Problem 1: Description about Beta function

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Introduction

Beta function is also known as Euler's integral of first kind and is very important in calculus and is basically an association between input and output values. The beta function is used to determine average time to complete some tasks in the time related problems. Beta function has a very close connection to the gamma function which also the generalisation of the factorial function. The beta function is defined as follows:

$$\beta(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$$

for Re (x) > 0 and Re (y) > 0

Domain

Re (x) > 0 and Re (y) > 0

Co-domain

All real numbers

$$(-\infty, \infty)$$

Characteristics

- The beta function is uniquely defined for positive numbers and complex numbers with positive real parts. It is approximated for other numbers.
- Beta function is symmetric,
$$\beta(x, y) = \beta(y, x)$$
- A key property of beta function is that it is closly related to the gamma function
$$\beta(x, y) = \frac{\gamma(x)\gamma(y)}{\gamma(x+y)}$$

Applications

- The Beta function was the first known 'Scattering' amplitude in String theory.
- In physics and string theory the beta function is used to calculate and reproduce scattering amplitudes in terms of the Regge trajectories.

References

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