GEETHANJALI COLLEGE OF ENGINEERING & TECHNOLOGY
(AUTONOMOUS)

CHEERYAL (V), KEESARA (M), MEDCHAL (Dist.), TELANGANA - 501 301

(Accredited by NBA, Approved by AICTE, New Delhi and Permanently Affiliated to JNTUH)

Accredited with "A" Grade NAAC

DEPARTMENT OF FRESHMAN ENGINEERING

www.geethanjaliinstitutions.com

2020-2021

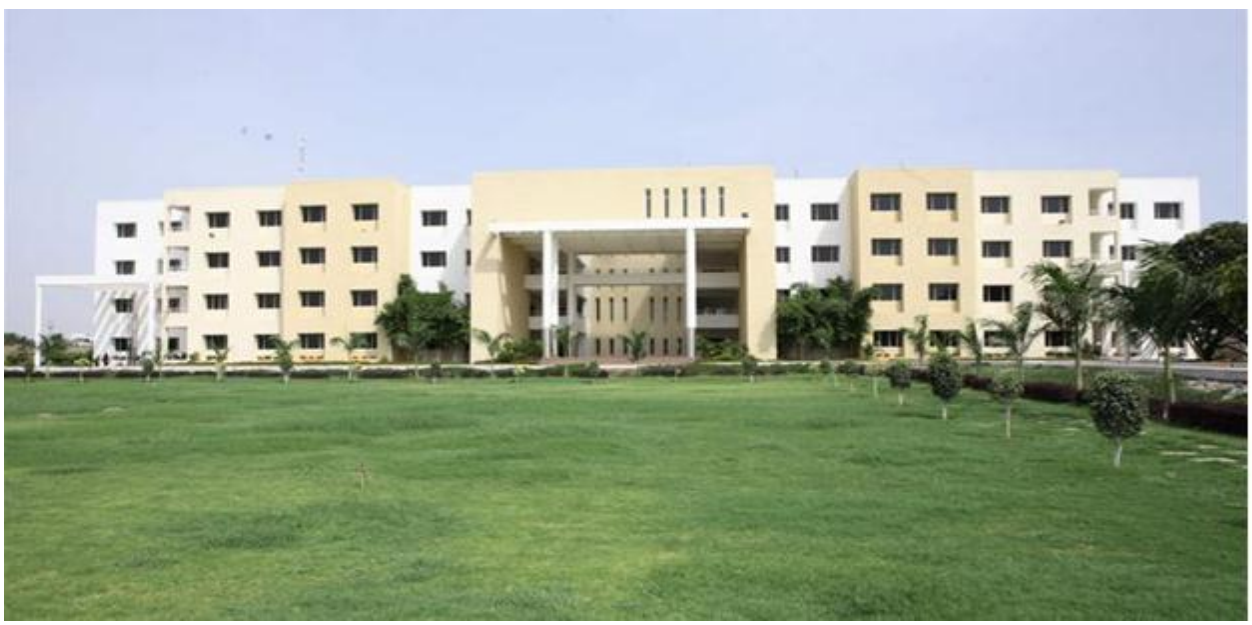
COURSE FILE (IT)

MULTI VARIABLE CALCULUS

(Subject Code: 20MA12001)

B.Tech I/II

Prepared by **Faculty of Mathematics**



GEETHANJALI COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF FRESHMAN ENGINEERING

Name of the Subject: MULTI VARIABLE CALCULUS

Subject Code: 20MA12001 **Programme: UG**

Branch: IT **Version No: 1**

Year: First Year **Updated on:26-03-2021**

Semester: II

Classification status (Unrestricted / Restricted)

Distribution List:

Prepared by: 1) Name : Faculty of Mathematics

2) Design : Assoc. Professor, Asst. Professors

3) Sign :

4) Date : 26-03-2021

Verified by : 1) Name :Dr. V. S. Triveni

2) Sign :

3) Design : Professor

4) Date :

*** <u>For Q.C Only.</u>**

1) Name :

2) Sign :

3) Design :

4) Date :

Approved by : (HOD) 1) Name :

2) Sign :

3) Date :

1. Syllabus copy of the course
2. Vision of the Department
3. Mission of the Department
4. PEOs, POs and PSOs
5. Course objectives and outcomes
6. Course mapping with POs and PSOs
7. Brief notes on the importance of the course, namely, course purpose:
   a. What role does this course play within the Program?
   b. How is the course unique or different from other courses of the Program?
   c. What essential knowledge or skills should they gain from this experience?
   d. What knowledge or skills from this course will students need to have mastered to perform well in future classes or later (Higher Education / Jobs)?
   e. Why is this course important for students to take?
   f. What is/are the prerequisite(s) for this course?
   g. When students complete this course, what do they need know or be able to do?
   h. Is there specific knowledge that the students will need to know in the future?
   i. Are there certain practical or professional skills that students will need to apply in the future?
   j. Five years from now, what do you hope students will remember from this course?
   k. What is it about this course that makes it unique or special?
   l. Why does the program offer this course?
   m. Why can't this course be "covered" as a sub-section of another course?
   n. What unique contributions to students' learning experience does this course make?
   o. What is the value of taking this course? How exactly does it enrich the program?

The "Course Purpose" describes how the course fits into the student's educational experience and curriculum in the program and how it helps in his/her professional career.

8. Prerequisites, if any
9. Instructional Learning Outcomes which refer to
   - Specific knowledge
   - Practical skills
   - Areas of professional development
   - Attitudes
   - Higher-order thinking skills, etc.

   which faculty members expect students to develop, learn, or master during a course.
10. Class Time Table
11. Individual Time Table
12. Lecture schedule with methodology being used/adopted for each session
13. Detailed notes
14. Additional topics
15. University Question papers of previous years
16. Question Bank
17. Assignment Questions
18. Unit wise Quiz Questions and long answer questions
19. Tutorial problems
20. Known gaps, if any and inclusion of the same in lecture schedule
21. Discussion topics, if any
22. References, Journals, websites and E-links if any
23. Quality Measurement Sheets
    a. Course End Survey
    b. Teaching Evaluation
24. Student List of the section
25. Group-wise students list for discussion topics, if any

1. **Syllabus:**

GEETHANJALI COLLEGE OF ENGINEERING AND TECHNOLOGY
(Autonomous)
Cheeryal (V), Keesara (M), Medchal Dist. -501 301, Telangana State
20MA12001 –MULTI VARIABLE CALCULUS (MATHEMATICS-II)
(CSE, CSE(DS), CSE(CS), CSE(AI&ML), CSE(IOT), IT, ECE, EEE, ME & CE)

I Year B.Tech. II Sem.
Prerequisite(s): 20MA11001-Basic Engineering Mathematics (Mathematics-I)

L	T	P/D	C
3	1	-/-	4

Course Objectives: Develop ability to

1. Compute partial derivatives, composite functions of several variables and apply the methods of differential calculus to optimize multivariable functions and evaluate improper integrals using Beta and Gamma functions.
2. Evaluate definite integrals to calculate surface and volume of revolutions of curves, multiple integrals and apply the same to solve engineering problems.
3. Explain properties of vector operators. To determine solenoidal/irrotational vectors, directional derivatives of vectors.
4. Determine the length of a curve, area between the surfaces and volumes of solids using vector integration.
5. Solve partial differential equations using method of separation of variables and their applications to solve heat and wave equations.

Course Outcomes (COs): At the end of course, the student would be able to

CO1: Apply the method of Lagrange Multipliers to solve such constrained optimization problems, evaluate improper integrals,
CO2: Compute surface areas and volumes of revolutions of curves using definite integrals, multiple (Double and Triple) integrals and apply the concepts of same to find the areas and volumes
CO3: Calculate scalar potential for a vector and directional derivative of a scalar point function.
CO4: **C**ompute length of a curve, area between the surfaces and volumes of solids using vector integrations.
CO5: Apply method of separation of variables to solve problems like one dimensional wave and heat equations that arise in engineering branches.

UNIT-I: Partial Differentiation, applications and Beta, Gamma Functions

Definitions of Limit and Continuity, Partial Differentiation, Euler’s Theorem, Total derivative, Jacobian, Functional dependence and independence, *Maxima and Minima of functions of two variables and three variables using method of Lagrange multiplier.

Improper Integrals: Beta and Gamma functions and their applications.

UNIT-II:Multiple Integrals and Applications of Integration

Applications of definite integrals to evaluate surface areas and volumes of revolutions of curves (Only in Cartesian coordinates).

Evaluation of Double Integrals (Cartesian and polar coordinates),change of order of integration (only Cartesian form).

Evaluation of Triple Integrals, change of variables (Cartesian to polar) for double integrals, (Cartesian to Spherical and Cylindrical polar coordinates) for triple integrals.

Applications: Areas (by double integrals) and volumes (by double integrals and triple integrals).

Unit-III:Vector Differentiation

Vector point functions and Scalar point functions, Gradient, Divergence and Curl, Directional derivatives, Tangent plane and Normal line, Vector Identities, Scalar potential function, Solenoidal and Irrotational vectors.

UNIT-IV:Vector Integration

Line, Surface and Volume Integrals. Fundamental theorems of Vector Integration: Green’s Theorem, Gauss divergence Theorem and Stokes Theorem (without proofs).

UNIT-V:Partial Differential Equations

Introduction and formation of Partial Differential Equations by elimination of arbitrary constants and arbitrary functions, solutions of first order Linear (Lagrangian) equation, Method of separation of variables for second order equations. Applications of Partial Differential Equations: One dimensional Wave equation, One dimensional Heat equation.

***Enlightenment with flowchart and algorithmic approach.

TEXT BOOKS:

1. Higher Engineering Mathematics, B.S. Grewal, Khanna Publishers, 44^{th} Edition, 2017.
2. Advanced Engineering Mathematics, Erwin Kreyszig, John Wiley & Sons, 10^{th} Edition, 2011.

REFERENCE BOOKS:

1. A Text book of Engineering Mathematics, N.P. Bali and Manish Goyal, Laxmi Publications, 10^{th} Edition, 2015.

2. Advanced Engineering Mathematics, H.K. Das, S. Chand and Company Ltd, 21st Edition, 2013.
3. Advanced Engineering Mathematics, Dr. A. B. Mathur and Prof. V.P. Jaggi, Khanna Publishers, 6th Edition, 2019.
4. Advanced Engineering Mathematics, R.K. Jain and S.R.K. Iyengar, Alpha Science International Limited, 4th Edition,2013.

2. Vision of the Department:

The department of Information Technology endeavors to bring out technically competent, socially responsible technocrats through continuous improvement in teaching learning processes and innovative research practices.

3. Mission of the Department:

1. Inculcate into the students, the technical and problem solving skills to ensure their success in their chosen profession.
2. Impart the essential skills like teamwork, lifelong learning etc. to the students which make them globally acceptable technocrats.
3. Facilitate the students with strong fundamentals in basic sciences, mathematics and Information Technology areas to keep pace with the growing challenges in field.
4. Enrich the faculty with knowledge in the frontiers of the Information Technology area.

4. Program Educational Objectives-PEOs:

PEO 1: To provide graduates with a good foundation in mathematics, sciences and engineering fundamentals required to solve engineering problems that will facilitate them to find employment in industry and / or to pursue postgraduate studies with an appreciation for lifelong learning.

PEO 2: To provide graduates with analytical and problem solving skills to design algorithms, other hardware / software systems, and inculcate professional ethics, inter-personal skills to work in a multi-cultural team.

PEO 3: To facilitate graduates to get familiarized with the art software / hardware tools, imbibing creativity and innovation that would enable them to develop cutting-edge technologies of multi-disciplinary nature for societal development.

Program Outcomes (POs):

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Specific Outcomes (PSOs):

PSO1: Demonstrate competency in Programming and problem solving skills and apply those skills in solving computing problems

PSO2: Select appropriate programming languages, Data structures and algorithms in combination with modern technologies and apply them in developing innovative solutions .
PSO3: Apply techniques of data modeling, analysis and visualization which include statistical techniques to solve real world problems delivering actionable insights for decision making.

5. Course Objectives: Develop ability to

1. Compute partial derivatives, composite functions of several variables and apply the methods of differential calculus to optimize multivariable functions and evaluate improper integrals using Beta and Gamma functions.
2. Evaluate definite integrals to calculate surface and volume of revolutions of curves,multiple integrals and apply the same to solve engineering problems.
3. Explain properties of vector operators. To determine solenoidal/irrotational vectors, directional derivatives of vectors.
4. Determine the length of a curve, area between the surfaces and volumes of solids using vector integration.
5. Solve partial differential equations using method of separation of variables and their applications to solve heat and wave equations.

Course Outcomes (COs):

At the end of course, the student would be able to

CO1. Apply the method of Lagrange Multipliers to solve such constrained optimization problems, evaluate improper integrals,

CO2. Compute surface areas and volumes of revolutions of curves using definite integrals, multiple (Double and Triple) integrals and apply the concepts of same to find the areas and volumes

CO3. Calculate scalar potential for a vector and directional derivative of a scalar point function.

CO4. **C**ompute length of a curve, area between the surfaces and volumes of solids using vector integrations.

CO5. Apply method of separation of variables to solve problems like one dimensional wave and heat equations that arise in engineering branches.

6. Mapping of Course Outcomes with POs and PSOs:

CO-PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	3	2	-	-	2	-	-	-	-	-	3	2	-	-
CO2	2	2	2	-	-	2	-	-	-	-	-	2	2	-	-

CO3	2	2	2	-	-	-	-	-	-	-	-	2	2	-	-
CO4	3	3	2	-	-	-	-	-	-	-	-	2	2	-	-
CO5	2	2	2	-	-	2	-	-	-	-	-	2	2	-	-

-	No Correlation	1	Slight(low)	2	Moderate(medium)	3	Substantial (High)

7. Brief note on the importance of the Course:

The aim of this course is to impart basic knowledge of Mathematics and its further applications in various disciplines of Engineering and Technology. This course will also help in logical development of mind.

By making use of critical analysis in mathematics students would sharpen their reasoning and intellectual abilities. This would certainly help in problem solving situations in real life.

The course gives an opportunity to interweave physical interpretation and engineering applications, hence stirring the student's imagination and motivation and reducing the separation between theory and practice. This would certainly help in problem solving situations in real life. A study of Mathematics is highly useful to engineering students as it gives a foundation of all basic principles, which are more essential in their core subjects.

This course gives the fundamental concepts related to Partial differential equations, vector calculus and integral calculus which are very useful for the understanding of engineering applications. The introduction of vector field theory in the second semester enables the knowledge of the student to use the concepts of applied science courses in fluid and solid mechanics, transport phenomena, and electromagnetism, etc.in the coming second year, third year.

Application of partial differential equations: To solve problems based on heat equation, wave equation in engineering branches such as Electrical, Electronic Communication, Mechanical and Computer Science branches.

a) What role does this course play within the programme?

This course plays a major role in the program, as it provides mathematical formation for the physical interpretation to find solution. Students acquire knowledge to find solution to a problem using appropriate method.

The study of Partial differential Equations, Vector calculus and Multivariable calculus are very useful and have many applications in Engineering.

b) How this course is unique or different from other courses of the programme.

This course gives all the basic concepts which are useful to formulate and solve and analyze engineering problems.

c) What essential knowledge or skills should they gain from this experience?

