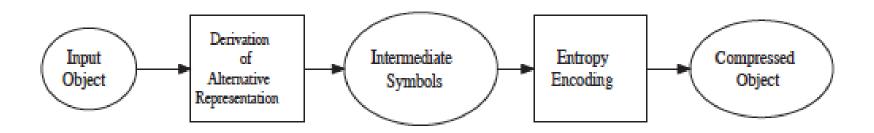


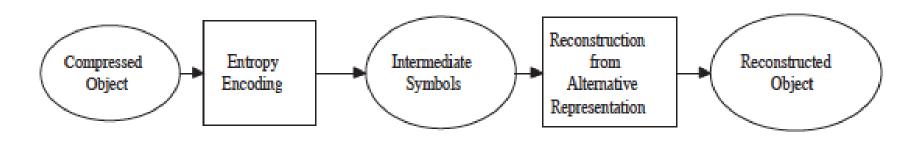
Processing in block DCT domain

Jayanta Mukhopadhyay Dept. of Computer Science and Engg.

"Image and video processing in the compressed domain", CRC Press, 2011.

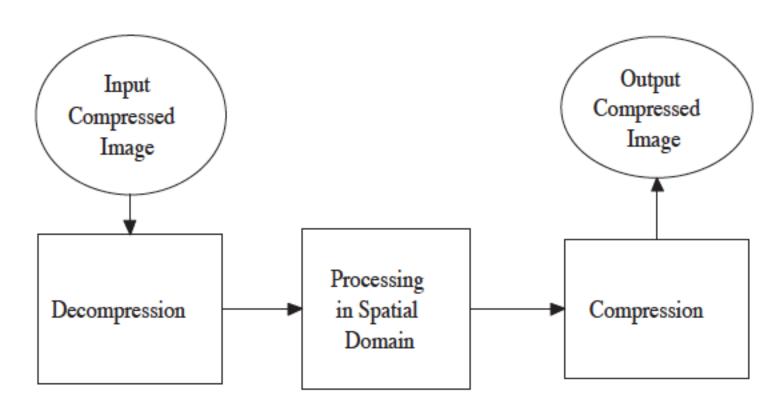
Image compression and decompression



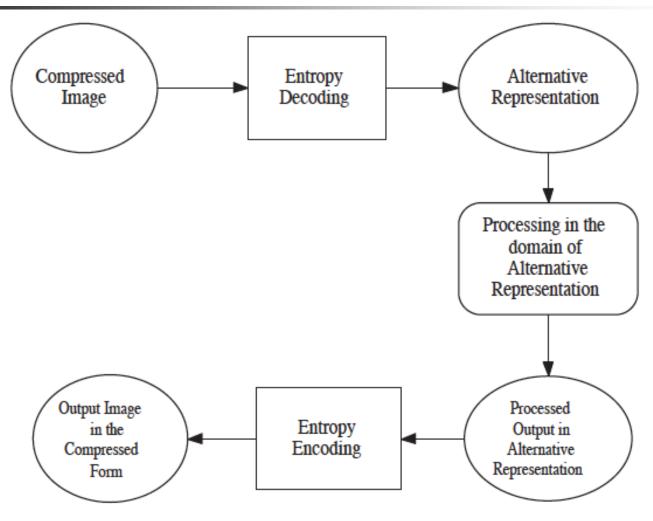




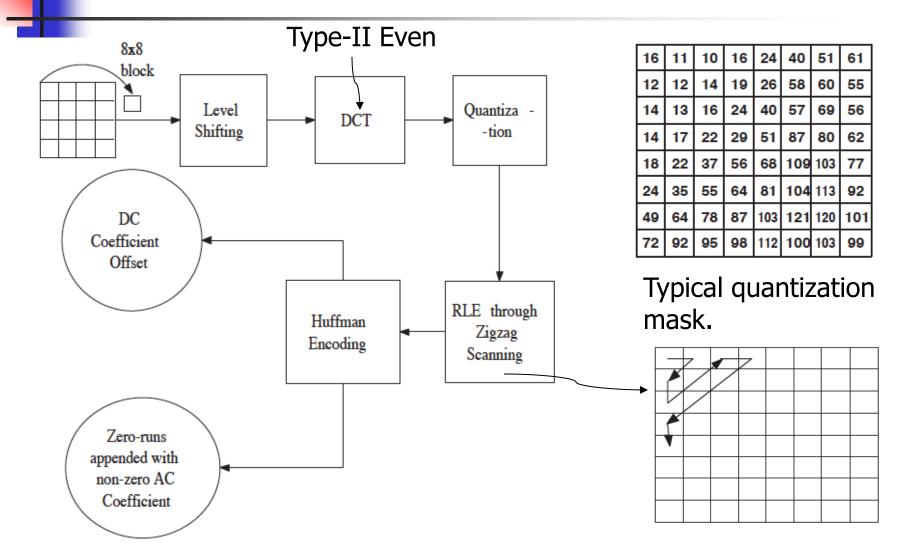
Processing with compressed image: Spatial domain approach



