Stereo Geometry

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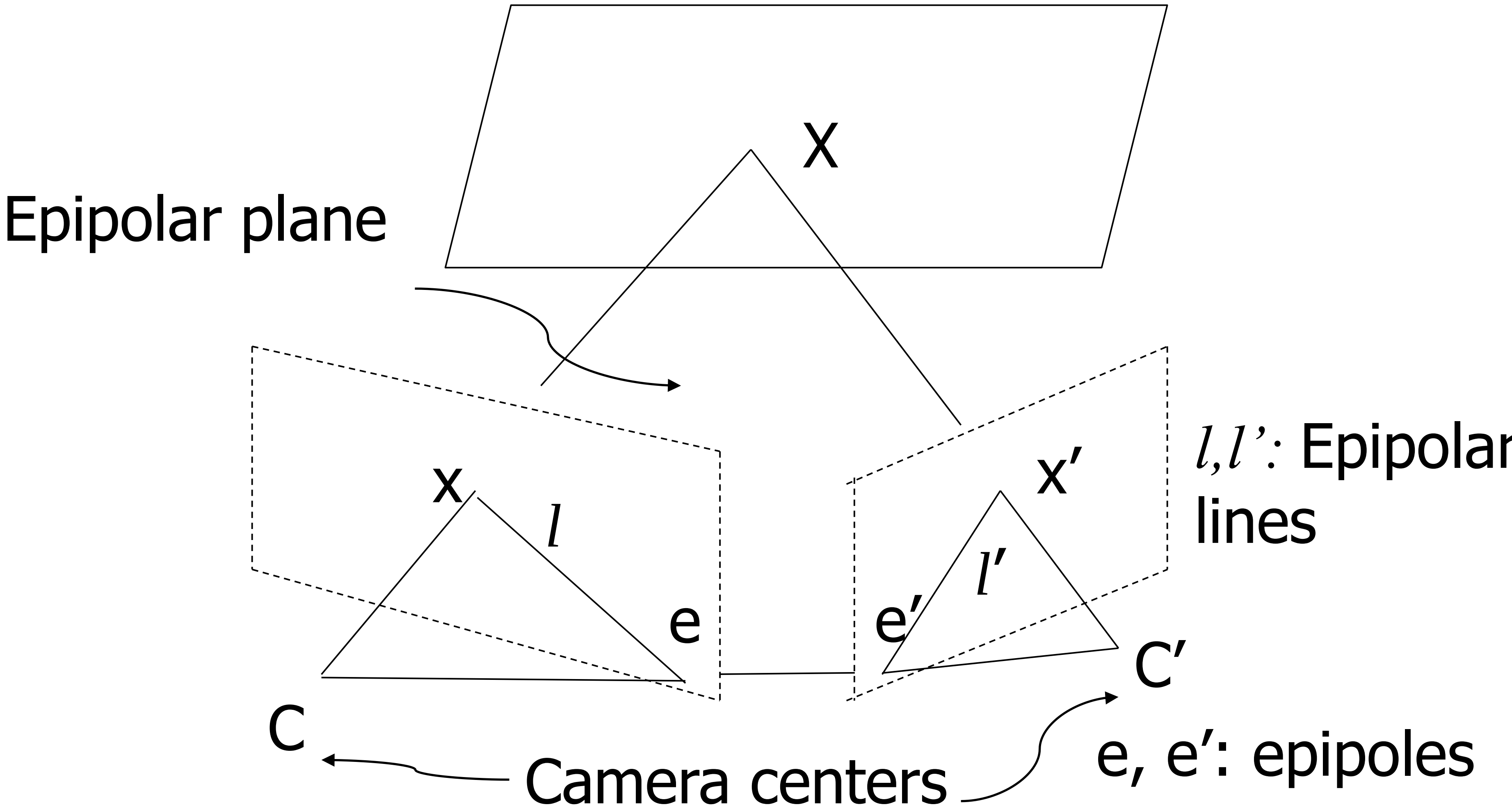
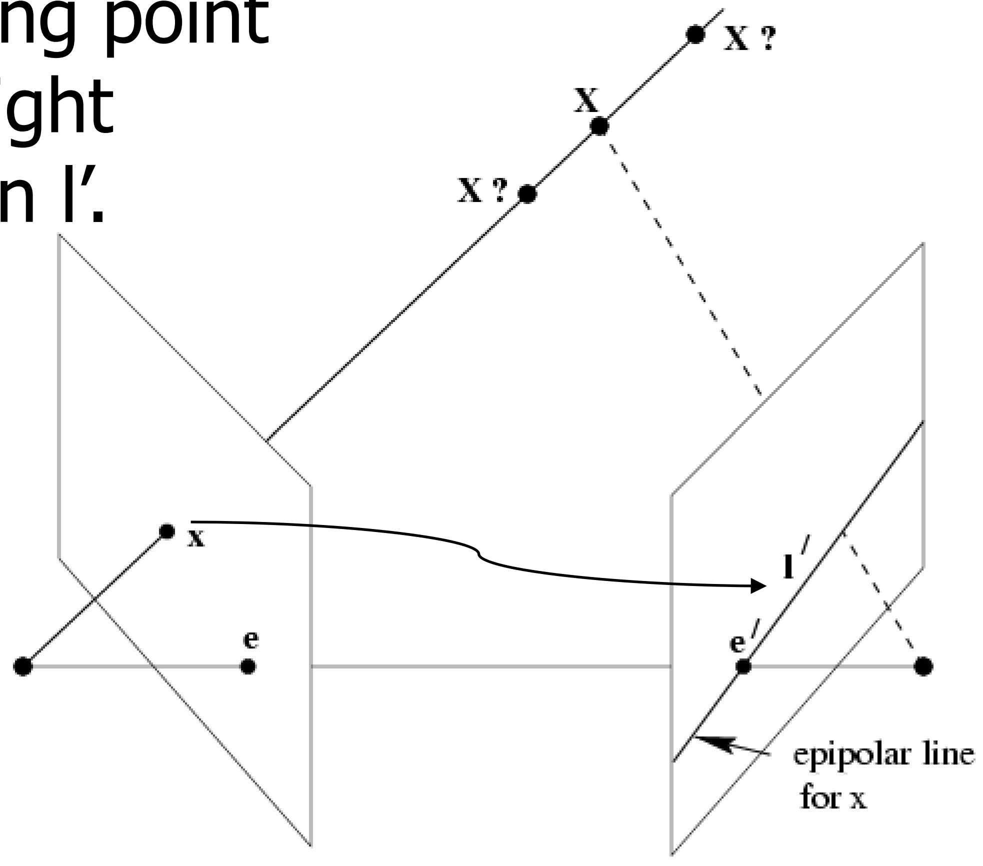
Stereo Set-up

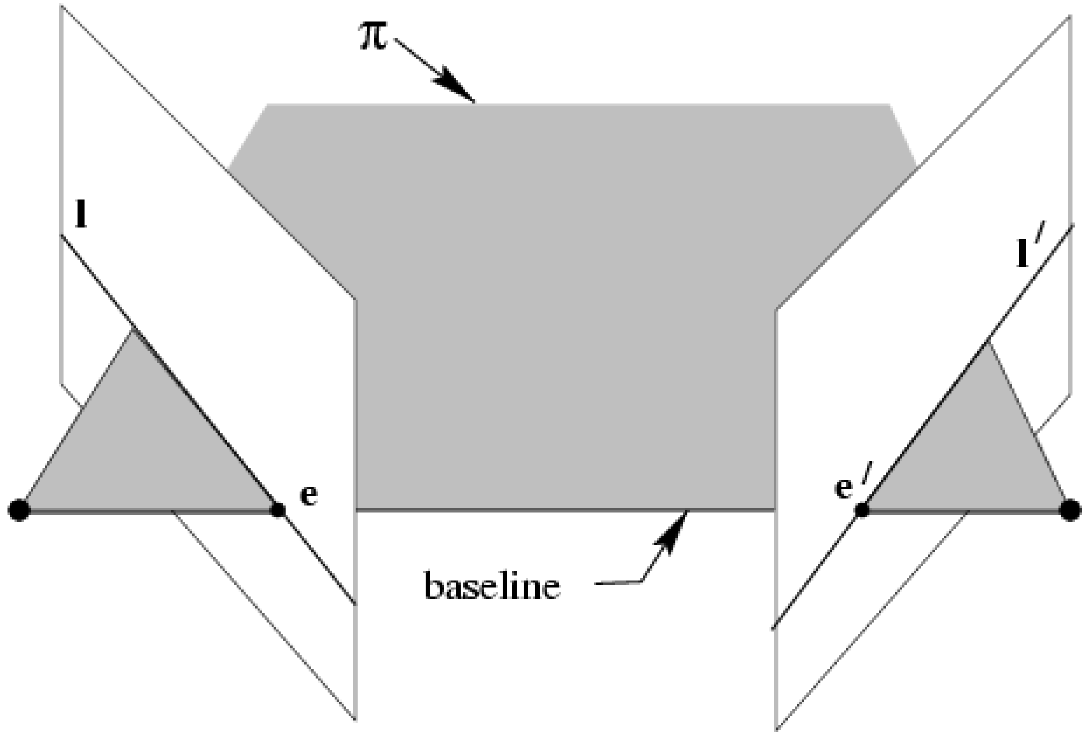


Epipolar geometry

Corresponding point of x in the right image lies on l′.



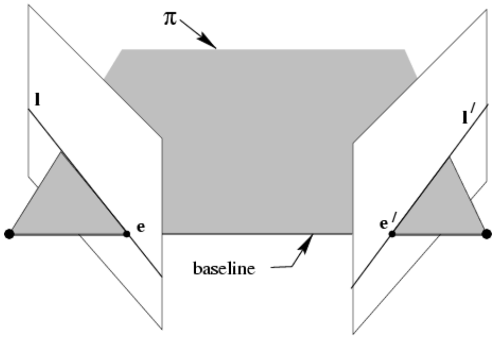
Epipolar geometry



All points on π project on l and l'

From Hartley and Zisserman, “Multiple view geometry in computer vision”, Cambridge Univ. Press (2000)

Epipolar geometry

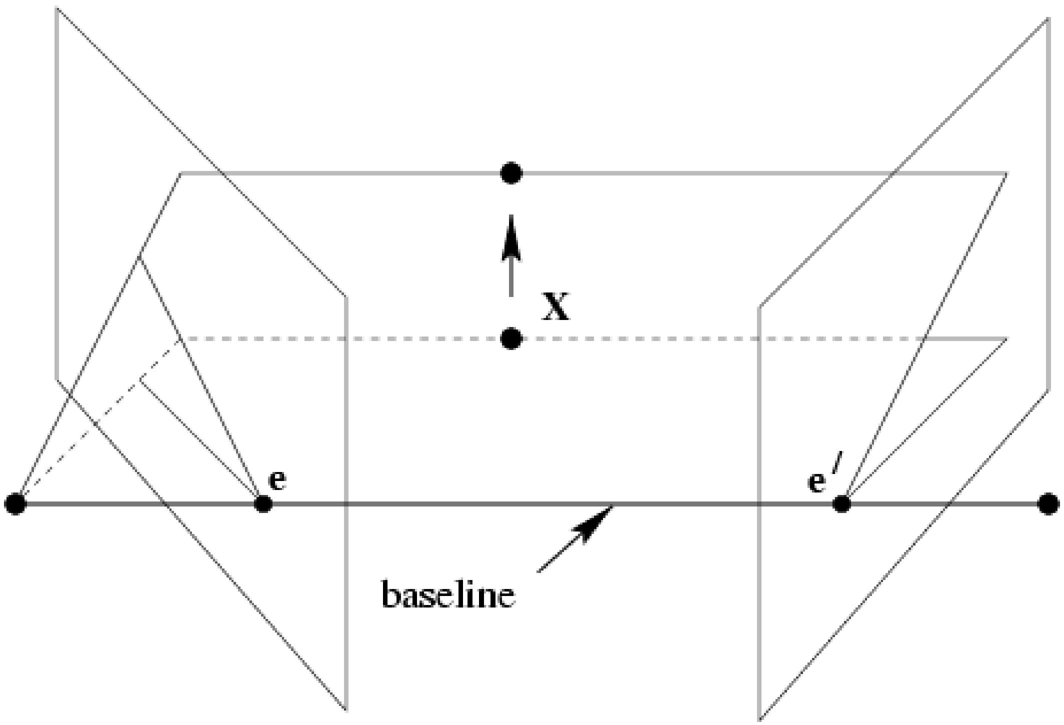


Epipoles e,e'
= intersection of baseline with image plane
= projection of projection center in other image
= vanishing point of camera motion direction

