Image Transforms: A brief introduction

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"Image and video processing in the compressed domain", CRC Press, 2011.

$$f(x,y)=\sum_j \sum_i \lambda_{ij} b_{ij}(x,y)$$

Image Transform

- Image in continuous form: $f(x,y)$: A 2-D function, where (x,y) in R^2.
- Let B be a set of basis functions:

Properties of basis functions can be extended in the analysis.

$B=\{b_i(x,y) \mid i=..,-1,0,1,2,3,\ldots.\}$, $b_i(x,y)$ in R or C.

- Let $f(x,y)$ be expanded using B as follows:

$$f(x,y)=\sum_i \lambda_i b_i(x,y)$$

Coefficients of transform

The **transform** of f w.r.t. B is given by $\{\lambda_i | i=\cdots.-1,0,1,2,3,\ldots\}$.

Indexing may be multidimensional say, λ_{ij}.

Orthogonal Expansion and 1-D Transforms

$$f(x) = \sum_i \lambda_i b_i(x)$$

- Inner product: $$< f, g >= \int f(x) g^*(x) dx$$

- Orthogonal expansion: If B satisfies :

$$\begin{aligned} < b_i.b_j > &= 0, \quad for\ i \neq j, \\ &= c_i, \quad Otherwise\ (for\ i = j),\ where\ c_i > 0. \end{aligned}$$

comma

- Transform coefficients in O.E.: $\lambda_i = \frac{1}{c_i} < f.b_i >$
- If c_i=1, it becomes orthonormal expansion.

Forward transform $\lambda_i =< f.b_i >$

- Inverse transform: $$f(x) = \int_{i=-\infty}^{\infty} \lambda_i.b_i(x) di$$

Fourier transform

Complete base

$$B = \{e^{-j\omega x} | -\infty < \omega < \infty\}$$

Orthogonality:

$$\int_{-\infty}^{\infty} e^{j\omega x} dx = \begin{cases} 2\pi\delta(\omega), & \text{for } \omega = 0, \\ 0, & \text{Otherwise.} \end{cases}$$

Fourier Transform:

$$\mathbb{F}(f(x)) = \hat{f}(j\omega) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$$

Inverse Transform:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(j\omega).e^{j\omega x} d\omega$$

Full reconstruction

$$e^{-j\omega x} = \cos(\omega x) - j\sin(\omega x)$$

$$\hat{f}(j\omega) = \int_{-\infty}^{\infty} f(x)(\cos(\omega x) - j\sin(\omega x))\, dx$$

$$C = \{\cos(\omega x) | -\infty < \omega < \infty\}$$

$$S = \{\sin(\omega x) | -\infty < \omega < \infty\}$$

Orthogonal

But not complete!

Even and odd functions

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

- Even: $f(-x)=f(x)$ for all x.
- Odd: $f(-x)=-f(x)$ for all x. → $f(0)=0$.
- For even $f(x)$: $\int_{-\infty}^{\infty} f(x)(\sin(\omega x))\,dx = 0$
- For odd $f(x)$: $\int_{-\infty}^{\infty} f(x)(\cos(\omega x))\,dx = 0$
- Full reconstruction possible with cosines (sines) only if it is even (odd).

Discrete representation

- Discrete representation of a function:

$$f(n) = \{f(nX_0) | n \in \mathbb{Z}\}$$

Set of integers

Sampling interval

- Can be considered as a vector in an infinite dimensional vector space.
- In our context, it is of a finite dimensional space, e.g. $\{f(n), n=0,1,..N-1\}$, or
- $\boldsymbol{f}=[f(0)\ f(1)\ \ldots\ f(N-1)]^T$.