Processing with compressed image: Compressed domain approach



JPEG: Baseline scheme

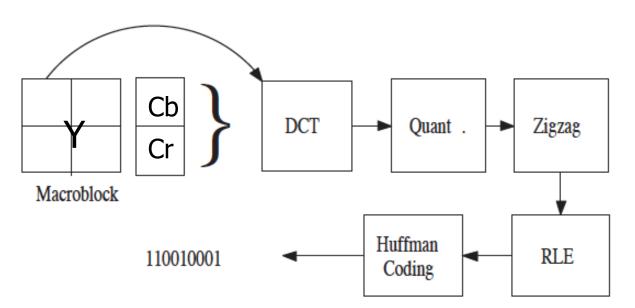


Color encoding in JPEG

Y-Cb-Cr color space:

$$Y = 0.502G + 0.098B + 0.256R,$$

 $Cb = -0.290G + 0.438B - 0.148R + 128,$
 $Cr = -0.366G - 0.071B + 0.438R + 128.$



Motivations

- Computation with reduced storage.
- Avoid overhead of entropy decoding and encoding.
- Exploit spectral factorization for improving the quality of result and speed of computation.

Typical Applications

- Resizing.
- Filtering.
- Enhancement.



Image Resizing

Image Halving

Use of linear and distributive properties.

$$DCT(\mathbf{x_d}) = \sum_{i=0}^{1} \sum_{i=0}^{1} DCT(p_i)DCT(\mathbf{x_{ij}})DCT(p_i^T).$$

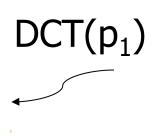
Not so sparse matrix multiplication!

 $DCT(p_0)$: Not so sparse.

No gain!

```
0.453
        0.204
               -0.034
                        0.010
                                 -0.006
                                           0.014
                                                  -0.041
        0.49
                                                  -0.098
                  0
                          0
                               0
                                     0
                                             0
              0.237
-0.159
        0.388
                       -0.041
                                   0.027
                                          -0.098
                                                  -0.077
               0.462
                         0
                                     0
                                          -0.191
                                                     0
0.106
       -0.173
              0.355
                        0.204
                               0 -0.136
                                          -0.147
                                                   0.034
          0
                  0
                        0.416
                               0 -0.278
                                            0
                                                     0
-0.090
        0.136
                                           0.072
                                                  -0.027
               -0.173
                        0.360
                                  -0.240
```

0 0 0 0.500-0.4530.2040.010 -0.0140.034-0.006-0.041-0.490 0 0.098 0 0.1590.388-0.237-0.0410.098-0.0770.0270 0.4620 0 -0.1910 -0.106-0.173-0.3550.2040 -0.1360.1470.0340 -0.4160.2780 0.0900.1360.1730.360-0.240-0.072



Typical result:



Original



Bi-linear



Linear and distributive method

1

2D DCT: Sub-band relation

$$\begin{array}{rcl} x_{LL}(m,n) & = & \frac{1}{4}\{x(2m,2n) + x(2m+1,2n) \\ & + x(2m,2n+1) + x(2m+1,2n+1)\}, \ 0 \leq m,n \leq \frac{N}{2} - 1. \end{array}$$

Low-pass truncated approximation:

$$X(k,l) = \begin{cases} 2\overline{X_{LL}}(k,l), & k,l = 0,1,\dots,\frac{N}{2} - 1, \\ 0, & \text{otherwise.} \end{cases}$$



Block composition and decomposition

$$X^{(N)} = C_N \mathbf{x}$$

 To convert M adjacent N-point DCT blocks to a single MxN-point DCT block.

$$X^{(MN)} = A_{(M,N)}[X_0^{(N)}X_1^{(N)}\dots X_{M-1}^{(N)}]^T,$$

$$A_{(M,N)} = C_{MN}\begin{bmatrix} C_N^{-1} & 0_N & 0_N & \cdots & 0_N & 0_N \\ 0_N & C_N^{-1} & 0_N & \cdots & 0_N & 0_N \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_N & 0_N & 0_N & \cdots & C_N^{-1} & 0_N \\ 0_N & 0_N & 0_N & \cdots & 0_N & C_N^{-1} \end{bmatrix},$$
 NxN zero matrix

2D DCT: Block composition and decomposition

$$X^{(LN\times MN)} = A_{(L,N)} \begin{bmatrix} X_{0,0}^{(N\times N)} & X_{0,1}^{(N\times N)} & \cdots & X_{0,M-1}^{(N\times N)} \\ X_{1,0}^{(N\times N)} & X_{1,1}^{(N\times N)} & \cdots & X_{1,M-1}^{(N\times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N\times N)} & X_{L-1,1}^{(N\times N)} & \cdots & X_{L-1,M-1}^{(N\times N)} \end{bmatrix} A_{(M,N)}^{T}$$

$$\left[\begin{array}{ccccc} X_{0,0}^{(N\times N)} & X_{0,1}^{(N\times N)} & \cdots & X_{0,M-1}^{(N\times N)} \\ X_{1,0}^{(N\times N)} & X_{1,1}^{(N\times N)} & \cdots & X_{1,M-1}^{(N\times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N\times N)} & X_{L-1,1}^{(N\times N)} & \cdots & X_{L-1,M-1}^{(N\times N)} \end{array} \right] = A_{(L,N)}^{-1} X^{(LN\times MN)} A_{(M,N)}^{-1^T}$$



Useful conversion for halving or doubling 8-point DCT blocks.

