Chapter 4

F=ma

4.1 Introduction

Newton's laws

In the preceding two chapters, we dealt with *kinematics*. We took the motions of objects as given and then looked at positions, velocities, and accelerations as functions of time. We weren't concerned with the forces that caused the objects' motions. We will now deal with *dynamics*, where the goal is to understand *why* objects move the way they do. This chapter and the following ones will therefore be concerned with force, mass, energy, momentum, etc.

The motion of any object is governed by Newton's three laws:

- First Law: "A body moves with constant velocity (which may be zero) unless acted on by a force."

  If you think hard about this law, it seems a bit circular because we haven't defined what a force is. But if you think harder, there is in fact some content there. See Section 3.1 in Morin (2008) for a discussion of this.

- Second Law: "The rate of change of the momentum of a body equals the force acting on the body."

  In cases where the mass of the body doesn't change (we'll deal with the more general case in Chapter 6), this law becomes

  $$\mathbf{F} = m\mathbf{a}. \tag{4.1}$$

  This is a vector equation, so it is really three equations, namely $F_x = ma_x$, $F_y = ma_y$, and $F_z = ma_z$.

- Third Law: "Given two bodies A and B, if A exerts a force on B, then B exerts an equal and opposite force on A."

  As we'll see in Chapter 6, this law basically postulates conservation of momentum. There is a great deal of physical content in this law; it says that things don't happen in isolation by magic. Instead, if an object feels a force, then there must be another object somewhere feeling the opposite force.

The second law, $\mathbf{F} = m\mathbf{a}$, is the one we'll get the most mileage out of. The unit of force is called a newton (N), and from $F = ma$ we see that $1\,\text{N} = 1\,\text{kg}\,\text{m}/\text{s}^2$. A few types of forces (gravitational, tension, normal, friction, and spring) come up again and again, so let's take a look at each of these in turn.

Gravity

The gravitational force on an object near the surface of the earth is proportional to the mass of the object. More precisely, the force is mg downward, where $g = 9.8\,\text{m/s}^2$. This mg force is a special case of the more general gravitational force we will encounter in Chapter 11. Substituting mg for the force F in Newton's second law quickly gives $mg = ma \implies a = g$. That is, all objects fall with the same acceleration (in the absence of air resistance).

Tension

If you pull on a rope, the rope pulls back on you with the same force, by Newton's third law. The magnitude of this force is called the tension in the rope. As far as the direction of the tension goes, the question, "At a given interior point in the rope, which way does the tension point?" can't be answered. What we *can* say is that the tension at the given point pulls leftward on the point just to its right, and pulls rightward on the point just to its left. Equivalently, the tension pulls leftward on someone holding the right end of the rope, and it pulls rightward on someone holding the left end of the rope.

Normal force

Whereas a tension arises from a material resisting being stretched, a normal force arises from a material resisting being compressed. If you push leftward on the right face of a wooden block, then the normal force from the block pushes rightward on your hand. And similarly, the leftward force from your hand is itself a normal force pushing on the block. In the case where you push inward on the ends of an object shaped like a rod, people sometimes say that there is a "negative tension" in the rod, instead of calling it a normal force at the ends. But whatever name you want to use, the rod pushes back on you at the ends. If instead of a rigid rod we have a flexible rope, then the rope can support a tension, but not a normal force.

Friction

The friction force between two objects is extremely complicated on a microscopic scale. But fortunately we don't need to understand what is going on at that level to get a rough handle on friction forces. To a good approximation under most circumstances, we can say the following things about kinetic friction (where two objects are moving with respect to each other) and static friction (where two objects are at rest with respect to each other).

- Kinetic friction: If there is slipping between two objects, then to a good approximation under non-extreme conditions, the friction force is proportional to the normal force between the objects, with the constant of proportionality (called the *coefficient of kinetic friction*) labeled as μ_k:
$$F_k = \mu_k N. \tag{4.2}$$
The friction force is independent of the contact area and relative speed. The direction of the friction force on a given object is opposite to the direction of the velocity of that object relative to the other object.

- Static friction: If there is no slipping between two objects, then to a good approximation under non-extreme conditions, the *maximum value* of the friction force is proportional to the normal force between the objects, with the constant of proportionality (called the *coefficient of static friction*) labeled as μ_s:
$$F_s \leq \mu_s N. \tag{4.3}$$
As in the kinetic case, the friction force is independent of the contact area. Note well the *equality* in Eq. (4.2) and the *inequality* in Eq. (4.3). Equation (4.3) gives only an *upper limit* on the static friction force. If you push on an object with a force that is smaller

than $\mu_s N$, then the static friction force is exactly equal and opposite to your force, and the object stays at rest.[1] But if you increase your force so that it exceeds $\mu_s N$, then the maximum friction force isn't enough to keep the object at rest. So it will move, and the friction force will abruptly drop to the kinetic value of $\mu_k N$. (It turns out that μ_k is always less than or equal to μ_s; see Problem 4.1 for an explanation why.)