Students acquire knowledge in mathematics and their applications, apply them to solve engineering problems and analyze the solution.

d) What knowledge or skills from this course will students need to have mastered to

perform well in future classes or later (Higher education /Jobs)
An ability to analyze and interpret the physical data into mathematical model.

e) **Why is this course important for student to take?**
This course provides systematic methods to formulate and solve engineering problems. Without this course student cannot solve the problems which occur in engineering programs.

f) **What is / or the prerequisite(s) for this course**
The prerequisites for this course are the basics of differential, integral calculus and vector field theory.

g) **When students complete this course what do they need to know or be able to do?**
The students can design models for the specific purpose and interpret the results.

h) **Is there specific knowledge that the students will need to know in the future.**
The students need to learn some theoretical aspects of mathematics so that they can develop applications and upgrade their knowledge and technical skills.

i) **Are there certain practical or professional skills that students will need to apply in the future?**
Students need to apply their scientific, mathematical knowledge and problems solving skills in their profession.

j) **Five years from now, what do you hope students will remember from this course?**
We hope the students utilize the concepts mathematics by acquiring practical knowledge and skills to design and build structures, machines, devices, systems, materials and processes that safely serve as solutions to the needs of society.

k) **What is it about this course that makes it unique or special?**
This course is specifically designed as per the needs of engineering students to learn problem solving methods and their applications.

l) **Why does the program offer this course?**
The program offers this course as this course is very essential for various disciplines of Engineering and Technology.

m) **Why can't this course be covered as a sub section of another course?**
As this course deals with topics like integral, multi variable calculus and vector calculus including partial differential equations, it cannot be messed up as a sub section of another course.

n) What unique contribution to students learning experience does this course make?

The unique contributions to the students are that each topic is designed in such a way that the undergraduate learns problem solving skills using different techniques.

o) What is the value of taking this course?

This course helps in improving problem solving and logical thinking skills. This enriches the student to develop Lifelong Learning skills and attitudes.

UNIT-I: Partial Differentiation, applications and Beta, Gamma Functions

Importance: In mathematical analysis, and applications in geometry, appliedmathematics engineering, natural sciences, and economics, a function of several real variables or real multivariate function is a function with more than one argument, with all arguments being real variables. This concept extends the idea of a function of a real variable to several variables. The "input" variables take real values, while the "output", also called the "value of the function", may be real or complex.

To define special functions known as Beta and Gamma functions to evaluate some integral problems.

***Applications*:**Partial differentiation is a great tool when the function has several independent variables. The application of Jacobian is to find whether the given functions are dependent or independent and Improper integrals are very common in probability and statistics; also, the Laplace transform, the Fourier transform and many special functions like Beta and Gamma are defined using improper integrals, which appear in a lot of problems and computations.

UNIT-II: Multiple Integrals and Applications of Integration

***Importance*:** Multiple integrals are applicable to find surface area and volume in a region.

***Applications*:**

- In electromagnetism, Maxwell's equations can be written using multiple integrals to calculate the total magnetic and electric fields.
- In mechanics, the moment of inertia is calculated as the volume integral (triple integral) of the density weighed with the square of the distance from the axis
- Now the applications of multiple integrals in mechanical engineering are the basic applications of them i.e. to find areas and volumes of various bodies just by taking a little part of them into consideration and also in calculating the mass.
- And this is applicable in various fields like while preparing a machine, or the parts to be fitted in any machine its size and volume etc. are very important.

Unit-III: Vector Differentiation

***Importance*:** Vector calculus (or vector analysis) is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields and fluid flow.

***Applications*:** To apply vectors in higher dimensional space in experimental data, storage and warehousing, electrical circuits, graphical images, economics, mechanical systems**.**

Unit-IV: Vector Integration

Importance: Vector calculus (or vector analysis) is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields and fluid flow.
Applications: To apply vectors in higher dimensional space in experimental data, storage and warehousing, electrical circuits, graphical images, economics, mechanical systems**.**

UNIT-V: Partial Differential Equations

Importance: Partial Differential Equations are used to formulate problems involving functions of several variables, and are either solved by hand, or used to create a relevant computer model.
Applications:Used to describe a wide variety of phenomena such as sound, heat, electrostatics, Electrodynamics, fluid dynamics, elasticity or quantum mechanics.

8. Prerequisite(s):

Basic Engineering Mathematics (20MA11001)

9. Instructional Learning Outcomes:

- **Specific knowledge**

  Students acquire the specific knowledge on applications of mathematics to real world problems. In the field of computing, Partial differential equation are used in engineering problems. The course is used in the study of quantum mechanics, vibration analysis, control theory etc. The Gamma function is applicable in quantum physics, fluid dynamics. Partial differential equations are useful to formulate physical problems in terms of differential equations and solve them choosing the method appropriate for specific problem.

- **Practical skills**

  The students can apply mathematical techniques to design mathematical models to real world problems and interpret the results.
  Students need to apply their scientific, mathematical and practical knowledge skills to design and build structures, machines, devices, systems, materials and processes that safely serve as solutions to the needs of society.

- **Areas of professional development**

  Ability to identify a problem, formulates, solve and analyze its solution, which are very necessary for the professional development.

- **Attitudes**

The course helps the student to improve logical thinking and provides a tool to learn latest technologies.

- **Higher-order thinking skills etc.**

  Higher Order thinking skills like Creativity, Designing, Analysis, Data interpretation, Problem solving, Optimal Utilisation of the knowledge gained etc., expected from the students.

UNIT-I: Partial Differentiation, applications and Beta, Gamma functions

Students that successfully complete this course will be able to:

- Evaluate maximum and minimum of functions which consists more than one independent variables
- Apply the concept of partial differentiation to functions having more than one variable.
- Evaluate definite Improper integrals using Beta and Gamma functions

UNIT-II: Multiple Integrals and Applications of Integration

Students that successfully complete this course will be able to:

- Transforming line integrals, double and triple integrals into one another in solving mathematical models of engineering applications.

UNIT-III:Vector Differentiation

Students that successfully complete this course will be able to:

- Knowledge of the Differential operators like divergence, gradient and curl.
- Calculate angle between the vectors, directional derivative of the vectors.

UNIT – IV:Vector Integration

Students that successfully complete this course will be able to:

- Evaluate line, surface and volume integrals.
- Apply Gauss Divergence, Green’s and Stoke’s theorems to find surface and volume of the given region.

UNIT –V: Partial Differential Equations

Students that successfully complete this course will be able to:

- Form Partial Differential Equations by eliminating arbitrary constants and functions.
- Solve Linear Partial Differential Equations using Lagrange’s method.
- Solve second order Partial Differential Equations using method of separation of variables.
- Apply Partial Differential Equations in one dimensional Wave equation.
- Apply Partial Differential Equations in one dimensional Heat equation.

10. Class Time-tables:

<table>
<tr><td colspan="6">Geethanjali College of Engineering and Technology</td></tr>
<tr><td colspan="6">Department of Freshman Engineering</td></tr>
<tr><td colspan="6">Time Table (Online Classes)</td></tr>
<tr><td colspan="4">Year/Sem/Branch & Sec: I-B.Tech II-Semester, IT</td><td colspan="2">Acad. Year : 2020 -21 W.E.F 30-03-2021</td></tr>
<tr><td colspan="4">Class Incharge: Ms. Fathima</td><td>Version-1</td><td></td></tr>
<tr><td>Time</td><td>09.00-10.30</td><td></td><td>11.00-12.30</td><td>12.30-1.30</td><td>1.30-3.00</td></tr>
<tr><td>Period</td><td>1</td><td rowspan="7">BREAK</td><td>2</td><td rowspan="7">LUNCH</td><td>3</td></tr>
<tr><td>Monday</td><td>PPS-II</td><td>SD</td><td>EG</td></tr>
<tr><td>Tuesday</td><td>MVC</td><td>DM</td><td>SD lab</td></tr>
<tr><td>Wednesday</td><td>SD</td><td>BEE</td><td>PPS-II Lab</td></tr>
<tr><td>Thursday</td><td>DM</td><td>MVC</td><td>BEE lab</td></tr>
<tr><td>Friday</td><td>BEE</td><td>PPS-II</td><td>EG</td></tr>
<tr><td>Saturday</td><td>–</td><td>–</td><td>–</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td>S.No</td><td colspan="3">Subject(T/P)</td><td>Course Code</td><td>Faculty Name</td></tr>
<tr><td>1</td><td colspan="3">Semiconductor Devices(SD)</td><td>20PH12001</td><td>Dr. P. Raju</td></tr>
<tr><td>2</td><td colspan="3">Multi Variable Calculus(MVC)</td><td>20MA12001</td><td>Dr. Mahesh</td></tr>
<tr><td>3</td><td colspan="3">Programming for Problem Solving-II(PPS-II)</td><td>20CS12001</td><td>Ms.Fathima</td></tr>
<tr><td>4</td><td colspan="3">Basic Electrical Engineering(BEE)</td><td>20EE12001</td><td>Mr. K. Jayakar Babu</td></tr>
<tr><td>5</td><td colspan="3">Engineering Graphics(EG)</td><td>20ME12002</td><td>Mr.G. Raju, Mr. G. Sampath</td></tr>
<tr><td>6</td><td colspan="3">Discrete Mathematics(DM)</td><td>20CS12002</td><td>Ms. K. Vandhana</td></tr>
<tr><td>7</td><td colspan="3">Semiconductor Devices Lab(SD Lab)</td><td>20PH12L01</td><td>Dr.B. Mamatha, Ms. Ch. Kalyani</td></tr>
<tr><td>8</td><td colspan="3">Programming for Problem Solving-II LAB (PPS-II LAB)</td><td>20CS12L01</td><td>Ms.Fathima</td></tr>
</table>

9	**Basic Electrical Engineering Lab(BEE LAB)**	**20EE12L01**	**Mr. K. Jayakar Babu, Mr. S. Hareesh Reddy**
10	**Mentors**		

11. Individual Time Tables:

Faculty: Dr. G. Mahesh

Time	09.00-10.30		11.00-12.30	12.30-1.30	1.30-3.00
Period	**1**	**BREAK**	**2**	**LUNCH**	**3**
Monday			**CSE-DS**		
Tuesday	**IT**				
Wednesday					
Thursday			**IT**		
Friday	**CSE-DS**				
Saturday					

12. Lecture Schedule:

Period	*Topic to be covered*	*Instructional Learning Outcomes: Student would be able to*	*BTL*	*Mode of teaching*	*Remarks*
	UNIT I: Partial Differentiation, applications and Beta, Gamma functions				
1	Limit, Continuity, Partial Differentiation,	Learn basic concepts	1	BB/PPT/ Clip board	
2	Total Derivatives, Functions of several variables	Apply the concept of partial differentiation to functions having more than one variable.	3	BB/PPT/ Clip board	
3	Functional dependence-Jacobian	Explain the relation between dependent functions	2	BB/PPT/ Clip board	
4	Problems Practice		3	BB/PPT/ Clip board	
5	Maxima and Minima of functions of two variables without constraints	Evaluate maximum and minimum of functions which consists more than one independent variables	4	BB/PPT/ Clip board	
6	Maxima and Minima of functions of two variables with constraints	Evaluate maximum and minimum of functions which consists more than one	4	BB/PPT/ Clip board	

		independent variables			
7	Problems Practice		3	BB/PPT/ Clip board	
8	Method of Lagrange multipliers	Evaluate maximum and minimum of functions which consists more than one independent variables	4	BB/PPT/ Clip board	
9	Beta function and its properties	Learn definitions and properties of Beta functions	1	BB/PPT/ Clip board	
10	Evaluation of improper integrals usingBeta functions	Apply Beta function to evaluate integrals	3	BB/PPT/ Clip board	
11	Gamma function and its properties	Learn definitions and properties of Gamma functions	3	BB/PPT/ Clip board	
12	Relation between Beta and Gamma functions	Apply relation to evaluate integrals	3	BB/PPT/ Clip board	
13	Problems Practice		3	BB/PPT/ Clip board	
14	Evaluation of improper integrals using Gamma functions Relation between Beta and Gamma functions	Apply Beta and Gamma functions to evaluate integrals	3	BB/PPT/ Clip board	
15	Applications of definite integrals to evaluate surface areas and volumes of revolutions of curves	Evaluate surface areas and volumes of revolutions of curves	4	BB/PPT/ Clip board	
	Unit-II Multiple Integrals and Applications of Integration				
16	Multiple integrals – double	Evaluate double integrals	4	BB/PPT/ Clip board	
17	Double integrals in polar coordinates	Evaluate double integrals in polar coordinates	4	BB/PPT/ Clip board	
18	Change of variables	Evaluate double integrals by changing variables	4	BB/PPT/ Clip board	
19	Multiple Integrals-triple	Evaluate Triple integrals	4	BB/PPT/ Clip board	
20	Change of Order of Integration	Evaluate integrals by changing order	4	BB/PPT/ Clip board	
21	Change of variables	Evaluate Triple integrals by changing variables	4	BB/PPT/ Clip board	
22	Areas by double integrals	Apply the concepts to find areas.	3	BB/PPT/ Clip board	
23	Problems Practice		3	BB	
24	Volume by double and Triple integrals	Apply the concepts to find volumes.	3	BB/PPT/ Clip board	
25	Problems Practice		3	BB/PPT/ Clip board	

	Unit-III **Vector Differentiation**				
26	Vector Algebra	Learn basic concepts	1	BB/PPT/ Clip board	
27	Scalar point function and vector point function, Gradient - Directional derivatives	Knowledge of the Differential operators on scalar function	3	BB/PPT/ Clip board	
28	Problems Practice		3	BB/PPT/ Clip board	
29	Divergence - Curl and their related properties	Knowledge of the Differential operators on vector function	3	BB/PPT/ Clip board	
30	Laplacian operator-Solenoidal and irrotational vectors, Scalar Potential function	Acquire the knowledge of identifying the potential function for a given vector function	4	BB/PPT/ Clip board	
31	Problems Practice		3	BB/PPT/ Clip board	
	Unit-IV **Vector Integration**				
32	Line integral – work done	Evaluate line integral	3	BB/PPT/ Clip board	
33	Surface integrals	Evaluate surface integral	4	BB/PPT/ Clip board	
34	Volume integrals	Evaluate volume integral	4	BB/PPT/ Clip board	
35	Green's theorem	Application of vector integrals	4	BB/PPT/ Clip board	
36	Stoke's theorem	Application of vector integrals	4	BB/PPT/ Clip board	
37	Gauss Divergence theorems	Application of vector integrals	4	BB/PPT/ Clip board	
38	Problems Practice		3	BB/PPT/ Clip board	
	Unit-V **Partial Differential Equations**				
39	Introduction and Formation of partial differential equation	Learn the procedure of formation of partial differential equation	3	BB/PPT/ Clip board	
40	Formation of partial differential equation by elimination of arbitrary constants	Learn the formation of partial differential equation by eliminating of arbitrary constants	3	BB/PPT/ Clip board	
41	Formation of partial differential equation by elimination of arbitrary functions	Learn the formation of partial differential equation by eliminating of arbitrary functions	3	BB/PPT/ Clip board	
42	solutions of first order linear (Lagrange) equation	Solve Linear Partial Differential Equations using Lagrange's	4	BB/PPT/ Clip board	

		method			
43	Method of separation of variables for second order equations	Solve Non – Linear Partial Differential Equations using Method of separation of variables	4	BB/PPT/ Clip board	
44	Problems Practice		3	BB/PPT/ Clip board	
45	One dimensional wave equation	Apply Partial Differential Equations in one dimensional wave equations.	4	BB/PPT/ Clip board	
46	One dimensional Heat equation	Apply Partial Differential Equations in one dimensional heat equations.	4	BB/PPT/ Clip board	
47	Practice problems	Solve problems on wave equation	4	BB/PPT/ Clip board	
48	Old Question papers	Review of previous question papers		BB/PPT/ Clip board	