Composition

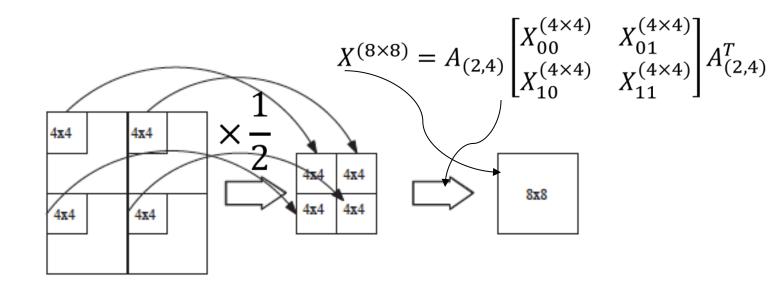
$$X^{(8\times8)} = A_{(2,4)} \begin{bmatrix} X_{00}^{(4\times4)} & X_{01}^{(4\times4)} \\ X_{10}^{(4\times4)} & X_{11}^{(4\times4)} \end{bmatrix} A_{(2,4)}^{T}$$

Decomposition

$$\begin{bmatrix} X_{00}^{(4\times4)} & X_{01}^{(4\times4)} \\ X_{10}^{(4\times4)} & X_{11}^{(4\times4)} \end{bmatrix} = A_{(2,4)}^{-1} X^{(8\times8)} A_{(2,4)}^{-T}$$

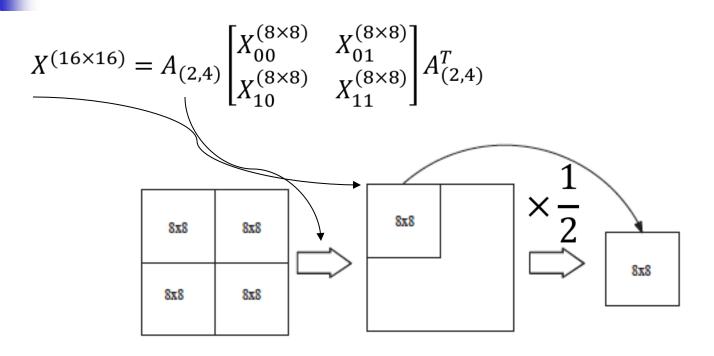


Image Halving: Approximation followed by Composition (IHAC)



$$X_{ij}^d = \frac{1}{2} \left[X_{ij}(k,l) \right]_{0 \leq k,l \leq 3}. \qquad Y = A_{(2,4)} \left[\begin{matrix} X_{00}^d & X_{01}^d \\ X_{10}^d & X_{11}^d \end{matrix} \right] A_{(2,4)}^T$$

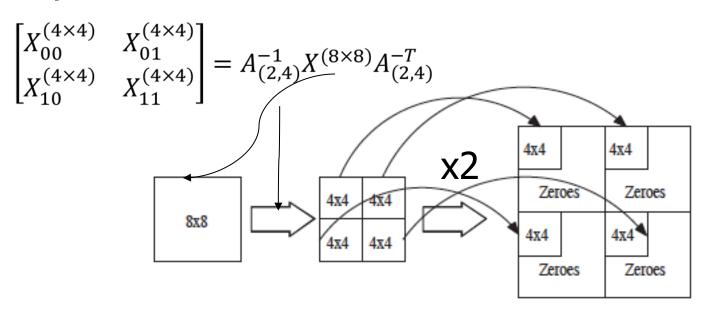
Image Halving: Composition followed by Approximation (IHAC)



$$Z = A_{(2,8)} \begin{bmatrix} X_{00} & X_{01} \\ X_{10} & X_{11} \end{bmatrix} A_{(2,8)}^T \qquad Y = \frac{1}{2} [Z(k,l)] \ 0 \le k,l \le 7.$$



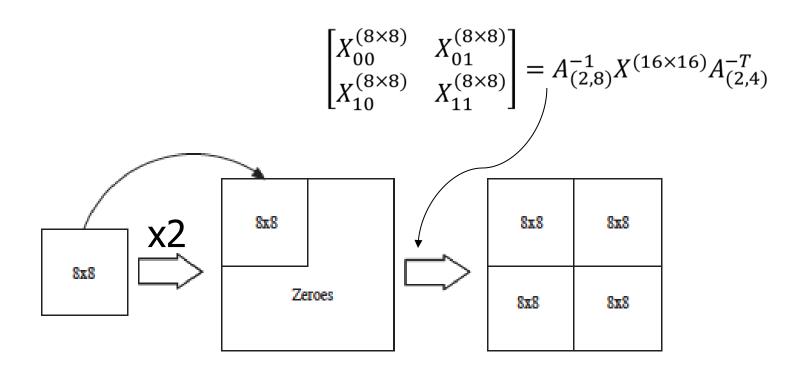
Image Doubling: Decomposition followed by Approximation (IDDA)



$$\begin{bmatrix} Y_{00}^d & Y_{01}^d \\ Y_{10}^d & Y_{11}^d \end{bmatrix} = A_{(2,4)}^T X A_{(2,4)} \qquad Y_{00} = 2 \begin{bmatrix} Y_{00}^d & 0_4 \\ 0_4 & 0_4 \end{bmatrix}.$$



Image Doubling: Approximation followed by Decomposition (IDAD)









IDDA



IDAD



4

Resizing with integral factors

To convert NxN block to LNxMN block.