The expressions in Eqs. (4.2) and (4.3) will of course break down under extreme conditions (large normal force, high relative speed, pointy shapes). But they work fairly well under normal conditions. Note that the coefficients of kinetic and static friction, μ_k and μ_s, are properties of *both* surfaces together. A single surface doesn't have a coefficient of friction. What matters is how two surfaces interact.

Spring force

To a good approximation for small displacements, the restoring force from a spring is proportional to the stretching distance. That is, $F = -kx$, where k is the *spring constant.* A large value of k means a stiff spring; a small value means a weak spring. The reason why the $F = -kx$ relation is a good approximation for small displacements in virtually any system is explained in Problem 10.1.

If x is positive then the force F is negative; and if x is negative then F is positive. So $F = -kx$ does indeed describe a restoring force, where the spring always tries to bring x back to zero. The $F = -kx$ relation, known as *Hooke's law,* breaks down if x is too large, but we'll assume that it holds for the setups we're concerned with.

The tension and normal forces discussed above are actually just special cases of spring forces. If you stand on a floor, the floor acts like a very stiff spring. The matter in the floor compresses a tiny amount, exactly the amount that makes the upward "spring" force (which we call a normal force in this case) be equal to your weight.

We'll always assume that our springs are massless. Massive springs can get very complicated because the force (the tension) will in general vary throughout the spring. In a massless spring, the force is the same everywhere in it. This follows from the reasoning in Problem 4.3(a).

Centripetal force

The centripetal force is the force that keeps an object moving in a circle. Since we know from Eq. (3.7) that the acceleration for uniform (constant speed) circular motion points radially inward with magnitude $a = v^2/r$, the centripetal force likewise points radially inward with magnitude $F = ma = mv^2/r$. This force might be due to the tension in a string, or the friction force acting on a car's tires as it rounds a corner, etc. Since $v = r\omega$, we can also write F as $mr\omega^2$.

The term "centripetal" isn't the same type of term as the above "gravity," "tension," etc. descriptors, because the latter terms describe the *type* of force, whereas "centripetal" simply describes the *direction* of the force (radially inward). The word "centripetal" is therefore more like the words "eastward" or "downward," etc. For example, we might say, "The downward force is due to gravity," or "The centripetal force is due to the tension in a string." The centripetal force is *not* a magical special new kind of force. It is simply one of the standard forces (or a combination of them) that points radially inward, and whose magnitude we know always equals mv^2/r.

Free-body diagrams

The "**F**" in Newton's second law in Eq. (4.1) is the *total* force on an object, so it is important to determine what all the various forces are. The best way to do this is to draw a picture. The picture of an object that shows all of the forces acting on it is called a *free-body diagram.* More precisely,

[1]The friction force certainly can't be *equal* to $\mu_s N$ in this case, because if you push rightward with a very small force, then the leftward (incorrect) $\mu_s N$ friction force would cause there to be a nonzero net force, which would hurl the object leftward back toward you!

a free-body diagram shows all of the *external* forces (that is, forces due to other objects) acting on a given object. There are undoubtedly also *internal* forces acting within the object; each atom might be pushing or pulling on the atom next to it. But these internal forces cancel in pairs (by Newton's third law), so they don't produce any acceleration of the object. Only external forces can do that. (We'll assume we have a rigid object, so that the distance between any two given points remains fixed.)

In simple cases (for example, ones involving only one force), you can get away with not drawing a free-body diagram. But in more complicated cases (for example, ones involving forces pointing in various directions), a diagram is absolutely critical. A problem is often hopeless without a diagram, but trivial with one.

The length of a force vector is technically a measure of the magnitude of the force. But when drawing a free-body diagram, the main point is just to indicate all the forces that exist. In general we don't yet know the relative sizes, so it's fine to give all the vectors the same length (unless it's obvious that a certain force is larger than another). Also, the exact location of each force vector isn't critical (at least in this chapter), although the most sensible thing to do is to draw the vector near the place where the force acts. However, when we discuss torque in Chapter 7, the location of the force *will* be important.

Note that due to Newton's third law, for every force vector that appears in the free-body diagram for one object, there is an opposite force vector that appears in the free-body diagram for another object. An example involving two blocks on a table is shown in Fig. 4.1. If a person applies a force F to the left block, then the two free-body diagrams are shown (assume there is no friction from the table). Note that the force pushing the right block rightward is *only* the normal force between the blocks, and *not* the applied force F. True, the N force wouldn't exist without the F force, but the right block feels only the N force; it doesn't care about the original cause of N.

