E-V1

Prove that for all integers m and n, if m mod 5 = 2 and n mod 5 = 1, then mn mod 5 = 2.

SOLUTION:
Assume that (m mod 5) = 2 and (n mod 5) = 1.

Then, by multiplying the two, we have
(m mod 5)(n mod 5) = 2.

Now, we can take the modulo 5 on both sides.
(m mod 5)(n mod 5) mod 5 = 2 mod 5
(m mod 5)(n mod 5) mod 5 = 2

However, by a property of modular multiplication, this can be simplified to
(mn mod 5) = 2
which is the exact same as our conclusion.

Therefore if m mod 5 = 2 and n mod 5 = 1, then it holds mn mod 5 = 2. QED

M-V1

Let m and n be integers. If m + n > 100, then m > 40 or n > 60.

SOLUTION:

Assume that m <= 40 and n <= 60.

The maximum value of m is 40, and the maximum value of n is 60.
Therefore, the maximum value of m + n is 100.

This means the statement m + n > 100 must be false.
It was shown that if m <= 40 and n <= 60, then m + n <= 100. Therefore, by contraposition, if m > 40 or n > 60, then m + n > 100. QED

H-V1

Prove that for any integers m and n, $7m + 28n \neq 1000$.

SOLUTION:

Assume that $7m + 28n = 1000$.

Dividing by 7, we have $m + 4n = 1000/7$

Since m is an integer, and 4n is an integer because n is an integer, $m + 4n$ should be equal to an integer. However, 1000/7 is NOT an integer.

This contradicts our original assumption, therefore for any integers m and n, $7m + 28n \neq 1000$.
QED