Discrete Transform

- For n-dimensional vector X any linear transform, e.g. $Y_{mx1}=B_{mxn}X_{nx1}$
- Has inverse transform if B is square of size (nxn) and invertible.
- Rows of B are called basis vectors.
- $Y(i)=<\boldsymbol{b}_i^{*T}.\ X>$

  dot product or inner product.

$$B=\begin{bmatrix}\boldsymbol{b}_0^{*T}\\ \boldsymbol{b}_1^{*T}\\ \vdots\\ \boldsymbol{b}_n^{*T}\end{bmatrix}$$

- Orthogonality condition:

$$\begin{aligned}<\boldsymbol{b}_i^{*T}.\boldsymbol{b}_j> &= 0 \; if \; i \neq j\\ &= c_i, \quad otherwise\end{aligned}$$

Discrete Fourier Transform (DFT)

$$b_k(n) = \frac{1}{\sqrt{N}} e^{j2\pi \frac{k}{N} n}, \text{ for } 0 \le n \le N-1, \text{ and } 0 \le k \le N-1.$$

$$\hat{f}(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi \frac{k}{N} n} \text{ for } 0 \le k \le N-1.$$

$$\hat{f}(N+k) = \hat{f}(k)$$

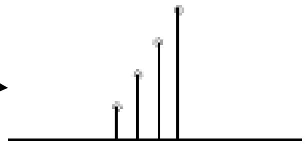
k/N: Normalized frequency

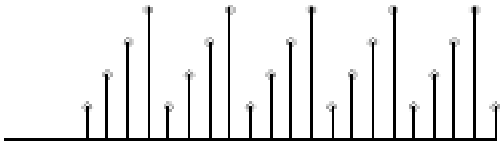
$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} \hat{f}(k) e^{j2\pi \frac{k}{N} n} \text{ for } 0 \le n \le N-1.$$

Hermitian Transpose

$$\mathbf{X} = \mathbf{F}\mathbf{x} \qquad \mathbf{F} = \left[e^{-j2\pi \frac{k}{N} n} \right]_{0 \le (k,n) \le N-1}. \qquad \mathbf{F}^{-1} = \frac{1}{N} \mathbf{F}^H$$

A single period





$f(n+N)=f(n)$

Fundamental frequency: $1/(NX_0)$

DFT: Fourier series of a periodic function

Generalized Discrete Fourier Transform (GDFT)

$$\mathbf{F}_{\alpha,\beta} = \left[e^{-j2\pi \frac{k+\alpha}{N}(n+\beta)} \right]_{0 \le (k,n) \le N-1}$$

$$\begin{aligned}
\mathbf{F}_{0,0}^{-1} &= \frac{1}{N}\mathbf{F}_{0,0}^{H} &&= \frac{1}{N}\mathbf{F}_{0,0}^{*}, \\
\mathbf{F}_{\frac{1}{2},0}^{-1} &= \frac{1}{N}\mathbf{F}_{\frac{1}{2},0}^{H} &&= \frac{1}{N}\mathbf{F}_{0,\frac{1}{2}}^{*}, \\
\mathbf{F}_{0,\frac{1}{2}}^{-1} &= \frac{1}{N}\mathbf{F}_{0,\frac{1}{2}}^{H} &&= \frac{1}{N}\mathbf{F}_{\frac{1}{2},0}^{*}, \text{ and} \\
\mathbf{F}_{\frac{1}{2},\frac{1}{2}}^{-1} &= \frac{1}{N}\mathbf{F}_{\frac{1}{2},\frac{1}{2}}^{H} &&= \frac{1}{N}\mathbf{F}_{\frac{1}{2},\frac{1}{2}}^{*}.
\end{aligned}$$