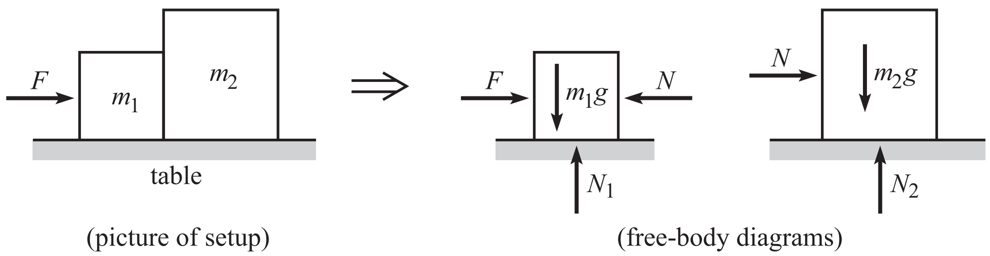


Figure 4.1

If we happened to be concerned also with the free-body diagram for the person applying the force F, then we would draw a force F acting leftward, along with a downward mg gravitational force and an upward normal force from the ground (and also probably a rightward friction force from the ground). Likewise, the free-body diagram for the table would involve gravity and the N_1 and N_2 normal forces pointing downward, along with upward normal forces from the ground at the bases of the legs. But if we're concerned only with the blocks, then all of this other information is irrelevant.

If the direction of the acceleration of an object is known, it is often helpful to draw the acceleration vector in the free-body diagram. But you should be careful to draw this vector with a dotted line or something similar, so that you don't mistake it for a force. Remember that although F equals ma, the quantity ma is *not* a force; see the last section of this introduction.

Atwood's machines

The name *Atwood's machine* is the term used for any system of pulleys, strings, and masses. Although a subset of these systems is certainly very useful in everyday life (a "block and tackle" enables you to lift heavy objects; see Problem 4.4), the main reason for all the Atwood's problems in this chapter is simply that they're good practice for drawing free-body diagrams and applying $F = ma$. An additional ingredient in solving any Atwood's problem is the so-called "conservation of string" relation. This is the condition that the length of any given string doesn't change. This constrains the motion of the various masses and pulleys. A few useful Atwood's facts that come up again and again are derived in Problem 4.3. In this chapter we will assume that all strings and pulleys are massless.

The four forces

Having discussed many of the forces we see in everyday life, we should make at least a brief mention of where all these forces actually come from. There are four known fundamental forces in nature:

- Gravitational: Any two masses attract each other gravitationally. We are quite familiar with the gravitational force due to the earth. The gravitational force between everyday-sized objects is too small to observe without sensitive equipment. But on the planetary scale and larger, the gravitational force dominates the other three forces.

- Electromagnetic: The single word "electromagnetic" is indeed the proper word to use here, because the electric and magnetic forces are two aspects of the same underlying force. (In some sense, the magnetic force can be viewed as a result of combining the electric force with special relativity.) Virtually all everyday forces have their origin in the electric force. For example, a tension in a string is due to the electric forces holding the molecules together in the string.

- Weak: The weak force is responsible for various nuclear processes; it isn't too important in everyday life.

- Strong: The strong force is responsible for holding the protons and neutrons together in a nucleus. Without the strong force, matter as we know it wouldn't exist. But taking the existence of matter for granted, the strong force doesn't show up much in everyday life.

ma is not a force!

Newton's second law is "F equals ma," which says that ma equals a force. Does this imply that ma *is* a force? Absolutely not. What the law says is this: Write down the sum of all the forces on an object, and also write down the mass times the acceleration of the object. The law then says that these two quantities have the same value. This is what a physical law does; it says to take two things that aren't obviously related, and then demand that they are equal. In a simple freefall setup, the $F = ma$ equation is $mg = ma$, which tells us that a equals g. But just because $a = g$, this doesn't mean that the two side of $mg = ma$ represent the same type of thing. The left side is a force, the right side is a mass times an acceleration. So when drawing a free-body diagram, you should *not* include ma as one of the forces. If you do, you will end up double counting things. However, as mentioned above, it is often helpful to indicate the acceleration of the object in the free-body diagram. Just be careful to distinguish this from the forces by drawing it with a dotted line.

4.2 Multiple-choice questions

4.1. Two people pull on opposite ends of a rope, each with a force F. The tension in the rope is

(a) $F/2$ (b) F (c) $2F$

4.2. You accelerate the two blocks in Fig. 4.2 by pushing on the bottom block with a force F. The top block moves along with the bottom block. What force directly causes the top block to accelerate?

(a) the normal force between the blocks
(b) the friction force between the blocks
(c) the gravitational force on the top block
(d) the force you apply to the bottom block

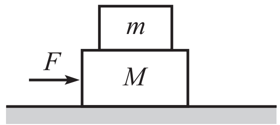


Figure 4.2

4.3. Three boxes are pushed with a force F across a frictionless table, as shown in Fig. 4.3. Let N_1 be the normal force between the left two boxes, and let N_2 be the normal force between the right two boxes. Then

(a) $F = N_1 = N_2$
(b) $F + N_1 = N_2$
(c) $F > N_1 = N_2$
(d) $F < N_1 < N_2$
(e) $F > N_1 > N_2$

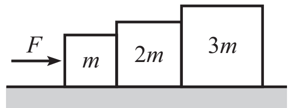


Figure 4.3

4.4. Two blocks with masses 2 kg and 1 kg lie on a frictionless table. A force of 3 N is applied as shown in Fig. 4.4. What is the normal force between the blocks?

