Ball and Beam System, Part 2

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The open transfer function relating the motor gear angle θ and the ball position a is given as:

$$H(s) = \frac{A(s)}{\Theta(s)} = -\frac{mg\left(\frac{d}{L}\right)}{\left(m + \frac{J}{R^2}\right)s^2}, \tag{1}$$

The parameters used in the transfer function are defined as follows:

- m: Mass of the ball
- g: Acceleration due to gravity
- d: Distance from the motor gear center to the pin joint of the lever arm
- L: Length of the lever arm
- J: Moment of inertia of the ball
- R: Radius of the ball
- s: Laplace transform variable

Parameter Values for the Ball-and-Beam System

We assume suitable values for the parameters used in the transfer function of the ball-and-beam system as follows:

- Mass of the ball, m: $0.0023\,\text{kg}$
- Radius of the ball, R: $0.02\,\text{m}$
- Lever arm offset, d: $0.07\,\text{m}$
- Length of the beam, L: $0.25\,\text{m}$
- Ball's moment of inertia, J:

$$J = \frac{2}{5}mR^2 = \frac{2}{5}(0.0023)(0.02)^2 = 0.000000368\,\text{kg}\cdot\text{m}^2$$

Transfer Function Calculation

The transfer function is given by:

$$H(s) = -\frac{mg\left(\frac{d}{L}\right)}{\left(m + \frac{J}{R^2}\right)s^2} \tag{2}$$

The acceleration due to gravity:

$$g \approx 9.81\,\text{m/s}^2 \tag{3}$$

Substituting the parameter values, the transfer function simplifies to:

$$H(s) \approx \frac{1.96}{s^2} \tag{4}$$

Transfer Function

Given the transfer function:

$$H(s) = \frac{1.96}{s^2} \tag{5}$$

Task 6: Root Locus and Bode Plot of Ball-Beam System

6.1 Using MATLAB, Plot the Following for the System's Open-Loop Transfer Function

The open-loop transfer function of the ball-beam system is given as:

$$H(s) = \frac{1.96}{s^2}. \tag{6}$$

A. Root Locus

Using MATLAB, the following plots are generated for the open-loop transfer function:

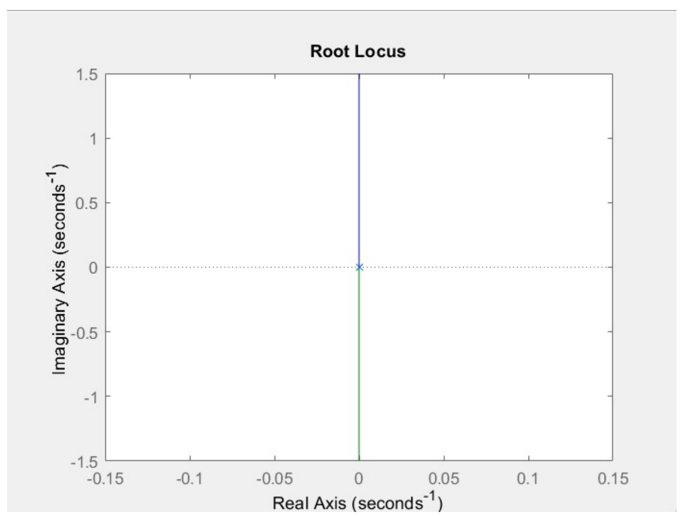

Fig. 1: Root Locus

Comment on the system's stability.

- The system has two poles at the origin, indicating **unstable**.

B. Bode Plot

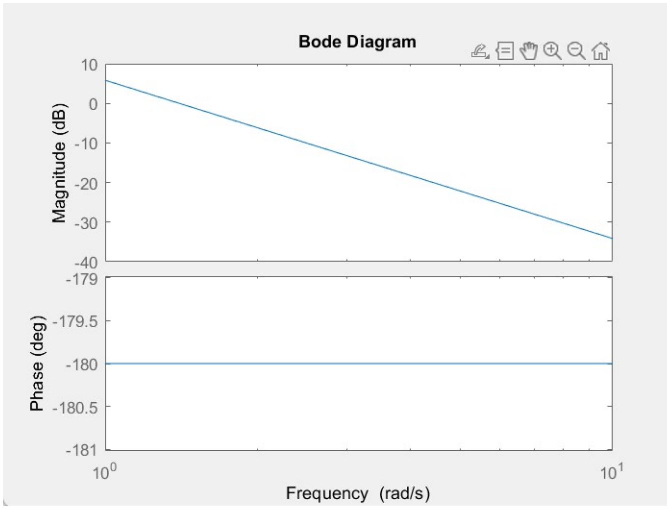

Fig. 2: Bode Plot

```
Gain Margin: 1 dB
Phase Margin: 0 degrees
```

Fig. 3: Phase Margin and Gain Margin

C. Task 6.2

Design Criteria

The design criteria are as follows:

- Less than 5% overshoot.
- Settling time less than 3 seconds within a 2% tolerance band.