13. Detailed Notes:

- Hard copy available

14. Additional topics:

- Vector Algebra

15. University Question papers of previous year and Model Papers:

AR20

Code No:20MA12001

Geethanjali College of Engineering and Technology (Autonomous), Hyderabad

B.Tech I Year II Semester Examinations, Jun 2021

Course: Multi Variable Calculus

Branch: ECE / EEE / CSE/ CSE (AI&ML, IoT, CS, DS) / ME / CE/ IT

Model Paper-I

Time: 3 hours **Max. Marks: 70**

Part–A (10 x 2 = 20)

1. Answer all Questions

a) If u= e^x sin y, v= e^x cos y. Find $\frac{\partial(u,v)}{\partial(x,y)}$. [2M]

b) Prove that $\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^4\theta d\theta = \pi/32$ [2M]

c) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\, dxdy$ [2M]

d) Over the region bounded by hyperbola $xy = 4$, $y = 0$; $x = 1, x = 4$ evaluate $\iint x^2 \, dx\, dy$ [2M]

e) Show that $\boldsymbol{v} = (2x + 3y)\boldsymbol{i} + (x - y)\boldsymbol{j} - (x + y + z)\boldsymbol{k}$ is Solenoidal. [2M]

f) Find curl$\bar{f}$ where $\bar{f} = grad\,(x^3 + y^3 + z^3 - 3xyz)$. [2M]

g) Evaluate $\int_c \bar{F}.\overline{dr}$, where $\bar{F} = x^2 i + y^3 j$ and the curve c of the arc of the parabola $y = x^2$ in the xy plane from $(0,0)$ to $(1,1)$. [2M]

h) State Stokes theorem. [2M]

i) Form the partial differential equation by eliminating the arbitrary constants a, b from $(x - a)^2 + (y - b)^2 = r^2$. [2M]

j) Solve $px - qy = z$ [2M]

Part–B (10 x 5 = 50)

2. a) If u=f(r, s, t) where $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ [5M]

b) Find three positive numbers whose sum is 100 and whose product is maximum [5M]

(OR)

3. a) Show that $B\left(m, \frac{1}{2}\right) = 2^{2m-1} B(m, m)$ [5M]

b) Show that $\Gamma\left(\frac{1}{2}\right)\Gamma(2n) = 2^{2n-1}\Gamma(n)\Gamma\left(n + \frac{1}{2}\right)$ [5M]

4. a) By changing the order of integration, Evaluate $\int_0^3 \int_1^{\sqrt{(4-y)}} (x + y)\, dx dy$ [5M]

b) Evaluate $\iiint z^2 \, dx\, dy\, dz$ taken over the volume bounded by the surfaces $x^2 + y^2 = a^2, x^2 + y^2 = z$ and z=0 [5M]

(OR)

5. Find the volume of the greatest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ [10M]

6. a) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + 2z^2$ at the point $(1,1,2)$ in the direction of $gradf$. [5M]

b) Find div $\bar{f}$ where $\bar{f} = r^n \bar{r}$ find n if it is Solenoidal? [5M]

(OR)

7. a) Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential. [5M]

b) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $|\bar{\boldsymbol{r}}| = r$. [5M]

8. a) If $\bar{f} = \cos y\, i - x\sin y j$ evaluate $\int_c \bar{f}\,.d\bar{r}$ where C is the curve $y^2 = 1 - x^2$ in the xy-plane from (1,0) to (0,1). [5M]

   b) Evaluate the line integral $\int_c (x^2+xy)dx + (x^2 + y^2)dy$ where c is the square formed by the lines $x = \pm 1$ and $y = \pm 1$. [5M]

(OR)

9. Verify Gauss Divergence theorem for $\overline{F} = x^3 i + y^3 j + z^3 k$ taken over the cubebounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$. [10M]

10. a) Form the partial differential equation by eliminating the arbitrary functions from $z = (x + y)\Phi(x^2 - y^2)$ [5M]

    b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ [5M]

(OR)

11. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x, 0) = 3\sin\pi x$, $u(0, t) = 0, u(1, t) = 0, 0 < x < 1, t > 0$. [10M]

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Geethanjali College of Engineering and Technology

I B.Tech (EEE/CSE/IT) I Semester (Regular/Supplementary) End Examinations, Nov 2020

MATHEMATICS – II

Time: 2 hours

Max. Marks: 70M

5 X 14M = 70M

Answer All Questions

1 Show that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} B(m+1, n+1)$.

OR

2 If $f(x) = \log x$ and $g(x) = x^2$ in [a,b] with b>a>1, using Cauchy's Theorem prove that $\frac{\log b - \log a}{b-a} = \frac{a+b}{2c^2}$.

3 if $u = f(r)$ and $x = r\cos\theta, y = r\sin\theta$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

OR

4 Find the maximum and minimum values of the function $f(x,y) = x^3 y^2 (1-x-y)$.

5 Find the area of the region bounded by the parabolas $y^2 = 4ax \text{ and } x^2 = 4ay$.

OR

6 Find the volume of the greatest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

7 Find the directional derivative of $xyz^2 + xz$ at (1, 1, 1) in a direction of the normal to the surface $3xy^2 + y = z$ at (0, 1, 1).

OR

8 If $\overline{F} = 2xz\bar{i} - x\bar{j} + y^2\bar{k}$ evaluate $\int_V \overline{F}.dv$ where V is the region bounded by the surfaces $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$.

9 Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

OR

10 Derive the general solutions of One Dimensional Heat Equation.

16. Question Bank:

Long answer questions:

Unit-I

1. If $u = f(x, y, z)$ where $r = \frac{x}{y}$, $s = \frac{y}{z}$ and $t = \frac{z}{x}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
2. If $u = f(x - y, y - z, z - x)$ then evaluate $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
3. If $x^x y^y z^z = e$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(xlogex)^{-1}$.
4. If $z = f(x + ay) + \Phi(x - ay)$ show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.
5. If $u = x + y + z$; $y + z = uv$; $z = uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.
6. If $u = x^2 - y^2, v = 2xy \; and \; x = rcos\theta, y = rsin\theta$ then evaluate $J\left(\frac{u,v}{r,\theta}\right)$.
7. If $u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. Hence prove that u and v are functionally dependent. Find the functional relation between them.
8. If $u = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
9. If $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$, then prove that u, v and w are functionally dependent and find the relationship among them.
10. Examine the function for extreme values $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
11. Discuss the maximum and minimum of $f(x, y) = x^3y^2(1 - x - y)$.
12. Evaluate the maximum of $x^2y^3z^4$ subject to the condition $2x + 3y + 4z = a$.
13. Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$ using Lagrange's method of undetermined multipliers.
14. Divide 24 into three parts such that the product of the first, square of the second and cube of the third is maximum.
15. A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.
16. Evaluate $\int_0^{\frac{\pi}{2}} (\sqrt{tan\theta} + \sqrt{sec\theta})\, d\theta$.
17. Prove that $\int_0^1 x^m (logx)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
18. Show that $B\left(m, \frac{1}{2}\right) = 2^{2m-1} B(m, m)$.
19. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where $m > 0, n > 0$.
20. Show that $\Gamma\left(\frac{1}{2}\right)\Gamma(2n) = 2^{2n-1}\Gamma(n)\Gamma\left(n + \frac{1}{2}\right)$.
21. If $m > 0, n > 0$ then prove that $\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$.

22. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function and evaluate $\int_0^1 x^7(1-x^5)^{14}dx$.

23. Show that $2\beta(m,n) = \int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$.

24. Show that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$ and deduce that $\int_0^{\pi/2} sin^n\theta\, d\theta = \int_0^{\pi/2} cos^n\theta\, d\theta = \frac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{n+2}{2}\right)}$.

25. Prove that $\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}}dx \times \int_0^1 \frac{1}{\sqrt{(1+x^4)}}dx = \frac{\pi}{4\sqrt{2}}$.

26. Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$.

Unit-II

1. Evaluate $\int_0^2 \int_0^x e^{x+y}dydx$.
2. Evaluate $\int_0^4 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$.
3. Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\, dxdy$.
4. Evaluate $\iint y\, dx\, dy$ over the region R, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$
5. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$ and show the region of integration.
6. Evaluate $\iint xy(x+y)dx\, dy$ over the region bounded by $y = x^2$ and $y = x$.
7. Evaluate $\iint(x^2+y^2)\, dxdy$ in the positive quadrant for $x + y \leq 1$
8. Evaluate $\iint r^2\sin\theta\ dr\, d\theta$ over the semi-circle $r = 2a\cos\theta$ above the initial line.
9. Evaluate $\int_0^{\pi/4} \int_0^{asin\theta} \frac{r}{\sqrt{a^2-r^2}}drd\theta$.
10. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^\infty \frac{r}{(r^2+a^2)^2}\ dr\, d\theta$.
11. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dydx$ by changing into polar coordinates
12. Evaluate the double integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\, dx\, dy$ by transforming intopolar coordinates.
13. By changing order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\ dx\, dy$.
14. By changing into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\, dx\, dy$ over the region between the circles $x^2+y^2 = a^2$ and $x^2+y^2 = b^2$ where $b > a$.
15. By Changing the order of integration, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2dydx$
16. Find the area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$
17. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Unit-III

1. If $\bar{\boldsymbol{r}} = x\boldsymbol{i} + y\boldsymbol{j} + zk, |\bar{\boldsymbol{r}}| = r$ then show that $gradr^n = nr^{n-2}\bar{r}$.
2. Find the directional derivative of $f(x,y,z) = xy^2 + 4xyz + z^2$ at the point $(1,2,3)$ in the direction of $3i + 4j - 5k$.
3. Find the directional derivative of the given scalar function $f(x,y,z) = xyz$ at the point $(1,4,3)$ in the direction of the line from $(1,2,3)$ to $(1,-1,-3)$. also calculate the magnitude of the maximum directional derivative.
4. Find the directional derivative of $f = 5x^2y - 5y^2z + 2.5z^2x$ at the point $(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$.
5. Find the directional derivative of $f(x,y,z) = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1,1,1,)$.
6. Find the directional derivative of $\nabla.\nabla f$ at the point $(1,-2,1)$ in the direction of normal to the surface $xy^2z = 3x+z^2$ where $f = 2x^3y^2z^4$.
7. If $\bar{r}$ is the position vector of any point in space, then prove that $r^n\bar{r}$ is irrotational.
8. Show that $\nabla f(r) = \frac{f^I(r)}{r}\bar{r}$ where $\bar{r} = xi + yj + zk$.
9. Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential.
10. Find div $\bar{f}$ where $\bar{f} = r^n\bar{r}$ find n if it is Solenoidal?
11. Prove that div (gradr^m) = $m(m+1)r^{m-2}$.
12. Evaluate $\nabla[r\,\nabla(\frac{1}{r^3})]$ = where r= $\sqrt{x^2 + y^2 + z^2}$.
13. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $|\bar{\boldsymbol{r}}| = r$.
14. Let $f(x,y,z)$ be a solution of the Laplacian equation $\nabla^2 f = 0$, then show that ∇f is a vector which is both irrotational and Solenoidal.

UNIT-IV

1. Evaluate $\int_c \bar{F}.\overline{dr}$, where $\bar{F} = x^2i + y^3j$ and the curve c of the arc of the parabola $y = x^2$ in the xy plane from $(0,0)$ to $(1,1)$.
2. If $\bar{F} = yi - xj$, evaluate $\int_c \bar{F}.\overline{dr}$ from $(0,0)$ to $(1,1)$ along the straight lines from $(0,0)$ and $(1,1)$.
3. Find the work done in moving a particle in the force field (a) $\bar{f} = 3x^2$ i +j+z k (b) $\bar{f}$ = s$3x^2$ i +(2xz-y) j+zk along the straight line from $(0,0,0)$ to $(2,1,3)$.
4. If $\bar{f}$ = (5xy-6x^2)i + $(2y - 4x)j$, Evaluate $\int_c \bar{f}$ .d$\bar{r}$ where C is the curve in the xy plane y=x^3 from (1,1) to (2,8).
5. Evaluate the line integral $\int_c(x^2$+xy) dx + $(x^2 + y^2)$dy where c is the square formed by the lines x$\pm$1 and y=$\pm$1.
6. If $\bar{f}$ = $\cos y\, i - xsinyj$ evaluate $\int_c \bar{f}$ .d$\bar{r}$ where C is the curve y^2=1-x^2 in the xy plane from $(1,0)$ to $(0,1)$.

7. Evaluate greens theorem $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$ where C is bounded by $y = x^2$ and $y^2 = x$
8. Verify Gauss divergence theorem for $\bar{F} = (x^3 - yz)\, i - 2x^2yj + zk$ taken over the surface of the cube bounded by the planes $x = y = z = a$ and coordinate planes.
9. Verify Gauss divergence theorem for $\bar{F} = x^3$ i + y^3j + z^3k taken over the surface of the cube bounded by the planes $x = y = z = 0\ and x = y = z = a$.
10. Verify Gauss divergence theorem for $\bar{F} = x^2$ i + y^2j + z^2k taken over the surface of the solid cut off by the plane $x + y + z = a$ in the first octant.
11. Verify Greens theorem in a plane $\oint(3x^2 - 8y^2)\, dx$ +(4y -6xy) dy where C is the region bounded by y = $\sqrt{x}$ and y = x^2
12. Verify Greens theorem for $\int_c[(3x^2 - 8y^2)dx + (4\text{y} - 6\text{xy})dy]$ where C is the region bounded by $x = y = 0\ and x + y = 1$.
13. Verify Greens theorem in a plane $\int_c[(x^2 - xy^3)dx + (y^2 - 2xy)\, dy]$ where C is the square with vertices (0,0), (2,0), (2,2), (0,2).
14. Apply Greens theorem to evaluate $\oint_c(2x^2 - y^2)\, dx + (y^2 + x^2)\, dy$ where c is the boundary of the area enclosed by the x-axis and upper half of the circle $y^2 + x^2 = a^2$.
15. Verify stokes theorem for $\bar{F} = x^2 i + xyj$ integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0\ , y = 0, x = a, y = a$.
16. Verify stokes theorem for $\bar{F}$= (2x-y)i -yz^2 j -y^2zk over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection on xy −plane.

Unit-V

1. Form the partial differential equation by eliminating the arbitrary functions from
   (i) $z = (x + y)\Phi(x^2 - y^2)$. (ii)$f(x^2 + y^2, z - xy) = 0$
2. Form the partial differential equation by eliminating the arbitrary constants from
   (i) $z = alog\left\{\frac{b(y-1)}{1-x}\right\}$ (a,b) (ii)$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (a,b,c)
3. Form the partial differential equation by eliminating the arbitrary functions from
   (i) $F(xy + z^2, x + y + z) = 0$ (ii)$z = f(2x + y) + g(3x - y)$
4. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$
5. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u where u(x, 0) = 6e^{-3x}$
6. Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x, 0) = 4\, e^{-x}$
7. A tightly stretched string with fixed end points x=0 and x=1 is initially in a position given by$y = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find displacement $y(x, t)$
8. An insulator rod of length L has its ends A and B maintained at 0^0C and 100^0 C separately until steady state condition prevails. If B is suddenly reduced to 0^0C, find the temperature at a distance x from A at time t.