(a) 0 (b) 0.5 N (c) 1 N (d) 2 N (e) 3 N

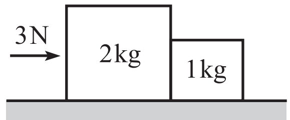


Figure 4.4

4.5. In the system shown in Fig. 4.5, the ground is frictionless, the blocks have mass m and $2m$, and the string connecting them is massless. If you accelerate the system to the right, as shown, the tension is the same everywhere throughout the string connecting the masses because

(a) the string is massless
(b) the ground is frictionless
(c) the ratio of the masses is 2 to 1
(d) the acceleration of the system is nonzero
(e) The tension is the same throughout any string; no conditions are necessary.

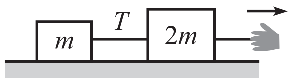


Figure 4.5

4.6. You are in a plane accelerating down a runway during takeoff, and you are holding a pendulum (say, a shoe hanging from a shoelace). The string of the pendulum

(a) hangs straight downward
(b) hangs downward and forward, because the net force on the pendulum must be zero
(c) hangs downward and forward, because the net force must be nonzero
(d) hangs downward and backward, because the net force must be zero
(e) hangs downward and backward, because the net force must be nonzero

4.7. When you stand at rest on a floor, you exert a downward normal force on the floor. Does this force cause the earth to accelerate in the downward direction?

(a) Yes, but the earth is very massive, so you don't notice the motion.
(b) Yes, but you accelerate along with the earth, so you don't notice the motion.
(c) No, because the normal force isn't a real force.
(d) No, because you are also pulling on the earth gravitationally.
(e) No, because there is also friction at your feet.

4.8. If you stand at rest on a bench, the bench exerts a normal force on you, equal and opposite to your weight. Which force is related to this normal force by Newton's third law?

(a) the gravitational force from the earth on you

(b) the gravitational force from you on the earth

(c) the normal force from you on the bench

(d) none of the above

4.9. The driver of a car steps on the gas, and the car accelerates with acceleration a. When writing down the horizontal $F = ma$ equation for the car, the "F" acting on the car is

(a) the normal force between the tires and the ground

(b) the friction force between the tires and the ground

(c) the force between the driver's foot and the pedal

(d) the energy obtained by burning the gasoline

(e) the backward friction force that balances the forward ma force

4.10. The static friction force between a car's tires and the ground can do all of the following *except*

(a) speed the car up

(b) slow the car down

(c) change the car's direction

(d) It can do all of the above things.

4.11. A car is traveling forward along a road. The driver wants to arrange for the car's acceleration to point diagonally backward and leftward. The driver should

(a) turn right and accelerate

(b) turn right and brake

(c) turn left and accelerate

(d) turn left and brake

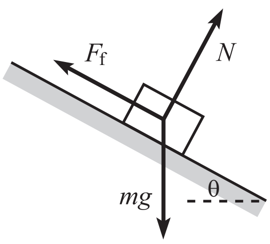


Figure 4.6

4.12. A block is at rest on a plane inclined at angle θ. The forces on it are the gravitational, normal, and friction forces, as shown in Fig. 4.6. These are *not* drawn to scale. Which of the following statements is *always* true, for any θ?

(a) $mg \le N$ and $mg \le F_f$

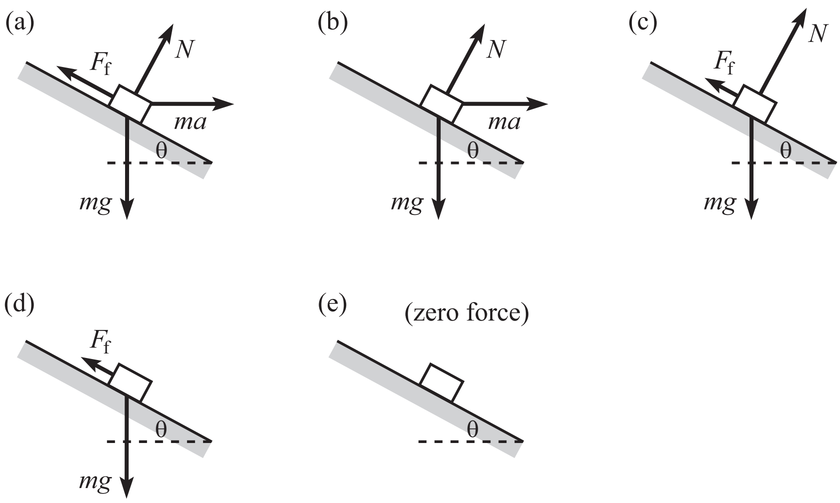
(b) $mg \ge N$ and $mg \ge F_f$

(c) $F_f = N$

(d) $F_f + N = mg$

(e) $F_f > N$ if $\mu_s > 1$

4.13. A block sits on a plane, and there is friction between the block and the plane. The plane is accelerated to the right. If the block remains at the same position on the plane, which of the following pictures might show the free-body diagram for the block? (All of the vectors shown are forces.)



