·8
Manganin	+ 0·47	Carbon	+20
Nichrome	+ 2·0	Doped Crystals	100−5000
Constantan	+ 2·1		
Soft iron	+ 4·2		

Example 25·7. A resistance wire strain gauge uses a soft iron wire of small diameter. The gauge factor is +4·2. Neglecting the piezoresistive effects, calculate the Poisson's ratio.

Solution. The gauge factor is given by Eqn. 25·30, $G_f = 1 + 2\nu + \dfrac{\Delta\rho/\rho}{\epsilon}$

If piezoresistive effect is neglected, the gauge factor is given by Eqn. 25·32 as : $G_f = 1 + 2\nu$

$\therefore$ Poisson's ratio $\nu = \dfrac{G_f - 1}{2} = \dfrac{4{\cdot}2 - 1}{2} = 1{\cdot}6.$

Example 25·8. A compressive force is applied to a structural member. The strain is 5 micro-strain. Two separate strain gauges are attached to the structural member, one is a nickel wire strain gauge having a gauge factor of −12·1 and the other is nichrome wire strain gauge having a gauge factor of 2. Calculate the value of resistance of the gauges after they are strained. The resistance of strain gauges before being strained is 120 Ω.

Solution. According to our convention, the tensile strain taken as positive while the compressive strain is taken as negative. Therefore, strain $\epsilon = -5 \times 10^{-6}$ (1 micro strain = 1 μm/m)

Now $\Delta R/R = G_f \epsilon$ (See Eqn. 25·29)

$\therefore$ Change in value of resistance of nickel wire strain gauge : $\Delta R = G_f \epsilon \times R$

$$= (-12{\cdot}1) \times (-5 \times 10^{-6}) \times 120 = 7{\cdot}26 \times 10^{-3}\ \Omega = 7{\cdot}26\ \text{m}\Omega.$$

Thus there is an increase of 7·26 mΩ in the value of resistance.

For nichrome, the change in the value of resistance is :

$$\Delta R = (2) \times (-5 \times 10^{-6}) \times 120 = -1{\cdot}2 \times 10^{-3}\ \Omega = -1{\cdot}2\ \text{m}\Omega.$$

Thus with compressive strain, the value of resistance gauge shows a decrease of 1·2 mΩ in the value of resistance.

25·19 Types of Strain Gauges

There are three types of strain gauges :

(*i*) Wire wound strain gauges, (*ii*) Foil type strain gauges,

and (*iii*) Semiconductor strain gauges.

25·20. Resistance Wire Strain Gauges

Resistance wire strain gauges are used in two forms These two forms are :

(*i*) Unbonded, and (*ii*) Bonded.

25·20·1. Unbonded Strain Gauges.

An unbonded strain gauge is shown in Fig. 25·15. This gauge consists of a wire stretched between two points in an insulating medium such as air. The diameter of the wire is about 25 μm and can be strained depending on the way a spring flexure element moves.

In Fig. 25·15 the flexture element is connected via a rod to a diaphragm which is used for pressure measurement. The wires are tensioned to avoid buckling when they experience compressive forces.

The unbonded strain gauges are usually connected in a bridge circuit. With no load applied to the strain gauges, the bridge is balanced.

When an external load is applied, the resistance of strain gauges change causing an unbalance of the bridge circuit which results in an output voltage. This voltage is proportional to the strain. A displacement of the order of 50 μm may be detected with these strain gauges.

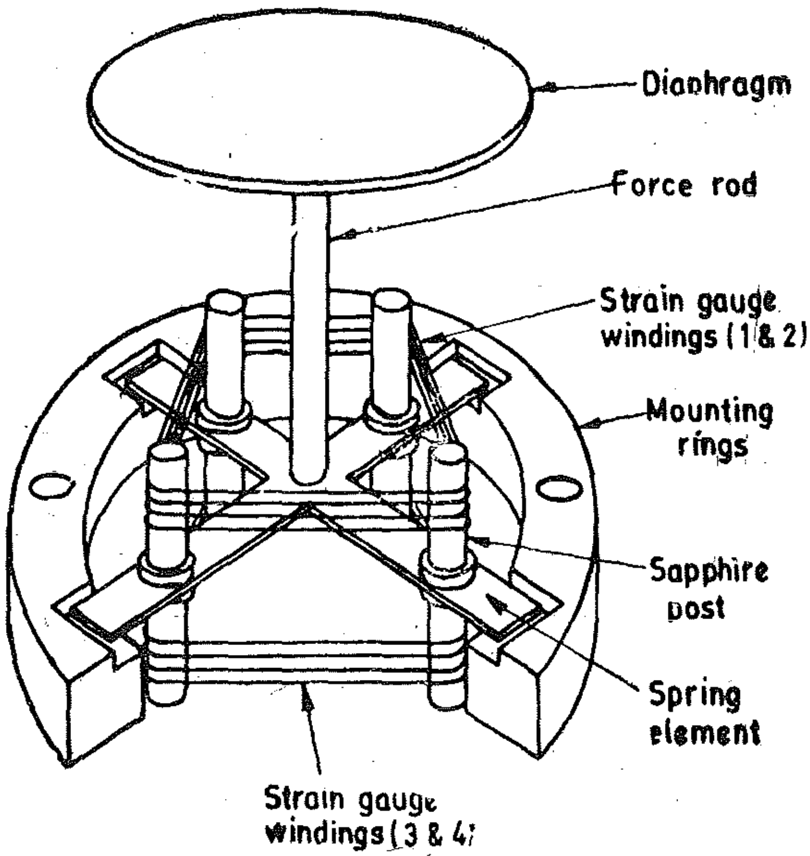


Fig. 25·15. Unbonded resistance wire strain gauge.

25·20·2. Bonded Resistance Wire Strain Gauges.