ROOT LOCUS

The design requirements on the root locus observed using the `sgrid` command. This command creates a grid that shows constant damping ratio ζ and natural frequency ω_n. These values are calculated using the following equations, based on the maximum overshoot (Mp) and settling time (T_s):

$$Mp = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$T_s = \frac{4}{\zeta\omega_n}$$

Since our our design criteria is settling time (T_s) less than 3 seconds and overshoot (Mp) less than 5

By soving above equations with respect to our design criteria, we will get

$$\zeta \approx 0.7, \quad \omega_n \approx 1.905\,\text{rad/s}$$

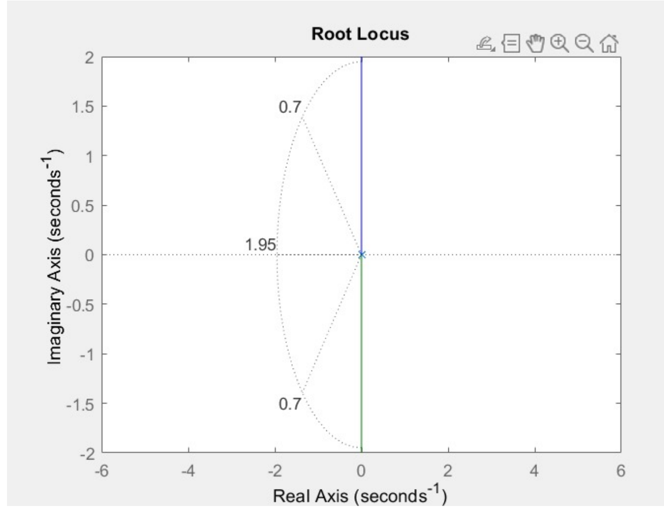

Fig. 4: Design criteria

From the above graph The region between the two dotted diagonal lines shows where the percent overshoot is less than 5%. The area outside the curved line represents locations where the settling time is under 3 seconds. However, no part of the plot meets the design criteria indicated by these lines. To fix this and move the root locus into the left-hand plane for stability, we can recommend the **lead compensator** to achieve the desired performance.

BODE PLOT

- From the Fig.3 (Bode plot) shows that the phase margin is zero, indicating that our system is unstable.
- The phase margin tells us how much the open-loop phase shift needs to change to make the closed-loop system unstable. If the phase margin is zero, it means the system is unstable.
- To make the system stable, we need to increase the phase margin. A **lead compensator** recommended to achieve the desired performance.

TASK 7: CONTROLLER DESIGN USING ROOT LOCUS AND BODE PLOT APPROACHES

I. ROOT LOCUS OF THE SYSTEM WITH THE COMPENSATOR.

A first-order lead compensator moves the root locus towards the left-hand side of the s-plane. The general form of a lead compensator is:

$$G_c(s) = K_c\frac{(s+z_0)}{(s+p_0)}, \tag{7}$$

where $|z_0| < |p_0|$, and the magnitude of z_0 is less than the magnitude of p_0. We designed the plant with the controller and plotted the root locus. To achieve this, we place the zero z_0 close to the origin to effectively cancel a pole. To pull the root locus further into the left-hand plane, the pole p_0 of the compensator is located to the left of the origin. Considering the following values for z_0 and p_0:

$$z_0 = 0.02, \quad p_0 = 5$$

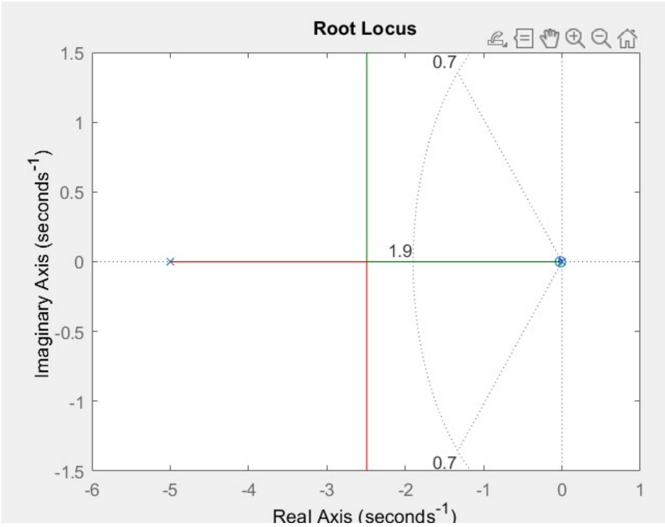

Fig. 5: Root locus design

Now to get gain, we need to select a point in the graphics window ,then we will get

```
selected_point =

  -3.2093 + 0.5806i


k =

    3.1515


poles =

   -2.8415
   -2.1382
   -0.0203
```

Fig. 6: Gain selection

II. Bode plot of the system with the compensator.

A phase-lead compensator is a first-order system that can be written as:

$$C(s) = K\left(\frac{1+Ts}{1+aTs}\right)$$

where T and a are design parameters.

The phase-lead compensator adds a positive phase to the system in the frequency range between $\frac{1}{aT}$ and $\frac{1}{T}$, which are called the corner frequencies. The maximum phase that a single phase-lead compensator can provide is 90 degrees.

In this design, we aim for a percent overshoot of less than 5, which corresponds to a damping ratio $\zeta = 0.7$. Typically, 100ζ gives the minimum phase margin required to achieve the desired overshoot. Therefore, we need a phase margin greater than 70 degrees.

To design the compensator, we follow these steps:

1) **The required phase margin:** We need the compensator to provide at least 80 degrees of phase margin.
2) **The center frequency:** The compensator adds phase around a center frequency, which is close to the system's bandwidth frequency ω_{bw}. Using the relation between bandwidth frequency and settling time, we approximate $\omega_{bw} \approx 1.92$ rad/s. Therefore, we select the center frequency to be close to 1 rad/s.
3) **The constant a:** The value of a determines the spacing between the compensator's zero and pole to achieve the desired phase margin. It is calculated using the following formula:

$$a = \frac{1-\sin\phi}{1+\sin\phi}$$

Determining T and aT

The parameters T and aT can be calculated using the following equations:

$$T = \frac{1}{\omega\sqrt{a}}$$

$$aT = \frac{\sqrt{a}}{\omega}$$

Here, ω is the center frequency, and a is the parameter calculated from the desired phase margin.
For a phase margin of 70 degrees and a center frequency $\omega = 1$ rad/s, the calculated values are:

$$aT = 0.0875, \quad T = 11.43$$

where ϕ is the desired phase margin. For a phase margin of 80 degrees, a is approximately 0.00766.
We can now add our lead controller to the system and observe the bode plot.

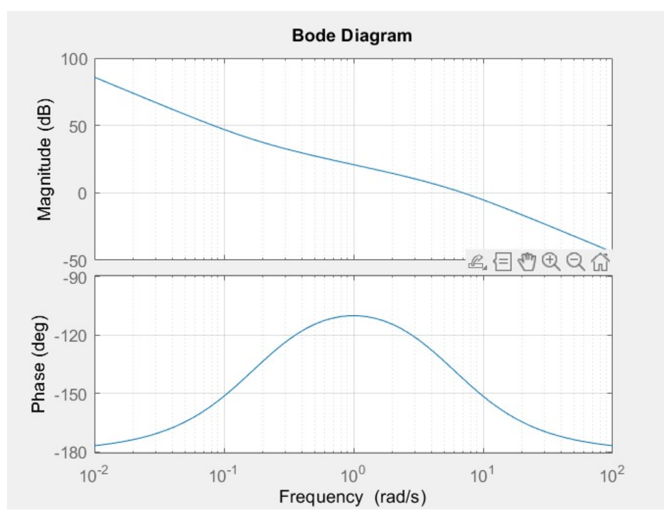

Fig. 7: Bode Diagram

From the above Bode Plot, we can observe that the phase margin is around 70 degrees

III. Pole-Zero Placement Strategy

The pole-zero placement strategy plays a critical role in both the root locus and Bode plot of a control system. Below is a discussion of the strategy and its effects:

IV. Pole-Zero Placement Strategy

The pole-zero placement in compensator design significantly impacts both the root locus and the Bode plot:

- **Root Locus:** Placing the compensator zero close to a plant pole cancels its effect, moving the root locus toward the left-hand side of the s-plane. The compensator pole, located farther left, pulls the locus further left, improving system stability and response speed.
- **Bode Plot:** The compensator zero adds positive phase over a specific frequency range, enhancing phase margin. The compensator pole limits this phase boost and ensures proper high-frequency roll-off, maintaining system stability.