4.14. A block with mass m sits on a frictionless plane inclined at angle θ, as shown in Fig. 4.7. If the plane is accelerated to the right with the proper acceleration that causes the block to remain at the same position with respect to the plane, what is the normal force between the block and the plane?

(a) mg (b) $mg\sin\theta$ (c) $mg/\sin\theta$ (d) $mg\cos\theta$ (e) $mg/\cos\theta$

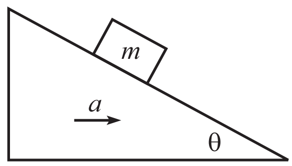


Figure 4.7

4.15. A bead is arranged to move with *constant* speed around a hoop that lies in a vertical plane. The magnitude of the net force on the bead is

(a) largest at the bottom

(b) largest at the top

(c) largest at the side points

(d) the same at all points

4.16. A toy race car travels through a loop-the-loop (a circle in a vertical plane) on a track. Assuming that the speed at the top of the loop is above the threshold to remain in contact with the track, the car's acceleration at the top is

(a) downward and larger than g

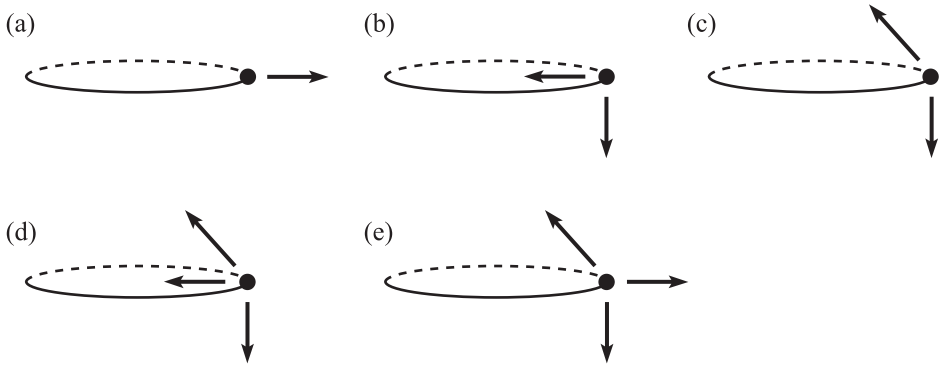
(b) downward and smaller than g

(c) zero

(d) upward and smaller than g

(e) upward and larger than g

4.17. A plane in a holding pattern is flying in a horizontal circle at constant speed. Which of the following free-body diagrams best illustrates the various forces acting on the plane at the instant shown? (See Problem 4.20 for a quantitative treatment of this setup.)



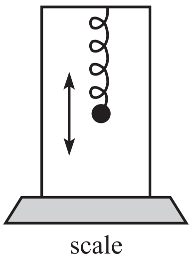


Figure 4.8

4.18. A mass hangs from a spring and oscillates vertically. The top end of the spring is attached to the top of a box, and the box is placed on a scale, as shown in Fig. 4.8. The reading on the scale is largest when the mass is

(a) at its maximum height
(b) at its minimum height
(c) at the midpoint of its motion
(d) All points give the same reading.

4.19. A spring with spring constant k hangs vertically from a ceiling, initially at its relaxed length. You attach a mass m to the end and bring it down to a position that is $3mg/k$ below the initial position. You then let go. What is the upward acceleration of the mass right after you let go?

(a) 0 (b) g (c) $2g$ (d) $3g$ (e) $4g$

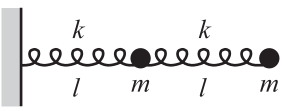


Figure 4.9

4.20. Two springs both have spring constant k and relaxed length zero. They are each stretched to a length ℓ and then attached to two masses and a wall, as shown in Fig. 4.9. The masses are simultaneously released. Immediately afterward, the magnitudes of the accelerations of the left and right masses are, respectively,

(a) $2k\ell/m$ and $k\ell/m$
(b) $k\ell/m$ and $2k\ell/m$
(c) $k\ell/m$ and $k\ell/m$
(d) 0 and $2k\ell/m$
(e) 0 and $k\ell/m$

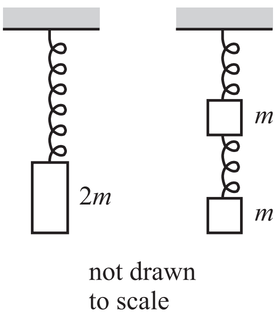


Figure 4.10

4.21. A mass $2m$ suspended from a given spring causes it to stretch relative to its relaxed length. The mass and the spring are then each cut into two identical pieces and connected as shown in Fig. 4.10. Is the bottom of the lower mass higher than, lower than, or at the same height as the bottom of the original mass? (This one takes a little thought.)