9. Solve $\frac{\partial u}{\partial x}$=4$\frac{\partial u}{\partial y}$ given that u(0,y) =8e^{-3u}
10. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$
11. Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions
$u(x,0)=3sin\pi x, u(0,t)=0, u(1,t)=0, 0<x<1, t>0.$
12. A string is stretched and fastened to two points at x=0 and x=L. Motion is started by displacing the string into the form $y=k(lx-x^2)$ from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t.
13. Solve $(mz-ny)p+(nx-lz)q=ly-mx$
14. Solve $(z^2-2yz-y^2)p+(xy+zx)q=xy-zx$

Short Answer Questions (2 marks):

Unit-I

1. Write the conditions of maximum and minimum.
2. Define Functional Dependence and Functional Independence.
3. Define Jacobian of u, v .and write the properties of the Jacobian.
4. If u= e^x sin y, v= e^x cosy. Find $\frac{\partial(u,v),}{\partial(x,y)}$.
5. Discuss the maximum and minimum of $x^2+y^2+6x+12$.
6. Find the maximum value & minimum value of $x+y+z$ subject to the condition

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

7. Decompose positive number into three positive terms such that their product is a maximum.
8. Find the maximum and minimum values of $f(x)=3x^4-2x^3-6x^2+6x+1$.
9. Find the minimum value of $x^2+y^2+z^2$ given that xyz=a^3.
10. Find the first and second order partial derivatives of $log(x^2+y^2)$.
11. If $u=x^2-2y, v=x+y+z, w=x-2y+3z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
12. If x=$\frac{u^2}{v}$,y=$\frac{v^2}{u}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.
13. If u=$\frac{x-y}{x+y}$,v=$\frac{xy}{(x+y)^2}$ find the functional relation between them.
14. If u=log($\frac{x^4+y^4}{x+y}$) then find x$\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.
15. Prove that $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}=\frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$.
16. Show that $\int_0^\infty \frac{x^{m-1}}{(x+a)^{m+n}}dx=a^{-n}B(m,n)$
17. Prove that $\int_0^1 x^{n-1}\left(\log\frac{1}{x}\right)^{\mathrm{m}-1}dx=\frac{\Gamma(\mathrm{m})}{n^m}.m>0,n>0$
18. Show that $\int_0^\infty x^{2n-1}e^{-ax^2}dx=\frac{\Gamma(\mathrm{n})}{2a^n},a>0,n>0$

19. Evaluate $\int_0^\infty \sqrt{x}e^{-x^2}dx$

20. Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$

21. Compute $\Gamma(\frac{11}{2})$

22. Evaluate $\int_0^{\pi/2} sin^5x\, cos^{7/2}xdx.$

23. Prove that $\beta(m,n) = \beta(m+1,n) = \beta(m,n+1)$

24. Define β and Γ functions.

25. Prove that $\beta(m,n) = \beta(n,m)$

26. Show that $\int_a^b (x-a)^m(b-x)^n dx = (b-a)^{m+n+1}\beta(m+1,n+1), m > 0, n > 0.$

27. Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}}dx = \frac{B(m,n)}{a^n(1+a)^m}$

28. Evaluate $\int_0^1 \frac{x^2dx}{\sqrt{1-x^5}}$ and $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Beta function.

29. Show that $\int_{-1}^1 (1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n-1}B(m,n)$

30. Show that $\Gamma(1) = 1$

31. If n is a non-negative integer, then prove that $\Gamma(n+1) = n!$

32. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and $\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$

33. Show that $\Gamma(n) = \int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx, n > 0$

34. Evaluate $\int_0^{\pi/2} \sqrt{cot\theta}\, d\theta$

35. Evaluate $\int_0^\infty a^{-bx^2}dx$ and $\int_0^\infty 3^{-4x^2}dx$

36. Evaluate $\int_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx$

37. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{7}{2}}\theta\cos^{\frac{3}{2}}\theta d\theta$

38. Prove that $\int_0^\infty e^{-y^{1/m}}dy = m\Gamma(m)$

39. Find the value of $\beta\left(\frac{1}{4},\frac{1}{4}\right)$

40. Find the value of $\int_0^\infty e^{-kx}x^{n-1}\,dx$

41. Find the value of $\int_0^\infty e^{-x^2}\,x\,dx$

42. Prove that $\int_{-\infty}^0 e^{-x^2}dx = \int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ and $\int_{-\infty}^\infty e^{-x^2}dx = \sqrt{\pi}$

43. Evaluate $\int_0^2 x(8-x^3)^{\frac{1}{3}}dx.$

44. Evaluate $\int_0^1 x^{5/2}(1-x^2)^{3/2}dx.$

45. Evaluate $\int_0^1 x^4(1-x)^2dx.$

46. Evaluate $\int_0^\infty x^6e^{-2x}dx$ and $\int_0^\infty e^{-4x}x^{3/2}dx.$

47. Find the value of $\Gamma(5/4)\Gamma(3/4).$

Unit-II

1. Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dxdy$
2. Evaluate $\iint_r ydxdy$ where R is the domain bounded by $y-axis$, the curve $y=x^2$ and line$x+y=2$ in first quadrant.
3. Evaluate $\iint r^3 drd\theta$ over the area included between the circles $r=2\,sin\theta$ and

$$r=4\sin\theta$$

4. Show that $\int_0^{4a} \int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2} dxdy = 8a^2\left(\frac{\pi}{2}-\frac{5}{3}\right)$
5. By using the transformation $x+y=u,\ y=uv$ show that

$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}}\,dydx = \frac{1}{2}\,(e-1)$

6. Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dydx$
7. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the double integral.
8. By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{(4-y)}} (x+y)\,dxdy$
9. Change the order of integration and evaluate $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} f\,(x,y)\,dxdy$
10. By changing the order of integration, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y\sqrt{x^2+y^2}\,dydx$
11. Evaluate $I=\int_2^1 \int_1^2 \int_2^3 xyzdxdydz$
12. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdz\,dydx$
13. Evaluate $\int_0^1 \int_y^1 \int_0^{1-x} xdzdxdy$
14. Find the area enclosed by the parabolas $x^2=yandy^2=x$
15. Using double integration determine the area of the region bounded by curves

$y^2=4ax, x+y=3aandy=0$

16. Evaluate $\iiint z^2 dxdydz$ taken over the volume bounded by surface

    $x^2+y^2=zandz=0$

17. Using double integration determine the area of the region bounded by curves

    $y^2=4ax, x+y=3aandy=0$

Unit-III

1. Find the gradient of the function $f(x,y,z)=\ x^2y^2+xy^2-z^2$ at (3,1,1).
2. Define Solenoidal vector and give an example.
3. Define irrotational vector and give an example.
4. Find the divergence of the vector field $\boldsymbol{v}=(x^2y^2-z^3)\boldsymbol{i}+2xyz\boldsymbol{j}+e^{xyz}\boldsymbol{k}$.
5. Find the unit normal vector to the given surface $y^2=16x$ at the point (4,8).
6. Show that $\boldsymbol{v}=e^{x+y-2z}(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$ is Solenoidal.
7. Show that $\boldsymbol{v}=(3x^2y^2z^4)\boldsymbol{i}+2x^3yz^4\boldsymbol{j}+(4x^3y^2z^3)\boldsymbol{k}$is irrotational.

8. Find$div v = xe^{-y}i + 2ze^{-y}j + xy^2k$.
9. Find$curl v = xe^{-y}i + 2ze^{-y}j + xy^2k$.
10. Find constants a, b & c if the vector $\vec{f} = (2x + 3y + az)i + (bx + 2y + 3z)j + (2x + cy + 3z)k$ is Irrotational.
11. Evaluate $\nabla^2 log r$ where $r = \sqrt{x^2 + y^2 + z^2}$.
12. Find div $\bar{f}$ when $\bar{f}$= grad ($x^3 + y^3 + z^3$ -3xyz).
13. If $\bar{f}$ = (x+3y)i + y-2z)j +(x+pz)k is solenoidal find p.
14. Find curl$\bar{f}$ where $\bar{f} = grad$ ($x^3 + y^3 + z^3$ -3xyz).
15. If $\bar{f}$ = (x+y+1) i + j –(x+y) k then show that $\bar{f}$. curl$\bar{f}$ =0.
16. Verify that $div(curl\boldsymbol{v}) = 0$ for $\boldsymbol{v} = (x^2 - y^2)\boldsymbol{i} + 4xy\boldsymbol{j} + (x^2 - xy)\boldsymbol{k}$
17. Verify that $curl(grad f) = \bar{0}$ for $f = x + y - 2z^2$.
18. Show that $grad\left(\frac{1}{r}\right) = \frac{-\boldsymbol{r}}{r^3}$.
19. If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ then show that $\nabla.\boldsymbol{r} = 3$ and $\nabla \times \boldsymbol{r} = \bar{0}$.

UNIT IV

1. Define Line Integral.
2. Define Surface Integral.
3. Define Volume Integral.
4. State Stokes theorem.
5. State Gauss Divergence theorem.
6. State Green's theorem.

Unit-V

1. Form the partial differential equation by eliminating the arbitrary constants
   i. $z = ax + by + a^2 + b^2$.
   ii. $(x-a)^2 + (y-b)^2 = r^2$
   iii. $(x-a)^2 + (y-b)^2 = z^2 cot^2\alpha$
   iv. $\log(az - 1) = x + ay + b$
   v. $z = (x^2 + a)(y^2 + b)$
2. Form the partial differential equation by eliminating the arbitrary function/functions
   (i) $z = f(x^2 - y^2)$.
   (ii) $z = f(sinx + cosy)$
   (iii) $z = yf(x) + xg(y)$
   (iv) $z = y^2 + 2f\left(\frac{1}{x} + logy\right)$
   (v) $z = (x+y)\Phi(x^2 - y^2)$
   (vi) $z = f(y)\Phi(x + y)$
3. Solve $px - qy = z$
4. Solve $ptanx + qtany = tanz$
5. Solve $y^2zp + x^2zq = y^2x$
6. Solve $x(y-z)p + y(z-x)q = z(x-y)$

7. Solve $p\sqrt{x}+q\sqrt{y}=\sqrt{z}$.
8. Solve $xp + yq = 3z$
9. Write the solution of method of separation of variables
10. Define Lagrange's linear equation
11. Write Lagrange's Subsidiary equations
12. Write the three possible solutions of $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
13. Write the three possible solutions of $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$
14. Solve $(x + y)(p - q) = z$
15. Solve $p + q = 1$
16. Solve $x^2 p + y^2 q = z^2$
17. Define Wave equation
18. Define Heat equation
19. Solve $px - qy = y^2 - x^2$
20. Solve $yq - xp = z$

17. Assignment Questions:

Assignment-1 (UNIT-I)

1. If $z = f(x + ay) + \Phi(x - ay)$ show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

2. (A) If u=f(r, s, t) where $r = \frac{x}{y}$, $s= \frac{y}{z}$, $t = \frac{z}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

(B) Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the function $u = \tan^{-1}\frac{x}{y}$

3. (A) Discuss the maximum and minimum of $f(x,y) = x^3 y^2 (1 - x - y)$.

(B) Divide 24 into three parts such that the product of the first, square of the second and cube of the third is maximum.

4. If $u = \frac{x+y}{1-xy}, v = \tan^{-1}x + \tan^{-1}y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. Hence prove that u and v are functionally dependent. Find the functional relation between them.

5. (A) Prove that $\int_0^{\frac{\pi}{2}} \sin^2\theta \cos^4\theta d\theta = \pi/32$

(B) Prove that $\int_0^1 x^m (logx)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$

Assignment-2 (UNIT-II)

1. Evaluate$\iint xy(x+y)dx\,dy$ over the region bounded by $y=x^2$ and $y=x$.

2. Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)\,dxdy$
3. Evaluate $\iint(x^2+y^2)\,dxdy$ in the positive quadrant for $x+y\ \le 1$
4. Evaluate $\iint r^3 dr\,d\theta$ over the area included between the circles $r=2\ sin\theta$ and $r=4\sin\theta$
5. (A) Evaluate the following integral by transforming in to polar coordinates$\int_0^a\int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\ dxdy$

   (B) By changing the order of integration, evaluate $\int_0^3\int_1^{\sqrt{(4-y)}}(x+y)\ dxdy$

Assignment-3 (UNIT-III)

1. Show that the vector $(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ is irrotational and find its scalar potential.
2. (A) Find div $\bar{f}$ where $\bar{f}=r^n\bar{r}$ find n if it is Solenoidal?

   (B) Prove that div (gradr^m) = $m\ (m+1)\ r^{m-2}$.

3. If$\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$, $|\bar{\boldsymbol{r}}|=r$ then show that $grad\ \left(\frac{1}{r}\right)=\frac{-\bar{\boldsymbol{r}}}{r^2}$.
4. Find the directional derivative of the given scalar function $f(x,y,z)=xyz$ at the point $(1,4,3)$ in the direction of the line from $(1,2,3)$ to $(1,-1,-3)$. also calculate the magnitude of the maximum directional derivative.
5. Show that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$ where $|\bar{\boldsymbol{r}}|=r$.

Assignment-4 (UNIT-IV)

1. Verify the Green's theorem in a plane for $\int_c (y-\sin x)dx+\cos x dy$ over the curve $x=0$, $x=\pi/2,\ y=(\pi/2)\ x$
2. Verify the Stoke's theorem for F = (2 x — $y)\ i$ — $y\ z^2\ J$ — $y^2 zk$ over the upper half of the surface $x^2+y^2+z^2=1$ bounded by its projection on xy-plane.
3. Verify the divergence theorem for F $=x^2$ i $+$ zj $+$ yzk taken over the cube bounded by $x=0, x=1, y=0, y=1\ , z\ =\ 0, z\ =\ 1$.
4. Verify Greens theorem in a plane $\int_c[(x^2-xy^3)dx+(y^2-2xy)\ dy]$ where C is the square with vertices $(0,0),(2,0),(2,2),(0,2)$.
5. Verify Gauss divergence theorem for $\bar{F}$ = $(x^3-yz)\ i\ -2x^2y\ j\ +zk$ taken over the surface of the cube bounded by the planes $x=y=z=a$ and coordinate planes.