LN x MN block
$$X(k,l) = \left\{ \begin{array}{l} \sqrt{LM} X_{LL}(k,l), & k,l=0,1,\ldots,N-1,\\ 0, & otherwise. \end{array} \right.$$

LxM D/S (LMDS) 1. Merge LxM adjacent DCT blocks.

$$Z = A_{(L,N)} \begin{bmatrix} X_{0,0} & X_{0,1} & \cdots & X_{0,(M-1)} \\ X_{1,0} & X_{1,1} & \cdots & X_{1,(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{(L-1),0} & X_{(L-1),1} & \cdots & X_{(L-1),(M-1)} \end{bmatrix} A_{(M,N)}^{T}$$

2. Sub-band approximation to a NxN DCT block.

$$Y = \sqrt{\frac{1}{LM}} \left[Z(k,l) \right]_{0 \le k,l \le N-1}$$

LxM U/S (LMUS)

1. Convert NxN to LNxMN block

$$\hat{B} = \left[\begin{array}{ccc} \sqrt{LM}B & 0_{(N,(M-1)N)} \\ 0_{((L-1)N,N)} & 0_{((L-1)N,(M-1)N)} \end{array} \right] \begin{array}{c} \text{compute} \\ \text{exploiting large} \\ \text{blocks of} \end{array}$$

Efficiently blocks of zeroes.

2. Decompose into LxM NxN blocks.

An example: 3x2 D/S and U/S







Arbitrary Resizing (P/Q x R/S)

- U/S-D/S Resizing Algorithm (UDRA)
 - U/S by PxR
 - D/S by QxS
- D/S-U/S Resizing Algorithm (DURA)
 - D/S by QxS
 - U/S PxR

HDTV (1080x920) to NTSC (480x640)



UDRA





DURA





More general sub-band relation

X: DCT block of QNxSN

Y: DCT block of PNxRN

$$\begin{aligned} \mathbf{Y} &= \sqrt{\frac{PR}{QS}} \left[\mathbf{X} \right]_{0 \leq i < PN, 0 \leq j < RN}, & P \leq Q, \, R \leq S, \\ &= \sqrt{\frac{PR}{QS}} \begin{bmatrix} \left[\mathbf{X} \right]_{0 \leq i < QN, 0 \leq j < RN} \\ 0_{(P-Q)N \times RN} \end{bmatrix}, & P > Q, \, R \leq S, \\ &= \sqrt{\frac{PR}{QS}} \left[\left[\mathbf{X} \right]_{0 \leq i < PN, 0 \leq j < SN} & 0_{PN \times (R-S)N} \right], & P \leq Q, \, R > S, \\ &= \sqrt{\frac{PR}{QS}} \begin{bmatrix} \left[\mathbf{X} \right]_{0 \leq i < QN, 0 \leq j < SN} & 0_{QN \times (R-S)N} \\ 0_{(P-Q)N \times SN} & 0_{(P-Q)N \times (R-S)N} \end{bmatrix}, & P > Q, \, R > S. \end{aligned}$$

Truncated DCT block of X or padded with zeroes, if required,

$$\mathbf{Y} = \sqrt{\frac{PR}{QS}} \hat{\mathbf{X}}$$

- 1. Merge QxS input DCT blocks to form X.
- 2. Take PxR blocks to form Y with the sub-band approximation.
- 3. Decompose Y into PxR NxN blocks.

Hybrid Resizing: A few examples



 $\frac{2}{3} \times \frac{4}{5}$



 $\frac{2}{3} \times \frac{3}{2}$



 $\frac{3}{2} \times \frac{5}{4}$



 $\frac{3}{2} \times \frac{2}{3}$.



DCT Domain Filtering



Filtering a finite sequence

Input: x(n), n=0,1, ..., N-1 \tag{Pad with zeroes at}

Filter: h(n): n=0,1,2,...,L-1 undefined points.

$$y(n) = \sum_{m=0}^{N-1} x(m)h(n-m), \qquad n = 0,1,...(N+L-1)$$

$$n=0,1,\dots(N+L-1)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(L-1) \\ \vdots \\ y(N-1) \\ \vdots \\ y(N+L-1) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & \cdots & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ h(L-1) & h(L-2) & \cdots & h(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \vdots & \cdots & h(0) \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & h(L-1) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(L-1) \\ \vdots \\ x(N-1) \end{bmatrix}. DCT(\mathbf{y}) = DCT(H_l^{(N+L-1)\times N})DCT(\mathbf{x}).$$

$$\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(L-1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$DCT(\mathbf{y}) = DCT(H_l^{(N+L-1)\times N})DCT(\mathbf{x})$$



Filtering with periodic extension of input sequence and response

$$x(n+N)=x(n)$$

Input: x(n), n=0,1,...,N-1 \tag{Periodically extended}

Filter: h(n): n=0,1,2,...,N-1 with period N.

Periodically extended with period N.

Circular convolution

$$y(n) = x(n) \circledast h(n) = \sum_{m=0}^{n} x(m)h(n-m) + \sum_{m=n+1}^{N-1} x(m)h(n-m+N).$$

$$H_{c}^{(NxN)}$$

Can be made equiv. to. linear conv. by zero padding of x(n) and h(n).

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & \cdots & h(1) \\ h(1) & h(0) & \cdots & h(2) \\ \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & \cdots & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$



Filtering with antiperiodic extension

Input: x(n), n=0,1,...,N-1 Antiperiodic

Filter: h(n): n=0,1,2,...,N-1 extension extended with antiperiod N.