(a) higher
(b) lower
(c) same height

4.22. What is the conservation-of-string relation for the Atwood's machine shown in Fig. 4.11? All of the accelerations are defined to be positive upward.

(a) $a_3 = -2(a_1 + a_2)$

(b) $a_3 = -(a_1 + a_2)$

(c) $a_3 = -(a_1 + a_2)/2$

(d) $a_3 = -(a_1 + a_2)/4$

(e) $a_3 = -2a_2$

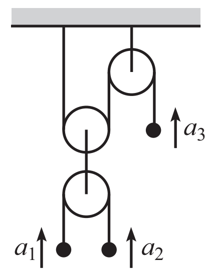


Figure 4.11

4.23. What is the conservation-of-string relation for the Atwood's machine shown in Fig. 4.12? All of the accelerations are defined to be positive upward.

(a) $a_3 = -a_1 - a_2$

(b) $a_3 = -2a_1 - 2a_2$

(c) $a_3 = -4a_2$

(d) $2a_3 = -a_1 - a_2$

(e) $4a_3 = -a_1 - a_2$

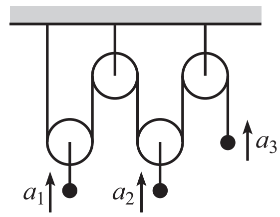


Figure 4.12

4.24. What is the conservation-of-string relation for the Atwood's machine shown in Fig. 4.13? All of the accelerations are defined to be positive upward.

(a) $a_1 = a_3$

(b) $a_1 = -a_3$

(c) $a_2 = -(a_1 + a_3)/2$

(d) $a_2 = -(a_1 + a_3)$

(e) $a_2 = -2(a_1 + a_3)$

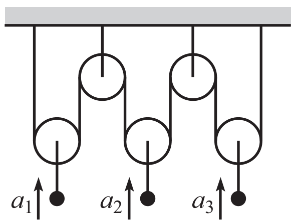


Figure 4.13

4.3 Problems

The first five problems are foundational problems.

4.1. **Coefficients of friction**

Explain why the coefficient of static friction, μ_s, must always be at least as large as the coefficient of kinetic friction, μ_k.

4.2. **Cutting a spring in half**

A spring has spring constant k. If it is cut in half, what is the spring constant of each of the resulting shorter springs?

4.3. **Useful Atwood's facts**

In the Atwood's machine shown in Fig. 4.14(a), the pulleys and strings are massless (as we will assume in all of the Atwood's problems in this chapter). Explain why (a) the tension is the same throughout the long string, as indicated, (b) the tension in the bottom string is twice the tension in the long string, as indicated, and (c) the acceleration of the right mass is negative twice the acceleration of the left mass.

Also, in Fig. 4.14(b), explain why (d) the acceleration of the left mass equals negative the average of the accelerations of the right two masses.

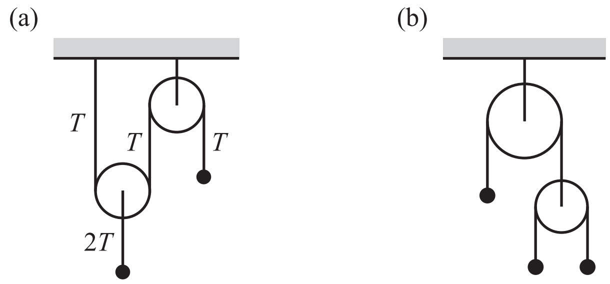


Figure 4.14

4.4. **Block and tackle**

(a) What force on the rope must be exerted by the person in Fig. 4.15(a) in order to hold up the block, or equivalently to move it upward at constant speed? The rope wraps twice around the top of the top pulley and the bottom of the bottom pulley. (Assume that the segment of rope attached to the center of the top pulley is essentially vertical.)

(b) Now consider the case where the person (with mass m) stands on the block, as shown in Fig. 4.15(b). What force is now required?

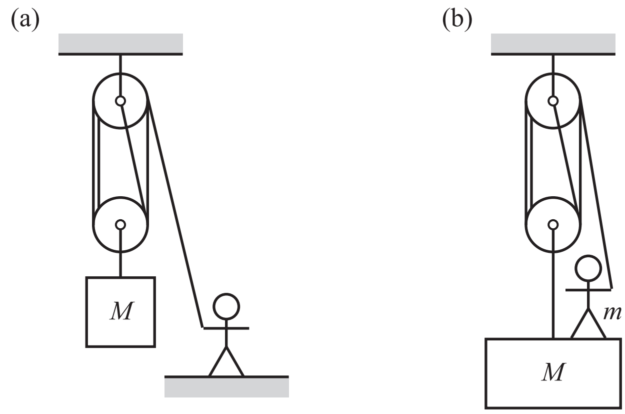


Figure 4.15

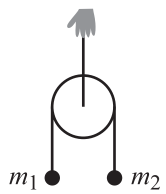


Figure 4.16

4.5. **Equivalent mass**

In Fig. 4.16 you support the pulley system, with your hand at rest. If you have your eyes closed and think that you are instead supporting a single mass M at rest, what is M in terms of m_1 and m_2? Is M simply equal to $m_1 + m_2$?