Assignment-5 (UNIT-V)

1. Form the partial differential equations by eliminating the arbitrary constants:
   (i) $(x-a)^2+(y-b)^2=z^2cot^2\alpha$ (ii) $z=(x^2+a)(y^2+b)$
2. Form the partial differential equations by eliminating the arbitrary functions:
   (i)$f(x^2+y^2, x^2-z^2)=0$ (ii) $F(xy+z^2, x+y+z)=0$
3. Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$
4. Solve $4\frac{\partial y}{\partial x}+\frac{\partial y}{\partial x}=3u$, given $u=3e^{-y}-5e^{-5y}$ when $x=0$ by method of separation of variables.
5. Solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ given that
   $u(0,t)=0, u(L,t)=0, t>0$ and $u(x,0)=3sin\left(\frac{\pi x}{L}\right), 0<x<L$

18. Unit wise Quiz questions:

Unit-I

Multiple Choice Questions:

1. If $x=r\cos\theta$, $y=r\sin\theta$ then $\frac{\partial x}{\partial r},\frac{\partial y}{\partial \theta}$ are [ ]
   (a) $\cos\theta, r\cos\theta$ (b) $\cos\theta, \sin\theta$ (c) $\cos\theta, \sec\theta$ (d)$\sin\theta, r\cos\theta$
2. If $Z=J(u,v|x,y)$, then $J(x,y|u,v)$= [ ]
   (a)Z (b)$1/Z$ (c) 1 (d) 0
3. If u=$x\cos y$, $v=y\sin x$ then $\frac{\partial(u,v)}{\partial(x,y)}$ is [ ]
   (a) $cosysinx+xysinycosx$ (b) $cosysinx-xysinycosx$
   (c) $cosycosx+sinysinx$ (d) $cosycosx-xysinysinx$
4. Are $u=x\sqrt{1-x^2}, v=2x$functionally dependent? If so, what is J$(u,v|x,y)$? [ ]
   (a) yes,1 (b) yes, 0 (c) No, 0 (d) No, 1
5. If $u(1-v)=x, uv=y$, then J$(u,v|x,y)\times J(x,y|u,v)$= [ ]
   (a) 0 (b) 1 (c)xy (d) $x-y$
6. If z is a homogeneous function of degree "n", then $x^2z_{xx}+2xyz_{xy}+y^2z_{yy}$= [ ]
   (a) nz (b) 0 (c) $n(n-1)z$ (d) z
7. If $f=x^2+y^2$then $\frac{\partial^2 f}{\partial x\partial y}$ = [ ]
   (a) 1 (b) 0 (c) -1 (d) 2
8. The minimum value of $x^2+y^2+z^2$ given that $x+y+z=3a$ is [ ]
   (a) 3a (b) $4a^2$ (c) $\frac{1}{3}a^2$ (d) $3a^2$
9. If $u=sin(ax+by+cz)$ then $\frac{\partial u}{\partial x}$= [ ]
   (a) $acos(ax+by+cz)$ (b) $asin(ax+by+cz)$
   (c) $bcos(ax+by+cz)$ (d) $bsin(ax+by+cz)$

10. If $u = x^2 - 2y, v = x + y$ then $J(\frac{u,v}{x,y})$= [ ]

(a)$(x + 1)^2$ (b)$2(x + 1)$ (c)$3(x + 1)$ (d)$4(x + 1)$

11. Laplace equation in two dimensions is [ ]

(a) $u_{xx} + u_{yy} = 0$ (b)${u_x}^2 + {u_y}^2 = 0$ (c) $u_{xy} + u_{xx} = 0$ (d) $u_{xx} - u_{yy} = 0$

12. The function has minimum value for [ ]

(a)$ln - m^2 > 0, l > 0$ (b)$ln - m^2 = 0$ (c)$ln - m^2 < 0, l < 0$ (d)$ln - m^2 > 0, l < 0$

13. If $u = sin(x + y)$ then $\frac{\partial u}{\partial y}$= [ ]

(a)$sinx$ (b)$cos(x + y)$ (c)$tan(x + y)$ (d)$sin(x + y)$

14. $\int_0^{\pi/2} sin^7 x$= [ ]

(a)16/35 (b) 35/16 (c)18/40 (d) 40/18

15. $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ = [ ]

(a)$\Gamma(\frac{3}{4})\Gamma(\frac{1}{2})$ (b) $\Gamma(\frac{3}{4})\Gamma(\frac{1}{4})$ (c) $\frac{\frac{1}{4}\Gamma(\frac{1}{4})\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}$ (d) $\frac{\frac{1}{2}\Gamma(\frac{3}{4})\Gamma(\frac{1}{2})}{\Gamma(\frac{5}{4})}$

16. $\Gamma(-\frac{1}{2})$= [ ]

(a) $-2\sqrt{\pi}$ (b) $2\sqrt{\pi}$ (c)$\sqrt{\pi}$ (d) $-\sqrt{\pi}$

17. The value of $\beta(1,2) + \beta(2,1)$ = [ ]

(a)1 (b) 2 (c)3 (d) 4

18. The value of $\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$ is [ ]

(a)2π (b) π (c) $\sqrt{2}\pi$ (d) $\sqrt{\frac{\pi}{2}}$

19. $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}$ [ ]

(a)$\frac{1}{2}\beta\left(\frac{2}{3},\frac{1}{2}\right)$ (b)$\frac{1}{3}\beta\left(\frac{2}{3},\frac{1}{2}\right)$ (c)$\frac{1}{7}\beta\left(\frac{2}{7},\frac{1}{2}\right)$ (d)$\frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$

20. $\int_0^\infty x^3 e^{-x^3}\, dx$= [ ]

(a)$\frac{1}{6}\Gamma(\frac{1}{3})$ (b) $\frac{1}{3}\Gamma(\frac{1}{3})$ (c)$\frac{1}{9}\Gamma(\frac{1}{3})$ (d) $\frac{1}{12}\Gamma(\frac{1}{3})$

21. $\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx, m > 0, n > 0$= [ ]

(a)$\beta(m,n)$ (b) $2\beta(m,n)$ (c)$\frac{1}{2}\beta(m,n)$ (d) $4\beta(m,n)$

22. $\int_0^{\pi/2} cos^n\theta d\theta$= [ ]

(a)$\frac{1}{2}\beta\left(\frac{n+1}{2},\frac{1}{2}\right)$ (b) $2\,\beta\left(\frac{n+1}{2},\frac{1}{2}\right)$ (c)$2\beta\left(\frac{1}{2},\frac{n+1}{2}\right)$ (d) $\frac{1}{2}\beta\left(\frac{1}{2},\frac{n+1}{2}\right)$

23. In terms of β function $\int_0^{\pi/2} sin^7\theta\sqrt{cos\theta}d\theta$= [ ]

(a)$\frac{1}{2}\beta\left(4,\frac{3}{4}\right)$ (b) $\frac{1}{2}\beta\left(\frac{1}{4},\frac{3}{4}\right)$ (c)$\frac{1}{2}\beta\left(\frac{3}{4},\frac{5}{4}\right)$ (d) $\frac{1}{2}\beta\left(2,\frac{1}{4}\right)$

24. The value of $\Gamma(1)$= [ ]

(a)1/2 (b) 1 (c)2 (d) 3

25. $\int_0^1 x^3(1-x^2)^{3/2}dx$= [ ]

(a) $2\beta(4,5/2)$ (b) $\beta(2,1/2)$ (c)$\beta(4,5/2)$ (d) $2\beta(2,1/2)$

26. If n is a positive integer then $\Gamma(\, n+1) =$ [ ]

(a)n (b) 2n (c)n/2 (d) 3n

27. $\int_0^{\pi/2} tan^{1/2}\theta d\theta$= [ ]

(a)$\sqrt{\pi/2}$ (b)$\pi/\sqrt{2}$ (c)$\pi/2$ (d) π

28. $\int_{-\infty}^0 e^{-x^2}dx$= [ ]

(a)2π (b) π (c) $\sqrt{2}\pi$ (d) $\frac{\sqrt{\pi}}{2}$

29. The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$= [ ]

(a)π (b) 2π (c)3π (d) 4π

30. The value of $\int_0^{\infty}\frac{x^3}{(1+x)^7}dx$= [ ]

(a)1/20 (b) 1/40 (c)1/60 (d) 1/80

31. The value of $\int_0^{\pi/2} sin^p\theta\, cos^q\theta d\theta$ in terms of Beta function is [ ]

(a)$\frac{1}{2}\beta\left(\frac{p+1}{4},\frac{q+1}{4}\right)$ (b)$\frac{1}{2}\beta(p,q)$ (c)$\frac{1}{2}\beta\left(\frac{p}{2},\frac{q}{2}\right)$ (d) $\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

32. The value of $\Gamma(1/2)$= [ ]

(a) $-2\sqrt{\pi}$ (b) $2\sqrt{\pi}$ (c)$\sqrt{\pi}$ (d) $-\sqrt{\pi}$

33. $\beta(p+1,q)+\beta(p,q+1)$= [ ]

(a) $2\beta(p,q)$ (b) $\frac{1}{2}\beta(p,q)$ (c)$\frac{1}{4}\beta(p,q)$ (d) $\beta(p,q)$

Fill in the Blanks:

1. If $u = x^y$ then $\frac{\partial u}{\partial x}$ =---------------------
2. If $u = x^3 + y^3 - 3axy$ then $\frac{\partial^2 u}{\partial x \partial y}$= ----------------------------
3. If $u = 3x + 5y$ and $v = 4x - 3y$ then $\frac{\partial(u,v)}{\partial(x,y)}$= --------------------
4. Two functions u and v are said to be functionally dependent if $\frac{\partial(u,v)}{\partial(x,y)}$= --------------------
5. If $x = e^u$and $y = e^u sinv$ then $\frac{\partial(x,y)}{\partial(u,v)}$= -----------------------------
6. If $u = x^2 + 2xy + y^2 + x + y$ then $\frac{\partial^2 u}{\partial x^2}$ = ------------------
7. If $z = \sin^{-1}\frac{x^2+y^2}{x+y}$then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ = ------------------
8. If $u = e^x siny, v = e^x cosy$ then$\frac{\partial(u,v)}{\partial(x.y)}$= -----------------
9. If $u = xy$ then $\frac{\partial u}{\partial y}$= ------------------------
10. If $u = \frac{xy}{x+y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$= --------------------------
11. If $z = log(x^3 + y^3 - x^2y - xy^2)$ then$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ is ----------------------
12. If $u = \frac{x}{y}$ and $v = \frac{(x+y)}{x-y}$ then J($\frac{u,v}{x,y}$)= ---------------------------------------
13. $\int_0^1 x^{p-1}(1-x^2)^{q-1}dx$ where $p > 0, q > 0$ in terms of Gamma function is____________
14. The value of $\int_0^\infty e^{-x^2}\, dx$ is_____________________
15. The value of $\frac{\beta(m+1,n)}{\beta(m,n)}, m > 0, n > 0$ = __________________
16. In terms of β function $\int_0^{\pi/2} sin^n\theta d\theta =$___________
17. $B(m,m)$= ________________
18. $\int_0^\infty e^{-x^3} dx =$ __________
19. $\int_0^{\frac{\pi}{2}} sin^4\theta\, cos^2\theta\; d\theta =$____________
20. The value of $\int_{-\infty}^\infty e^{-x^2} dx$ = _____________
21. $\int_0^{\pi/2} sin^3 x\, cos^{5/2} x\; dx =$__________
22. The value of $\int_0^\infty x^6 e^{-2x}\; dx$=______________
23. The value of Γ (m)Γ (m + $\frac{1}{2}$) is _____________
24. $\int_0^{\pi/2} \frac{d\theta}{sin^{1/2}\theta}$=___________
25. The value of $\int_0^\infty e^{-kx} x^{n-1}\, dx(n > 0, k > 0$ is ____________
26. Γ (7/2) = _____________
27. Express $\int_0^1 x^m(1-x^2)^n dx$ if $m > -1, n > -1$ in terms of Beta function _____________

28. The value of $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ where $m > 0, n > 0$ in terms of beta function is ________

29. Relation between Beta and Gamma functions is __________

30. The value of $\int_0^{\pi/2} sin^p\theta \, cos^q\theta \, d\theta$ in terms of Gamma function is ___________

Unit-II

Multiple choice questions:

1. $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dxdydz =$ [ ]
   A. 12 B. 24 C.36 D. 48
2. $\int_o^1 \int_0^1 \int_0^1 e^{x+y+z} dxdydz$= [ ]
   A. $(e-1)^2$ B. $(e-1)^{-1}$ C.$(e-1)^3$ D. $(e-1)^{-2}$
3. $\int_0^1 \int_0^{\pi} \int_0^{2\sin\theta} drd\theta dz$ = [ ]
   A.2 B.8 C.6 D. 4
4. The volume of a solid V in Cartesian coordinates is given by [ ]
   A.$\iiint_V dxdydz$ B.$\iiint_V xyzdxdydz$
   C.$\iiint_V xdxdydz$ D. $\iiint_V ydxdydz$
5. $\int_0^1 \int_1^2 xydydx =$ [ ]
   A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{1}{4}$ D. 4
6. $\int_0^2 \int_0^x (x+y)dxdy =$ [ ]
   A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{1}{4}$ D. 4
7. What is the result of the integration $\int_3^4 \int_1^2 (x^2+y)dydx$? []
   A. $\frac{83}{6}$ B.$\frac{83}{3}$ C. $\frac{86}{6}$ D. $\frac{83}{2}$
8. $\int_1^2 \int_1^2 (x-y)dxdy =$ []
   A. 0 B. 1 C.-1 D. 2
9. The value of $\int_0^1 \int_0^{1-x} xydydx$ is given by []
   A.$-\frac{1}{24}$ B.$\frac{1}{24}$ C. $-\frac{1}{4}$ D. $\frac{1}{2}$
10. Evaluate $\int_0^1 \left[\int_x^{\sqrt{x}} xydy\right] dx$ []
   A.$-\frac{1}{8}$ B.$\frac{1}{24}$ C. $-\frac{1}{4}$ D. $\frac{1}{2}$
11. The value of $\int_0^1 dx \int_0^1 (x+y)dy$ is []
   A. 0 B. -1 C.1 D. 2
12. $\int_{-1}^{2} \int_{x^2}^{x+2} dydx =$ []
   A.$\frac{9}{2}$ B.$\frac{9}{4}$ C. $\frac{3}{2}$ D. $\frac{1}{2}$

13. The order of integration reversed for $\int_0^1 \int_1^{e^x} dydx$ []

A. $\int_0^1 \int_1^{e^x} dxdy$ B. $\int_1^e \int_{logy}^1 dxdy$ C. $\int_0^1 \int_0^{e^x} dydx$ D. $\int_e^1 \int_1^{logy} dxdy$

14. $\int_0^a \int_0^{\sqrt{ay}} xydxdy =$ []

A. $\frac{a^4}{6}$ B. $\frac{a^4}{3}$ C. $\frac{a^4}{2}$ D. $-\frac{a^4}{3}$

15. The volume of the tetrahedron bounded by the surfaces $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is

A. $\frac{abc}{2}$ B. $\frac{abc}{4}$ C. $\frac{abc}{6}$ D. $\frac{abc}{3}$ []

16. The volume of the tetrahedron bounded by the surfaces $x = 0, y = 0, z = 0$ and $x + y + z = 1$ is

A. $\frac{1}{2}$ B. $\frac{1}{6}$ C. $\frac{3}{5}$ D. $\frac{1}{3}$ []

17. $\int_0^1 \int_0^2 \int_1^2 x^2yzdxdydz =$ []

A. $\frac{1}{2}$ B. $\frac{1}{6}$ C. 1 D. $\frac{1}{3}$

18. $\int_0^{\pi} \int_0^{asin\theta} rdrd\theta =$ []

A. $\frac{\pi a^2}{2}$ B. $\frac{\pi a^2}{4}$ C. $-\frac{\pi a^2}{2}$ D. $-\frac{\pi a^2}{4}$

19. $\iint xydxdy$ over the region bounded by $x-$ axis, ordinate at $x = 2a$ and the parabola $x^2 = 4ay$ is