An epipolar plane = plane containing baseline (1-D family)

An epipolar line = intersection of epipolar plane with image (always come in corresponding pairs)

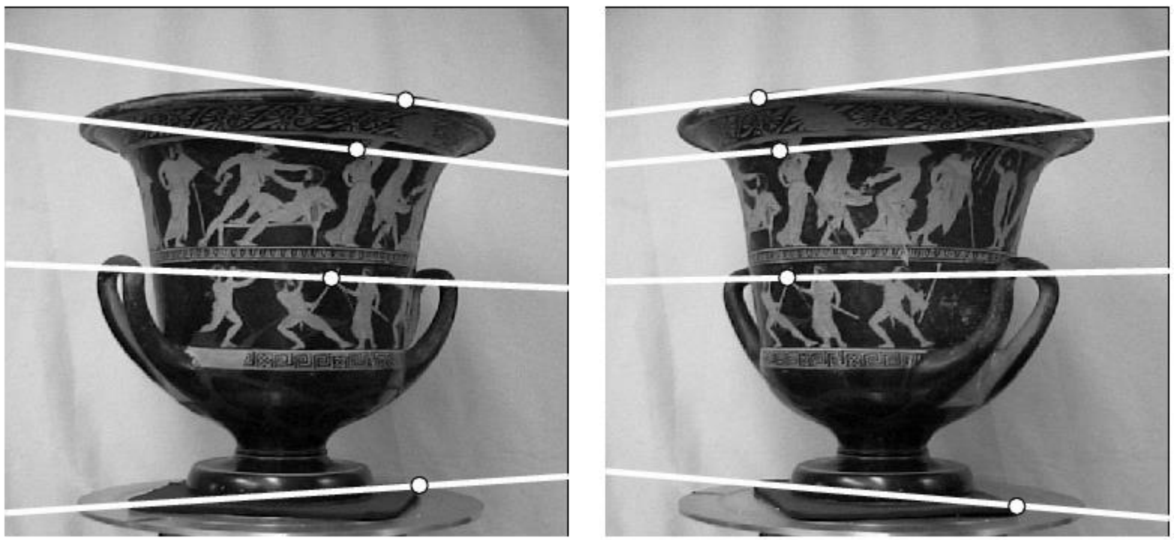
Epipolar geometry

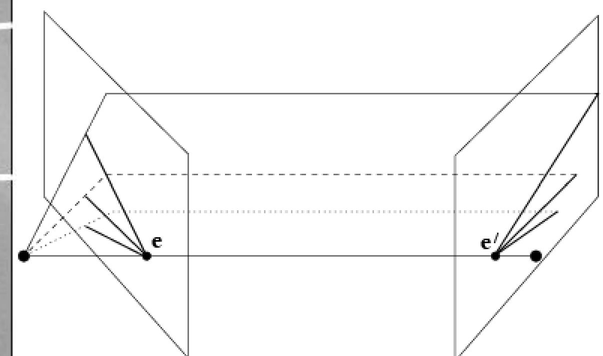


Family of planes π and lines l and l'
Intersection in e and e'

From Hartley and Zisserman, "Multiple view geometry in computer vision", Cambridge Univ. Press (2000)

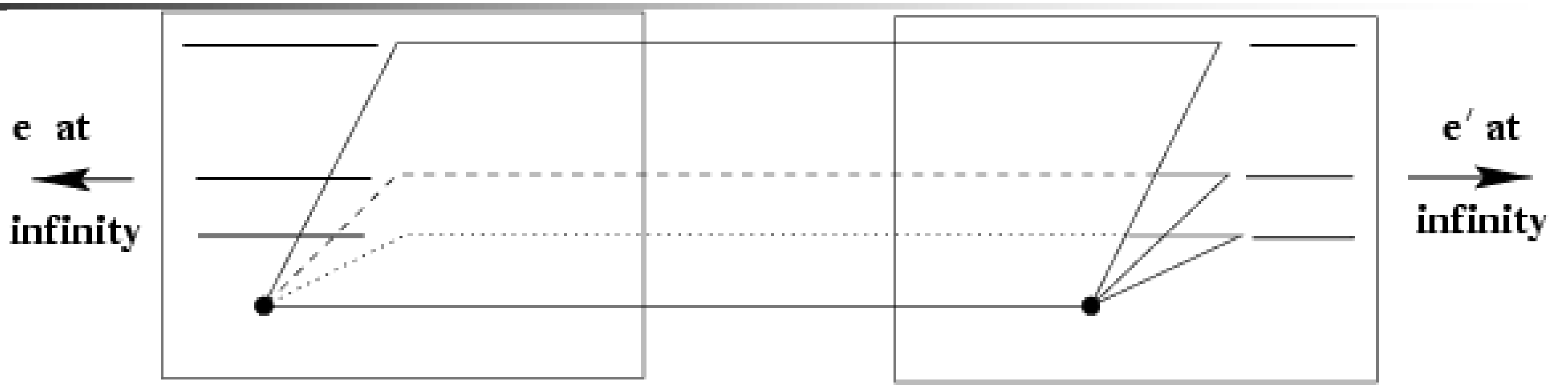
Example: converging cameras

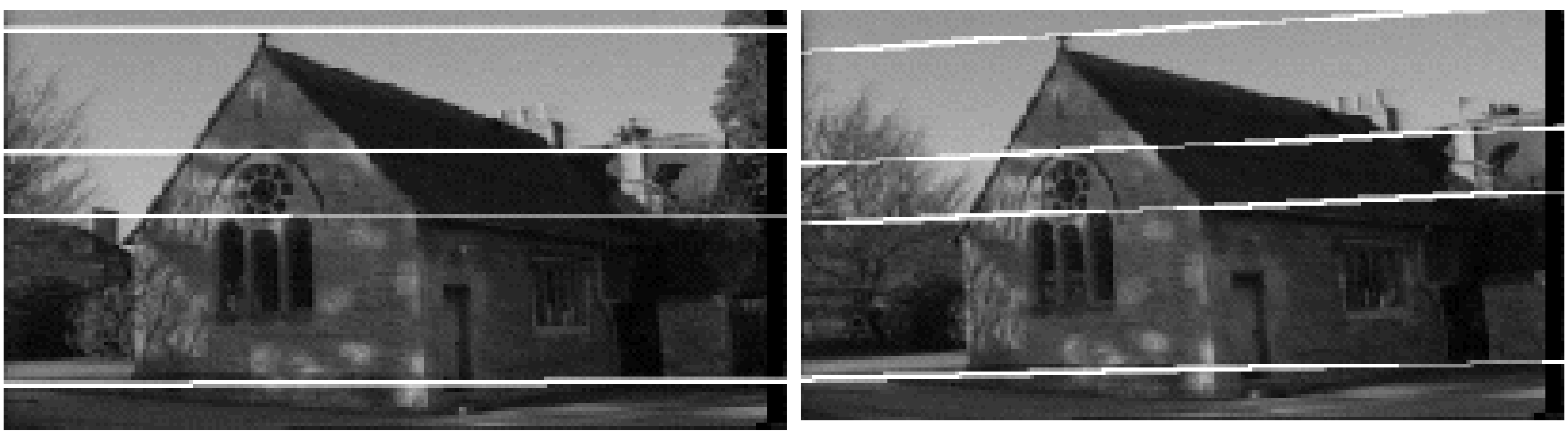




From Hartley and Zisserman, "Multiple view geometry in computer vision", Cambridge Univ. Press (2000)

Example: motion parallel with image plane





From Hartley and Zisserman, “Multiple view geometry in computer vision”, Cambridge Univ. Press (2000)

Epipolar Geometry

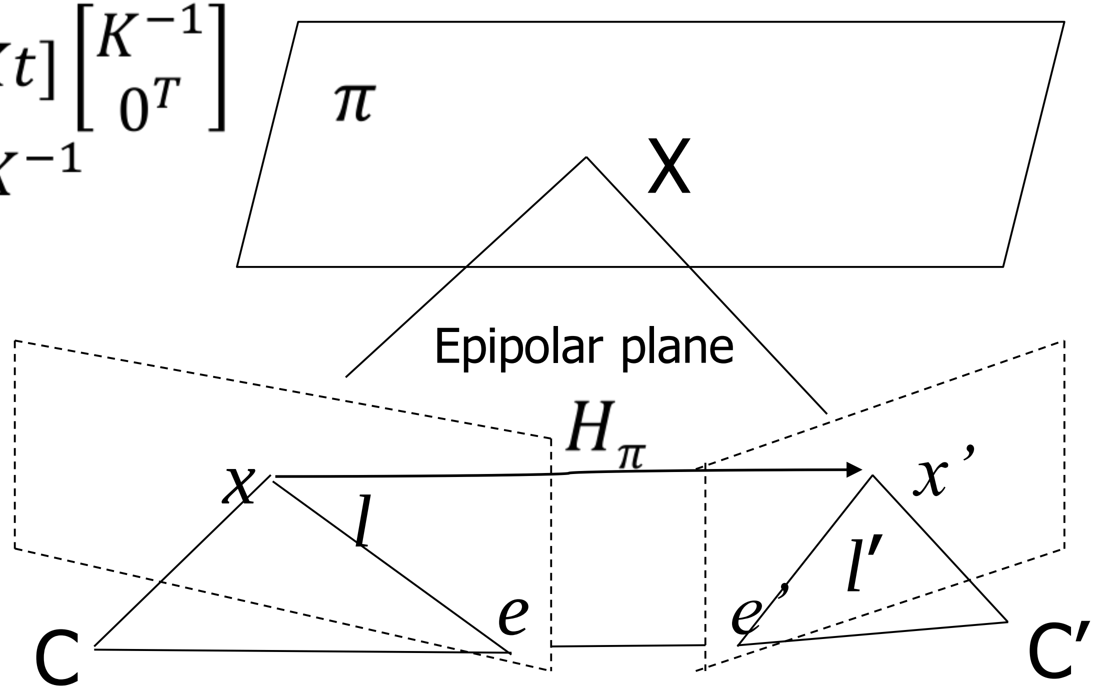
$$[e']_\times = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix}$$

$$H_\pi = [K'R|Kt]\begin{bmatrix} K^{-1} \\ 0^T \end{bmatrix} = K'RK^{-1}$$

$$x' = P'X = P'P^+x$$

$$P^+ = \begin{bmatrix} K^{-1} \\ 0^T \end{bmatrix}$$

$$l' = e' \times x' = [e']_\times x' = [e']_\times H_\pi x = Fx$$



$P=K[I \mid 0]$ $\quad$ $P'=K'[R \mid t]$

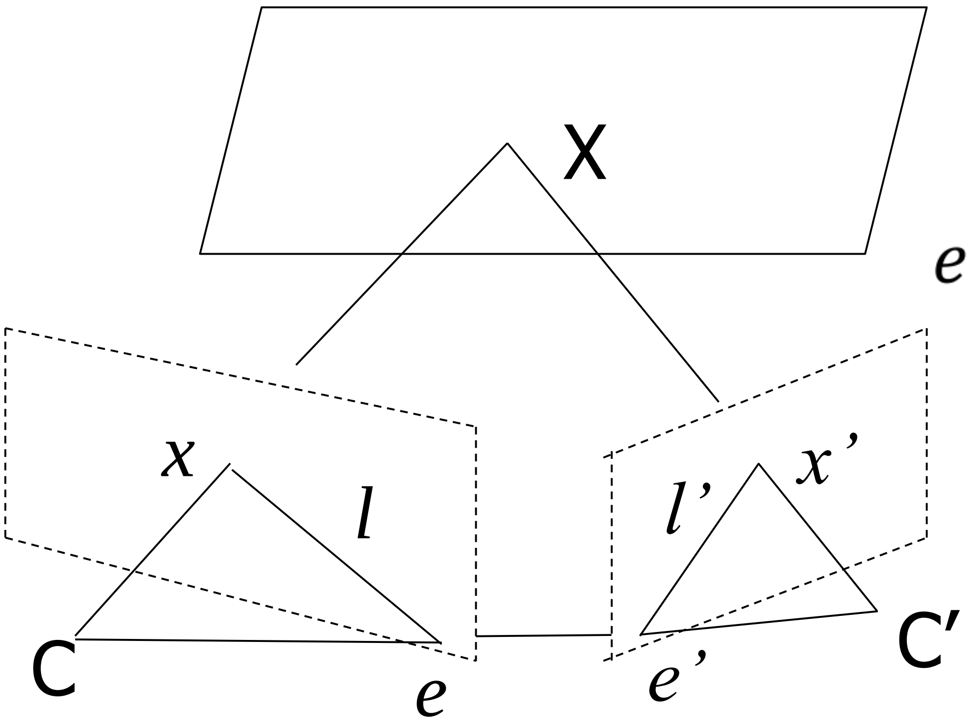
$F=[e']_x K'RK^{-1}$

Coplanar: X, x, x', C, C′, e, e', l, l'