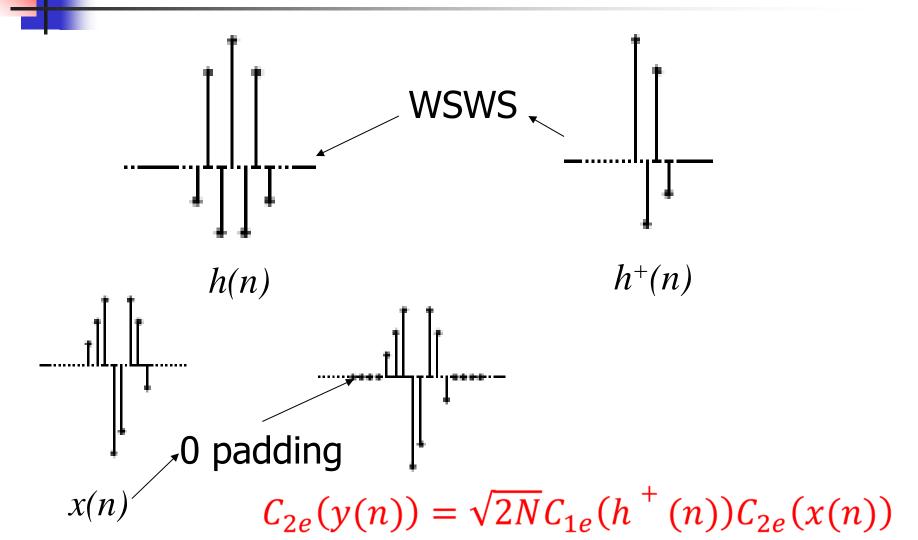
Skew Circular convolution

$$y(n) = x(n) \circledast h(n) = \sum_{m=0}^{n} x(m)h(n-m) - \sum_{m=n+1}^{N-1} x(m)h(n-m+N)$$

Can be made equiv. to. linear conv. by zero padding of x(n) and h(n).

$$H_s^{N \times N} = \begin{bmatrix} h(0) & -h(N-1) & \cdots & -h(1) \\ h(1) & h(0) & \cdots & -h(2) \\ \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & \cdots & h(0) \end{bmatrix}$$

Symmetric convolution and CMP for type II DCT



CONVOLUTION-MULTIPLICATION PROPERTY

$$x(n) \longrightarrow h(n) \longrightarrow y(n)=h(n)*x(n)$$

•Symmetric convolution operation

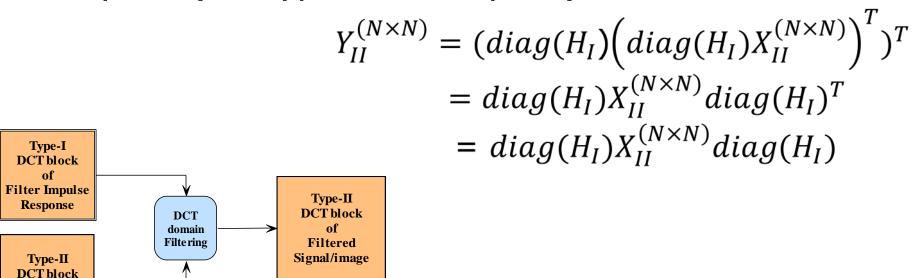
$$X_{II}(k) \longrightarrow H_{I}(k) \longrightarrow Y_{II}(k) = H_{I}(k).X_{II}(k)$$

$$Y_{II}^{(N)} = \begin{bmatrix} H_{I,0} & 0 & 0 & 0 \\ 0 & H_{I,1} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & & & & \\ 0 & H_{I,N-1} \end{bmatrix} X_{II}^{(N)} \qquad diag(H_{I})$$
Diagonal matrix formed by $H_{I}(k)$

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Filtering in 2-D block DCT space

Given, **type-I DCT** of the impulse response of a filter and an input in **type-II DCT space**, filtered output can be transformed in the same space (i.e. type-II DCT space).



of Signal/image

Separable response: h(x,y)=h(x)h(y)

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BOUNDARY EFFECT IN BLOCK DCT DOMAIN

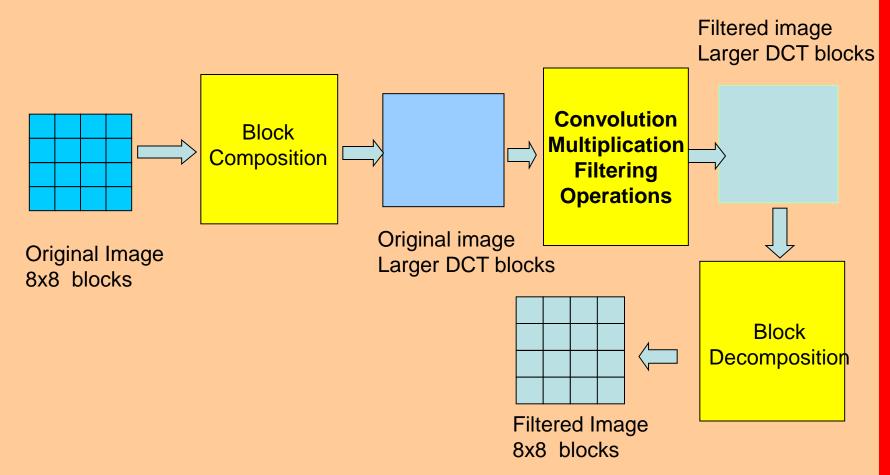


Linear Convolution



Block Symmetric Convolution

Computation with larger blocks: The BFCD filtering Algorithm



1

Composite operation

 Block composition + Multiplication with filter coefficients + Block decomposition can be expressed by a composite matrix for the linear operation.