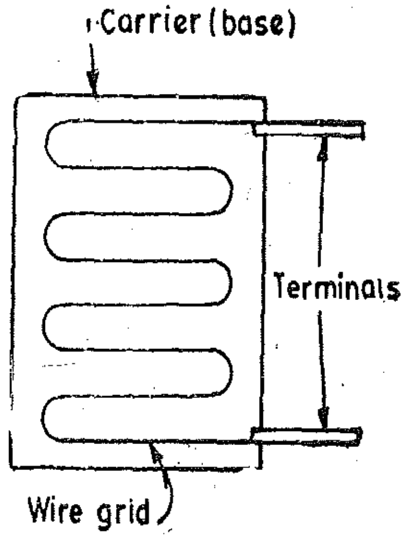
A resistance wire strain gauge consists of a grid of fine resistance wire of about 25 μm (0·025 mm) in diameter or less. The grid of fine wire is cemented to a carrier (base) which may be thin sheet of paper or to a very thin bakelite sheet or to a sheet of teftlon. The wire is covered on top with a sheet of thin material so that it is not damaged mechanically. The spreading of the wire permits a uniform distribution of stress. The carrier is bonded with an adhesive material to the structure under study. This permits a good transfer of strain from carrier to wires. The most commonly used form of resistance wire strain gauges is shown in Fig. 25·16.

The size of the strain gauges varies with application. They can be as small as 3 mm by 3 mm square Usually they are larger, but seldom more than 2·5 mm long and 12·5 mm wide

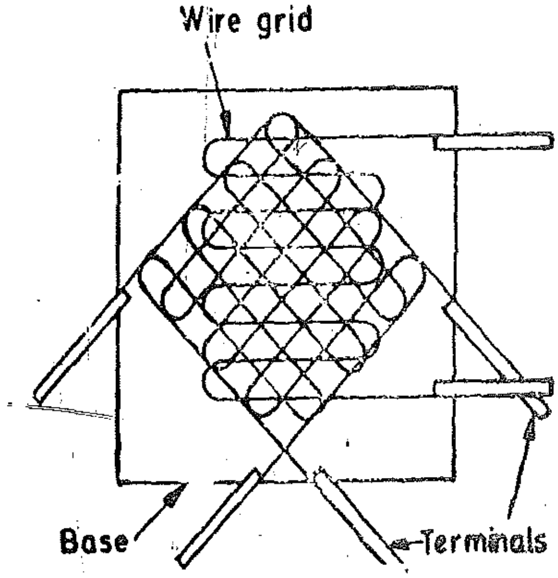
For excellent and reproducible results it is desirable that the resistance wire strain gauges should have the following characteristics :

(*i*) The strain gauge should have a high value of gauge factor G_f. A high value of gauge factor indicates a large change in resistance for a particular strain resulting in high sensitivity.

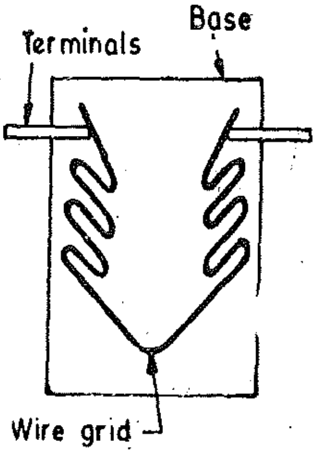
(*ii*) The resistance of the strain gauge should be as high as possible since this minimizes the effects of undesirable variations of resistance in the measurement circuit. Although undesirable, but necessary, are the resistance of connecting leads and terminals, etc. Typical resistances of strain gauges are 120 Ω, 350 Ω and 1000 Ω. Although a high resistance value of strain gauges is desirable from the point of view of swamping out the effects of variations of resistance in other parts of the bridge circuit in which they are invariably used but it results in lower sensitivity.



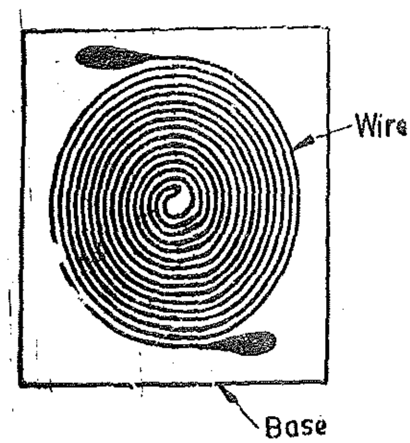
(*a*) Linear strain gauge.



(*b*) Rosette.



(*c*) Torque gauge.



(*d*) Helical gauge.

Fig. 25·16. Resistance wire strain gauge.

Thus in order to get high sensitivity higher bridge voltages have to be used. The bridge voltage is limited by the maximum current carrying capacity of the wires which is typically 30 mA.

(*iii*) The strain gauges should have a low resistance temperature co-efficient. This is essential to minimize errors on account of temperature variations which affect the accuracy of measurements.

(*iv*) The strain gauge should not have any hysteresis effects in its response.

(*v*) In order to maintain constancy of calibration over the entire range of the strain gauge, it should have linear characteristics *i.e.*, the variations in resistance should be a linear function of the strain.

(*vi*) The strain gauges are frequently used for dynamic measurements and hence their frequency response should be good. The linearity should be maintained within accuracy limits over the entire frequency range.

The desirable characteristics of resistance wire strain gauges are listed above but no single material is able to satisfy all the properties since they are, many a times, conflicting in nature. A number of metals and their alloys have been used in making resistance wire strain gauges. Some popular metals and alloys used for construction of resistance wire strain gauges are listed in Table 25·4 along with their properties.

TABLE 25·4

Materials for Strain Gauges

Material	*Composition*	*Gauge Factor*	*Resistivity* Ω m	*Resistance Temperature Co-efficient* $\Omega/\Omega-°C$	*Upper Temperature* °C
Nichrome	Ni : 80% Cr : 20%	2·5	100×10^{-8}	$0·1\times10^{-3}$	1200
Constantan	Ni : 45% Cu : 55%	2·1	48×10^{-8}	$\pm0·02\times10^{-3}$	400
Isoelastic	Ni : 36% Cr : 8% Mo : 0·5% etc.	3·6	105×10^{-8}	$0·175\times10^{-3}$	1200
Nickel	—	−12	$6·5\times10^{-8}$	$6·8\times10^{-3}$	—
Platinum	—	4·8	10×10^{-8}	$4·0\times10^{-3}$	—

Base (Carrier) Materials. Several types of base or carrier materials are used to support the wires. Impregnated paper is used for room temperature applications. The range of some of the other materials are :

Epoxy : −200°C to 150°C
Bakelite impregnated cellulose or glass fibre filled materials :
up to 200°C for continuous operation, and up to 300°C for limited operation.