Epipolar Geometry

Fundamental Matrix: F

$$l' = Fx$$



$$e = PC'$$

$$e' = P'C$$

$$e = -KR^T t \equiv KR^T t$$

$$e' = K't$$

$$l' = Fx$$

$$\Longrightarrow x'^T l' = 0$$

$$\Longrightarrow x'^T F x = 0$$

$$\Longrightarrow x^T F^T x' = 0$$

$$C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$P = K[\, I \mid 0 \,]$

$P' = K'[\, R \mid t \,]$

$$C' = \begin{bmatrix} -R^T t \\ 1 \end{bmatrix}$$

Fundamental and Essential Matrices

$P= K[I \mid 0]$

$P'= K'[R \mid t]$

$$F = [e']_\times P'P^+$$
$$= [K't]_\times K'RK^{-1}$$

$$F = [m]_\times M$$

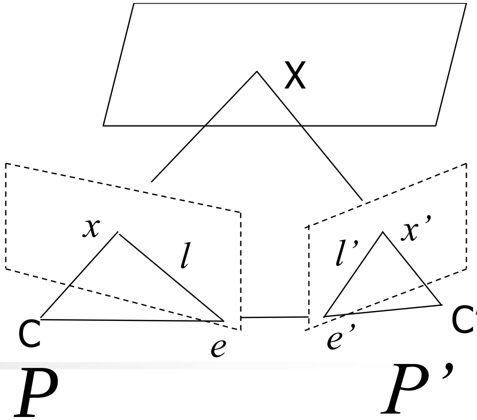
Say, $P=[I \mid 0]$, i.e. $K=I$

$P'=K'[R \mid t]=[K'R \mid K't]=[M \mid m]$

$for\ K = I\ \ and\ K' = I,$ $\qquad F = [t]_\times R$

Essential Matrix (E)

Fundamental Matrix: Properties



$$x'^T F x = x^T F^T x' = 0, \forall (x', x)$$

Transpose:

If F is fundamental matrix of (P,P'), F^T for (P',P).

Epipolar lines: For x, epipolar line $l' = Fx$.

For x', epipolar line $l = F^T x'$.

F is a correlation. → Rank deficient. → Inverse does not exit.

Epipoles: $e'^T (Fx) = e^T (F^T x') = (Fe)^T x' = 0$

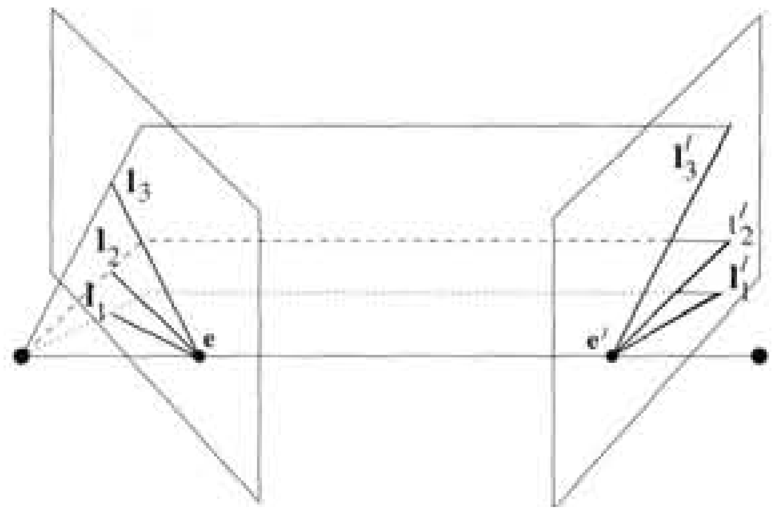
$e'^T F = 0$ → e' is left NULL vector of F.

$Fe = 0$ → e is the right NULL vector of F.

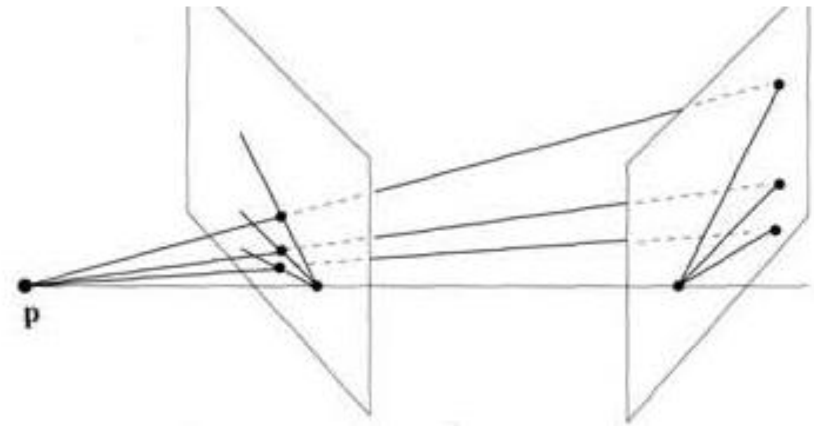
Rank deficient:

det(F)=0 and F is a projective element → 7 d.o.f.

Epipolar line homography



l_i and l_i' form pencil of planes with axis the baseline



l_i and l_i' related by a perspectivity with centre at any point **p** on the baseline → 1D homography.

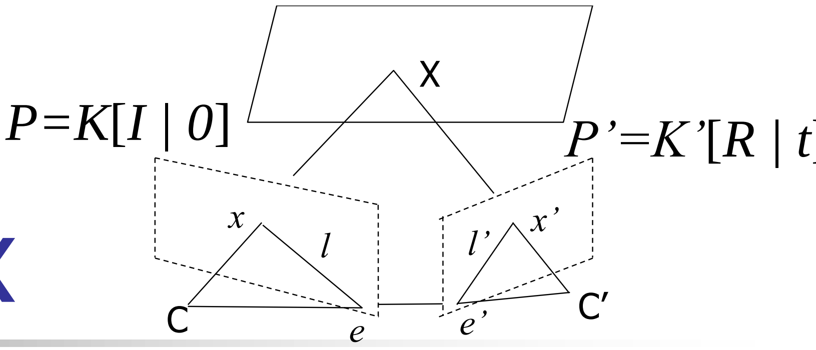
From Hartley and Zisserman, "Multiple view geometry in computer vision", Cambridge Univ. Press (2000)

Epipolar line homography

Let (l,l') be corresponding epipolar lines and k is any line not passing through epipole e.
Then, $l' = F\,[k]_\times\, l$

Proof: $k \times l = p$ (point of intersection)
As p lies on l, $Fp = l'$
$\rightarrow l' = F\,k \times l = F\,[k]_\times\, l$ *(Q.E.D.)*
As $e^T e > 0$,
the line e does not pass through the epipole e.
$\rightarrow l' = F\,[e]_\times\, l$

Essential Matrix



Stereo geometry for calibrated cameras.

$x_c = K^{-1}x$ $\quad x'^{T}_{c}Ex_c=0$ $\quad x'_c=K'^{-1}x'$

Coordinates in calibrated image planes.

$x'^{T}Fx=0$

$\rightarrow (K'x_c')^{T}F(Kx_c)=0$

$\rightarrow x'^{T}_{c}K'^{T}FKx_c=0$

$\rightarrow x'^{T}_{c}(K'^{T}FK)x_c=0$

E

$E=K'^{T}FK$

$F=K'^{-T}EK^{-1}$

$=K'^{-T}[t]_{\times}R\,K^{-1}$

$E=[t]_{\times}R$

6 parameters

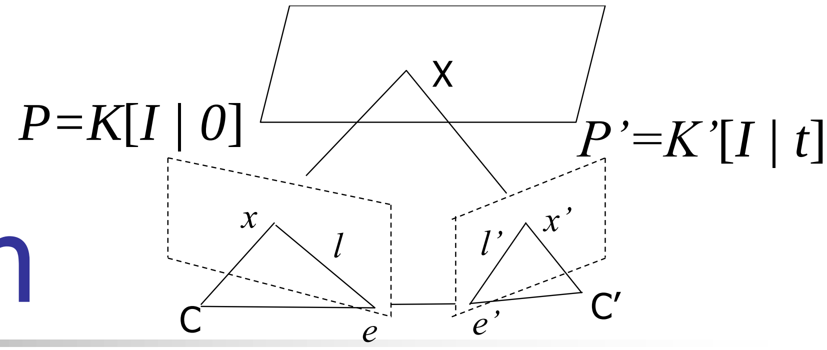
d.o.f.=5

$Ee_c=e'_cE=0$

Rank: 2

det(E)=0

Pure translation



$\mathrm{F}=[e']_{\times}K'IK^{-1}$
$=[e']_{\times}K'K^{-1}$

camera translation $||^{l}$ to x-axis, $e'=[1\ 0\ 0]^{\mathrm{T}}$

$$\rightarrow F=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

For $K=K'$, $F=[e']_{\times}$ $\quad [e']_{\times}=\begin{bmatrix}0 & -e_z & e_y\\ e_z & 0 & -e_x\\ -e_y & e_x & 0\end{bmatrix}$

$\rightarrow x'^{\mathrm{T}}Fx=0$

$\rightarrow y'=y$

$$x\equiv PX=[K\ |\ 0]\begin{bmatrix}\tilde{X}\\ 1\end{bmatrix}=K\tilde{X}$$

$$K^{-1}x\equiv\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}\qquad \tilde{X}=ZK^{-1}x$$

$$K=\begin{bmatrix}f & 0 & 0\\ 0 & f & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$Z=\frac{f\,t_x}{x'-x}$$

$$x'\equiv P'X=[K'\ |\ K't]\begin{bmatrix}\tilde{X}\\ 1\end{bmatrix}=K'\tilde{X}+K't$$
$$=K'ZK^{-1}x+K't$$
$$=Z\left(K'K^{-1}x+\frac{K't}{Z}\right)$$

For $K=K'$, $\quad x'\equiv Z\left(x+\frac{Kt}{Z}\right)$

$$\rightarrow x'=x+\frac{Kt}{Z}$$

General Motion of Camera

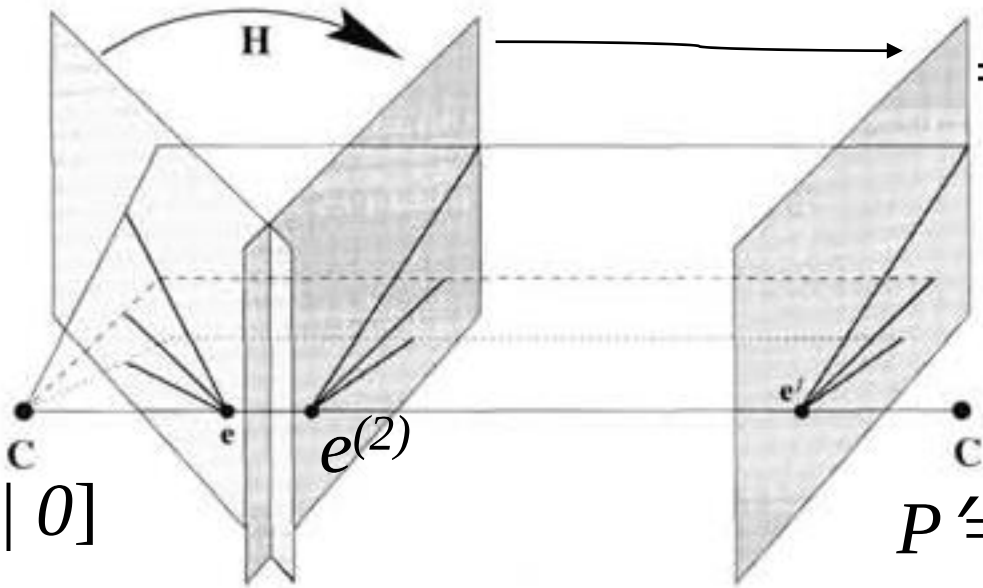
$$x^{(2)} = K[R|0]X$$
$$= KR[I|0]X$$
$$= KRK^{-1}K[I|0]X$$
$$= KRK^{-1}x$$