In 1-D: For filtering 3 NxN adjacent DCT blocks:

$$U^{(3N\times 3N)} = A_{(3,N)}^T \mathbb{D}(\{\sqrt{6N}C_{3N}^I\mathbf{h}^+\}_0^{3N-1})A_{(3,N)}$$

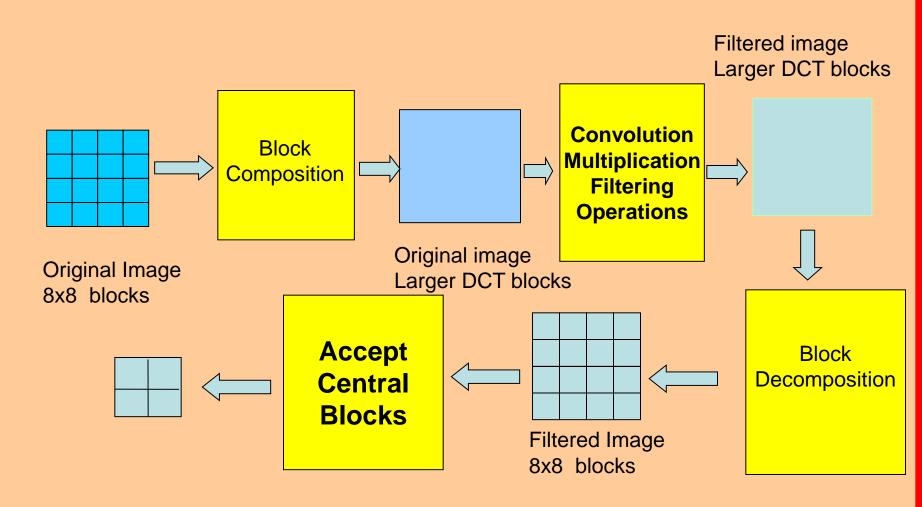
$$\begin{bmatrix} Y_1^{(N)} \\ Y_2^{(N)} \\ Y_3^{(N)} \end{bmatrix} = U\begin{bmatrix} X_1^{(N)} \\ X_2^{(N)} \\ X_3^{(N)} \end{bmatrix} \text{ Decomp. Multiplication Composition}$$

In 2-D

$$\begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & Y_{13}^{(N\times N)} \\ Y_{21}^{(N\times N)} & Y_{22}^{(N\times N)} & Y_{23}^{(N\times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N\times N)} & X_{12}^{(N\times N)} & X_{13}^{(N\times N)} \\ X_{21}^{(N\times N)} & X_{22}^{(N\times N)} & X_{23}^{(N\times N)} \end{bmatrix} U^T$$

$$\begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & Y_{13}^{(N\times N)} & X_{12}^{(N\times N)} & X_{13}^{(N\times N)} \\ X_{21}^{(N\times N)} & X_{22}^{(N\times N)} & X_{23}^{(N\times N)} \end{bmatrix} U^T$$

Exact Computation: The Overlapping and Save (OBFCD) filtering Algorithm



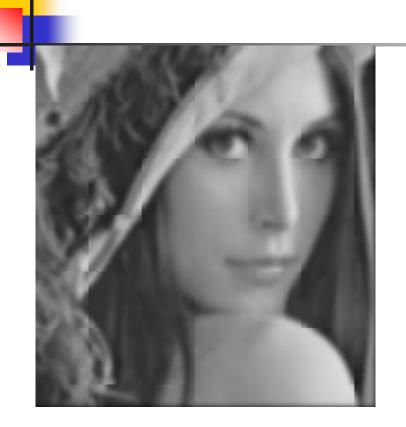
1

Overlap and Save Strategy

$$\begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & Y_{13}^{(N\times N)} \\ Y_{21}^{(N\times N)} & Y_{22}^{(N\times N)} & Y_{23}^{(N\times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N\times N)} & X_{12}^{(N\times N)} & X_{13}^{(N\times N)} \\ X_{21}^{(N\times N)} & X_{22}^{(N\times N)} & X_{23}^{(N\times N)} \end{bmatrix} U^T \begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & X_{13}^{(N\times N)} & X_{22}^{(N\times N)} & X_{23}^{(N\times N)} \\ X_{31}^{(N\times N)} & X_{32}^{(N\times N)} & X_{33}^{(N\times N)} \end{bmatrix} U^T$$

Save only the central block.