A. $\frac{a^2}{2}$ B. $\frac{a^2}{8}$ C. $\frac{a^4}{3}$ D. $\frac{a^2}{4}$ []

20. $\int_1^0 \int_0^1 (x + y)dxdy =$

A. 2 B. -2 C. -1 D. 1 []

21. $\int_0^1 dx \int_0^x e^{y/x}\, dy =$

A. $\frac{e}{2}$ B. $\frac{e-1}{2}$ C. $-\frac{e}{2}$ D. $\frac{e-1}{3}$ []

22. $\int_0^1 \int_0^x e^x\, dxdy =$

A. 2 B. -2 C. -1 D. 1 []

23. $\int_0^2 \int_0^{x^2} x(x^2 + y^2)dxdy =$

A. $\frac{32}{3}$ B. $\frac{64}{3}$ C. $\frac{84}{3}$ D. $\frac{1}{3}$ []

24. $\int_0^{\pi} \int_0^{a\cos\theta} r\sin\theta drd\theta =$

A. $\frac{a^2}{6}$ B. $\frac{a^2}{3}$ C. $\frac{a^4}{3}$ D. $\frac{a^2}{4}$ []

25. In Polar coordinates $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy =$

A. $\int_0^{\frac{\pi}{2}} \int_0^\infty e^{-r^2} r\,drd\theta$ B. $\int_0^{\frac{\pi}{4}} \int_0^\infty e^{-r^2} r\,drd\theta$

C. $\int_0^{\frac{\pi}{2}} \int_0^\infty e^{-r^2}\,drd\theta$ D. $\int_0^{\frac{\pi}{4}} \int_0^\infty e^{-r^2} r\,drd\theta$ []

26. Using double integral, The volume of the tetrahedron bounded by the surfaces $x = 0, y = 0, z = 0$ and $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ is

A. *4* B. *6* C. $\frac{1}{6}$ D. $\frac{1}{3}$ []

27. By changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx =$

A. $\int_0^1 \int_0^{\sqrt{1-y^2}} y^2 dxdy$ B. $\int_0^1 \int_0^{\sqrt{1+y^2}} y^2 dxdy$ [ ]

C. $\int_0^2 \int_0^{\sqrt{1-y^2}} y^2 dxdy$ D. $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} y^2 dxdy$

28. By changing the order of integration $\int_0^1 \int_0^{2-x} xydydx =$

A. $\int_0^1 \int_0^{2-y} xydxdy$ B. $\int_0^2 \int_0^{2-y} xydxdy$ [ ]

C. $\int_0^1 \int_0^{2+y} xydxdy$ D. $\int_0^2 \int_0^{2+y} xydxdy$

29. The value of $\int\int xy\,dx\,dy$ taken over the positive quadrant of the circle $x^2+y^2 = 1$ is

A. $\int_0^1 \int_0^{\sqrt{1-x^2}} xy\,dydx$ B. $\int_0^1 \int_0^{\sqrt{1+x^2}} xy\,dydx$ [ ]

C. $\int_0^2 \int_0^{\sqrt{1-x^2}} xy\,dydx$ D. $\int_{-1}^1 \int_0^{\sqrt{1+x^2}} xy\,dydx$

30. The value of $\int\int (x + y)\,dxdy$ taken over the positive quadrant of the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$ is

A. 4 B.5 C.6 D.8 [ ]

Unit-III

Multiple Choice Questions:

1. If $\bar{a}$ is a constant vector then $curl(\bar{r} \times \bar{a}) =$ [ ]
   A. $2\bar{a}$ B. $-2\bar{a}$ C. $\bar{a}$ D. $-\bar{a}$
2. If $\phi = x^3 + y^3 + z^3 - 3xyz$ then curl(grad ϕ) = [ ]
   A. $\bar{0}$ B. $\bar{r}$ C. $6x+6y+6z$ D. $x+y+z$
3. If $(x^2 + 4x)\bar{\imath} - 2xy\bar{\jmath} + pz\bar{k}$ is solenoidal, then $p =$ [ ]
   A. 4 B. -4 C. 6 D.2
4. Unit normal to the surface $\phi = xy + z$ at the point (-1, 1, 2) is [ ]
   A. $\frac{\bar{\imath}-\bar{\jmath}+\bar{k}}{\sqrt{3}}$ B. $\frac{-\bar{\imath}+\bar{\jmath}+\bar{k}}{\sqrt{3}}$ C. $\frac{-\bar{\imath}+\bar{\jmath}+2\bar{k}}{\sqrt{6}}$ D. $\frac{\bar{\imath}-\bar{\jmath}+2\bar{k}}{\sqrt{6}}$
5. The directional derivative of $\phi = x + y + z - 2$ at $(1, 2, 1)$ in the direction of $2\bar{\imath} + 3\bar{\jmath} - \bar{k}$ is [ ]
   A. $\frac{-6}{\sqrt{14}}$ B. $\frac{-4}{\sqrt{14}}$ C. $\frac{7}{\sqrt{14}}$ D. $\frac{4}{\sqrt{14}}$

6. If $\phi = 2xz^4 - x^2y$, find $|\nabla\phi|$ at the point (2, -2, -1) [ ]
A. $2\sqrt{31}$ B. $3\sqrt{31}$ C. $2\sqrt{93}$ D. $\sqrt{93}$
7. If $\bar{f} = (x^2 + 4x)\bar{\imath} - 2xy\bar{\jmath} + pz\bar{k}$ is solenoidal, then $p =$ ____ [ ]
A.0 B.– 4 C.$2x - 4$ D. 4
8. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and if $(r^n\bar{r})$ is solenoidal then $n =$______ [ ]
A. 3 B. -3 C. 1 D. None
9. If $\phi = x^2 + y^2 + z^2 - 3xyz$ then div(grad ϕ) =_____________ [ ]
A. $\bar{0}$ B. $6x + 6y + 6z$ C. $x + y + z$ D. None
10. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ then $\nabla^2\left(\frac{1}{r}\right) =$______________ [ ]
A. 0 B. 3x C. 2x D.3(x+y+z)
11. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ then $\nabla^2(log r) =$______________ [ ]
A. 0 B. $1/r^2$ C. $x + y + z$ D. None
12. If $\bar{f} = x^3\bar{\imath} + 5xy^2\bar{\jmath} + 2z\bar{k}$ then $curl\ \bar{f}$=_____________ [ ]
A. $\bar{0}$ B. $-5y^2\bar{k}$ C. $5y^2\bar{k}$ D. $(5y^2 - 3x^2)\bar{k}$
13. If $\phi = x^3 + y^3 + z^3 - 3xyz$ then curl(grad ϕ) =__________ [ ]
A. $\bar{0}$ B. $6x + 6y + 6z$ C. $x + y + z$ D. None
14. If $\bar{a}$ is a constant vector then $curl(\bar{r} \times \bar{a}) =$______________ [ ]
A. $2\bar{a}$ B. $-2\bar{a}$ C. $\bar{a}$ D. $-\bar{a}$
15. If $\bar{f} = x^2\bar{\imath} + y^2\bar{\jmath} + \mathrm{A}z\bar{k}$ is irrotational, then the value of A is? [ ]
A.1 B.2 C. 0 D.any value
16. The scalar potential of $\bar{f} = 3x^2\bar{\imath} + 3y^2\bar{\jmath} + 3z^2\bar{k}$ is ___________ [ ]
A. $3x^2 + 3y^2 + 3z^2 + c$ B. $x + y + z + c$ C.$x^3 + y^3 + z^3 + c$ D.3c
17. A vector point function $\bar{f}$ is said to be solenoidal if? [ ]
A. $curl\ \bar{f} = 0$ B. $div\ \bar{f} = 0$ C. $div\ \bar{f} = \infty$ D. $curl\ \bar{f} = \infty$
18. If $\bar{f}$ is a constant vector then $curl\ \bar{f}$=______[]
A. $\bar{0}$ B. ∞ C. Constant D. None
19. A vector point function $\bar{f}$ is said to be irrotational if? [ ]
A. $curl\ \bar{f} = \infty$ B. $div\ \bar{f} = 0$ C. $div\ \bar{f} = \infty$ D. $curl\ \bar{f} = 0$
20. For what values of a, b, c values the vector $\bar{f} = (2x + 3y + az)\bar{\imath} + (bx + 2y + 3z)\bar{\jmath} + (2x + cy + 3z)\bar{k}$ is irrotational? [ ]
A. a=2, b=3, c=2 B. a=2, b=2, c=3 C. a=2, b=3, c=3 D. a=1, b=3, c=3
21. If $\emptyset$ satisfies Laplacian equation, then $\nabla\emptyset$ is ________ [ ]
A. Solenoidal B. irrotational C. Both A&B D. None
22. Unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at the point $(2, 2, 3)$ is? [ ]
A. $\frac{i-j+3k}{\sqrt{11}}$ B.$\frac{i+j+3k}{\sqrt{11}}$ C. $\frac{i+j+3k}{\sqrt{10}}$ D.$\frac{i+j-3k}{\sqrt{11}}$
23. If $\bar{f} = grad\ (x^3 + y^3 + z^3 - 3xyz)$ then $div\ \bar{f}$ is _______ [ ]
A.6(x+y+z) B. x+y+z C.3(x+y+z) D. -6(x+y-z)
24. If $curl\ \bar{f} = \bar{0}$ then $\bar{f}$ is called? [ ]
A. Constant vector B. Irrotational vector C. Solenoidal vector D. cannot say
25. If $div\ \bar{f}$=0 then $\bar{f}$ is called [ ]
A. Constant vector B. Irrotational vector C. Solenoidal vector D. cannot say
26. For what values of a, b, c values the vector $\bar{f} = (x + 2y + az)\bar{\imath} + (bx - 3y - z)\bar{\jmath} + (4x + cy + 2z)\bar{k}$ is irrotational?[]
A. a=1, b=4, c=2 B. a=4, b=2, c=3 C. a=4, b=2, c=3 D. a=4, b=2, c=-1

27. If $\bar{f} = (x+3y)\bar{\imath} + (y-2z)\bar{\jmath} + (x+pz)\bar{k}$ is solenoidal, then the value of p is? [ ]
A.-1 B. -2 C. 1 D. 2
28. If $\emptyset = 3x^2y - y^3z^2$ then $grad\ \emptyset$ at point (1, -2, -1) is _________ [ ]
A. $-12\bar{\imath} - 9\bar{\jmath} - 16\bar{k}$ B. $-12\bar{\imath} + 9\bar{\jmath} - 16\bar{k}$ C. $12\bar{\imath} - 9\bar{\jmath} - 16\bar{k}$ D. $-12\bar{\imath} - 9\bar{\jmath} + 16\bar{k}$
29. Unit normal to the surface $\phi = x^2y + 2xz - 4$ at the point (2, -2, 3) is [ ]
A. $\frac{\bar{\imath}+2\bar{\jmath}-2\bar{k}}{3}$ B. $\frac{\bar{\imath}-2\bar{\jmath}+2\bar{k}}{3}$ C. $\frac{-\bar{\imath}+2\bar{\jmath}+2\bar{k}}{3}$ D. $\frac{\bar{\imath}-\bar{\jmath}+2\bar{k}}{\sqrt{6}}$
30. If $\bar{f} = (x+y+1)\bar{\imath} + \bar{\jmath} - (x+y)\bar{k}$, then $curl\ \bar{f}$ is _________ [ ]

A. $\bar{\imath} + \bar{\jmath} + \bar{k}$ B. $\bar{\imath} + \bar{\jmath} - \bar{k}$ C. $-\bar{\imath} + \bar{\jmath} - \bar{k}$ D. $\bar{\imath} + \bar{\jmath} + 2\bar{k}$

Fill in the blanks:

1. Unit normal to $x^2 + y^2 + z^2 = 5$ at $(0,\ 1,\ 2)$ is equal to _______________
2. Unit normal to the surface $z = x^2 + y^2$ at $(-1, -2,\ 5)$ is equal to _______
3. Unit normal to the surface $xy^3z^2 = 4$ at $(-1, -1,\ 2)$ is equal to _______
4. The directional derivative of $\Phi = xyz$ at the point $(1, 1, 1)$ in the direction $\hat{\imath}$ is ____
5. A vector point function $\bar{f}$ is said to be solenoidal if ___________
6. If $\vec{A}$ is such that $\nabla \times \vec{A}$ =0 then $\vec{A}$ is called _____________
7. If $\bar{a}$ and $\bar{b}$ are irrotational then $\bar{a} \times \bar{b}$ is ___________________
8. If $\bar{f} = xy^2\bar{\imath} + 2x^2yz\bar{\jmath} - 3yz^2\bar{k}$ then, $curl\ \bar{f}$=____________
9. If $f = x^2yz^2$, then ∇fat (2, 1, -1) is___________
10. If $\bar{r} = \mathrm{x}\bar{\mathrm{\imath}} + \mathrm{y}\bar{\mathrm{\jmath}} + \mathrm{z}\bar{\mathrm{k}}$, then $\nabla\left(\frac{1}{r}\right)$ =__________
11. If $\bar{r}$ is the position vector of any point in space, then $r^n\bar{r}$ is____________
12. If $\bar{r} = \mathrm{x}\bar{\mathrm{\imath}} + \mathrm{y}\bar{\mathrm{\jmath}} + \mathrm{z}\bar{\mathrm{k}}$, and if $r^n\bar{r}$ is solenoidal then n=_______
13. Physical interpretation of $\nabla\emptyset$ is that__________________
14. If $b = xy + yz + zx$ then $grad\ b$=______________
15. If $\bar{f} = grad(x^2 + y^2 + z^2 - 2xyz)$ then $div\ \bar{f}$ =______
16. $curl(grad\ \emptyset)$is always________________
17. $div.\left(curl\ \bar{f}\right)$is always_________________
18. If $\emptyset = ax^2 + by^2 + cz^2$ satisfies Laplacian equation, then a+b+c=__________
19. If $\emptyset$ satisfies Laplacian equation, then $\nabla\emptyset$ is ____________
20. If $\bar{f} = (x+2y+az)\bar{\imath} + (bx-3y-z)\bar{\jmath} + (4x+cy+2z)\bar{k}$ is irrotaional then, a=___, b=___, c=____
21. If θ be the angle between the two normal's, where $\overline{n_1} = \bar{\imath} + 4\bar{\jmath} - 4\bar{k}$,$\overline{n_2} = 3\bar{\imath} + 3\bar{\jmath} + 6\bar{k}$ thencos θ =_______
22. If θ be the angle between the two normal's, where $\overline{n_1} = 2\bar{\imath} - \bar{\jmath} - \bar{k}$, $\overline{n_2} = 4\bar{\imath} - \bar{\jmath} - 4\bar{k}$ then

$\cos\theta$ =_______

Unit-IV

Multiple Choice Questions:

