

A1 Assignment

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Q2) Propositional logic statements:

Charran
Shanllar

- ① Mythical $\rightarrow \neg$ Mortal
- ② \neg Mythical \rightarrow Mortal \wedge Mammal
- ③ \neg Mortal \vee Mammal \rightarrow Horned
- ④ Horned \rightarrow Magical

we will infer

- ⑤ \neg Mythical $\vee \neg$ Mortal — ①
- ⑥a Mythical \vee Mortal — ②
- ⑥b Mythical \vee Mammal — ②
- ⑦ \neg Mortal \vee Mammal — ⑤ \wedge ⑥b
- ⑧ Horned — ③ \wedge ⑦
- ⑨ Magical — ④ \wedge ⑧
- ⑩ Mythical — Modus ponens or any other rule doesn't satisfy the condition

"Hence the unicorn is both Magical & Horned"

Q3) (i) True

because False has no models & entails every sentence AND because True is True for all models & hence is entailed by every sentence.

(ii) False

(iii) True

because on L.H.S all the terms are considered as one model and which is also present in the 2 models on R.H.S

(iv) False

because one of the models of $A \Leftrightarrow B$ has both $A \in B$ & this does not satisfy the condition

(v) True

Because $A \Leftrightarrow B$ is on R.H.S which states one of the conjuncts in the def of $A \Leftrightarrow B$

(vi) $A \wedge B$ True

R.H.S is false because both the disjuncts are false i.e. A, B are true & C is false. So L.H.S is false

(vii) True

(viii) True; Because remaining a conjunct only allows more models

(ix) False, because remaining a conjunct only allows more models

(x) True, because it is satisfiable

(xi) True, because R.H.S is entailed by L.H.S so models

are those of $A \Leftrightarrow B$

(xii) True, Because $\frac{1}{2}$ of models satisfy $(A \Leftrightarrow B) \wedge C$ & there are same no. of models & non models

Q4) (a) True

if α is valid it is true in all models
hence α must be valid

(b) False

α trivially holds in every model of false

(c) True

(d) True

(e) False

Q5) (a) True & it follows monotonicity

(b) True if $(\beta \wedge r)$ is true in every model of α
then β & r are true in every model of α
so $\alpha \models \beta$ & $\alpha \models r$

(c) If $\alpha \models (\beta \vee r)$ then $\alpha \models \beta$ or $\alpha \models r$

False, consider $\beta = A$, $r = \neg A$

Q6) (a) False, only if B & C are false, which
occurs in 4 cases for $A \wedge D$

(b) False, only if A, B, C & D are false
occurs only in 1 case.

(c) the last four conjuncts specify a model in
which the 1st conjunct is false.

Q7) a) valid

- b) Neither
- c) Neither
- d) valid
- e) valid
- f) valid
- g) valid

Q8) CNF representations :-

$$S_1: (\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$$

$$S_2: (\neg E \vee D)$$

$$S_3: (\neg C \vee \neg F \vee \neg B)$$

$$S_4: (\neg E \vee B)$$

$$S_5: (\neg B \vee F)$$

$$S_6: (\neg B \vee C)$$

Q9) (a) (i) No, this sentence asserts, among other things, that all conservatives are radical, which is not what was stated.

(ii) Yes, this says that if a person is a radical then they are electable iff they are conservative.

(iii) No, this is equivalent to $\neg R \vee \neg C \vee E \vee \neg E$ which is a tautology, true under any assignment.

$$(b)(i) \equiv ((R \wedge E) \Rightarrow C) \wedge (C \Rightarrow (R \wedge E))$$

$$\equiv ((R \wedge E) \Rightarrow C) \wedge (C \Rightarrow R) \wedge (C \Rightarrow E)$$

$$\equiv R \Rightarrow ((E \Rightarrow C) \wedge (C \Rightarrow E))$$

$$\equiv \neg R \vee ((\neg E \vee C) \wedge (\neg C \vee E))$$

$$\equiv (\neg R \vee \neg E \vee C) \wedge (\neg R \vee \neg C \vee E)$$

(iii) True \Leftrightarrow True

$$R \Rightarrow ((C \Rightarrow E) \vee \neg E)$$

Q10) (a)

Goal \Rightarrow negated (A) / ($\neg A$) with last 2 clauses to produce $\neg C \wedge \neg D$ with $\neg C \wedge \neg D$ we can reduce $\neg A \wedge \neg B$ with all clauses we are able to produce an empty clause.

(b) First each 2-CNF clause has 2 phases to put literals. There are 2^n distinct literals. So there are $(2^n)^2$ syntactically distinct clauses & they are identical

$$C(2n, 2) = (2n)(2n-1)/2 = 2^{n^2-n} \text{ clauses}$$

with 2 diff literals. & all these clauses are semantically distinct except those are equivalent to true so we get 2^{n^2-2n+1} clauses with different literals & there are 2^n clauses with repeated literals. all distinct. So there are 2^{n^2+1} distinct clauses in all.

(c) Resolving 2-CNF clauses can't increase the clause size therefore resolution can generate only $O(n^2)$ distinct clauses before it terminates

(d) The no. of 3-CNF clauses is $O(n^3)$. so there won't be non polynomial complexity on the basis of the no. of diff clause by resolving two 3-CNF clauses can increase the clause size to 4 & so on. so clause size grows to $O(n)$ giving $O(n^r)$ possible clause.

(2) From the given fig shows that "There is no pit in $\{2,2\}$ ", "There is a wumpus in $\{1,3\}$ ".

If there is a pit in $\{2,2\}$, then there has to be breeze in $\{1,2\}$ but it is not there. So there is no pit in $\{2,2\}$.

$1,4$	$2,4$	$3,4$	$4,4$
$1,3$	$2,3$	$3,3$	$4,3$
$1,2$ <small>(AT) S ok</small>	$2,2$ <small>S ok</small>	$3,2$	$4,2$
$1,1$ <small>S ok</small>	$2,1$ <small>S ok</small>	$3,1$	$4,1$

As there is stench in $\{1,2\}$ there has to be a wumpus in $\{1,3\}$ or $\{2,2\}$. But if there is wumpus in $\{2,2\}$ then there has to be stench in $\{2,1\}$, but it is not there.

So, there is wumpus in $\{1,3\}$.
So, passed both of them.