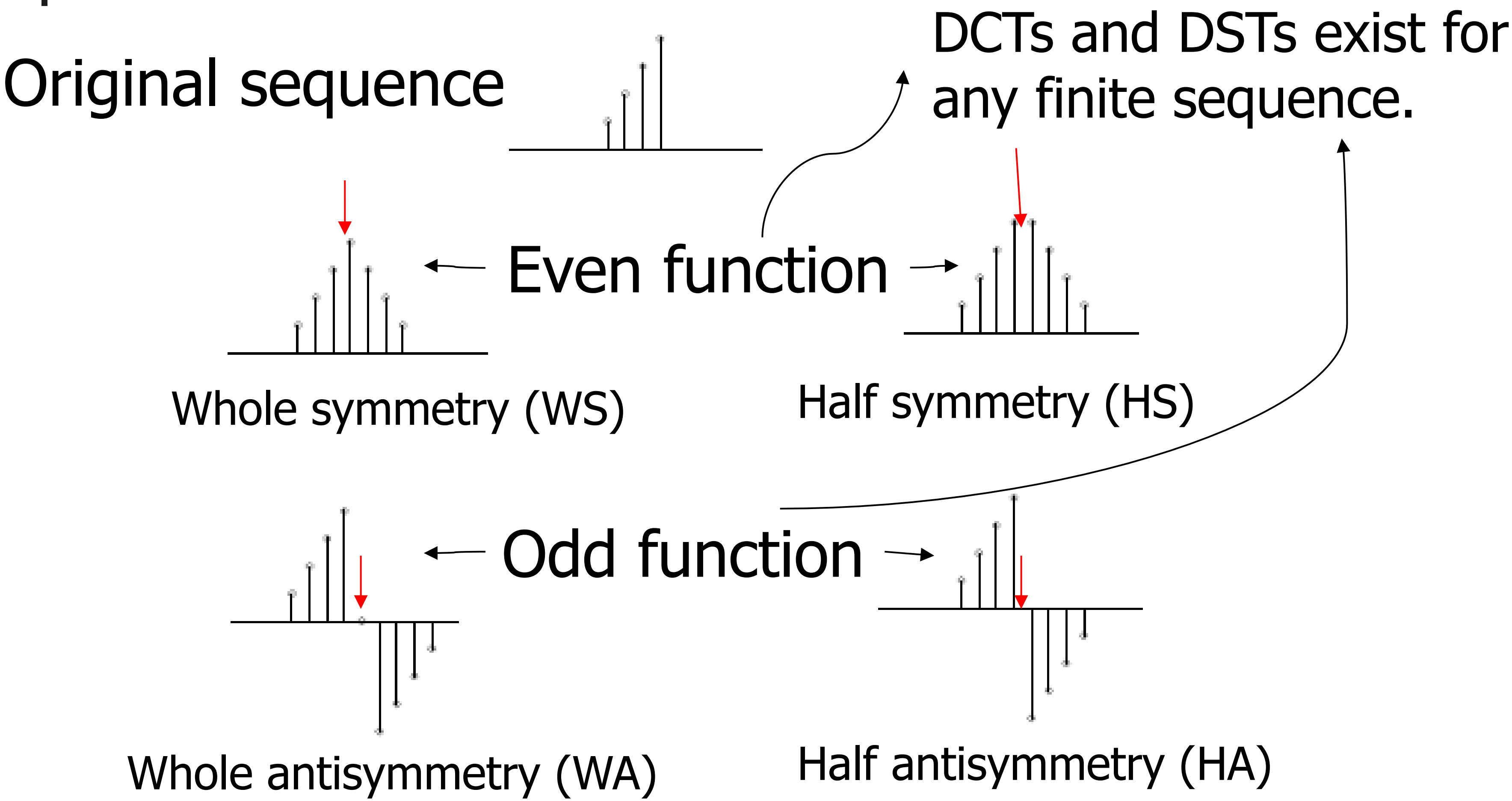
$$b_k^{(\alpha,\beta)}(n) = \frac{1}{\sqrt{N}} e^{j2\pi \frac{k+\alpha}{N}(n+\beta)}, \text{ for } 0 \le n \le N-1, \text{ and } 0 \le k \le N-1$$

$$\hat{f}_{\alpha,\beta}(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi \frac{k+\alpha}{N}(n+\beta)}, \text{ for } 0 \le k \le N-1$$

$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} \hat{f}_{\alpha,\beta}(k) e^{j2\pi \frac{k+\alpha}{N}(n+\beta)}, \text{ for } 0 \le n \le N-1$$

α	β	Transform name	Notation
0	0	Discrete Fourier Transform (DFT)	$\hat{f}(k)$
0	$\frac{1}{2}$	Odd Time Discrete Fourier Transform ($OTDFT$)	$\hat{f}_{0,\frac{1}{2}}(k)$
$\frac{1}{2}$	0	Odd Frequency Discrete Fourier Transform ($OFDFT$)	$\hat{f}_{\frac{1}{2},0}(k)$
$\frac{1}{2}$	$\frac{1}{2}$	Odd Frequency Odd Time Discrete Fourier Transform (O^2DFT)	$\hat{f}_{\frac{1}{2},\frac{1}{2}}(k)$