Rotational Homography

$H=KRK^{-1}$

$||^{l}$ stereo

$$x' = x^{(2)} + \frac{K't}{Z}$$
$$= KRK^{-1}x + \frac{K't}{Z}$$



$e^{(2)}$

$P=K[I \mid 0]$

$P'=K'[R \mid t]$

$$Z = \frac{K't}{x' - KRK^{-1}x}$$

From Hartley and Zisserman, "Multiple view geometry in computer vision", Cambridge Univ. Press (2000)

Estimation of Fundamental Matrix

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$\mathrm{x}'^{\mathrm{T}} \mathrm{F} \mathrm{x} = 0$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$[x'x, x'y, x', y'x, y'y, y', x, y, 1][f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}]^{\mathrm{T}} = 0$$

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \mathrm{f} = 0$$

$$\mathrm{A}\mathrm{f} = 0$$

- Solution up to scale.
- Minimum 8 point correspondences.
- Use of DLT (for 7 point correspondences from linear combination of smallest and second smallest eigen vectors.

Estimation of Fundamental Matrix

$$\begin{bmatrix} x_1x_1' & y_1x_1' & x_1' & x_1y_1' & y_1y_1' & y_1' & x_1 & y_1 & 1 \\ x_2x_2' & y_2x_2' & x_2' & x_2y_2' & y_2y_2' & y_2' & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx_n' & y_nx_n' & x_n' & x_ny_n' & y_ny_n' & y_n' & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

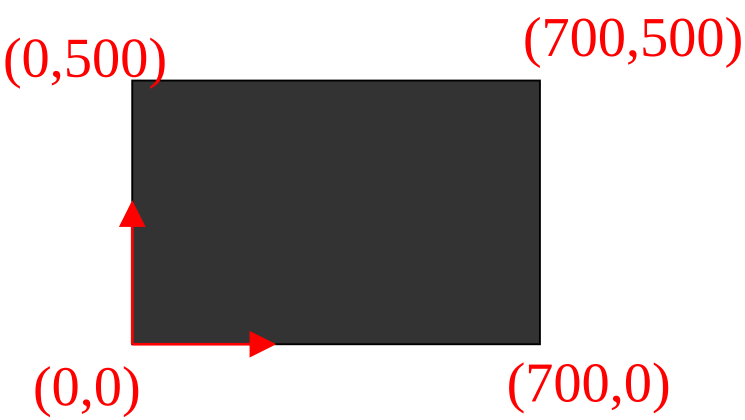
~10000 ~10000 ~100 ~10000 ~10000 ~100 ~100 ~100 1

!

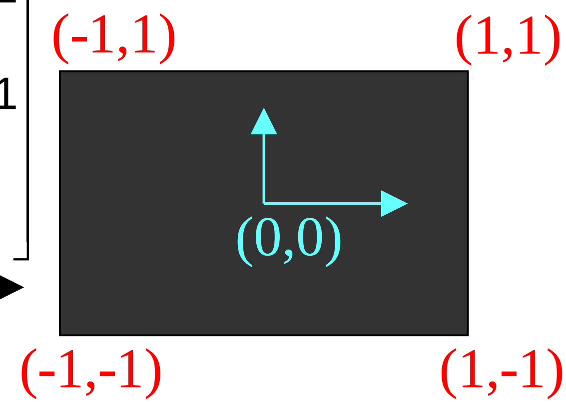
Orders of magnitude difference
Between column of data matrix
$\rightarrow$ least-squares yield poor results

The normalized 8-point algorithm

Transform image to
~[-1,1] x [-1,1]



$$\begin{bmatrix} \frac{2}{700} & 0 & -1 \\ & \frac{2}{500} & -1 \\ & & 1 \end{bmatrix}$$



Least squares yields good results
(Hartley, PAMI´97)

The singularity constraint

$$\det F = 0 \quad \text{rank } F = 2$$

SVD from linearly computed F matrix (rank 3)

$$F = U \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} V^T = U_1 \sigma_1 V_1^T + U_2 \sigma_2 V_2^T + U_3 \sigma_3 V_3^T$$

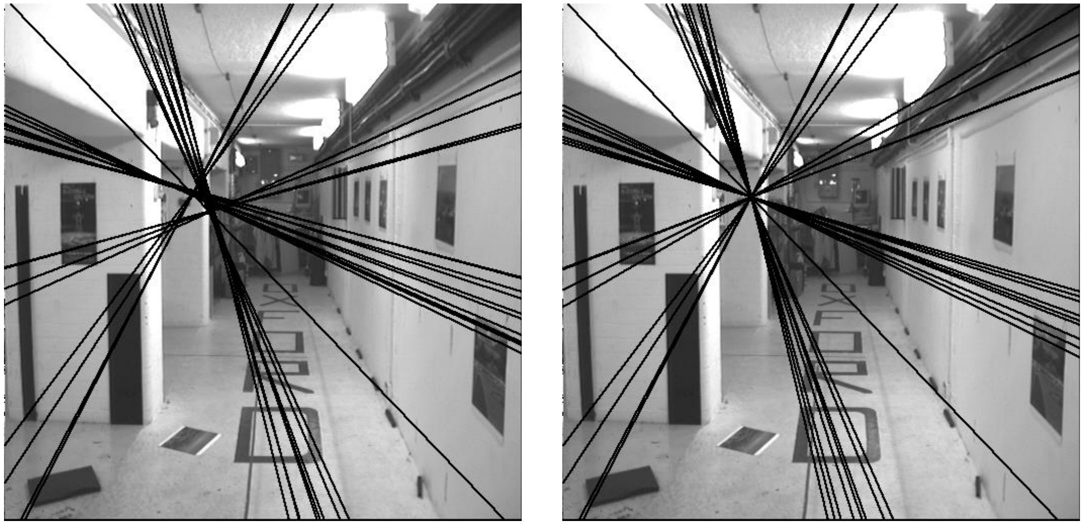
$$\min \| F - F' \|_F$$

Compute closest rank-2 approximation

$$F' = U \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix} V^T = U_1 \sigma_1 V_1^T + U_2 \sigma_2 V_2^T$$

The singularity constraint

Nonsingular F



Singular F

Non-singular F causes epipolar lines not converging.

From Hartley and Zisserman, “Multiple view geometry in computer vision”, Cambridge Univ. Press (2000)

The singularity constraint for Essential Matrix

$$\det(E)=0$$

Estimate $\hat{E}$ by any of the techniques used for F.

Perform SVD of $\hat{E}$.

$\hat{E}=UDV^T$ Where $D=\text{diag}(a,b,c)$ $a \geq b \geq c$

For essential matrix, two singular values are the same.

$$\Rightarrow \hat{E} = U\widehat{D}V^T \qquad where\ \widehat{D} = \left(\frac{a+b}{2}, \frac{a+b}{2}, 0\right)$$

The minimum case – 7 point correspondences

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_7 x_7 & x'_7 y_7 & x'_7 & y'_7 x_7 & y'_7 y_7 & y'_7 & x_7 & y_7 & 1 \end{bmatrix} \mathrm{f} = 0$$

$$\mathrm{A} = \mathrm{U}_{7x7} \mathrm{diag}(\sigma_1, \ldots, \sigma_7, 0, 0) \mathrm{V}_{9x9}^{\mathrm{T}}$$

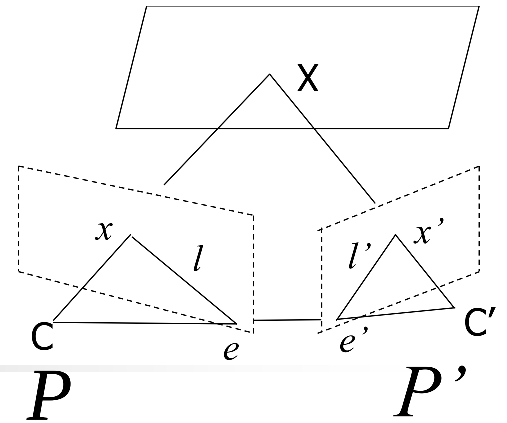
$F_1, F_2 \rightarrow$ Eigen vectors corresponding to two zero's. The solution is $F_1 + \lambda F_2$.

But $F_1 + \lambda F_2$ not automatically rank 2.

Solve for λ from $\det(F_1 + \lambda F_2) = 0$.

As it is a cubic polynomial, there are 1 or 3 solutions.

Parametric representation of F



Over parameterization: $F=[t]_x M$ ➔ $\{t,M\}$ → 12 params.

Epipolar parameterization:

$$\{a, b, c, d, e, f, \alpha, \beta\}$$

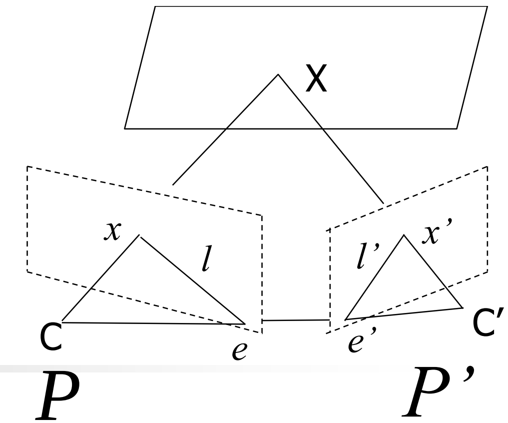
$$F = \begin{bmatrix} a & b & \alpha a + \beta b \\ c & d & \alpha c + \beta d \\ e & f & \alpha e + \beta f \end{bmatrix} \quad Right\ epipole: e' = [\alpha \quad \beta \quad -1]^T$$

Both epipoles as parameters $\{a, b, c, d, \alpha, \beta, \alpha', \beta'\}$

$$F = \begin{bmatrix} a & b & \alpha a + \beta b \\ c & d & \alpha c + \beta d \\ \alpha' a + \beta' c & \alpha' b + \beta' d & \alpha\alpha' a + \alpha\beta' c + \alpha'\beta b + \beta\beta' d \end{bmatrix}$$

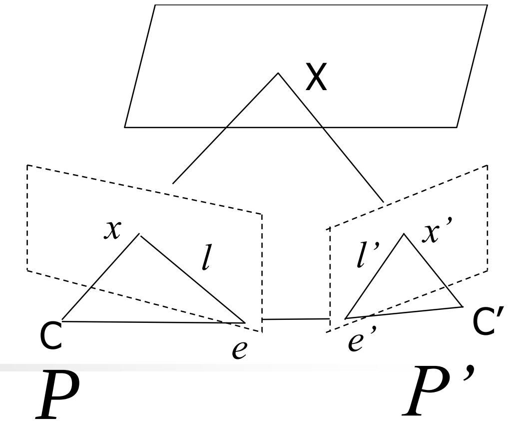
$$Epipoles: e' = [\alpha \quad \beta \quad -1]^T \qquad e = [\alpha' \quad \beta' \quad -1]^T$$