Adhesives. The adhesives act as bonding materials. Like other bonding operations, successful strain gauge bonding depends upon careful surface preparation and use of the correct bonding agent. In order that the strain be faithfully transferred on to the strain gauge, the bond has to be formed between the surface to be strained and the plastic backing material on which the gauge is mounted. It is important that the adhesive should be suited to this backing ; numerous materials are used for backings which require different adhesive materials. It is usually desirable that the adhesive material should be of quick drying type and also be insensitive to moisture in order to have good adherence.

Ethylcellucose cement, nitrocellucose cement, bakelite cement and epoxy cement are some of the commonly used adhesive materials. The temperature range up to which they can be used is usually below 175°C.

Leads. The leads should be of such materials which have low and stable resistivity and also a low resistance temperature co-efficient.

25·20·3. Foil Strain Gauges. This class of strain gaues is only an extension of the resistance wire strain gauges. The strain is sensed with the help of metal foils as against metal wires as in wire strain gauges. The metals and alloys used for the foil are listed in Table 25·4. Foil gauges have a much greater dissipation capacity as compared with wire wound gauges on account of their greater surface area for the same volume. For this reason they can be used for higher operating temperature range. Also the large surface area of foil gauges leads to better bonding. The bonded foil gauges are extensively used. Early strain gauges were made from fine wire, but these gauges have now been virtually superseded by foil gauges.

A typical foil gauge is shown in Fig. 25·17. Foil type strain gauges have similar characteristics to those of wire wound strain gauges and their gauge factors are typically the same as that of wire wound strain gauges. The advantage of foil type strain gauges is that they can be fabricated economically on a mass scale. The techniques used for fabrication are similar to electronic micro-circuitry *i.e.*, thick and or/thin-film technology commonly used for hybrid integrated circuits (ICs). This means that the foil strain gauges can be fabricated in any shape. The resistance film formed is typically 0·02 mm thick. The resistance elements are vacuum coated with ceramic film and are deposited on a plastic backing which provides insulation, and facilitates perfect bonding. In some cases the backing is strippable vinyle material that can be peeled off so as to cement the film directly on to the test unit with ceramic adhesive. The resistance value of foil gauges which are commercially available is between 50 and 1000 Ω.

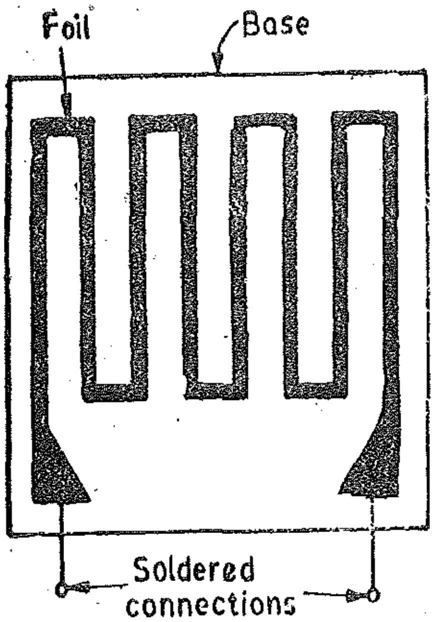


Fig. 2·417. Foil type strain gauge.

Some commonly used forms of foil gauges are shown in Fig. 25·18.

It is interesting to carry out simple calculations to find out what effect an applied stress has on the resistance of a metal strain gauge. Hook's law gives the relationship between stress and strain for linear stress strain curve (*i.e.*, for elastic limits) in terms of modulus of elasticity of material under tension. Hook's law may be written as :