Symmetric / Antisymmetric extension of a finite sequence

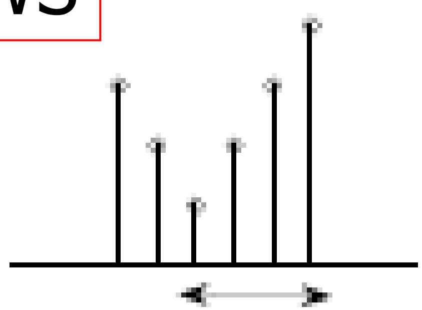


Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

- Types of symmetric / antisymmetric extensions at the two ends of a sequence and a type of GDFT→ DCTs / DSTs

WSWS



$F_{0,0}$

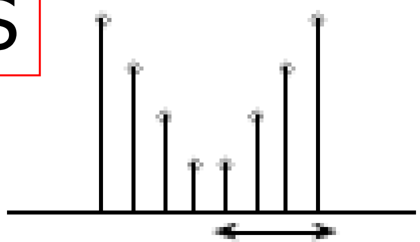
Type-I Even DCT

$$C_{1e}(x(n)) = X_{Ie}(k) = \sqrt{\frac{2}{N}}\alpha^2(k)\sum_{n=0}^{N} x(n)\cos\left(\frac{2\pi nk}{2N}\right), \; 0 \le k \le N.$$

Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

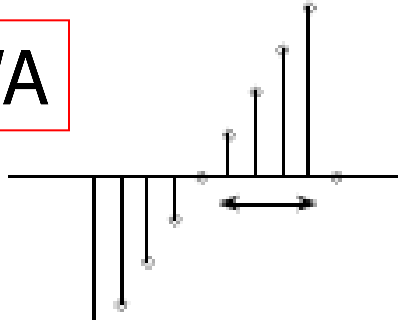
HSHS



$\xrightarrow{F_{0,1/2}}$ Type-2 Even DCT

$$C_{2e}(x(n)) = X_{IIe}(k) = \sqrt{\frac{2}{N}}\alpha(k)\sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi k(n+\frac{1}{2})}{2N}\right),\ 0 \le k \le N-1$$

WAWA



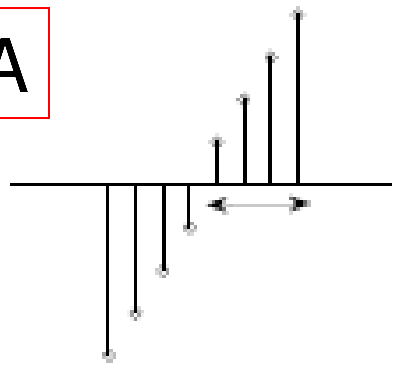
$\xrightarrow{jF_{0,0}}$ Type-1 Even DST

$$S_{1e}(x(n)) = X_{sIe}(k) = \sqrt{\frac{2}{N}}\sum_{n=1}^{N-1} x(n)\sin\left(\frac{2\pi kn}{2N}\right),\ 1 \le k \le N-1$$

Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

HAHA



$\xrightarrow{jF_{0,1/2}}$ Type-2 Even DST

$$S_{2e}(x(n)) = X_{sIIe}(k) = \sqrt{\frac{2}{N}}\alpha(k) \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi k(n+\frac{1}{2})}{2N}\right), \; 1 \le k \le N-1$$

There exist 16 different types of DCTs and DSTs.
Type-II Even DCT is used in signal, image, and video compression.

Convolution Multiplication Property (CMP)

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$$

$$F(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega)F(j\omega)$$

CMP for Fourier Transform

$$\widehat{f \circledast h}(k) = \hat{f}(k)\hat{h}(k)$$

CMP for DFT

CMP for DFT holds for circular convolution.

Linear convolution

$$f(n) \longrightarrow \boxed{h(n)} \longrightarrow \sum_{m=-\infty}^{\infty} f(m)h(n-m)$$

- Periodic convolution: Convolution between two finite sequences with periodic extension.
- It is defined if both have the same period, providing a periodic sequence with the same period.