Retrieving the camera matrices from F



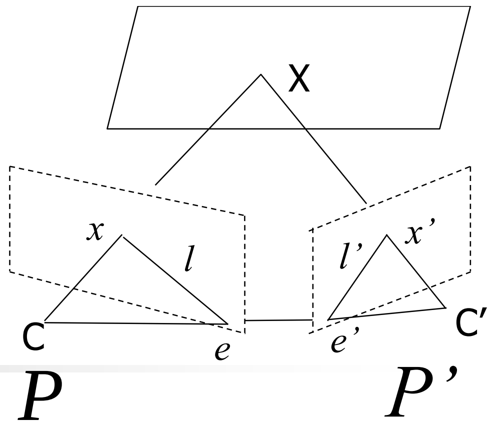
- F only depends on projective properties of P and P'.
- Independent of choice of world frame.
- $(P,P') \rightarrow F$ (unique)
- $F \rightarrow (P,P')$ (?)
- Given a homography H (4x4 non-singular matrix) in P^3, if $(P,P') \rightarrow$ F, then $(PH,P'H) \rightarrow F$.
  Proof: $PX \leftarrow\rightarrow P'X$
  $\Rightarrow (PH)(H^{-1}X) \leftarrow\rightarrow (P'H)(H^{-1}X)$
- *F does not uniquely map to (P,P').*

Retrieving the camera matrices from F



- $P=[I \mid 0]$ & $P'=[M \mid m]$ → $F=[m]_x M$.
- If F derived from both (P_1,P_1') and (P_2,P_2'), there exists 4x4 H s.t. $P_2=P_1H$ & $P_2'=P_1'H$.
- Suppose a rank 2 matrix F decomposed in two different ways, $F=[\mathbf{a}]_x A=[\mathbf{b}]_x B$. Then $\mathbf{a}=k\mathbf{b}$ and $B=k^{-1}(A+\mathbf{a}\mathbf{v}^T)$ for some non zero constants k and 3-vector $\mathbf{v}$.
- d.o.f. of P + d.o.f. of P'=22
- d.o.f. of H = 15
- d.o.f. of F=22-15=7

Retrieving the camera matrices from F

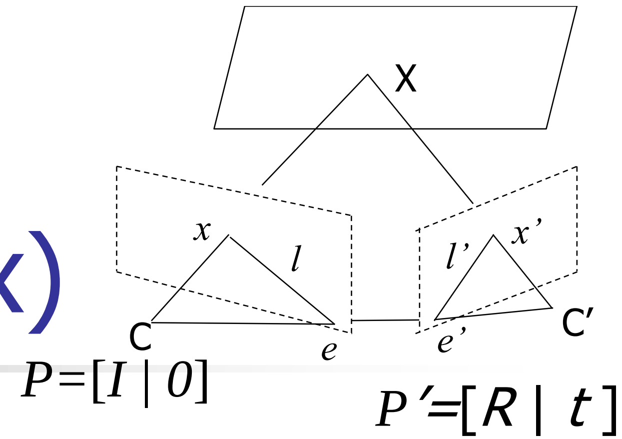


- F corresponds to (P,P'), iff P'^TFP is *skew symmetric*.

  Proof:For a skew symmetric matrix S, $X^TSX=0$, for all X. Now, $X^TP'^TFPX=(P'X)^TF(PX)=x'^TFx=0$ (for any X in P^3, as F is the fundamental matrix). …
- F corresponds to $P=[I \mid 0]$ & $P'=[SF \mid e']$, where e' is the right epipole of F s.t. $e'^TF=0$.
- A good choice of $S=[e']_x$.
- F → $([I \mid 0], [[e']_x F \mid e'])$

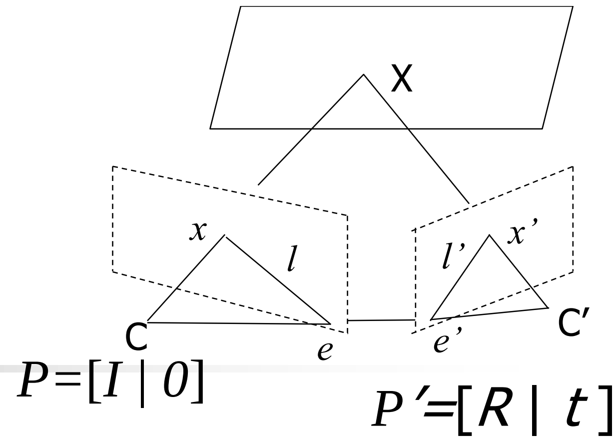
  ←→ $([I \mid 0], [[e']_x F + e' \mathbf{v}^T \mid ke'])$

The camera matrices from E (Essential matrix)



- E is an essential matrix iff two of its singular values are equal and the third one is zero.
- $E=[t]_x R$
- $[t]_x$ and R can be computed through decomposition of E s.t. $E=SR$, where S is a skew symmetric matrix and R is orthogonal.

Decomposition of E (Essential matrix)

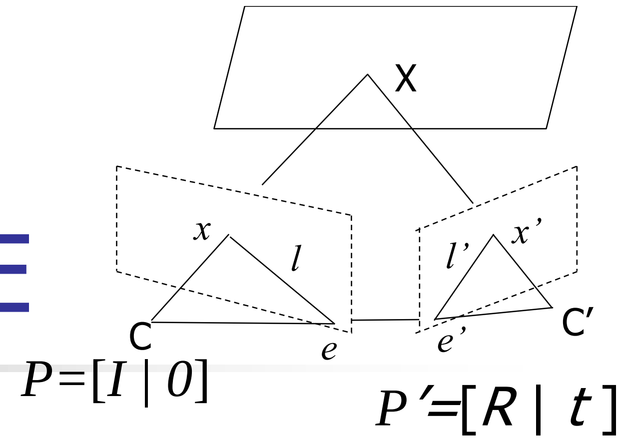


- SVD of $E=U\,\mathrm{diag}(1,1,0)\,V^T$
- Two possible decomposition of $E=SR$
- $S=UZU^T$ and $R=UWV^T$ or UW^TV^T

$$z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Any skew symmetric matrix S can be decomposed as $S=kUZU^T$
- W is orthogonal and $Z=\mathrm{diag}(1,1,0)W$.

Camera matrices from E



- SVD of $E=U\,\mathrm{diag}(1,1,0)\,V^T$
- Two possible decomposition of $E=SR$
- $S=UZU^T$ and $R=UWV^T$ or UW^TV^T

$$z=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

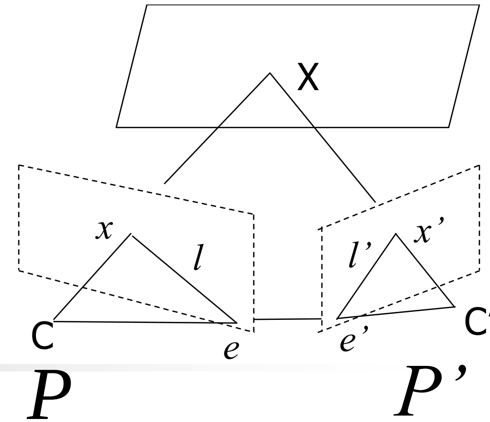
$$P'=[UWV^T \mid +u_3] \text{ or } [UWV^T \mid -u_3]$$
$$[UW^TV^T \mid +u_3] \text{ or } [UW^TV^T \mid -u_3]$$

$$W=\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

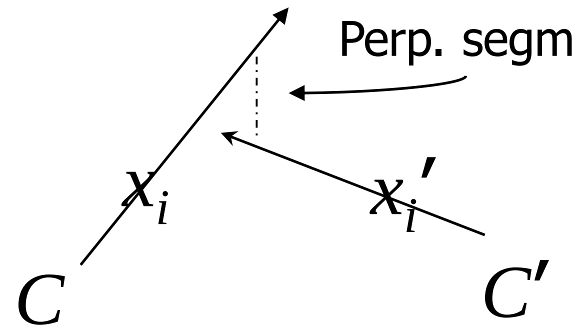
Last column of U.

Out of the four only one is valid for viewing a point from both the cameras. It is sufficient to test a single point for the above.

Computing scene points (structure)



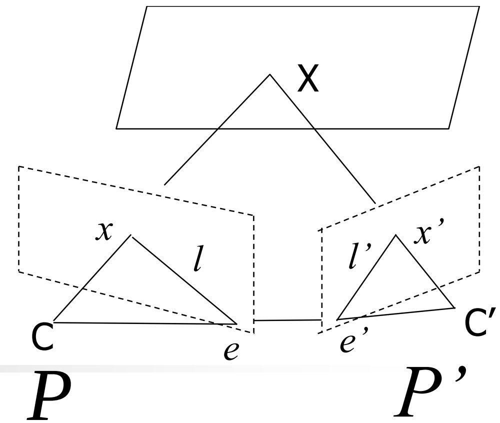
Given $x_i \leftarrow\rightarrow x_i'$, compute X.



1. Compute F.
2. Compute P and P'.
3. For each (x_i, x_i') compute X by triangulation.
   i. Compute intersection of Cx_i and $C'x_i'$.
   ii. Compute segment perpendicular to both.
   iii. Get the mid-point.

Not projective invariant, i.e. $(PH, P'H)$ does not give $H^{-1}X$.

Minimizing Reprojection Error



Given $x_i \leftrightarrow x_i'$, compute X.

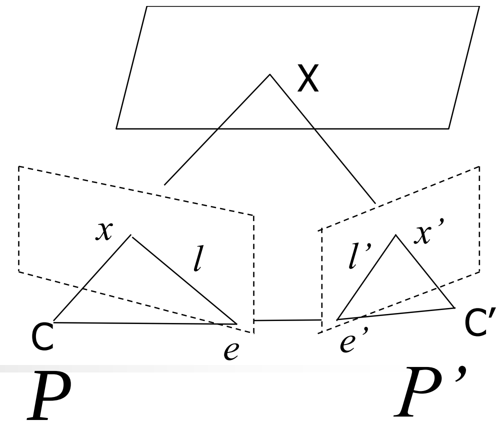
1. Estimate $\hat{X}$ s.t. $P\hat{X} = \hat{x}$ and $P'\hat{X} = \widehat{x'}$.
2. Minimize the reprojection error (E_{rp}).

$$E_{rp} = d(x, \hat{x})^2 + d\left(x', \widehat{x'}\right)^2$$

subject to $x'^T F x = 0$

Projective invariant.

Linear triangulation methods



Given $x_i \leftarrow\rightarrow x_i'$, compute X.

$$x \times PX = 0$$
$$x' \times P'X = 0$$

4 equations,
3 unknowns.
$[A]_{4x4}X=0$

Minimize $||AX||$
subject to $||X||=1$.

Use DLT.
Not projective invariant.

Generalize to multiview correspondences.