$$\text{Strain } \epsilon = s/E \qquad \text{...(25·36)}$$

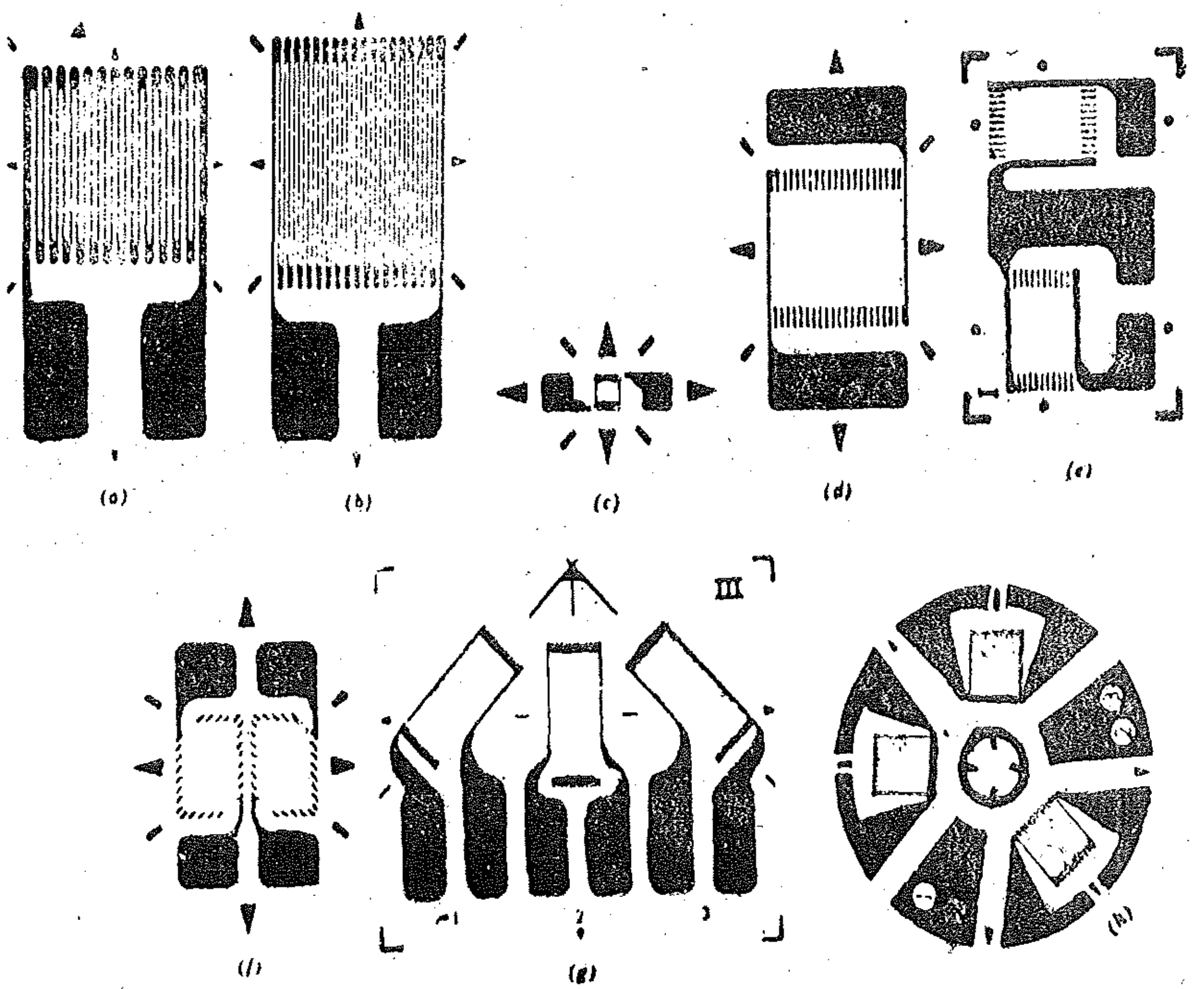


Fig. 25·18. Strain Gauges and Rosettes.

where E=modulus of elasticity.

The change in the value of resistance is quite small as is amply clear from the following example :

Example 25·9. A resistance wire strain gauge with a gauge factor of 2 is bonded to a steel structural member subjected to a stress of 100 MN/m². The modulus of elasticity of steel is 200 GN/m². Calculate the percentage change in the value of the gauge resistance due to the applied stress. Comment upon the results.

Solution. Strain $\epsilon=\frac{s}{E}=\frac{100\times10^6}{200\times10^9}=500\times10^{-6}$. (500 microstrain).

We have : $\frac{\Delta R}{R}=G_f\,\epsilon=2\times500\times10^{-6}=0{\cdot}001=0{\cdot}1\%$.

∴ The change in resistance is only 0·1%.

Comments. The above example illustrates that a very heavy stress of 100 MN/m² results in resistance change of only 0·1 per cent, which is by all means a very small change. This may present difficulties in measurement. Lower stresses produce still lower changes in resistance which may not be perceptible at all or the methods required to detect those changes may have to be highly accurate. To overcome this difficulty we must use strain gauges which have a high gauge factor and thus produce large changes in resistance when strained. These changes are easy to detect and measure with good degree of accuracy.

Example 25·10. A single strain gauge having resistance of 120 Ω is mounted on a steel cantilever beam at a distance of 0·15 m from the free end. An unknown force F applied at the free end produces a deflection of 12·7 mm of the free end. The change in gauge resistance is found to be 0·152 Ω. The beam is 0·25 m long with a width of 20 mm and a depth of 3 mm. The Young's modulus for steel is 200 GN/m². Calculate the gauge factor.

Solution. Moment of inertia of beam, $I=1/12\,(bd^3)=1/12\times0{\cdot}02\times(0{\cdot}003)^3=45\times10^{-12}$ m⁴.

Deflection $x=\frac{Fl^3}{3EI}$

∴ Force $F=\frac{3EIx}{l^3}=\frac{3\times200\times10^9\times45\times10^{-12}\times12{\cdot}7\times10^{-3}}{(0{\cdot}25)^3}=22$ N.

Bending moment at 0·15 m from free end $M=Fx=22\times0{\cdot}15=3{\cdot}3$ Nm.

Stress at 0·15 m from free end $s=\frac{M}{I}\cdot\frac{t}{2}=\frac{3{\cdot}3}{45\times10^{-12}}\times\frac{0{\cdot}003}{2}=110$ MN/m².

Strain $\epsilon=\frac{\Delta L}{L}=\frac{s}{E}=\frac{110\times10^6}{200\times10^9}=0{\cdot}55\times10^{-3}$

∴ Gauge factor$=\frac{\Delta R/R}{\Delta L/L}=\frac{0{\cdot}152/120}{0{\cdot}55\times10^{-3}}=2{\cdot}3$.

25·20·4. Semi-conductor Strain Gauges. It has been explained above in order to have a high sensitivity, a high value of gauge factor is desirable. A high gauge factor means a relatively higher change in resistance which can be easily measured with a good degree of accuracy.

Semiconductor strain gauges are used where a very high gauge factor and a small envelope are required. The resistance of the semi-conductors changes with change in applied strain. Unlike in the case of metallic gauges where the change in resistance is mainly due to change in dimensions when strained, the semi-conductor strain gauges depend for their action upon piezo-resistive effect *i.e.*, the change in the value of the resistance due to change in resistivity.

Semi-conducting materials such as silicon and germanium are used as resistive materials for semi-conductor strain gauges. A typical strain gauge consists of a strain sensitive crystal material and leads that are sandwiched in a protective matrix. The production of these gauges employs conventional semi-conductor technology using semi-conducting wafers or filaments which have a thickness of 0·05 mm and bonding them on a suitable insulating substrates, such as teflon. Gold leads are generally employed for making the contacts. Some of the typical semi-conductor strain gauges are shown in Fig 25·19. These strain gauges can be fabricated along with integrated circuit (IC) operational amplifiers which can act as pressure sensitive transducers.

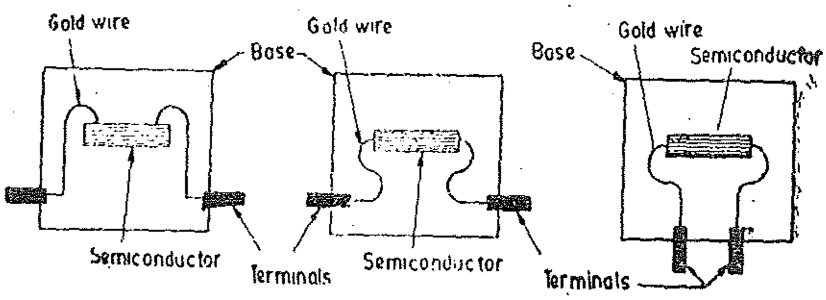


Fig. 25·19. Semi-conductor strain gauge.