1. If $\int \bar{f}(t)dt = \bar{F}(t) + \bar{c}$ then $\int_a^b \bar{f}(t)dt =$ __________ [ ]
   A) $\bar{F}(b) - \bar{F}(a)$ B) $\bar{F}(a) - \bar{F}(b)$ C) $\bar{f}(b) - \bar{f}(a)$ D) $\bar{f}(a) - \bar{f}(b)$
2. Any integral which is to be evaluated along a curve is called ___________ [ ]
   A) Surface integral B) Line integral C) Volume integral D) Contour integral
3. If $\bar{F}$ is the force vector acting on a particle moving along an arc then the total work done by $\bar{F}$ is given by ____________ integral. [ ]
   A) Surface integral B) Volume integral C) Line integral D) Contour integral
4. If $\bar{F}.d\bar{r} = (-12t^{11} + 4t^3 + 6t^2)dt$ and $t: 0 \to 1$then $\int_C \bar{F}.d\bar{r} =$ ________ [ ]
   A) 1 B) -1 C) -2 D) 2
5. Which of the following condition represents the work done is independent of the path? [ ]
   A) Curl $\bar{F} \neq 0$ B) Curl $\bar{F} < 0$ C) Curl $\bar{F} > 0$ D) Curl $\bar{F} = 0$
6. If $\bar{F}$ is a force vector acting on a particle along a curve C is a rectangle OPQR then $\int_C \bar{F}.d\bar{r} =$ _________ [ ]
   A) $\int_{OP} \bar{F}.d\bar{r} + \int_{PQ} \bar{F}.d\bar{r} + \int_{QR} \bar{F}.d\bar{r} + \int_{RO} \bar{F}.d\bar{r}$
   B) $\int_{OP} \bar{F}.d\bar{r} + \int_{PQ} \bar{F}.d\bar{r} - \int_{QR} \bar{F}.d\bar{r} - \int_{RO} \bar{F}.d\bar{r}$
   C) $\int_{OP} \bar{F}.d\bar{r} + \int_{PQ} \bar{F}.d\bar{r} + \int_{RQ} \bar{F}.d\bar{r} - \int_{RO} \bar{F}.d\bar{r}$
   D) $\int_{OP} \bar{F}Xd\bar{r} - \int_{PQ} \bar{F}Xd\bar{r} + \int_{QR} \bar{F}Xd\bar{r} + \int_{RO} \bar{F}Xd\bar{r}$
7. If $\bar{r} = xi + yj + zk$ then $\int_C \bar{r}.d\bar{r} =$ _________ [ ]
   A) x B) $\bar{r}$ C) 0 D) 1
8. $\int \bar{r} \times \bar{n}\ dS =$ ________ [ ]
   A) 0 B) 1 C) -1 D) -1
9. A necessary and sufficient condition for the line integral $\int_C A.dr = 0$ for every closed curve C is that ___________ [ ]
   A) $div\ A = 0$ B) $div\ A \neq 0$ C) $Curl\ A \neq 0$ D) $Curl\ A = 0$
10. $\int grad\ (x + y - z)d\bar{r} =$ __________ from $(0, 1, -1) to\ (1, 2, 0)$ [ ]
    A) 0 B) 2 C) 3 D) -3
11. Any integral which is to be evaluated over a surface is called ___________ [ ]
    A) Line integral B) Surface integral C) Volume integral D) Contour integral
12. $\int_C f\ \nabla f.d\bar{r} =$ _________ [ ]
    A) f B) $2f$ C) 1 D) 0
13. The surface integral $\int_S \bar{F}.\bar{n}ds$taken over a region R in xy plane is given by _______ [ ]
    A) $\iint_R \frac{\bar{F}.\bar{n}dxdz}{|\bar{n}.j|}$ B) $\iint_R \frac{\bar{F}.\bar{n}dydz}{|\bar{n}.i|}$ C) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.k|}$ D) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.i|}$
14. $\int_S \bar{F}.\bar{n}ds =$_________ over a region R in yz plane [ ]
    A) $\iint_R \frac{\bar{F}.\bar{n}dxdz}{|\bar{n}.j|}$ B) $\iint_R \frac{\bar{F}.\bar{n}dydz}{|\bar{n}.i|}$ C) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.k|}$ D) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.i|}$
15. $\int_S \bar{F}.\bar{n}ds =$_________ over a region R in xz plane [ ]
    A) $\iint_R \frac{\bar{F}.\bar{n}dxdz}{|\bar{n}.j|}$ B) $\iint_R \frac{\bar{F}.\bar{n}dydz}{|\bar{n}.i|}$ C) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.k|}$ D) $\iint_R \frac{\bar{F}.\bar{n}dxdy}{|\bar{n}.i|}$

16. To evaluate $\int_S \bar{F}.\bar{n}ds$ in xy plane, the integral limits of x and y when S is the portion of the plane $x+y+z=1$ included in the first octant are ____________ [ ]
A) $x: 0 \to 1, y: 0 \to 1-x$ B) $x: 0 \to 1, y: 0 \to 1$
C) $x: 0 \to 1, y: 0 \to x-1$ D) $x=0, y=1$

17. For any closed surface S, $\int_S Curl\,\bar{F}.\bar{n}ds$= ______________ [ ]
A) 0 B) $\bar{n}$ C) $\int_C \bar{F}.d\bar{r}$ D) $\bar{F}$

18. The integral limits of y are ________ to evaluate $\int_S \bar{F}.\bar{n}ds$ in yz plane, where S is surface $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$. [ ]
A) $y: 0 \to 16$ B) $y: 0 \to 4$ C) $y: 4 \to 0$ D) $y: 16 \to 0$

19. $\int_S x\,dydz + y\,dzdx + z\,dxdy =$ _______, where $S: x^2+y^2+z^2=a^2$ [ ]
A) 4π B) $4a$ C) $\frac{4}{3}\pi a^3$ D) $4\pi a^3$

20. Which of the following represents the Volume integral? [ ]
A) $\int_C \bar{F}.d\bar{r}$ B) $\int_S \bar{F}.\bar{n}ds$ C) $\int_V \bar{F}\,dv$ D) $\int_S \bar{F}.d\bar{r}$

21. To evaluate the Volume integral $\int_V \nabla.\bar{F}\,dv$, where V is the closed region bounded by $x=0, y=0, z=0, 2x+2y+z=4$, the limits of x are __________ [ ]
A) $x=0\ to\ 2$ B) $z=0\ to\ 4-2x-2y$
C) $y=0\ to\ 2-x$ D) $x=0\ to\ 2-y-\frac{z}{2}$

22. To evaluate the Volume integral $\int_V \nabla.\bar{F}\,dv$, where V is the closed region bounded by $x=0, y=0, z=0, 2x+2y+z=4$, the limits of y are __________ [ ]
A) $x=0\ to\ 2$ B) $z=0\ to\ 4-2x-2y$
C) $y=0\ to\ 2-x$ D) $y=0\ to\ 2-x-\frac{z}{2}$

23. To evaluate the Volume integral $\int_V \nabla.\bar{F}\,dv$, where V is the closed region bounded by $x=0, y=0, z=0, 2x+2y+z=4$, the limits of z are __________ [ ]
A) $x=0\ to\ 2$ B) $z=0\ to\ 4-2x-2y$
B) $y=0\ to\ 2-x$ D) $z=0\ to\ 2-y-\frac{x}{2}$

24. Transformation between Surface integral and Volume integral is given by _____ [ ]
A) Green's theorem B) Stoke's theorem
C) Volume integral theorem D) Gauss divergence theorem

25. Which of the following is the Gauss divergence theorem? [ ]
A) $\int_V div\,\bar{F}\,dv = \int_S \bar{F}.\bar{n}ds$ B) $\int_C Mdx + Ndy = \int_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$
C) $\int_C \bar{F}.d\bar{r} = \int_S Curl\,\bar{F}.\bar{n}ds$ D) $\int_C \bar{F}.d\bar{r} = \int_S \bar{F}.\bar{n}ds$

26. Transformation between Line integral and Double integral is given by _____ [ ]
A) Green's theorem B) Stoke's theorem
C) Volume integral theorem D) Gauss divergence theorem

27. Transformation between Line integral and Surface integral is given by _____ [ ]
A) Green's theorem B) Stoke's theorem
C) Volume integral theorem D) Gauss divergence theorem

28. Which of the following represents the Green's theorem? [ ]
A) $\int_V div\,\bar{F}\,dv = \int_S \bar{F}.\bar{n}ds$ B) $\int_C Mdx + Ndy = \int_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$
C) $\int_C \bar{F}.d\bar{r} = \int_S Curl\,\bar{F}.\bar{n}ds$ D) $\int_C \bar{F}.d\bar{r} = \int_S \bar{F}.\bar{n}ds$

29. Which of the following is the Stoke's theorem? [ ]

A) $\int_V div\,\bar{F}\,dv = \int_S \bar{F}.\bar{n}ds$ B) $\int_C Mdx + Ndy = \int_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$

C) $\int_C \bar{F}.d\bar{r} = \int_S Curl\,\bar{F}.\bar{n}ds$ D) $\int_C \bar{F}.d\bar{r} = \int_S \bar{F}.\bar{n}ds$

30. If $\bar{n}$ is the unit outward drawn normal to any closed surface then $\int_V div\,\bar{n}\,dv =$___ [ ]

A) S B) 2S C) V D) 3V

Fill in the Blanks:

1. $\int(\bar{r} + \bar{s})dt =$ ___________
2. If $\int_C \bar{v}.d\bar{r} = 0$ then $\bar{v}$ is called _______________
3. If $\int_C \bar{F}.d\bar{r} = 0$then the work done is ____________ of the path.
4. Any integral along an arc is called __________
5. If $\bar{F} = (x^2 + y^2)i - 2xyj$ and the curve C is the rectangle OPQR in xy plane bounded by O (0, 0), P (a, 0), Q (a, b) and R (0, b) then $\int_{OP} \bar{F}.d\bar{r} =$ ______
6. The curve C is the rectangle OPQR with $\int_{OP} \bar{F}.d\bar{r} = \frac{a^3}{3}$, $\int_{PQ} \bar{F}.d\bar{r} = -ab^2$, $\int_{QR} \bar{F}.d\bar{r} = \frac{-a^3}{3} - ab^2$and $\int_{RO} \bar{F}.d\bar{r} = 0$ then $\int_C \bar{F}.d\bar{r} =$_____
7. The work done by the force $(3x - 2y)i + (y + 2z)j - x^2k$ in moving along the straight line joining $(0, 0, 0)$ and $(1, 1, 1)$ is _____________
8. A necessary and sufficient condition that the line integral $\int_C \bar{A}.d\bar{r} = 0$ for every closed curve C is that _____________
9. If $\bar{F} = x^2y^2i + yj$ and the curve $C: y^2 = 4x$ in the xy plane from $(0, 0)$ to $(4, 4)$ then $\int_C \bar{F}.d\bar{r} =$__________
10. If C is a closed curve surrounding the area A, then $\int_C x\,dy - y\,dx =$ ______
11. The integral which is evaluated over a surface is called _________
12. If $\bar{F} = axi + byj + czk$ where a, b, c are constants then $\iint \bar{F}.\bar{n}\,ds$ where S is the surface of the unit sphere is _____________
13. If $\bar{F} = F_1i + F_2j + F_3k$ where F_1, F_2, F_3 are functions of x, y, z and $dv = dx\,dy\,dz$ then then the Volume integral is given by $\int_V \bar{F}\,dv =$ _______
14. Gauss divergence theorem is useful only for ____________ surfaces.
15. The theorem which gives the relation between the surface and volume integral is _________________
16. **F**rom Green's theorem,$\iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy =$ _____
17. **I**f $M = \sin y$, $N = x(1 + \cos y)$and $\int_C Mdx + Ndy = \pi a^2$ then by Green's theorem

$\int_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy =$ ________

18. By Stoke's theorem, $Curl\ grad\ \varphi =$ ________

19. $\int_C f\nabla f . d\bar{r} =$ ______

20. The theorem which gives the relation between Line integral and surface integral is

____

Unit-V

Multiple Choice Questions:

1. The order of the partial differential equation $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nu$ is [ ]

   a) 1 b) 2 c) 3 d) 0

2. The degree of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ is [ ]

   a) 0 b) 1 c) 2 d) 3

3. If the number of arbitrary constants to be eliminated is equal to the number of independent variables then the partial differential equation is of ------ order [ ]

   a) First b) Second c) Third d) Fourth

4. The partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by$ is [ ]

   a) $z = qx + py$ b) $z = px - qy$ c) $z = px + qy$ d) $z = qx - py$

5. The partial differential equation corresponding to the equation $z = f(x^2 + y^2)$ by eliminating the arbitrary function is [ ]

   a) $px + qy = 0$ b) $\frac{p}{y} = \frac{q}{x}$ c) $py = qx$ d) $px - qy = 0$

6. The Lagrange's subsidiary equation; where P, Q, R are functions of x, y and z is [ ]

   a) $Pdx + Qdy + Rdz = 0$ b) $dx + dy + dz = 0$ c) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ d) None

7. By eliminating 'a' and 'b' from $z = a(x + y) + b$, the partial differential equation formed is [ ]

   a) $\mathrm{p} = 2\mathrm{q}$ b) $px = qy$ c) $p = q$ d) $py = qx$

8. The general solution of $2p + 3q = 1$ is [ ]

   a) $\emptyset(x - 2z, 3x - 2y) = 0$ b) $\emptyset(x + z, 3y - 2z) = 0$

   c) $\emptyset(x, 3z - 2y) = 0$ d) $\emptyset(2z, 3x - 2y) = 0$

9. The general solution of $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is [ ]

   a) $2f\left(\frac{x}{y}, \frac{x}{z}\right)$ b) $f\left(\frac{x}{y}, \frac{x}{z}\right)$ c) $\frac{1}{2}f\left(\frac{x}{y}, \frac{x}{z}\right)$ d) $3f\left(\frac{x}{y}, \frac{x}{z}\right)$

10. The partial differential equation can be formed by eliminating the arbitrary functions from $z = f(x + at) + g(x - at)$ [ ]

    a) $\frac{\partial^2 z}{\partial t^2} = a^2\frac{\partial^2 z}{\partial x^2}$ b) $\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}$ c) $\frac{\partial z}{\partial t} = a^2\frac{\partial^2 z}{\partial x^2}$ d) $\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2}$

11. One of the solution of $y^2zp + x^2zq = y^2$ is given by [ ]

a) $x^3 - y^3 = c$b) $x^2 + y^2 = c$ c) $x^3 + y^3 = c$ d) $x^2 - y^2 = c$

12. One of the solution of $p + 5q = \tan(y - 5x)$ is given by [ ]

a) $y + 5x = c$ b) $y - 5x = c$c) $5y + x = c$ d) $5y - x = c$

13. The Lagrange's Multipliers for $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ are[ ]

a) $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$ b) $\frac{1}{x^3}, \frac{1}{y^3}, \frac{1}{z^3}$ c) 1,1,1 d) None of the above

14. The general solution of $z = px + qy + log\ pq$ is ----------- [ ]
a) $z = ax - by - \log ab$ b) $z = ax + by + \log ab$
c) $z = ax - by + \log ab$ d) $z = ax + by - \log ab$
15. The general solution of $p\ cotx + q\ coty = cotz$ is --------- [ ]
a) $f\left(\frac{cosy}{cosx}, \frac{cosz}{cosy}\right) = 0$ b) $f\left(\frac{siny}{sinx}, \frac{sinz}{siny}\right) = 0$
c) $f\left(\frac{cosx}{cosy}, \frac{cosy}{cosz}\right) = 0$ d) $f\left(\frac{y}{x}, \frac{z}{y}\right) = 0$
16. The partial differential equation by eliminating the arbitrary function from
$z = f(sinx + cosy)$ is ---------- [ ]
a) $p \sin y + q \cos x = 0$ b) $p \sin y - q \cos x = 0$
c) $p \cos y + q \sin x = 0$ d) $p \sin y + q \sin x = 0$
17. The general solution of $pe^y = qe^x$ is ----------- [ ]
a) $z = a(e^x - e^y) + c$ b) $z = a(e^x + 2e^y) + c$
c) $z = a(e^x + e^y) + c$ d) $z = a(2e^x + e^y) + c$
18. The partial differential equation of all spheres whose centers lie on the z-axis is ------
a) $py + qx = 0$ b) $px - qy = 0$
b) $px + qy = 0$ d) $py - qx = 0$ [ ]
19. The equation$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ is known as [ ]
a) One dimension Wave equation
b) One dimension Heat equation
c) Two-dimension Heat equation
d) Laplace equation
20. The equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is known as [ ]
a) One dimension Wave equation
b) One dimension Heat equation
c) Two-dimension Heat equation
d) Laplace equation

21. The compatible solution for one dimensional wave equation is [ ]
a) $u(x,t) = \sum_{n=1}^{\infty} \left[A_n \cos(\frac{n\pi at}{l}) B_n \sin(\frac{n\pi at}{l})\right] \sin(\frac{n\pi x}{l})$
b) $u(x,t) = \sum_{n=1}^{\infty} \left[A_n \sin(\frac{n\pi x}{l}) B_n \cos(\frac{n\pi x}{l})\right] \sin(\frac{n\pi at}{l})$

c) $u(x,t) = \sum_{n=1}^{\infty}\left[A_n\sin(\frac{n\pi x}{l})B_n\cos(\frac{n\pi x}{l})\right]\cos(\frac{n\pi at}{l})$
d) $u(x,t) = \sum_{n=1}^{\infty}\left[A_n\cos(\frac{n\pi at}{l})B_n\sin(\frac{n\pi at}{l})\right]\cos(\frac{n\pi x}{l})$

22. The solution of one dimensional heat equation, when $\lambda < 0$ is [ ]
a) $u(x,t) = a_1e^{-c^2\lambda^2t}(a_2e^{\lambda x} - a_3e^{-\lambda x})$, where c is positive real constant.
b) $u(x,t) = a_1e^{-c^2\lambda^2t}(a_2cos\ \lambda x + a_3sin\lambda x)$, where c is positive real constant.
c) $u(x,t) = a_1e^{c^2\lambda^2t}(a_2cos\ \lambda x - a_3sin\lambda x)$, where c is positive real constant.
d) $u(x,t) = a_1e^{c^2\lambda^2t}(a_2e^{\lambda x}\ + a_3e^{-\lambda x})$, where c is positive real constant.