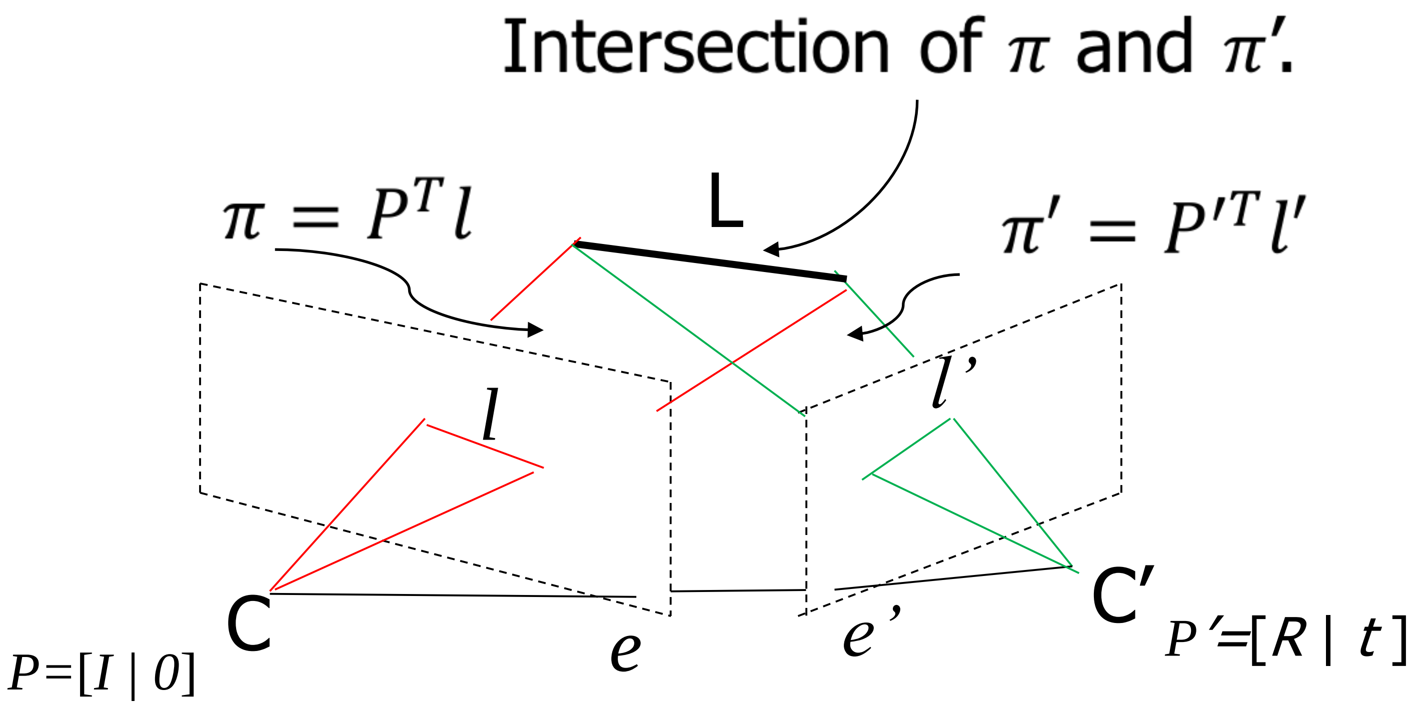
$$x_1 \leftarrow\rightarrow x_2 \leftarrow\rightarrow x_3$$
$$P_1 \quad P_2 \quad P_3$$

$$\Rightarrow$$

$$x_1 \times P_1 X = 0$$
$$x_2 \times P_2 X = 0$$
$$x_3 \times P_3 X = 0$$

6 equations
3 unknowns.

Line reconstruction



$$L = \begin{bmatrix} \pi \\ \pi' \end{bmatrix}$$

A convenient way of representing 3D line.

Plane Induced Homography

Proof:

$$x' = P'X = [A|a]X$$

Now, $x = PX = [I\ |0]X$

So any point in $\overrightarrow{CX}$ is $X = \begin{bmatrix} x \\ \rho \end{bmatrix}$

When it intersects π, $\pi^T \begin{bmatrix} x \\ \rho \end{bmatrix} = 0.$

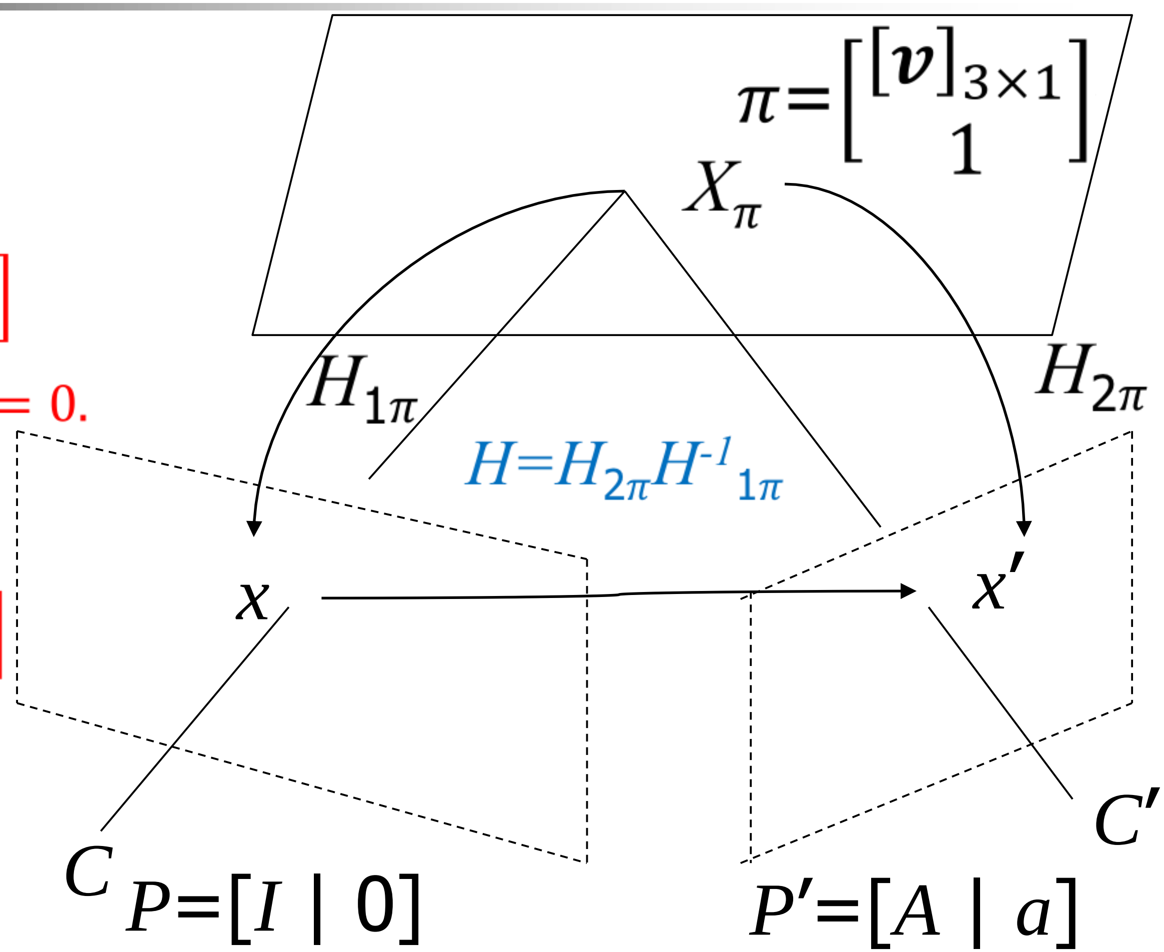
$$\Rightarrow\ \boldsymbol{v}^T x + \rho = 0$$

$$\Rightarrow\ \rho = -\boldsymbol{v}^T x$$

So, $x' = P'X = [A|a]\begin{bmatrix} x \\ -\boldsymbol{v}^T x \end{bmatrix}$

$= Ax - a\ \boldsymbol{v}^T x$

$=\underbrace{(A - a\ \boldsymbol{v}^T)}_{H}x$



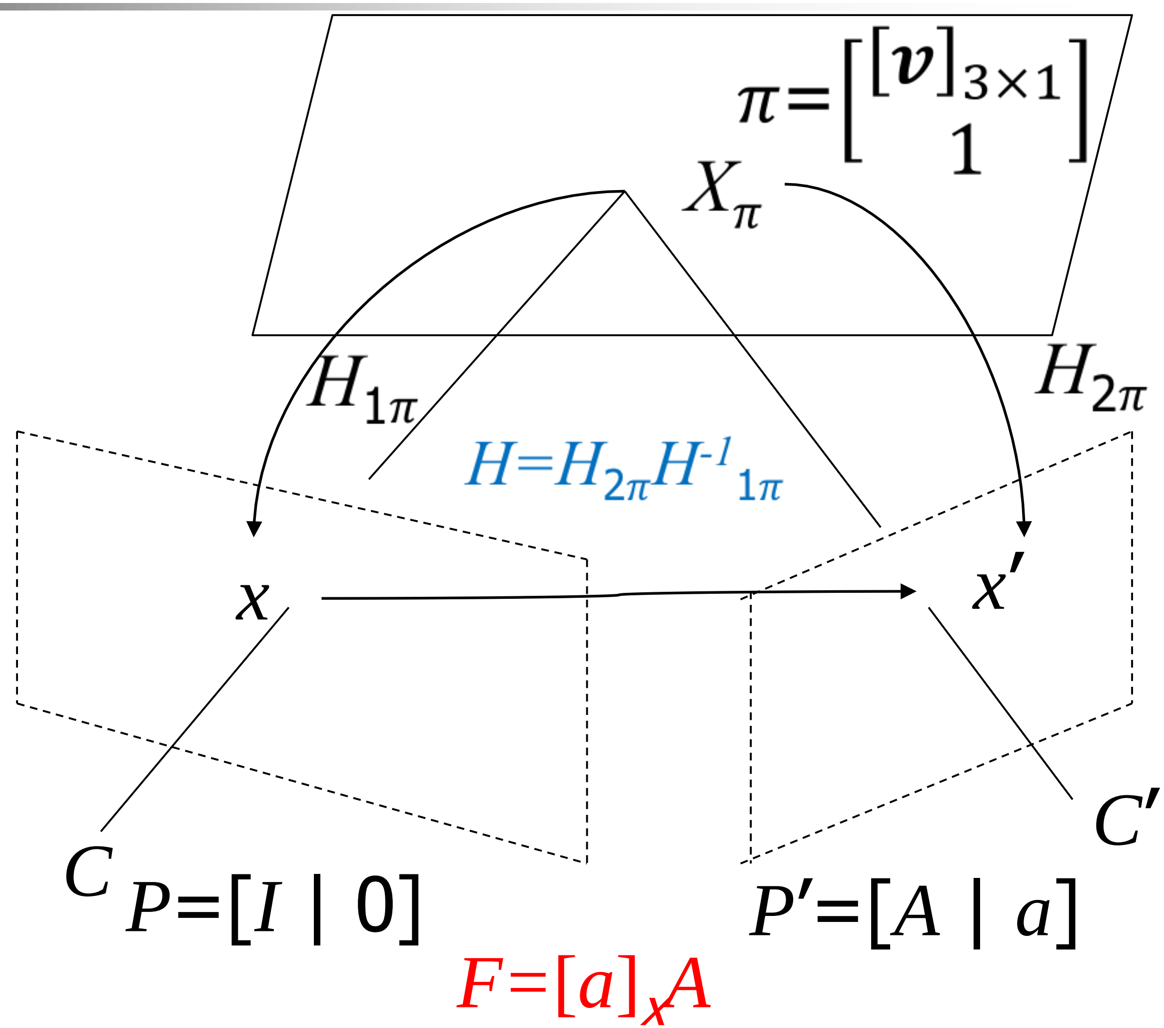
Plane Induced Homography

$(A - a\,\boldsymbol{v}^T)x$

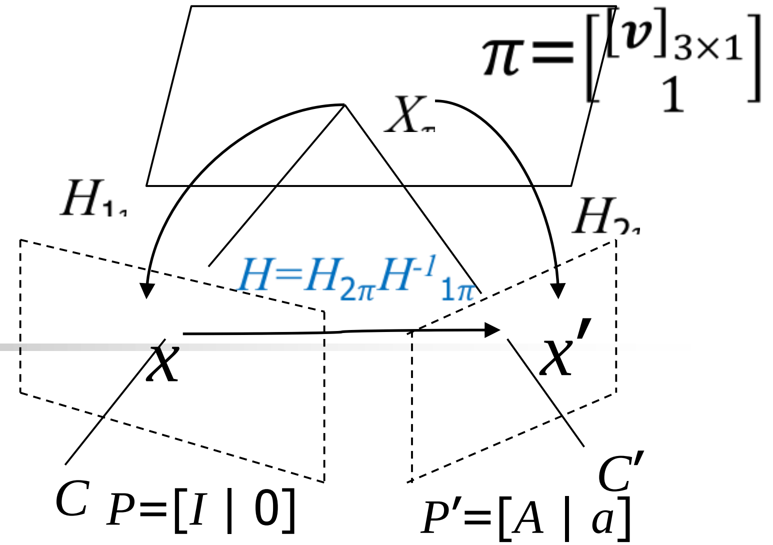
H

Given F, the three parameter family of homographies induced by a world plane $\pi(=[\boldsymbol{v}^T \quad 1])$ is $H=A - e'\,\boldsymbol{v}^T$

Where $[e']_x A=F$.



Plane induced homography



A transformation H between two stereo images is plane induced homography if F is decomposed into $[e']_x H$. Hence, $P=[I|0]$ & $P'=[H|e']$.

Plane at infinity

H is the transformation w.r.t. plane $[0\ 0\ 0\ 1]^T$ in the camera coordinate.