Circular Convolution

$$\begin{aligned} f \circledast h(n) &= \sum_{m=0}^{N-1} f(m)h(n-m), \\ &= \sum_{m=0}^{n} f(m)h(n-m) + \sum_{m=n+1}^{N-1} f(m)h(n-m+N). \end{aligned}$$

Antiperiodic extension and skew-circular convolution

- Antiperiodic function with an antiperiod N, if $f(x+N)=-f(x)$.
- An antiperiodic function of antiperiod N → a periodic function of period $2N$.
- Skew-circular convolution: convolution between two antiperiodic extended sequences of the same antiperiod.

$$\begin{aligned} f \circledS h(n) &= \sum_{m=0}^{N-1} f(m)h(n-m), \\ &= \sum_{m=0}^{n} f(m)h(n-m) - \sum_{m=n+1}^{N-1} f(m)h(n-m+N) \end{aligned}$$

CMPs for DCTs and DSTs

$$u(n) = x(n) \circledast y(n)$$

$$w(n) = x(n) \circledS y(n)$$

$$C_{1e}\left(u(n)\right) = \sqrt{2N}\, C_{1e}\left(x(l)\right) C_{1e}\left(y(m)\right)$$

$$C_{2e}\left(u(n)\right) = \sqrt{2N}\, C_{2e}\left(x(l)\right) C_{1e}\left(y(m)\right)$$

Input

Filter response

$$S_{2e}\left(u(n)\right) = \sqrt{2N}\, C_{2e}\left(x(l)\right) S_{1e}\left(y(m)\right)$$

$$S_{2e}\left(u(n)\right) = \sqrt{2N}\, S_{2e}\left(x(l)\right) C_{1e}\left(y(m)\right)$$

$$C_{3e}\left(w(n)\right) = \sqrt{2N}\, C_{3e}\left(x(l)\right) C_{3e}\left(y(m)\right)$$

Properties of Type-II DCT

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

$$X = C_N.\mathbf{x} \qquad C_N^{-1} = C_N^T$$

- Matrix form:

N-point DCT

$$C_N = \left[\sqrt{\frac{2}{N}}.\alpha(k) cos(\frac{\pi k(2n+1)}{2N}) \right]_{0 \le (k,n) \le N-1}.$$

- Each row is either symmetric (even row) or antisymmetric (odd row).

$$C_N(k, N-1-n) = \begin{cases} C_N(k,n) & for\ k\ even \\ -C_N(k,n) & for\ k\ odd \end{cases}$$

- Sub-band relationship:

$$\begin{aligned} x_L(n) &= \tfrac{1}{2}\{x(2n) + x(2n+1)\}, \\ x_H(n) &= \tfrac{1}{2}\{x(2n) - x(2n+1)\}, \quad n = 0, 1, ...\tfrac{N}{2} - 1. \end{aligned}$$

Type-II DCT: Sub-band relation

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

$$C_N = \left[\sqrt{\frac{2}{N}}.\alpha(k)cos(\frac{\pi k(2n+1)}{2N})\right]_{0\leq(k,n)\leq N-1}.$$

- Sub-band relationship:

$$\begin{aligned} x_L(n) &= \tfrac{1}{2}\{x(2n) + x(2n+1)\}, \\ x_H(n) &= \tfrac{1}{2}\{x(2n) - x(2n+1)\}, \quad n = 0, 1, \ldots.\frac{N}{2} - 1. \end{aligned}$$

$$X_L(k) = C_{2e}(x_L(n)) \quad S_H(k) = S_{2e}(x_H(n))$$

$$X(k) = \sqrt{2}\cos(\frac{\pi k}{2N})\overline{X_L}(k) + \sqrt{2}\sin(\frac{\pi k}{2N})\overline{S_H}(k), \; 0 \leq k \leq N-1,$$

$$\overline{X_L}(k) = \begin{cases} X_L(k), & 0 \leq k \leq \frac{N}{2} - 1, \\ 0, & k = \frac{N}{2}, \\ -X_L(N-k), & \frac{N}{2} + 1 \leq k \leq N-1. \end{cases} \qquad \overline{S_H}(k) = \begin{cases} S_H(k), & 0 \leq k \leq \frac{N}{2} - 1, \\ \sqrt{2}\sum_{n=0}^{\frac{N}{2}-1}(-1)^n x_H(n), & k = \frac{N}{2}, \\ S_H(N-k), & \frac{N}{2} + 1 \leq k \leq N-1. \end{cases}$$