Advantages : (*i*) Semi conductor strain gauges have the advantage that they have a high gauge factor of about ±130. This allows measurement of very small strains of the order of 0·01 microstrain.

(*ii*) Hysteresis characteristics of semi-conductor strain gauges are excellent. Some units maintain it to less than 0·05%.

(*iii*) Fatigue life is in excess of 10×10^6 operations and the frequency response is upto 10^{12} Hz.

(*iv*) Semi-conductor strain gauges can be very small ranging in length from 0·7 to 7 mm. They are very useful for measurement of local strains.

Disadvantages : (*i*) The major and serious disadvantage of semi-conductor strain gauges is that they are very sensitive to changes in temperature.

(*ii*) Linearity of the semi-conductor strain gauges is poor. The equation for the fractional change in resistance is :

$$\Delta R/R = As + B\,\epsilon^{-4} \text{ where } A \text{ and } B \text{ are constants.}$$

This gauge is rather non-linear at comparatively high strain levels. The gauge factor varies with strain. For example if the gauge factor is 130 at 0·2 per cent strain, then it is 112 at 0·4 per cent strain. The characteristics can be made linear by proper doping.

(*iii*) Semi-conductor strain gagues are more expensive and difficult to attach to the object under study.

25·21. Resistance Thermometers

The resistance of a conductor changes when its temperature is changed. This property is utilized for measurement of temperature.

The variation of resistance R with temperature T can be represented by the following relationship for most of the metals as :

$$R = R_0(1+\alpha_1 T+\alpha_2 T^2+\ldots\ldots+\alpha_n T^n+\ldots) \qquad \ldots(25\cdot32)$$

where R_0 = resistance at temperature $T=0$ and $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$ are constants.

The resistance thermometer uses the change in electrical resistance of conductor to determine the temperature. The requirements of a conductor material to be used in these thermometers are :

(*i*) the change in resistance of material per unit change in temperature should be as large as possible.

and (*ii*) the resistance of the materials should have a continuous and stable relationship with temperature.

The characteristics of various materials used for resistance thermometers are plotted in Fig. 25·20.

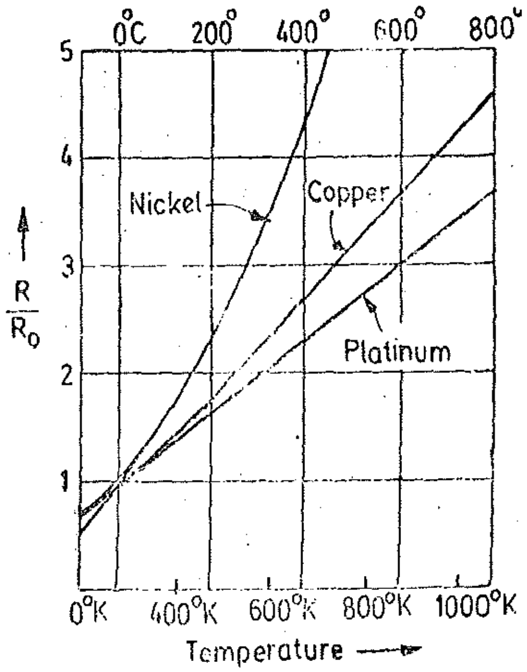


Fig. 25·20. Characteristics of materials used for resistance thermometers.

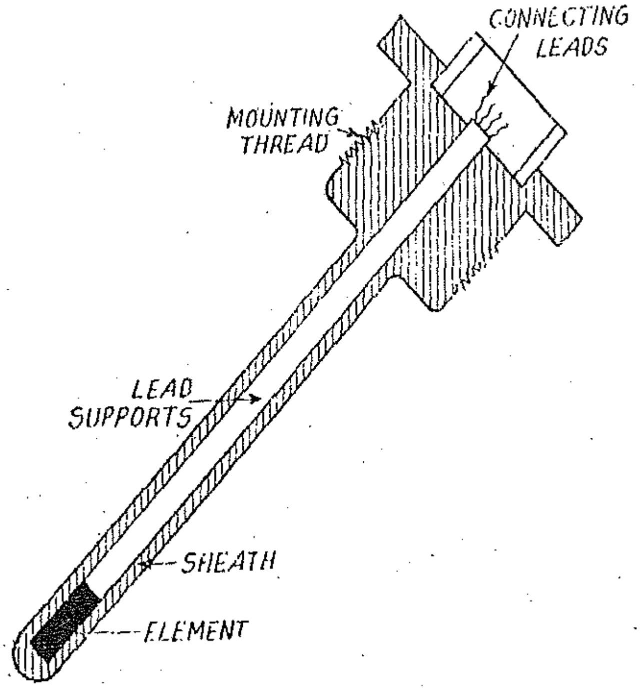


Fig. 25·21. Industrial platinum resistance thermometer

Metals commonly used for resistance thermometers are listed in Table 25·5 along with their salient properties.

TABLE 25·5

Metals Used for Resistance Thermometers

Metal	*Resistance temperature co-efficient* °C (*per cent*)	*Temperature range* °C		*Melting point* °C
		Min.	*Max.*	
Platinum	0·39	−260	1100	1773
Copper	0·39	0	180	1083
Nickel	0·62	−220	300	1455
Tungsten	0·45	−200	1000	3370

Platinum, nickel and copper are the most commonly used metals to measure temperature. The value of α_1 for platinum between $0-100$°C is about 0·004/°C. In fact the resistivity of platinum tends to increase less rapidly at higher temperatures than for other metals, and therefore it is most commonly used material for resistance thermometers. Platinum is commercially available in pure form and has a stability over a high range of temperature as shown in Table 25·5.

An examination of the resistance versus temperature curves of Fig. 25·22 show that the curves are nearly linear. In fact, when only short temperature spans are considered, the linearity is even more evident. This fact is employed to develop approximate analytical equations of resistance versus temperature of a particular metal.