Given $P=[I|0]$, $P'=[A|a]$, & a plane induced homography H, the plane can be recovered by solving $kH=A-a\mathbf{v}^T$, (linear equations for unknowns k and v).

Homography compatible stereo geometry

H is compatible iff $H^T F$ is skew symmetric, i.e.

$$H^T F + F^T H = 0$$

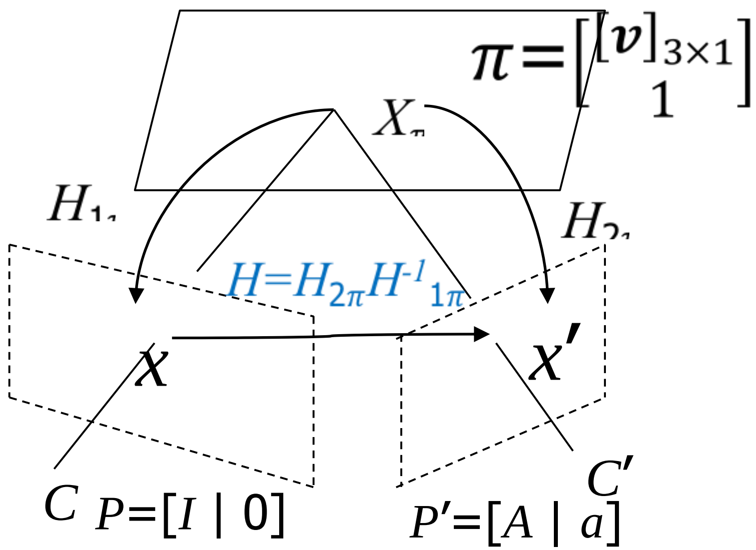
$x'^T F x = 0$

And, $x' = Hx$

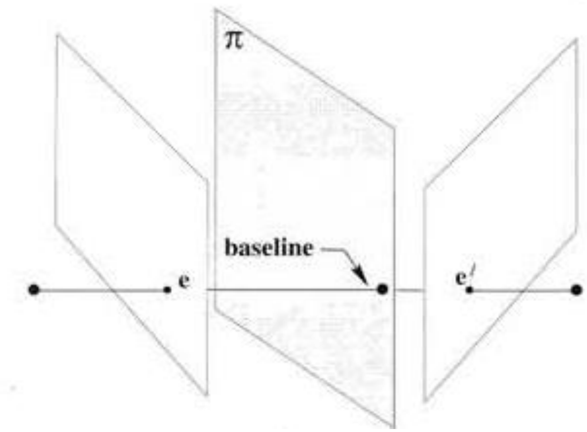
$\Rightarrow (Hx)^T F x = 0$

$\Rightarrow x^T H^T F x = 0$

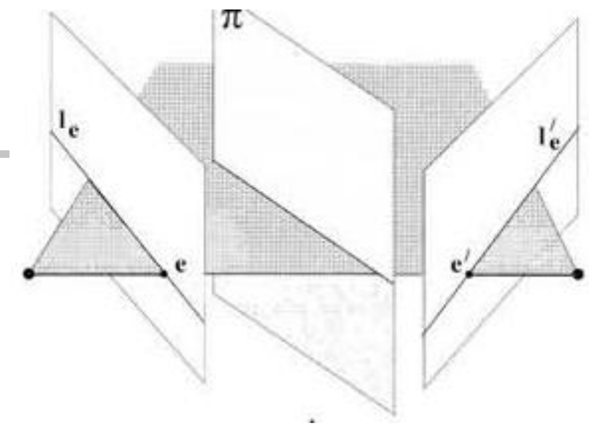
As this is true for all x, $H^T F$ is skew symmetric.



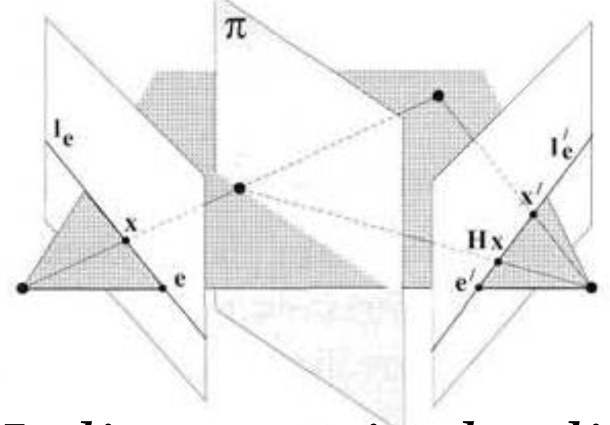
Plane induced H and epipolar constraints



Epipoles mapped by H, as e'=He, *since they are images of the point on the plane where the baseline intersects it.*



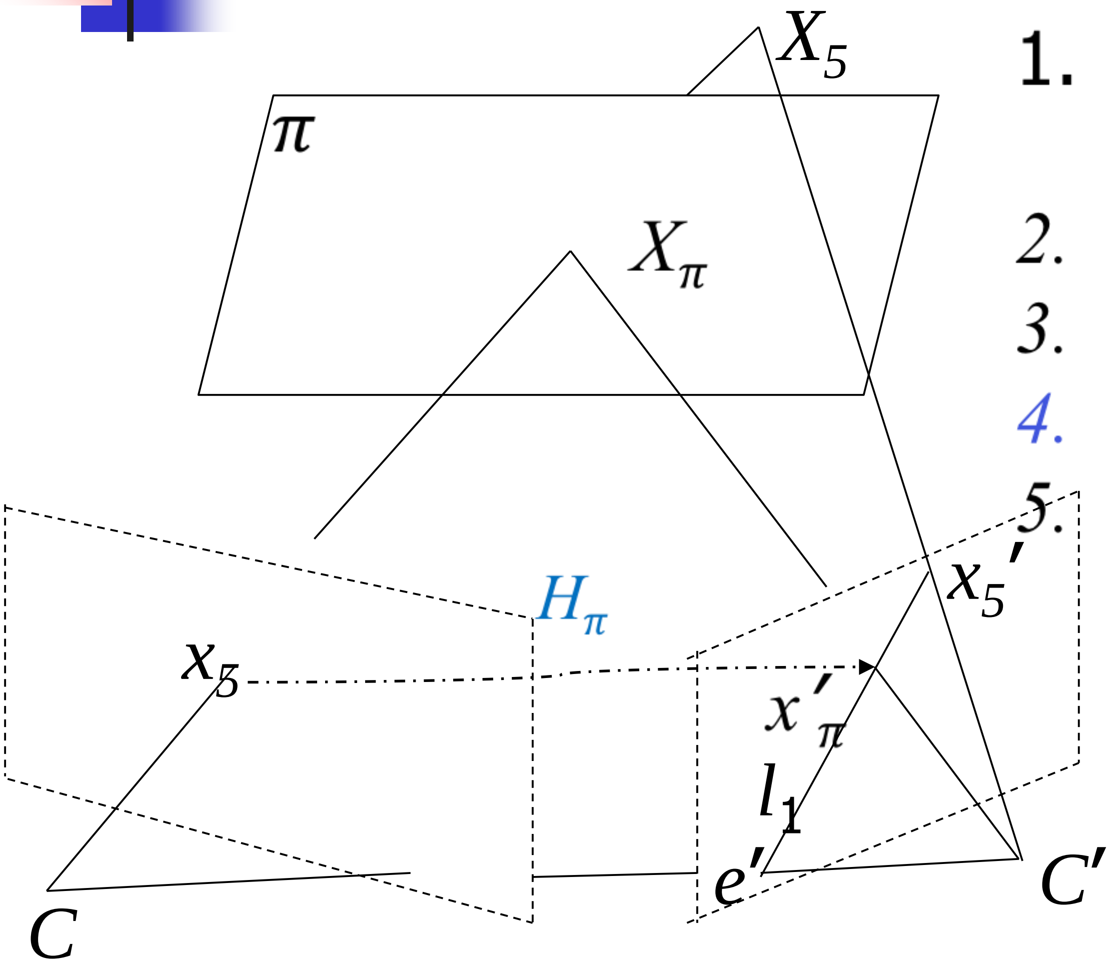
Epipolar lines are mapped by the homography as $H^T l'_e = l_e$.



Hx lies on epipolar line l_e'
so $l'_e = Fx = x' \times (Hx)$.

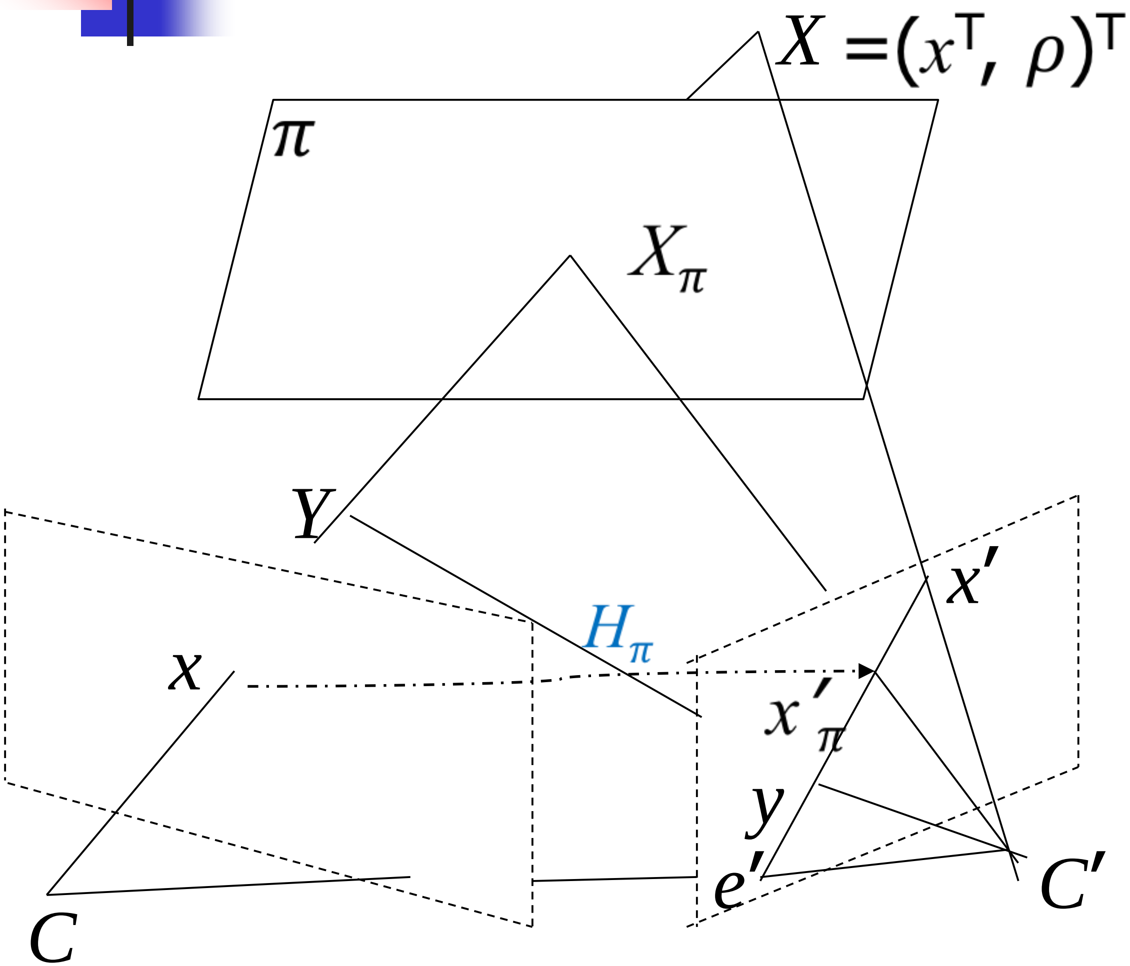
From Hartley and Zisserman, "Multiple view geometry in computer vision", Cambridge Univ. Press (2000)

Computing F from 6 points out of which 4 are coplanar



1. Use 4 coplanar points to compute H_π.
2. $l_1 = H_\pi(x_5) \times x_5'$
3. $l_2 = H_\pi(x_6) \times x_6'$
4. $e' = l_1 \times l_2$
5. $F = [e']_\times H_\pi$.