Filtered Images





BFCD Algorithm (5x5)

OBFCD Algorithm

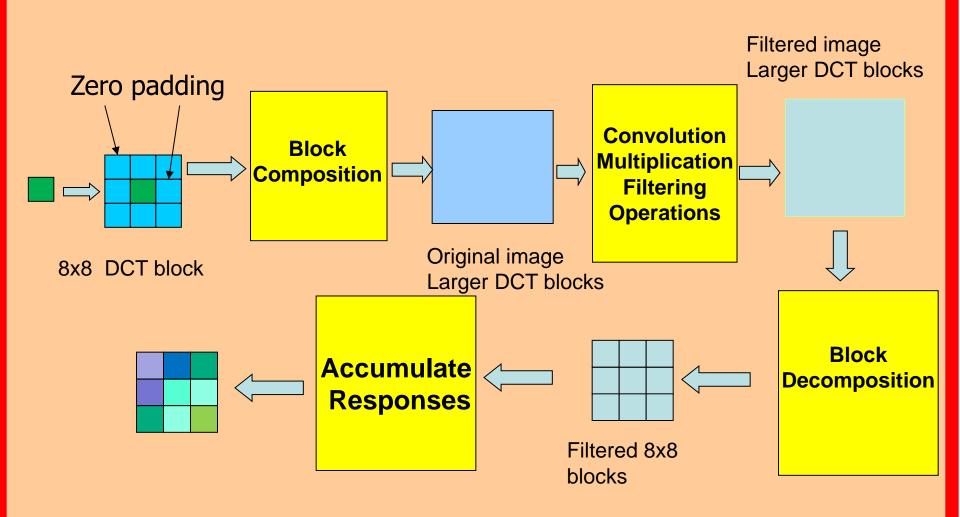
4

Overlap and add strategy

$$\begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & Y_{13}^{(N\times N)} \\ Y_{21}^{(N\times N)} & Y_{22}^{(N\times N)} & Y_{23}^{(N\times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N\times N)} & X_{12}^{(N\times N)} & X_{13}^{(N\times N)} \\ X_{21}^{(N\times N)} & X_{22}^{(N\times N)} & X_{23}^{(N\times N)} \end{bmatrix} U^T \begin{bmatrix} Y_{11}^{(N\times N)} & Y_{12}^{(N\times N)} & X_{13}^{(N\times N)} & X_{23}^{(N\times N)} \\ Y_{31}^{(N\times N)} & Y_{32}^{(N\times N)} & Y_{33}^{(N\times N)} \end{bmatrix} U^T$$

- For computing contribution of the central block X_{22} to neighboring blocks all other blocks in the input set to zeros.
- Add accumulated contribution at every block.

Exact Computation: The Overlapping and Add filtering Algorithm



Removal of Blocking Artifacts



Blocking artifacts
Compressed with quality
factor 10



Artifacts removed By filtering in DCT domain

Image Sharpening in DCT domain

$$X_s = X + k. (X - X_{lpf})$$



Lena



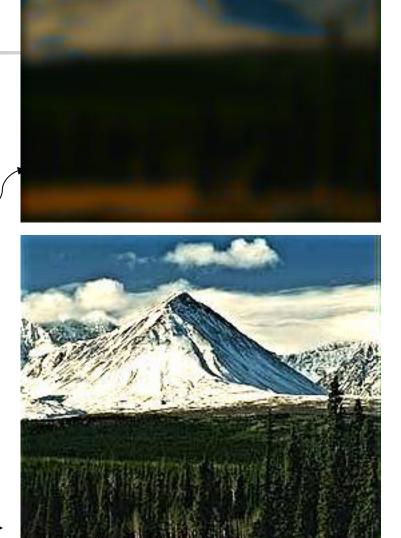
Peppers

Color Sharpening

Original



Filtered



Sharpened



Color Enhancement



Alpha Rooting

$$\widetilde{X}(i,j) = X(i,j) \left| \frac{X(i,j)}{Y_{max}} \right|^{\alpha-1}$$

Maximum Luminance

 Smaller values get magnified and larger values get reduced.





Original

AR

Multi-contrast Enhancement (MCE)

- X: 8x8 DCT block: Only AC Coefficients modified. Spectral energy: $E_k = \frac{1}{N_k} \Sigma_{i+j=k} |X(i,j)|,$
- Contrast measure: $c_k = \frac{E_k}{\sum_{k \in L} E_k}$.
- The coefficients of k-th band are scaled so that the resulting contrast becomes $w.c_k$, where w is a constant.
- By scaling coefficients in k-th band with $w.H_k$.

$$H_k = rac{\sum_{t < k} E_t}{\sum_{t < k} E_t}$$
, Spectral energy in the enhanced image



- Also modifies DC coefficient, by increasing its dynamic range.
- Expression for spectral energy slightly modified. $E_k = \sqrt{\frac{1}{N_k} \sum_{i+j=k} |X(i,j)|^2}.$

$$\eta(x) = \frac{(x^{\frac{1}{\gamma}} + (1 - (1 - x)^{\frac{1}{\gamma}}))}{2}, \quad 0 \le x \le 1.$$



Contrast: Definition

Let μ and σ denote the mean and standard deviation of an image. Contrast ζ of an image is defined here as:

$$\zeta = \frac{\mu}{\sigma}$$

Weber Law:
$$\zeta = \frac{\Delta L}{L}$$

where ΔL is the difference in luminance between a stimulus and its surround, and L is the luminance of the surround

Theorem on Contrast Preservation in the DCT Domain

 κ_d : the scale factor for the DC coefficient

 κ_a : the scale factor for the AC coefficients

$$Y_e(i,j) = \begin{cases} \kappa_d Y(i,j), & i=j=0 \\ \kappa_a Y(i,j), & otherwise \end{cases}$$

The contrast of the processed image : κ_a / κ_d times of the contrast of the original image.