25·21·1. Linear Approximation. A linear approximation means that we may develop an equation for a straight line which approximates the resistance versus temperature curve over a specified span. Fig 25·22 shows a curve for variation of resistance with temperature θ.

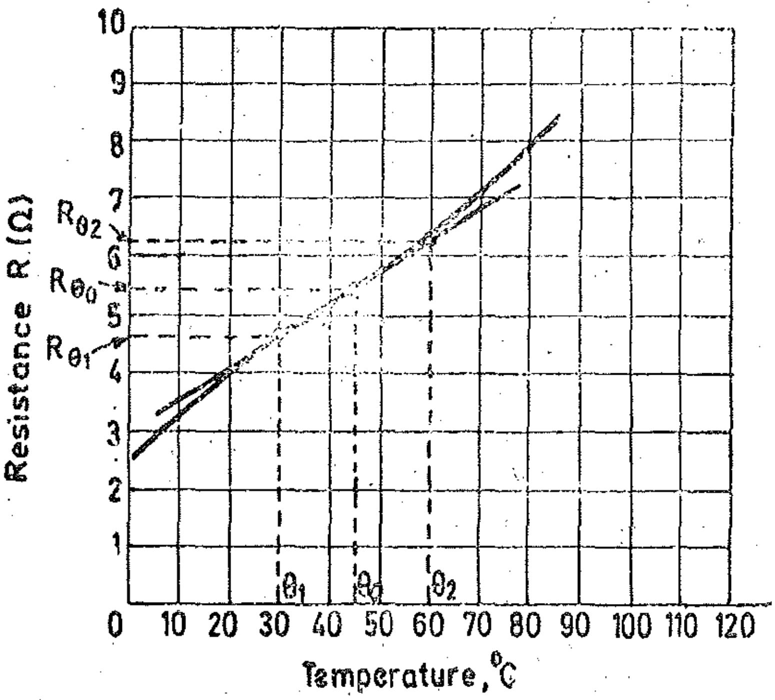


Fig. 25·22. A straight line representing an approximate relationship between resistance R and temperature θ.

Hence a straight line has been drawn between the points of the curve which represent temperatures θ_1 and θ_2 as shown, with θ_0 representing the midpoint temperature. The equation of this straight line is the linear approximation to the curve over the span θ_1 to θ_2. The equation of the straight line written as :

$$R_\theta = R_{\theta_0}(1+\alpha_{\theta_0}\Delta\theta) \quad \text{with } \theta_1<\theta_0<\theta_2. \qquad \ldots(25·33)$$

where R_θ = approximate resistance at temperature θ°Ω ;

R_{θ_0} = approximate resistance at temperature θ_0°; Ω

$\Delta\theta=\theta-\theta_0$ = change in temperature ; °C,

α_{θ_0} = resistance temperature co-efficient at temperature θ_0° ; °C.

The reason for using α_{θ_0} as the fractional slope of the resistance temperature curve is that this same constant can be used for conductors having of the same material with different dimensions. The value of α_{θ_0} can be found from the value of resistance and temperature as shown in Fig. 25·22.

In general :

$$\alpha_{\theta_0}=\frac{1}{R_{\theta_0}}\times(\text{slope at } \theta_0)=\frac{1}{R_{\theta_0}}\left[\frac{R_{\theta_2}-R_{\theta_1}}{\theta_2-\theta_1}\right] \qquad \ldots(25·34)$$

Example 25·11. Find the linear approximation for resistance between 30°C to 60°C using the resistance-temperature curve given in Fig. 25·22.

Solution. We have : $\theta_1=30$°C and $\theta_2=60$°C.

$$\therefore \quad \theta_0=\frac{\theta_1+\theta_2}{2}=\frac{30+60}{2}=45\text{°C}.$$

From Fig. 25·22, we have :

Resistance at 30°C, $R_{\theta_1}=4\cdot8\ \Omega$. Resistance at 45°C, $R_{\theta_0}=5\cdot5\ \Omega$,

and Resistance at 60°C, $R_{\theta_2}=6\cdot2\ \Omega$.

From Eqn. 25·34, $\alpha_{\theta_0}=\frac{1}{R_{\theta_0}}\times\frac{R_{\theta_2}-R_{\theta_1}}{\theta_2-\theta_1}=\frac{1}{5\cdot5}\times\frac{6\cdot2-4\cdot8}{60-30}=0\cdot0085\ \Omega/\Omega-{}^\circ C$

Hence the linear approximation is : $R_\theta=5\cdot5[1+0\cdot0085\,(\theta-45)]$

where θ is the unknown temperature in °C.

25·21·2. Quadratic Approximation. A quadratic approximation to the resistance-temperature curve is a more accurate representation of the resistance-temperature curve over a limited range of temperature. The quadratic approximation relationship includes both a linear term as in Eqn. 25·33 and in addition has a term which varies as the square of the temperature. This approximation is written as :

$$R_\theta=R_{\theta_0}[1+\alpha_1\Delta\theta+\alpha_2(\Delta\theta)^2] \quad ...(25\cdot35)$$

α_1= linear fractional change in resistance ; $\Omega/\Omega-{}^\circ C$,

and α_2= quadratic fractional change in resistance ; $\Omega/\Omega-({}^\circ C)^2$.

Values of α_1 and α_2 are found from tables or from graphs as indicated in example given below by using values of resistance and temperature at three different points. Two equations are formed and values of α_1 and α_2 can be found from these equations.

Example 25·12. Use the values of resistance *versus* temperature given in the table below to find the linear and quadratic approximations of resistance between 100°C and 130°C about a mean temperature of 115°C.

Temperature °C	90	95	100	105	110	115	120	125	130
Resistance Ω	562·66	568·03	573·40	578·77	584·13	589·48	594·84	600·18	605·52

Solution.

1. **Linear Approximation** $\theta_1=100°C$, $\theta_2=130°C$ and $\theta_0=115°C$.

Resistance at 100°C, $R_{\theta_1}=573\cdot40\ \Omega$. Resistance at 115°C, $R_{\theta_0}=510\cdot89\ \Omega$.

Resistance at 130°C, $R_{\theta_2}=605\cdot52\ \Omega$.