Type-II DCT: Sub band Approximation

$$\overline{X_L}(k) = \begin{cases} X_L(k), & 0 \le k \le \frac{N}{2}-1, \\ 0, & k = \frac{N}{2}, \\ -X_L(N-k), & \frac{N}{2}+1 \le k \le N-1 \end{cases}$$

- Sub-band approximation:

$$X(k) = \begin{cases} \sqrt{2}\cos(\frac{\pi k}{2N})\overline{X_L}(k), & k \in \{0, 1, \ldots\ldots, \frac{N}{2}-1\}, \\ 0, & \text{otherwise.} \end{cases}$$

- Low-pass truncated approximation:

$$X(k) = \begin{cases} \sqrt{2}\cdot\overline{X_L}(k), & k \in \{0, 1, \ldots\ldots, \frac{N}{2}-1\}. \\ 0, & \text{otherwise.} \end{cases}$$

Provide conversion from N-point DCT to N/2-point DCT and vice versa.

Block composition and decomposition

$$X^{(N)} = C_N \mathbf{x}$$

- To convert M adjacent N-point DCT blocks to a single MxN-point DCT block.

$$X^{(MN)} = A_{(M,N)}[X_0^{(N)} X_1^{(N)} \ldots X_{M-1}^{(N)}]^T,$$

Inversion decomposes a large block.

$$A_{(M,N)} = C_{MN} \begin{bmatrix} C_N^{-1} & 0_N & 0_N & \cdots & 0_N & 0_N \\ 0_N & C_N^{-1} & 0_N & \cdots & 0_N & 0_N \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_N & 0_N & 0_N & \cdots & C_N^{-1} & 0_N \\ 0_N & 0_N & 0_N & \cdots & 0_N & C_N^{-1} \end{bmatrix},$$

NxN zero matrix

A typical example

- Convert two 4-point DCT block to a single 8-point block

$$
\begin{aligned}
A_{(2,4)} &= C_8 . \begin{bmatrix} C_4^{-1} & 0_4 \\ 0_4 & C_4^{-1} \end{bmatrix} \\
&= \begin{bmatrix}
0.7071 & 0 & 0 & 0 & 0.7071 & 0 & 0 & 0 \\
0.6407 & 0.294 & -0.0528 & 0.0162 & -0.6407 & 0.294 & 0.0528 & 0.0162 \\
0 & 0.7071 & 0 & 0 & 0 & 0.7071 & 0 & 0 \\
-0.225 & 0.5594 & 0.3629 & -0.0690 & 0.225 & 0.5594 & -0.3629 & -0.069 \\
0 & 0 & 0.7071 & 0 & 0 & 0 & 0.7071 & 0 \\
0.1503 & -0.2492 & 0.5432 & 0.3468 & -0.1503 & -0.2492 & -0.5432 & 0.3468 \\
0 & 0 & 0 & 0.7071 & 0 & 0 & 0 & -0.7071 \\
-0.1274 & 0.1964 & -0.2654 & 0.6122 & 0.1274 & 0.1964 & 0.2654 & 0.6122
\end{bmatrix} .
\end{aligned}
$$

$$
X^{(8)} = A_{(2,4)} \begin{bmatrix} X_1^{(4)} \\ X_2^{(4)} \end{bmatrix} \qquad\qquad A_{(2,4)}^{-1} \begin{bmatrix} X_1^{(4)} \\ X_2^{(4)} \end{bmatrix} \rightarrow X^{(8)}
$$

$$f(x,y)=\sum_j \sum_i \lambda_{ij} b_{ij}(x,y)$$

2-D Transforms

- Easily extendable if basis functions are separable, i.e. $B=\{\, b_{ij}(x,y)=g_i(x).g_j(y)\}$.

They could be from two different sets, say $b(x,y)=g(x).h(y)$.