 $\kappa_d = \kappa_a = \kappa$ preserves the contrast.

Preservation of Colors in the DCT Domain

U, V: Blocks of DCT coefficients of C_b and C_r κ : Scale factor for the luminance component Y

$$\boldsymbol{U}_{e}(i,j) = \begin{cases} N(\kappa & (\frac{U(i,j)}{N} - 128)) + 128, i = j = 0 \\ \kappa U(i,j), & otherwise \end{cases}$$

$$V_e(i,j) = \begin{cases} N(\kappa & (\frac{V(i,j)}{N} - 128)) + 128, & i=j=0 \\ \kappa V(i,j), & otherwise \end{cases}$$

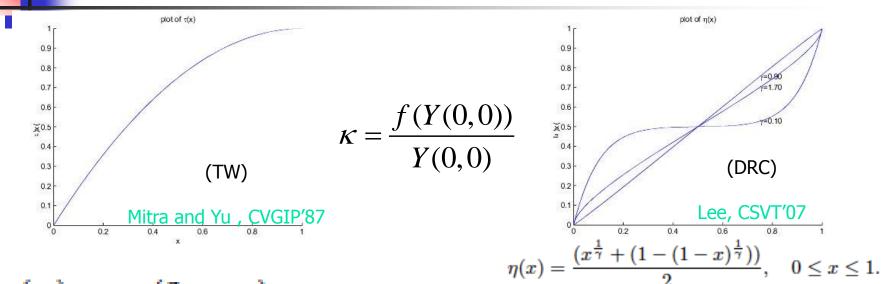
Color Enhancement by Scaling Coefficients (CES)

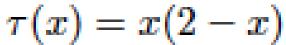
• Find the scale factor by mapping the DC coefficient with a monotonically increasing function. $1 \le \kappa \le \frac{B_{max}}{\mu + \lambda . \sigma}$ Max. intensity Scale factor

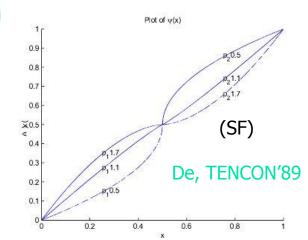
 Apply scaling to all other coefficients in all the components.

 For blocks having greater details judged by s.d., apply block decomposition and re-composition strategy.

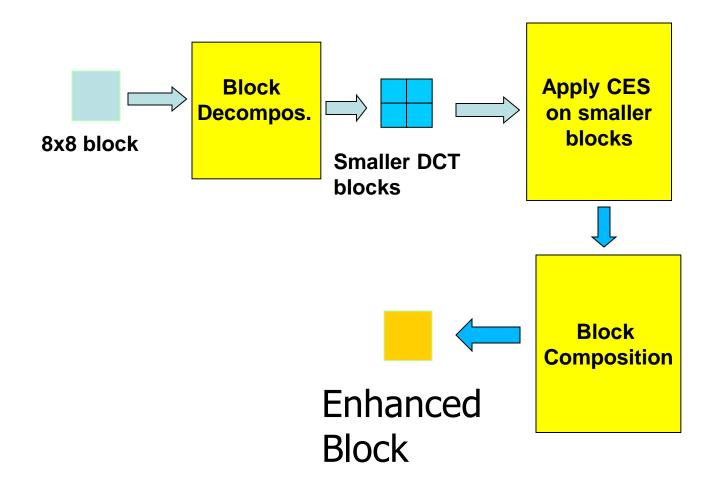
Mapping functions for adjusting the local background illumination







Enhancement of Blocks with more details







MCE



MCEDRC

CES

Some Results







original

AR

MCE







MCEDRC

TW-CES-BLK

MSR

Iterative Enhancement



Iteration no.=1





original



Iteration no.=3



Iteration no.=2



Iteration no.=4

Color constancy

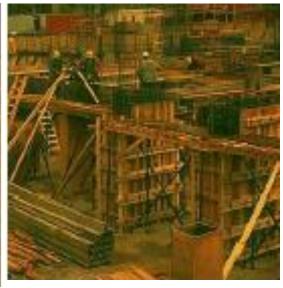
- Computation of color of illuminant
 - Avg. of colors (Gray world)
 - Maximum of each channel (White world)
 - Use of DC coefficient of DCT blocks in compressed domain.
- Diagonal Color correction: $(R_s, G_s, B_s) \rightarrow (R_d, G_d, B_d)$

Chromatic Shift in Y-Cb-Cr: $\begin{array}{ccc} Y_u &=& Y, \\ C_{bu} &=& C_b + C_{bd} - C_{bs}, \\ C_{ru} &=& C_r + C_{rd} - C_{rs}, \end{array}$

Color correction: Example







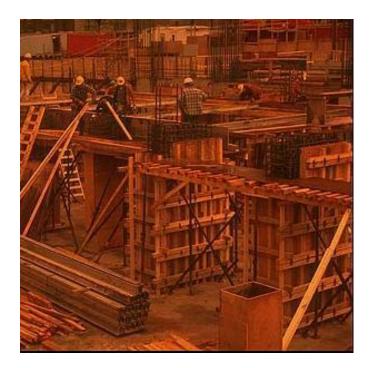
Original

Diagonal Correction

Chromatic Shift

Color constancy coupled enhancement

- Perform color correction
- Perform color enhancement



Enhancement without color correction