From Eqn. 25·34,

$$\alpha_1=\frac{1}{R_{\theta_0}}\times\frac{R_{\theta_2}-R_{\theta_1}}{\theta_2-\theta_1}=\frac{1}{589\cdot48}\times\frac{(605\cdot52-573\cdot40)}{(130-100)}=0\cdot00182\ \Omega/\Omega-{}^\circ C.$$

$\therefore$ The linear approximation is $R_\theta=589\cdot48[1+0\cdot0018(\theta-115)]$

2 Quadratic Approximation

The resistance at any temperature θ is given by Eqn. 25·35

$$R_\theta=R_{\theta_0}[1+\alpha_1\Delta\theta+\alpha_2(\Delta\theta)^2$$

We can find the quadratic terms in Eqn. 25·35 by forming two equations using two points about 115°C.

Therefore, $R_\theta = 589{\cdot}48\ \Omega$, $\theta_0 = 115°C$.

Now using 100°C and 130°C as the two points, we get two equations :

$$573{\cdot}40 = 589{\cdot}48[1+\alpha_1(100-115)+\alpha_2(100-115)^2]$$

and

$$605{\cdot}52 = 589{\cdot}48[1+\alpha_1(130-115)+\alpha_2(130-115)^2]$$

Solving the above equations we get :

$$\alpha_1 = 1{\cdot}59\times10^{-3}\ \Omega/\Omega-°C \quad \text{and} \quad \alpha_2 = 15{\cdot}098\times10^{-6}\ \Omega/\Omega-(°C)^2.$$

Hence, $R_\theta = 589{\cdot}48[1+1{\cdot}59\times10^{-3}(\theta-115)-15{\cdot}09\times10^{-6}(\theta-115)^2]$.

Example 25·13. (*a*) A Platinum resistance thermometer has a resistance of 100 Ω at 25°C. Find its resistance at 65°C. The resistance temperature co-efficient of Platinum is 0·00392 Ω/Ω−°C.

(*b*) If the thermometer has a resistance of 150 Ω, calculate the value of the temperature.

Solution. (*a*) Using the linear approximation, the value of resistance at any temperature is :

$$R_\theta = R_{\theta_0}[1+\alpha_{\theta_0}\Delta\theta]$$

∴ Resistance at 65°C, $R_{65} = 100[1+0{\cdot}00392(65-25)] = 115{\cdot}68\ \Omega$.

(*b*) Suppose θ is the unknown temperature.

∴ $150 = 100[1+0{\cdot}00392(\theta-25)]$ or $\theta = 152{\cdot}55\ \Omega$.

Example 25·14. A 10 Ω copper resistor at 20°C is to be used to indicate the temperature of bearings of a machine. What resistance should not be exceeded if the maximum bearing temperature is not to exceed 150°C. The resistance temperature co-efficient of copper is 0 00393 Ω/Ω−°C at 20°C.

Solution. The value of the resistance in case the temperature is not to exceed 150°C can be calculated as under :

$$R_{150} = 10[1+0{\cdot}00393(150-20)] = 15{\cdot}11\ \Omega.$$

Example 25·15. A temperature alarm unit with a time constant of 120 s is subjected to a sudden rise of temperature of 50°C because of a fire. If an increase of 30°C is required to actuate the alarm, what will be the delay in signalling the sudden temperature increase.

Solution. Assume the thermometer of to be first order system, the variation of indicated temperature θ to a step input temperature θ_0 is given by Eqn. 24·37.

$$\theta = \theta_0\left(1-e^{-t/\tau}\right) \quad \text{or} \quad 30 = 50\left(1-e^{-t/120}\right)$$

∴ $t = 110$ second.

Hence the alarm will be delayed by 110 second

25·22. Thermistors

Thermistors is a contraction of term "**Thermal Resistors**". They are essentially semi-conductors which behave as resistors with a high negative temperature co-efficient of resistance. In some cases the resistance of a thermistor at room temperature may decrease as much as 5 per cent for each 1°C rise in temperature. This high sensitivity to temperature changes make the thermistors extremely useful for precision temperature measurements, control and compensation. Thermistors are widely used in such applications especially in the temperature range of −60°C to +15°C. The resistance of thermistors ranges from 0·5 Ω to 0·75 MΩ.

25·22·1. Construction. Thermistors are composed of sintered mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in variety of sizes and shapes The thermistors may be in the form of beads, rods or discs. Commercial forms are shown in Fig. 25·23.

A thermistor in the form of a bead is smallest in size and the bead may have a diameter of 0·015 mm to 1·25 mm. Beads may be sealed in the tips of solid glass rods to form probes which may be easier to mount than the beads. Glass probes have a diameter of about 2·5 mm and a length which varies from 6 mm to 50 mm. Discs are made by pressing material under high pressure into cylindrical flat shapes with diameters ranging from 2·5 mm to 25 mm.

25·21·2. Resistance Temperature Characteristics of Thermistors. The mathematical expression for the relationship between the resistance of a thermistor and absolute temperature of thermistor is

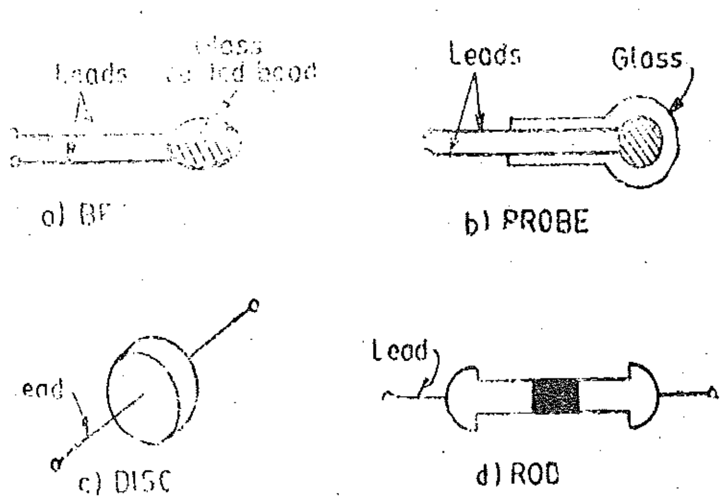


Fig. 25·23. Thermistors.

