1. Let R be the set of real numbers. Determine all functions $f : R \to R$ such that, for all real numbers z and y,

$$f(f(x)f(y)) + f(x+y) = f(xy) \tag{1}$$

.

2. A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, Ag, and the hunter's starting point, Bo, are the same. After $n-1$ rounds of the game, the rabbit is at point $An-$ and the hunter is at point $B-1$. In the nth round of the game, three things occur in order. (i) The rabbit moves invisibly to a point A, such that the distance between $An-1$ and A,, is exactly 1. (ii) A tracking device reports a point P, to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P and A, is at most 1. (iii) The hunter moves visibly to a point B, such that the distance between $Bu-1$ and Bn is exactly 1. Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10 rounds she can ensure that the distance between her and the rabbit is at most 1002

   (i) The rabbit moves invisibly to a point An such that the distance between $An-1$ and An is exactly 1.
   (ii) A tracking device reports a point Pa to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P, and An, is at most 1.
   (iii) The hunter moves visibly to a point B, such that the distance between $B-1$ and B, is exactly 1.

   Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10 rounds she can ensure that the distance between her and the rabbit is at most 100?

3. Let Rand S be different points on a circle and such that RS is not a diameter. Let E be the tangent line to 2 at R. Point T is such that S is the midpoint of the line segment RT. Point J is chosen on the shorter are RS of Q so that the circumcircle I of triangle JST intersects (at two distinct points. Let A

be the common point of I and that is closer to R. Line AJ meets again at K. Prove that the line KT is tangent to γ.

4. An integer $N \leq 2$ is given. A collection of $N(N + 1)$ soccer players, no two of whom are of the same height, stand in a row. Sir Alex wants to remove $N(N - 1)$ players from this row leaving a new row of $2N$ players in which the following V conditions hold

   (1) no one stands between the two tallest players,

   (2) no one stands between the third and fourth tallest players

   (3) no one stands between the two shortest players.

   Show that this is always possible.

5. An ordered pair (x, y) of integers is a primitive point if the greatest common divisor of r and y is 1. Given a finite set S of primitive points, prove that there exist a positive integer n and integers ao, 41, 4 such that, for each (x, y)4 in S, we have

$$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots + a_{n-1}xy^{n-1} + a_ny^n = 1. \qquad (2)$$