The following eight problems involve Atwood's machines. This large number of Atwood's problems shouldn't be taken to imply that they're terribly important in physics (they're not). Rather, they are included here because they provide good practice with $F = ma$.

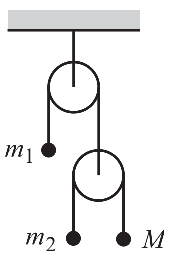


Figure 4.17

4.6. **Atwood's 1**

Consider the Atwood's machine shown in Fig. 4.17. The masses are held at rest and then released. In terms of m_1 and m_2, what should M be so that m_1 doesn't move? What relation must hold between m_1 and m_2 so that such an M exists?

4.7. **Atwood's 2**

Consider the Atwood's machine shown in Fig. 4.18. Masses of m and $2m$ lie on a frictionless table, connected by a string that passes around a pulley. The pulley is connected to another mass of $2m$ that hangs down over another pulley, as shown. Find the accelerations of all three masses.

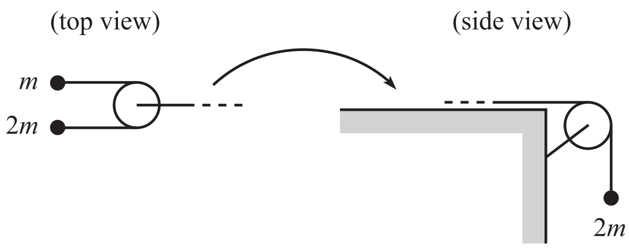


Figure 4.18

4.8. **Atwood's 3**

Consider the Atwood's machine shown in Fig. 4.19, with masses m, m, and $2m$. Find the acceleration of the mass $2m$.

4.9. **Atwood's 4**

In the Atwood's machine shown in Fig. 4.20, both masses are m. Find their accelerations.

4.10. **Atwood's 5**

Consider the triple Atwood's machine shown in Fig. 4.21. What is the acceleration of the rightmost mass? *Note*: The math isn't as bad as it might seem at first. You should take advantage of the fact that many of your $F = ma$ equations look very similar.

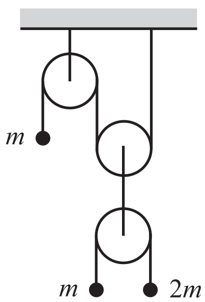


Figure 4.19

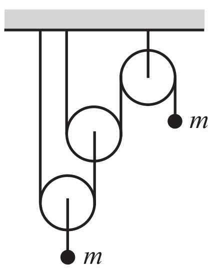


Figure 4.20

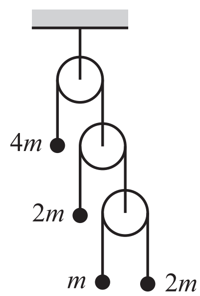


Figure 4.21

4.11. **Atwood's 6**

Consider the Atwood's machine shown in Fig. 4.22. The middle mass is glued to the long string. Find the accelerations of all three masses, and also the tension everywhere in the long string.

4.12. **Atwood's 7**

In the Atwood's machine shown in Fig. 4.23, both masses are m. Find their accelerations.

4.13. **Atwood's 8**

In the Atwood's machine shown in Fig. 4.24, both masses are m. The two strings that touch the center of the left pulley are both attached to its axle. Find the accelerations of the masses.

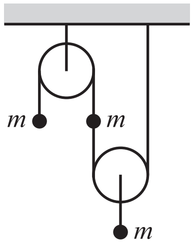


Figure 4.22

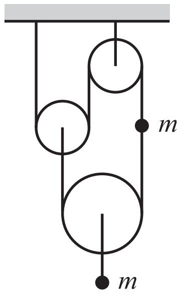


Figure 4.23

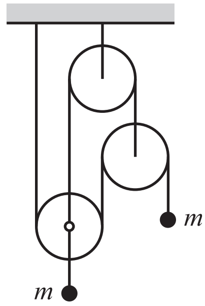
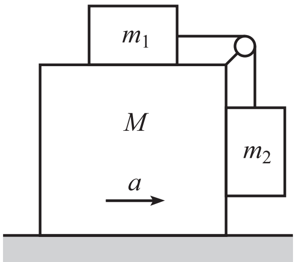


Figure 4.24



(side view)

Figure 4.25

4.14. **No relative motion**

All of the surfaces in the setup in Fig. 4.25 are frictionless. You push on the large block and give it an acceleration a. For what value of a is there no relative motion among the masses?