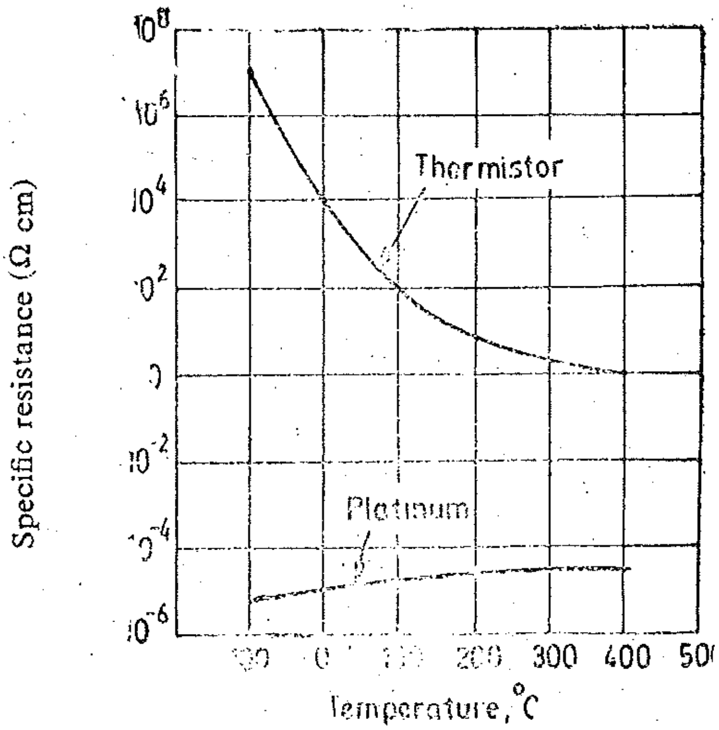


Fig. 25·24. Resistance-temperature characteristics of a typical thermistor and platinum.

$$R_{T1} = R_{T2} \exp.\left[\beta\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right] \quad \ldots(25·36)$$

where R_{T1} = resistance of the thermistor at absolute temperature T_1 ; K,

R_{T2} = resistance of the thermistor at absolute temperature T_2 ; K,

and β = a constant depending upon the material of thermistor, typically 3500 to 4500 K.

The resistance temperature characteristics of a typical thermistor are given in Fig. 25·24. The resistance temperature characteristics of Fig. 25·24 show that a thermistor has a very high negative temperature co-efficient of resistance, making it an ideal **temperature transducer.**

Fig. 25·24 also shows the resistance-temperature characteristics of platinum which is a commonly used material for resistance thermometers. Let us compare the characteristics of the two materials. Between -100°C and 400°C, the thermistor changes its resistivity from 10^{-7} to 1·0 Ω cm, a factor of 10^7, while platinum changes its resistivity by a factor of about 10 within the same temperature range. This explains the high sensitivity of thermistors for measurement of temperature.

The characteristics of thermistors are no doubt non-linear but a linear approximation of the resistance-temperature curve can be obtained over a small range of temperatures. Thus for a limited range of temperature, the resistance of a thermistor varies as given by Eqn. 25·33.

$$R_\theta = R_{\theta_0}[1+\alpha_{\theta_0}\Delta\theta]$$

A thermistor exhibits a negative resistance temperature co-efficient which is typically 0·05 Ω/Ω–°C.

In place of linear approximation, an approximate logarithmic relationship may be used for resistance-temperature relationship for a thermistor.

The relationship is : $R_T = aR_0 \, e^{b/T}$ ...(25·37)

where R_0=resistance at ice point ; Ω, R_0=resistance at temperature T ; K, and a, b are constants.

25·22·3. Applications of Thermistors. The applications of thermistors are :

(*i*) The major application of thermistors is in the field of measurement of temperature. The thermistor's large change of resistance with temperature provides good accuracy and resolution.

A typical thermistor with a resistance of 200 Ω at 25°C and a resistance temperature co-efficient of 0·039 Ω/Ω –°C shows a resistance change of 78 Ω/°C.

Thermistors can also be used for :

(*ii*) Temperature compensation in complex electronic equipment, magnetic amplifiers and instrumentation equipment. This is because thermistors possess a negative resistance temperature coefficient and therefore they can be used as compensators in electrical circuits, as in operation of computer circuits which are affected by temperature changes. An increased stability is obtained by using thermistors as compensating devices

(*iii*) Measurement of power at high frequencies. (*iv*) Measurement of thermal conductivity.

(*v*) Measurement of level, flow and pressure of liquids.

(*vi*) Measurement of composition of gases. (*vii*) Vacuum measurements.

(*vii*) Providing time delay.

Example 25·16. A thermistor has a resistance temperature co-efficient of −5% over a temperature range of 25°C to 50°C. If the resistance of the thermistor is 100 Ω at 25°C, what is the resistance at 35°C.

Solution. Resistance at a temperature of 35°C is :

$$R_{35} = 100\,[1 - 0{\cdot}05(35-25)] = 50\,\Omega$$

Example 25·17. A thermistor has a resistance of 3980 Ω at the ice point (0°C) and 794 Ω at 50 °C. The resistance-temperature relationship is given by $R_T = a\,R_0\,e^{b/T}$
Calculate the constants a and b.

Calculate the range of resistance to be measured in case the temperature varies from 40°C to 100°C.

Solution. The resistance at ice point R_0=3980 Ω.

Absolute temperature at ice point=273 K.

$$\therefore \quad 3980 = a \times 3980 \times e^{b/273} \quad \text{or} \quad 1 = a\,e^{b/273} \qquad ...(i)$$

Resistance at 50°C is R_T=794 Ω.

Absolute temperature corresponding to 50°C is T=273+50°=323 K.

$$\text{Hence,} \quad 794 = a \times 3980\,e^{b/323} = 3980\,a\,e^{b/323} \qquad ...(ii)$$

Solving (*i*) and (*ii*), we have $a = 30 \times 10^{-6}$ and b=2845.

Absolute temperature at 40°C=273+40=313 K

$$\therefore \quad \text{Resistance at } 40°\text{C} = 30 \times 10^{-6} \times 3980 \times e^{2845/313} = 1060\,\Omega.$$