

Q2) Propositional logic statements:

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- ① Mythical $\rightarrow \neg$ Mortal
- ② \neg Mythical \rightarrow Mortal \wedge Mammal
- ③ \neg mortal \vee Mammal \rightarrow Horned
- ④ Horned \rightarrow Magical

we will infer

- ⑤ \neg Mythical $\vee \neg$ Mortal — ①
- ⑥a Mythical \vee Mortal — ②
- ⑥b Mythical \vee Mammal — ②
- ⑦ \neg Mortal \vee Mammal — ⑤ & ⑥b
- ⑧ Horned — ③ & ⑦
- ⑨ Magical — ④ & ⑧
- ⑩ Mythical —

— Modus ponens or any
other rule doesn't satisfy
the condition

"Hence the unicorn is both Magical & Horned"

Q3) (i) True

because false has no models & entails
every sentence AND because true is true
for all models & hence is entailed by every
Sentence.

(ii) ~~True~~ False

(iii) True

because on L.H.S all the terms are considered as one model and which is also present in the 2 models on R.H.S

(iv) false

because one of the models of $A \leftrightarrow B$ has both $A \neq B$ & this doesnot satisfy the condition

(v) True

Because $A \leftrightarrow B$ is on R.H.S which states one of the conjuncts in the def of $A \leftrightarrow B$

(vi) ~~True~~ True

R.H.S is false because both the disjuncts are false i.e A, B are true & C is false. so L.H.S is false

(vii) True

(viii) True; Because remaining a conjunct only allows more models

(ix) False, because remaining a conjunct only allows more models

(x) True, because it is satisfiable

(xi) True, because R.H.S is entitles by L.H.S so models are those of $A \leftrightarrow B$

(xii) True, Because $\frac{1}{2}$ of models satisfy $(A \leftrightarrow B) \leftrightarrow C$ & there are same no. of models & non models

Q4) (a) True
if α is valid it is true in all models
hence α must be valid

(b) False
 α trivially holds in every model of false

(c) True

(d) True

(e) False

Q5) (a) True & it follows monotonicity

(b) True if $(\beta \wedge \gamma)$ is true in every model of ϕ
then β & γ are true in every model of ϕ
so $\alpha \models \beta$ & $\alpha \models \gamma$

(c) If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$

False, consider $\beta = A$, $\gamma = \underline{\neg A}$

Q6) (a) False, only if B & C are false, which
occurs in 4 cases for A & D

(b) False, only if A, B, C & D are false
occurs only in 1 case.

(c) the last four conjuncts specify a model in
which the 1st conjunct is false.

Q7) a) valid

b) Neither

c) Neither

d) valid

e) valid

f) valid

g) valid

Q8) CNF representations :-

$$S1: (\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$$

$$S2: (\neg E \vee D)$$

$$S3: (\neg C \vee \neg F \vee \neg B)$$

$$S4: (\neg E \vee B)$$

$$S5: (\neg B \vee F)$$

$$S6: (\neg B \vee C)$$

Q9) (a) (i) NO, the sentence asserts, among other things, that all conservatives are radical, which is not what was stated.

(ii) yes, this says that if a person is a radical then they are electable iff they are conservative

(iii) NO, this is equivalent to $\neg R \vee \neg C \vee E \vee \neg E$ which is a tautology, true under any assignment

$$(b) (i) \equiv ((R \wedge E) \Rightarrow C) \wedge (C \Rightarrow (R \wedge E))$$

$$\equiv ((R \wedge E) \Rightarrow C) \wedge (C \Rightarrow R) \wedge (C \Rightarrow E)$$

$$(ii) \equiv R \Rightarrow ((E \Rightarrow C) \wedge (C \Rightarrow E))$$

$$\equiv \neg R \vee ((\neg E \vee C) \wedge (\neg C \vee E))$$

$$\equiv (\neg R \vee \neg E \vee C) \wedge (\neg R \vee \neg C \vee E)$$

(iii) True \Leftrightarrow True

$$E \Rightarrow ((C \Rightarrow E) \vee \neg E)$$

Q10) (a)

Goal is neglected $(A) / (\neg A)$ with last 2 clauses to produce $\neg C \neq \neg D$ with $\neg C \neq \neg D$ we can reduce $\neg A \neq \neg B$ with all clauses we are able to produce an empty clause.

(b) First each 2-CNF clause has 2 phases to put literals. There are $2n$ distinct literals. So there are $(2n)^2$ syntactically distinct clauses & they are identical

$C(2n, 2) = (2n)(2n-1)/2 \approx 2n^2 - n$ clauses with 2 diff literals. & all these clauses are semantically distinct except those are equivalent to true so we get $2n^2 - 2n + 1$ clauses with different literals & there are $2n$ clauses with repeated literals. all distinct. So there are $2n^2 + 1$ distinct clauses in all.

(c) Resolving 2-CNF clauses can't increase the clause size therefore resolution can generate only $O(n^2)$ distinct clauses before it terminates

(d) The no. of 3-CNF clauses is $O(n^3)$. so there wont be non polynomial complexity on the basis of the no. of diff clause by resolving two 3-CNF clauses can increase the clause size to 4 & so on. so clause size grows to $O(n)$ giving $O(n^n)$ possible clause.

Q1) from the given fig shows that "There is no pit in $[2,2]$ ", "There is a wumpus in $[1,3]$ ".

if there is a pit in $[2,2]$, then there has to be breeze in $[1,2]$ but it is not there. so there is no pit in $[2,2]$.

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 5 OK	2,2 OK	3,2	4,2
4,1 OK	2,1 OK	3,1	4,1

As there is stench in $[1,2]$ there has to be a wumpus in $[1,3]$ or $[2,2]$. But if there is wumpus in $[2,2]$ then there has to be stench in $[2,1]$, but it is not there.

So, there is wumpus in $[1,3]$.

So, proved both of them.