1-D basis function

- B: Orthogonal if $G=\{g_i(x),\ i=1,2,..\}$ is orthogonal.
- B: Orthogonal and complete if G is so.
- Reuse of 1-D transform computation.

$$\lambda_{ij}=\sum_j g_j^*(y)\left(\sum_i f(x,y)g_i^*(x)\right)$$

2D Discrete Transform

$$Y_{mxn} = B_{mxm} X_{mxn} B_{nxn}^{\mathrm{T}}$$

- Use of separability:
  - Transform columns.
  - Transform rows.
- Input: X_{mxn} 1-D Transform Matrix: *B*
- Transform columns: $[Y_1]_{mxn} = B_{mxm} X_{mxn}$
- Transform rows: $Y_{mxn} = [B_{nxn} Y_1^{\mathrm{T}}]^{\mathrm{T}}$

$$= Y_1 B_{nxn}^{\mathrm{T}}$$

$$= B_{mxm} X_{mxn} B_{nxn}^{\mathrm{T}}$$

2D DCT

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}}, & \text{for } p = 0 \text{ or } N, \\ 1, & \text{otherwise.} \end{cases}$$

- Type-I:

$$X_I(k,l) = \frac{2}{N}.\alpha^2(k).\alpha^2(l).\sum_{m=0}^{M}\sum_{n=0}^{N}\left(x(m,n)\cos\left(\frac{m\pi k}{M}\right)\cos\left(\frac{n\pi l}{N}\right)\right), \quad 0 \le k \le M, 0 \le l \le N.$$

- Type-II

$$X_{II}(k,l) = \frac{2}{N}.\alpha(k).\alpha(l).\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left(x(m,n)\cos\left(\frac{(2m+1)\pi k}{2M}\right)\cos\left(\frac{(2n+1)\pi l}{2N}\right)\right), \quad 0 \le k \le M-1, 0 \le l \le N-1.$$

- Matrix Representation:

$$X = DCT(x) = C_M.\mathbf{x}.C_N^T$$

2D DCT: Sub-band relation

$$x_{LL}(m,n) = \tfrac{1}{4}\{x(2m,2n) + x(2m+1,2n) + x(2m,2n+1) + x(2m+1,2n+1)\},\ 0 \le m,n \le \tfrac{N}{2} - 1.$$

Sub-band approximation:

2D DCT of $x_{LL}(m,n)$

$$X(k,l) = \begin{cases} 2cos(\frac{\pi k}{2N})\cos(\frac{\pi l}{2N})\overline{X_{LL}}(k,l), & k,l = 0,1,\ldots\ldots,\frac{N}{2}-1 \\ 0, & \text{otherwise.} \end{cases}$$

Low-pass truncated approximation:

$$X(k,l) = \begin{cases} 2\overline{X_{LL}}(k,l), & k,l = 0,1,\ldots\ldots,\frac{N}{2}-1 \\ 0, & \text{otherwise.} \end{cases}$$

2D DCT: Block composition and decomposition

$$X^{(LN\times MN)} = A_{(L,N)} \begin{bmatrix} X_{0,0}^{(N\times N)} & X_{0,1}^{(N\times N)} & \cdots & X_{0,M-1}^{(N\times N)} \\ X_{1,0}^{(N\times N)} & X_{1,1}^{(N\times N)} & \cdots & X_{1,M-1}^{(N\times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N\times N)} & X_{L-1,1}^{(N\times N)} & \cdots & X_{L-1,M-1}^{(N\times N)} \end{bmatrix} A_{(M,N)}^{T}$$

$$\begin{bmatrix} X_{0,0}^{(N\times N)} & X_{0,1}^{(N\times N)} & \cdots & X_{0,M-1}^{(N\times N)} \\ X_{1,0}^{(N\times N)} & X_{1,1}^{(N\times N)} & \cdots & X_{1,M-1}^{(N\times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N\times N)} & X_{L-1,1}^{(N\times N)} & \cdots & X_{L-1,M-1}^{(N\times N)} \end{bmatrix} = A_{(L,N)}^{-1} X^{(LN\times MN)} A_{(M,N)}^{-1^T}$$

2D DCT: CMP

Circular convolution with
respective symmetric extensions.

$$C_{2e}\{x(m,n) \maltese h(m,n)\} = C_{2e}\{x(m,n)\}C_{1e}\{h(m,n)\}$$

$$C_{1e}\{x(m,n) \maltese h(m,n)\} = C_{2e}\{x(m,n)\}C_{2e}\{h(m,n)\}$$

Thank you!