4.15. **Slipping blocks**

A block with mass m sits on top of a block with mass $2m$ which sits on a table. The coefficients of friction (both static and kinetic) between all surfaces are $\mu_s = \mu_k = 1$. A string is connected to each mass and wraps halfway around a pulley, as shown in Fig. 4.26. You pull on the pulley with a force of $6mg$.

(a) Explain why the bottom block must slip with respect to the table. *Hint*: Assume that it doesn't slip, and show that this leads to a contradiction.

(b) Explain why the top block must slip with respect to the bottom block. (Same hint.)

(c) What is the acceleration of your hand?

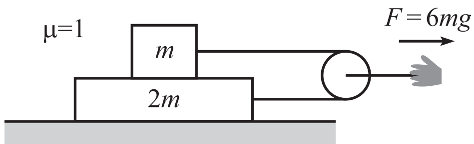


Figure 4.26

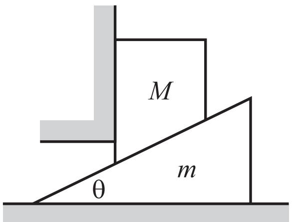


Figure 4.27

4.16. **Block and wedge**

A block with mass M rests on a wedge with mass m and angle θ, which lies on a table, as shown in Fig. 4.27. All surfaces are frictionless. The block is constrained to move vertically by means of a wall on its left side. What is the acceleration of the wedge?

4.17. **Up and down a plane**

A block with mass m is projected up along the surface of a plane inclined at angle θ. The initial speed is v_0, and the coefficients of both static and kinetic friction are equal to 1. The block reaches a highest point and then slides back down to the starting point.

(a) Show that in order for the block to in fact slide back down (instead of remaining at rest at the highest point), θ must be greater than 45°.

(b) Assuming that $\theta > 45°$, find the times of the up and down motions.

(c) Assuming that $\theta > 45°$, is the total up and down time longer or shorter than the total time it would take (with the same initial v_0) if the plane were frictionless? Or does the answer to this question depend on what θ is? (The solution to this gets a little messy.)

4.18. **Rope in a tube**

A rope is free to slide frictionlessly inside a circular tube that lies flat on a horizontal table. In part (a) of Fig. 4.28, the rope moves at constant speed. In part (b) of the figure, the rope is at rest, and you pull on its right end to give it a tangential acceleration. What is the direction of the net force on the rope in each case?

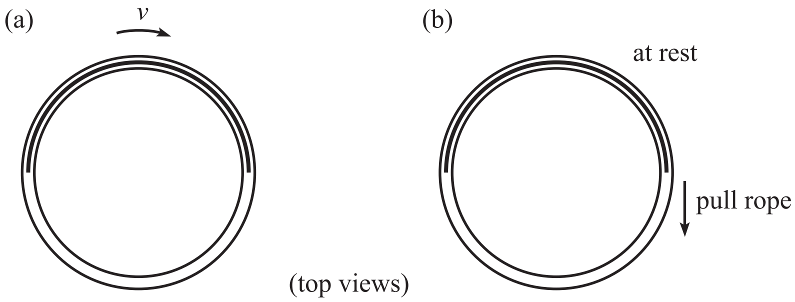


Figure 4.28

4.19. **Circling bucket**

You hold the handle of a bucket of water and swing it around in a vertical circle, keeping your arm straight. If you swing it around fast enough, the water will stay inside the bucket, even at the highest point where the bucket is upside down. What, roughly, does "fast enough" here mean? (You can specify the maximum time of each revolution.) Make whatever reasonable assumptions you want to make for the various parameters involved. You can work in the approximation where the speed of the bucket is roughly constant throughout the motion.

4.20. **Banking an airplane**

A plane in a holding patter flies at speed v in a horizontal circle of radius R. At what angle should the plane be banked so that you don't feel like you are getting flung to the side in your seat? At this angle, what is your apparent weight (that is, what is the normal force from the seat)?

4.21. **Breaking and turning**

You are driving along a horizontal straight road that has a coefficient of static friction μ with your tires. If you step on the brakes, what is your maximum possible deceleration? What is it if you are instead traveling with speed v around a bend with radius of curvature R?

4.22. **Circle of rope**

A circular loop of rope with radius R and mass density λ (kg/m) lies on a frictionless table and rotates around its center, with all points moving at speed v. What is the tension in the rope? *Hint*: Consider the net force on a small piece of rope that subtends an angle $d\theta$.

4.23. **Cutting the string**

A mass m is connected to the end of a massless string of length ℓ. The top end of the string is attached to a ceiling that is a distance ℓ above the floor. Initial conditions have been set up so that the mass swings around in a horizontal circle, with the string always making an angle θ with respect to the vertical, as shown in Fig. 4.29. If the string is cut, what horizontal distance does the mass cover between the time the string is cut and the time the mass hits the floor?

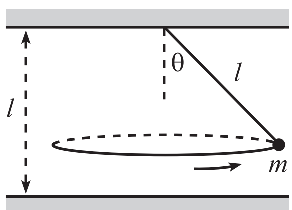


Figure 4.29