23. The solution of one dimensional wave equation, when $\lambda = 0$ is [ ]
a) $\cos(\frac{n\pi at}{l})$ b) $\sin(\frac{n\pi at}{l})$ c) 0 d) $\cos(\frac{n\pi x}{l})$

24. By using method of separation of variables, the solution of $u_x + u_y = 0$ is [ ]
a) e^{kx} b) e^{ky} c) $e^{k(x+y)}$ d) $e^{k(x-y)}$

25. By using method of separation of variables, the solution of $u_x = ku_y$, given that $u(0,y) = 8e^{-3y}$ is [ ]
a) $u(x,y) = 8e^{-3(kx+y)}$ b) $u(x,y) = 8e^{3(kx+y)}$
c) $u(x,y) = 8e^{kx+y}$ d) $u(x,y) = 8e^{kx-y}$

26. By using method of separation of variables, the solution of $u_x = 2u_y + u$, given that $u(1,0) = 6e^{-3x}$ is [ ]
a) $u(x,y) = 6e^{-3(x+y)}$ b) $u(x,y) = 6e^{3(x+y)}$
c) $u(x,y) = 6e^{-3x-2y}$ d) $u(x,y) = 6e^{3x+2y}$

27. In Method of Separation of Variables, the solution is [ ]
a) Product of single variable functions of each independent variable
b) Sum of single variable functions of each independent variable
c) Integration of single variable functions of each independent variable
d) None of the above

28. The solution of $\frac{\partial u}{\partial x} = 6\frac{\partial u}{\partial t} + u$ using method of separation of variables is [ ]
a) $u(x,t) = ce^{(1+6k)x+kt}$ b) $u(x,t) = ce^{(1-6k)x+kt}$
c) $u(x,t) = ce^{(1+6k)x-kt}$ d) None

29. By using method of separation of variables, the solution of $4u_x + u_y = 3u$ [ ]
a) $u(x,y) = Ce^{\frac{kx}{4}+(3-k)y}$ b) $u(x,y) = Ce^{\frac{kx}{4}-(3-k)y}$
c) $u(x,y) = Ce^{\frac{kx}{4}+(3+k)y}$ d) $u(x,y) = Ce^{-\frac{kx}{4}-(3-k)y}$

30. $\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x^2}$ is an example of [ ]
a) Non-Linear partial differential equation b) linear ordinary differential equation
c) Linear partial differential equation d) None of the above

Fill in the blanks:

1. If the number of arbitrary constants to be eliminated is equal to the number of independent variables then we get a partial differential equation of --------- order.

2. If the number of arbitrary constants to be eliminated is greater than the number of independent variables then we get a partial differential equation of --------- order.
3. The partial differential equation by eliminating the arbitrary constants a and b from $Z = ax + by$ is -----------
4. The partial differential equation by eliminating the arbitrary constants a and b from $Z = ax^2 + by^2$ is -----------
5. The partial differential equation by eliminating the arbitrary constants a and b from $Z = (x + a)(y + b)$ is -----------
6. The partial differential equation by eliminating the arbitrary constants a and b from $2Z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is -----------

7. The partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$ is ----------
8. The partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$ is ----------
9. The general solution of $2p + 3q = 1$ is -------
10. The general solution of $xp + yq = 3z$ is ---------
11. The general solution of $yzp - xzq = xy$ is --------
12. The general solution of $pq + p + q = 0$ is -----------
13. The general solution of $z = px + qy - 2pq$ is ---------
14. The general solution of $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is ---------
15. The general solution of $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$ is ---------
16. The general solution of $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$ is ---------
17. The general solution of $pyz + qzx = xy$ is ----------
18. The general solution of $dx = dy = dz$ is -------------
19. The general solution of $\sqrt{p} + \sqrt{q} = 1$ is -------------
20. The general solution of $p^2 + q^2 = m^2$ is -------------

19. Tutorial Problems:

UNIT-I

TUTORIAL-1

1. If $x^x y^y z^z = e$, then show that if $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x loge x)^{-1}$
2. If $u = x + y + z, y + z = uv, z = uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.
3. Find the rectangular parallelepiped of maximum volume that can be inscribed in a sphere.

TUTORIAL-2

1. Prove that $\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}}\, dx \times \int_0^1 \frac{1}{\sqrt{(1+x^4)}}\, dx = \frac{\pi}{4\sqrt{2}}$.
2. Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$
3. Show that $\int_0^\infty e^{-y^{1/n}}\, dy = n\,\gamma(n)$ hence evaluate $\int_0^\infty e^{-y^4}\, dy$

Unit –II

TUTORIAL-1

1. Evaluate $\iiint dx\, dy\, dz$ over the volume bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$.
2. Evaluate $\int\int xy\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.
3. Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$
4. By changing the order of integration, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx$

TUTORIAL-2

1. Evaluate $\iint r^3\, dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.
2. Evaluate $\iiint z^2\, dx\, dy\, dz\ taken\ over\ the\ volume\ bounded\ by\ surface$ $x^2 + y^2 = Z\ and\ z = 0$
3. **3.** (A) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\ dz\ dy\, dx$

   (B) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz\, dx\, dy\, dz$

UNIT-III

TUTORIAL-1

1. Find $curl\,\overline{f}$ where $\overline{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.
2. Find the directional derivative of the given scalar function $f(x,y,z) = xyz$ at the point $(1,4,3)$ in the direction of the line from $(1,2,3)$ to $(1,-1,-3)$ also calculate the magnitude of the maximum directional derivative.
3. Find the work done in moving a particle in the force field $\bar{F} = 3x^2 i + (2xz - y)\, j + zk$. Along the straight line from (0,0,0) to (2,1,3).
4. Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find Scalar potential.

TUTORIAL-2

1. Find div $\bar{f}$ where $\bar{f} = r^n\bar{r}$ and also find n if it is Solenoidal ?
2. Show that $grad\left(\frac{1}{r}\right) = \frac{-r}{r^3}$.
3. Evaluate ∇[r $\nabla(\frac{1}{r^3})$] = where r = $\sqrt{x^2+y^2+z^2}$.

UNIT-IV

TUTORIAL-1

1. State Green's theorem, Stokes theorem and gauss's Divergence theorem.
2. Apply Green's theorem to evaluate $\oint_C (3x^2-8y^2)dx + (4y-6xy)dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$.
3. Apply Greens theorem to evaluate $\oint_c (2x^2-y^2)\ dx + (y^2+x^2)\ dy$ where c is the boundary of the area enclosed by the x-axis and upper half of the circle $y^2 + x^2 = a^2$.

TUTORIAL-2

1. Use Gauss divergence theorem to evaluate $\iint_S \bar{F}.\ dS$ where $\bar{F} = x^3i + y^3j + z^3k$ and S is the surface of the sphere $x^2+y^2+z^2 = r^2$.
2. Verify stokes theorem for $\bar{F} = x^2\ i\ + x\ yj$ integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = a, y = a$.
3. Verify Gauss divergence theorem for $\bar{F} = x^2$ i + y^2j + z^2k taken over the surface of the solid cut off by the plane $x + y + z = a$ in the first octant.

UNIT-V

TUTORIAL-1

1. Form the partial differential equation by eliminating the arbitrary functions from
$$f(x^2+y^2, z-xy) = 0$$
2. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u where\ \ u(x,0) = 6e^{-3x}$
3. Solve $(mz-ny)p + (nx-lz)q = ly-mx$
4. Solve $(z^2-2yz-y^2)p + (xy+zx)q = xy-zx$

TUTORIAL-2

1. Solve $\frac{\partial u}{\partial x}$=4$\frac{\partial u}{\partial y}$ given that u(0,y) =8e^{-3u}
2. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$
3. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions
$$u(x,0) = 3sin\pi x, u(0,t) = 0, u(1,t) = 0, 0 < x < 1, t > 0.$$
4. Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ \ u(x,0) = 4\ e^{-x}$

20. Known Gaps if any:

- Limits and Continuity
- Maxima and Minima Single Variable Functions

21. Discussion Topics:

- Physical Applications of Maxima and Minima of functions
- Applications of Fourier Series and Transforms in real life

22. References, Journals, Websites and E-links if any:

Reference books:

1. A Text book of Engineering Mathematics, N.P. Bali and Manish Goyal, Laxmi Publications, 10th Edition, 2015.
2. Advanced Engineering Mathematics, H.K. Das, S. Chand and Company Ltd, 21st Edition, 2013.
3. Advanced Engineering Mathematics, Dr. A. B. Mathur and Prof. V.P. Jaggi, Khanna Publishers, 6th Edition, 2019.
4. Advanced Engineering Mathematics, R.K. Jain and S.R.K. Iyengar, Alpha Science International Limited,4thEdition,2013.

Journals:

1. Electronic Journal of Differential Equations
2. Journals of Differential Equations – Elsevier
3. http://www.ams.org/mathscinet/

Websites:

1. www.mathforum.org
2. www.intmath.com
3. www.sosmath.com
4. Mathworld.wolsrem.com
5. https://www.wolframalpha.com/
6. http://nptel.ac.in/
7. http://eqworld.ipmnet.ru/en
8. http://www.efunda.com/
9. http://www.mathalino.com/glossary
10. http://www.mathcentre.ac.uk/

23. Quality Measurement Sheets:

a. Course End Survey

b. Teaching Evaluation

24. Students List:

S.No	Roll Number	Student Name	S.No	Roll Number	Student Name
1	20R11A1201	Mr. ADDAKULA SIDDHARTHA	31	20R11A1231	Miss KODURU MADHU SRI
2	20R11A1202	Mr. ADEPU SAIKRISHNA	32	20R11A1232	Mr. KOLA DEEPAK GOUD
3	20R11A1203	Miss KURAPATI AKHILA	33	20R11A1233	Mr. KONADA SAGAR
4	20R11A1204	Miss AKITI SHRAVYA REDDY	34	20R11A1234	Mr. KORRA REVANTH
5	20R11A1205	Mr. AMANCHA RAHUL	35	20R11A1235	Mr. KULAL RAHUL
6	20R11A1206	Miss ARUKATLA USHA	36	20R11A1236	Miss MAMILLA SWETHA
7	20R11A1207	Mr. BALGONI SRUJAN KUMAR	37	20R11A1237	Mr. MACHERLA AMRUTHAMSH GOUD
8	20R11A1208	Miss BALMOOR DEEPANA	38	20R11A1238	Miss SATHVIKA MEDISETTI
9	20R11A1209	Mr. BHOOMPALLY MANIDEEP REDDY	39	20R11A1239	Mr. MENDRATHI AJAY KUMAR
10	20R11A1210	Miss CH LIKIESHA	40	20R11A1240	Miss METIKOTA KUSUMA
11	20R11A1211	Mr. CHETAN ROOP GUNDAKARAM	41	20R11A1241	Mr. MOHAMMED FERASAT
12	20R11A1212	Mr. CHITHA NAGAJASHWANTH	42	20R11A1242	Mr. MOHAMMED WASEEM TABREZ
13	20R11A1213	Miss DACHEPALLY SAHITHI	43	20R11A1243	Mr. MYADAM SUSHANTH
14	20R11A1214	Miss DAMASANI SNIGDHA SRI	44	20R11A1244	Mr. MYLA RAJESH
15	20R11A1215	Mr. DHARANI KRISHNA CHAITANYA	45	20R11A1245	Miss P VARSHA SREE
16	20R11A1216	Mr. E V ADITYA KRISHNA	46	20R11A1246	Mr. PADIGELA NITHIK
17	20R11A1217	Mr. G ABHINAY REDDY	47	20R11A1247	Mr. PARSI HARSHITH
18	20R11A1218	Mr. G SRUJAN KUMAR	48	20R11A1248	Mr. PEDDI VAMSHI KRISHNA
19	20R11A1219	Mr. GUDURU SIDHARTHA VASHISTA	49	20R11A1249	Mr. POCHABOINA VAMSHI YADAV
20	20R11A1220	Miss GUJJA MADHURI	50	20R11A1250	Mr. PRASHANT PALLAV
21	20R11A1221	Mr. GUJJARI SACHITH	51	20R11A1251	Miss R SHIVYA
22	20R11A1222	Mr. JAGADEESWAR MARELLA	52	20R11A1252	Miss S SHIVAPRIYA
23	20R11A1223	Mr. JAMDADE SWAPNIL	53	20R11A1253	Miss SHAGA SHIVANI
24	20R11A1224	Miss JANGAM SWETHA	54	20R11A1254	Miss SHIVANI GITTI
25	20R11A1225	Miss KANAGALA ABHINAYA	55	20R11A1255	Mr. SOMPALLI TEJESH
26	20R11A1226	Mr. KASARAMA YOUGISH	56	20R11A1256	Mr. SYED KHAWJA MOHIUDDIN
27	20R11A1227	Mr. KASULANATI SAI SHARAVANA SHARMA	57	20R11A1257	Miss TATIKONDA BHARGAVI
28	20R11A1228	Miss KATTA SRINITHYA	58	20R11A1258	Miss THARRE NIKITHA
29	20R11A1229	Miss KESIREDDY AMITHA	59	20R11A1259	Miss THOTA VAISHNAVI
30	20R11A1230	Miss KIRTI PUSHP	60	20R11A1260	Mr. YELICHARLA SAI PRAVEEN

25. Group-wise Students List for Discussion Topics:

Section	Roll No. of the student		Batch
	from	To	
IT	20R11A1201	20R11A1205	I
	20R11A1206	20R11A1210	II
	20R11A1211	20R11A1215	III
	20R11A1216	20R11A1220	IV
	20R11A1221	20R11A1225	V
	20R11A1226	20R11A1230	VI
	20R11A1231	20R11A1235	VII
	20R11A1236	20R11A1240	VIII
	20R11A1241	20R11A1245	IX
	20R11A1246	20R11A1250	X
	20R11A1251	20R11A1255	XI
	20R11A1256	20R11A1260	XII