Projective depth



Projective depth w.r.t. the plane

$$x' = H_\pi x + \rho e'$$

For images of points lying on the plane, ρ=0.

In front of the plane towards camera (say y), $\rho < 0$.

Behind the plane away from camera (say x'), ρ >0.

Given F and 3 point correspondences, compute H.

First Method

1. Obtain $(P=[I|0], P'=[A|a])$ from F and construct 3 scene points, X_1, X_2, & X_3.
2. Obtain plane $(v^T,1)^T$.
3. Compute $H=A-av^T$

Second Method

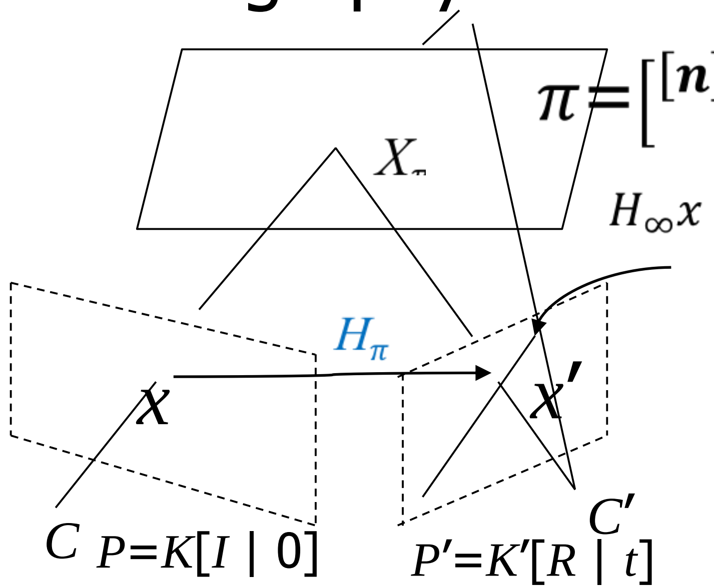
1. Obtain (e,e') from F.
2. Use 3 correspondences + (e,e'), to obtain H.

Any 3 points can bipartition the image space, w.r.t. the plane formed by them.

Infinite Homography

$H = (A - a\,\boldsymbol{v}^T)$
For P=[I |0], P'=[A | a]
and plane=[$\boldsymbol{v}^T$ 1]^T

Homography induced by the plane at α.



$$\pi = \begin{bmatrix} [\boldsymbol{n}]_{3\times 1} \\ d \end{bmatrix}$$

$$H_\pi = (K'R - e'\boldsymbol{v}^T)K^{-1}$$

$$\text{where } v = \frac{n}{d} \ and\ e' = K't$$

$$H_\pi = K'\left(R - \frac{\boldsymbol{tn}^T}{d}\right)K^{-1}$$

$$= K'RK^{-1} - K'\frac{\boldsymbol{tn}^T}{d}K^{-1}$$

As $d \rightarrow \infty$, $H_\infty = K'RK^{-1}$

$$x' = K'RK^{-1}x + \frac{K't}{Z}$$

Vanishing point

$$= H_\infty x + \frac{K't}{Z}$$

As $Z \rightarrow \infty$,
x' is the image of point on π_∞.

H_α and Vanishing points

- H_α maps vanishing points between two images.
- H_α can be computed by identifying three non-collinear vanishing points given F or from 4 vanishing points.
- Let $P=[M\,|m]$, $P'=[M'\,|\,m']$, $X=[x_\alpha{}^T\ 0]^T$ (a point at infinity).

$$x=PX=M\,x_\alpha$$
$$x'=P'X=M'x_\alpha$$
$$x'=M'M^{-1}x \Rightarrow H_\alpha=M'M^{-1}$$

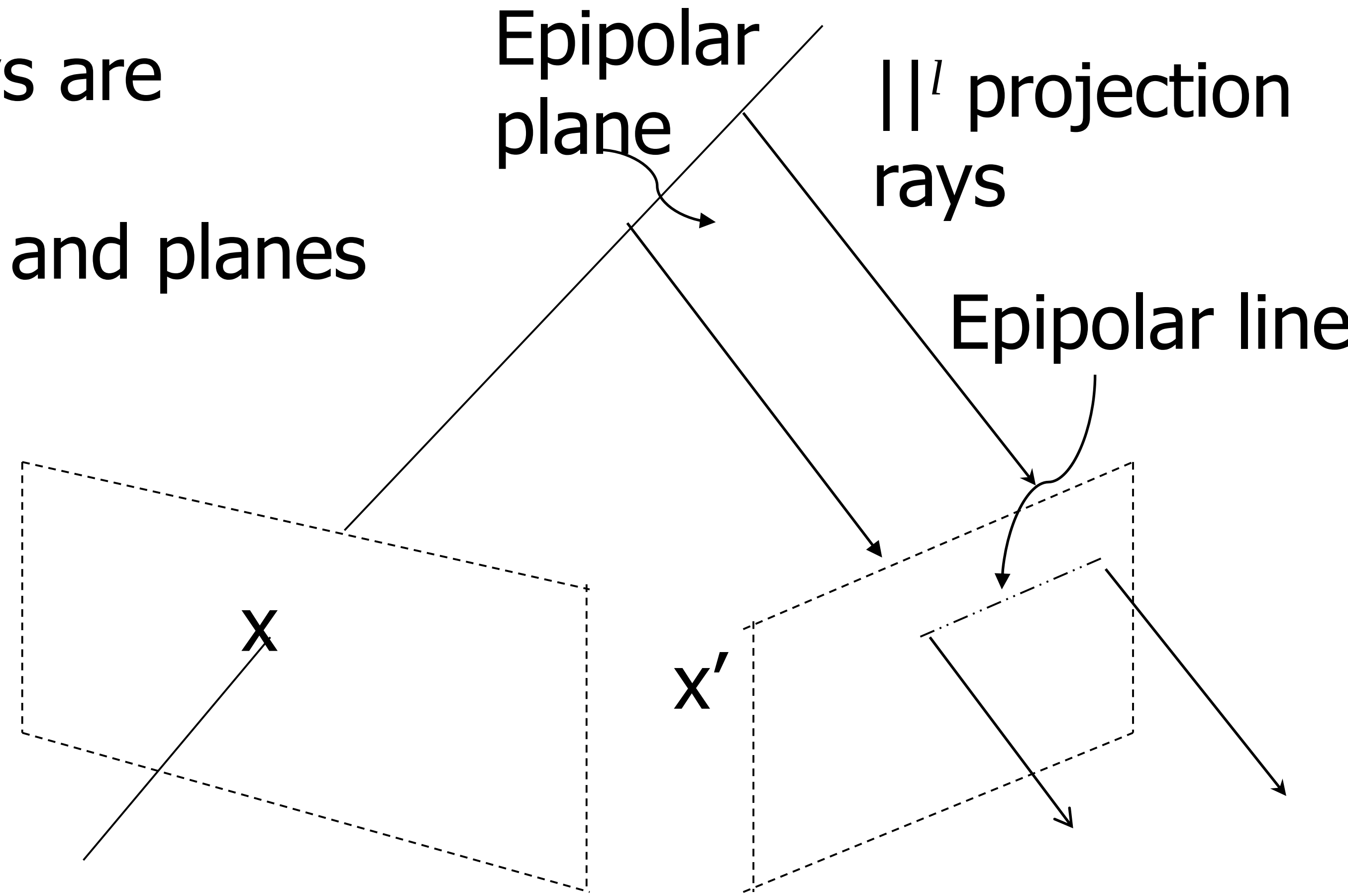
Affine epipolar geometry

- Projection rays are parallel.
- Epipolar lines and planes are parallel.

Form of F:

$$\begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{bmatrix}$$

a,b,c,d,e all non-zero.



As epipolar lines are parallel, epipoles are in the form $[e_1\ e_2\ 0]^T$

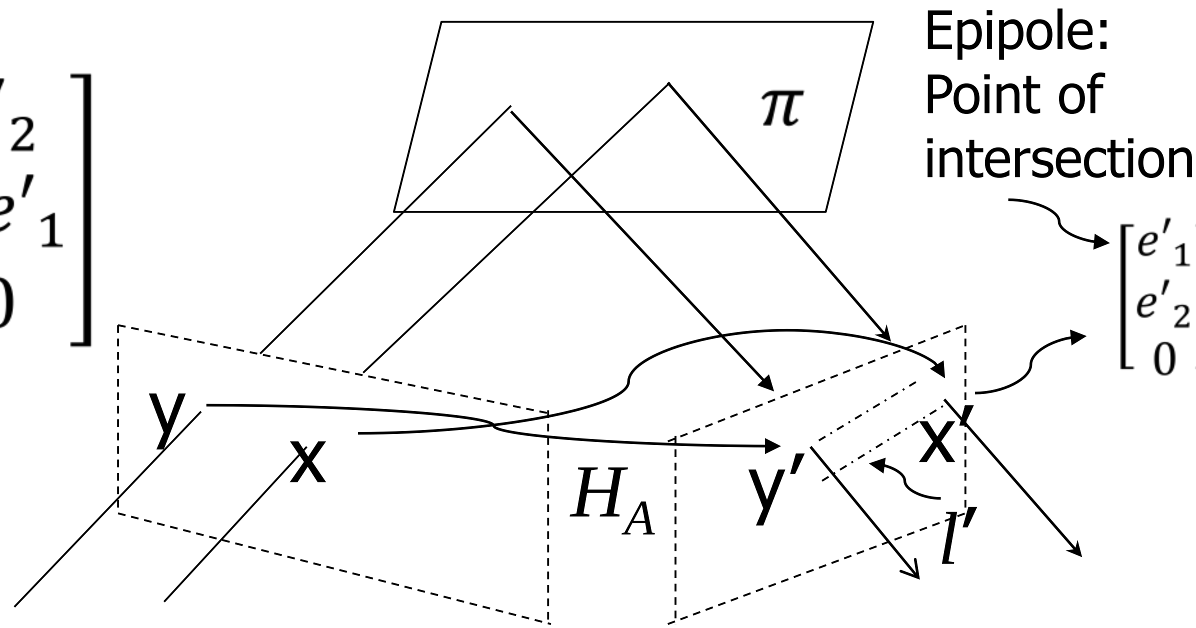
Affine stereo

$$l' = e' \times H_A x = [e']_\times H_A x$$

$$\begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{bmatrix} \Rightarrow F_A = [e']_\times H_A$$

$$[e']_\times = \begin{bmatrix} 0 & 0 & e'_2 \\ 0 & 0 & -e'_1 \\ -e'_2 & e'_1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \boldsymbol{b} \\ -\boldsymbol{b}^T & 0 \end{bmatrix}$$

$$H_A = \begin{bmatrix} A & \boldsymbol{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$$



F_A: d.o.f.: 4

$$F_A = \begin{bmatrix} 0 & \boldsymbol{b} \\ -\boldsymbol{b}^T A & -\boldsymbol{b}^T \boldsymbol{t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & e'_2 \\ 0 & 0 & -e'_1 \\ e & d & c \end{bmatrix}$$

Left epipole: $[-d\ e\ 0]^T$

right epipole: $[-b\ a\ 0]^T$

Estimating F_A

$$F_A = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{bmatrix}$$

Epipolar lines:

$$l' = F_A \boldsymbol{x} = [a \quad b \quad ex + dy + c]^T$$
$$l = {F_A}^T \boldsymbol{x}' = [e \quad d \quad ax' + by' + c]^T$$

Point correspondence: *Reduced to a single linear equation.*

$$x'^T F_A x = 0 \Longrightarrow ax' + by' + ex + dy + c = 0$$

$$[A]_{N\times 5} \boldsymbol{f}_{5\times 1} = 0$$ Solve using DLT.

Minimum 4 point correspondences required to get F_A.

Singularity constraint is satisfied by the structure of F_A.

Estimating F_A (another approach)

$$F_A = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ e & d & c \end{bmatrix}$$

1. Compute H_A using 3 point-correspondences.
2. $l' = H_A x_4' \times x_4'$ (say, $[l_1\ l_2\ l_3]^T$)
3. Get e' from l' as $[l_2\ {-l_1}\ 0]$
4. $F_A = [e']_{\times} H_A$

Thank you!