Thus, careful placement of the compensator pole and zero balances stability, transient response, and frequency performance.

V. System's closed-loop response (with the compensator) for a step input

A. Root locus

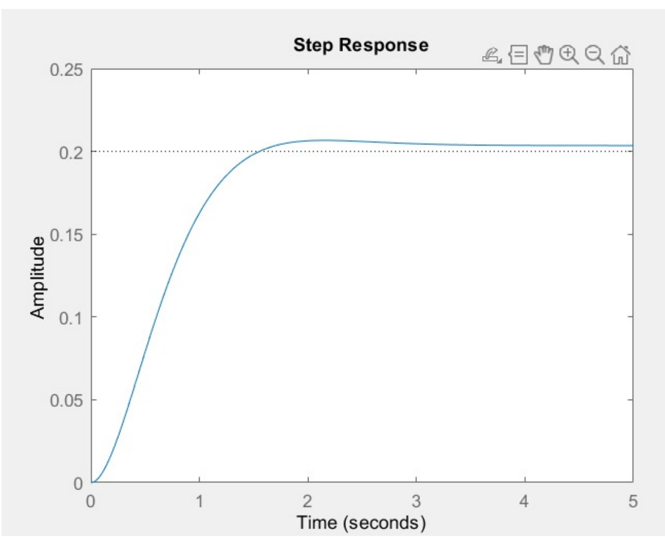

Fig. 8: Closed Loop response

This plot shows that when the system receives a 0.20-m step input, almost both the settling time and percent overshoot design conditions are met.

B. Frequency Response

After many trails Our system met required design critera for

phase margin = 82

w = 2.5,

K = 1

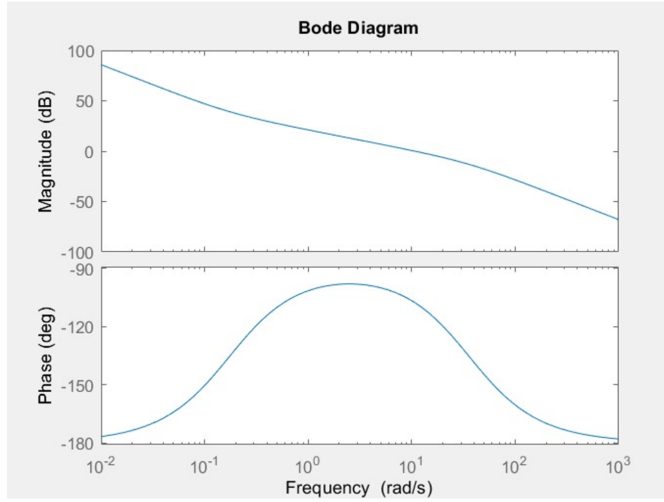

Fig. 9: Bode Diagram

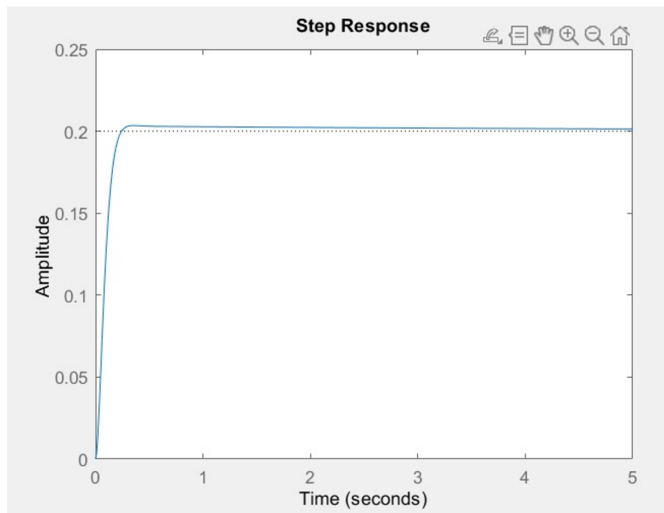

Fig. 10: Frequency response for a step input

This plot shows that when the system receives a 0.20-m step input, almost both the settling time and percent overshoot design conditions are met.