N3

$u_{tt} = u_{xx} + (t^2 - x)^2$

$u|_{t=0} = 2 \quad u_t|_{t=0} = e^x \sin x$

$u|_{x=0} = t \quad (u_x - hu_x)|_{x=l} = -1$

$w(x,t) = t + \frac{-1+ht}{1-lh} x$

$v_{tt} = v_{xx} + x^2 + t^4 - 2t^2 x$

$v|_{x=0} = 0 \qquad (v_x - hv)|_{x=l} = 0$

$v_{t=0} = 2 + \frac{x}{1-lh}$

$v_t|_{t=0} = e^x \sin x$

$X'' + \lambda^2 X = 0$

при $\lambda = 0 \quad X = C_1 x + C_2 \quad C_1 = C_2 = 0 \Rightarrow$ нет

нетрив реш.

$\lambda \neq 0$

$X = A\cos\lambda x + B\sin\lambda x$

$A = 0 \qquad \operatorname{tg}\lambda_n l = \frac{\lambda_n}{h}$

$X_n = \sin\lambda_n x \qquad \|X_n\|^2 = \frac{l}{2} - \sin\frac{2\lambda_n l}{4\lambda_n}$

$\sum T_n''(t)\sin\lambda_n x + \sum \lambda_n T_n(t)\sin\lambda_n x$

$+ \sum (f_n - 2t^2 g_n + h_n t^4)\sin\lambda_n x$

$f_n = \frac{1}{\|X_n\|^2}\int_0^l x^2 \sin\lambda_n x\,dx = \frac{1}{\|X_n\|^2}\left(\frac{(2-\lambda_n^2 l^2)(-1)^n - 2}{\lambda_n^3}\right)$

$g_n = \frac{1}{\|X_n\|^2}\int_0^l x\sin\lambda_n x\,dx = \frac{1}{\|X_n\|^2}\frac{(-1)^{n+1} l}{\lambda_n}$

$h_n = \frac{1}{\|X_n\|^2}\left(\frac{(-1)^{n+1} + 1}{\lambda_n}\right)$

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$$T_n'' + \lambda_n^2 T_n = f_n - 2t^2 g_n + h_n t^4$$

$$T_n = A_n \cos\lambda_n t + B_n \sin\lambda_n t + f_n/\lambda_n$$

$$-\frac{2g_n t^2}{\lambda_n} + \frac{h_n t^4}{\lambda_n} + \frac{4g_n}{\lambda_n^2} - \frac{12h_n t^2}{\lambda_n^2} + \frac{24h_n}{\lambda_n^3}$$

$$T_n(0) = 2 + \frac{x}{1-e^h} = 2h_n + \frac{g_n}{1-e^h} = A_n + \frac{f_n}{\lambda_n} + \frac{4g_n}{\lambda_n^2} + \frac{24h_n}{\lambda_n^3}$$

$$A_n = h_n\left(2 - \frac{24}{\lambda_n^3}\right) + g_n\left(\frac{1}{1-e^h} - \frac{4}{\lambda_n^2}\right) - \frac{f_n}{\lambda_n}$$

$$T_n'(0) = \lambda_n B_n = \frac{1}{\|X_n\|^2}\int_0^l e^x \sin x \sin\lambda_n x \, dx$$

$$U_n = T_n(t) \cdot X_n(x)$$

№4

$u_{tt} = u_{xx} + t^3 + x^4 t^2 + e^x$

$u|_{t=0} = x^2 \sin x \qquad u_t|_{t=0} = \cos x$

$u|_{x=0} = t^2 \qquad u_x|_{x=l} = 1$

$u = v + w$

$w(x,t) = t^2 + x$

$v_{tt} = v_{xx} + t^3 + x^4 t^2 + e^x + 2$

$v|_{x=0} = 0 \; ; \; v_x|_{x=l} = 0$

$v|_{t=0} = x^2 \sin x - x \qquad v_t|_{t=0} = \cos x$

$X'' + \lambda^2 X = 0$

• $\lambda = 0 \quad X = C_1 x + C_2; \; C_1 = C_2 = 0$ нет нетрив реш.

• $\lambda \neq 0$

$X = A\cos \lambda x + B \sin \lambda x$

$A = 0 \qquad \lambda B \cos \lambda l = 0$

$\lambda l = \frac{\pi}{2} + \pi n$

$\lambda_n = \frac{\pi(1+2n)}{l}$

$X_n = \sin \lambda_n x$

$\|X_n\|^2 = l/2$

$\sum T_n^{4}(t) \sin \lambda_n x + \sum \lambda_n^2 T_n(t) \sin \lambda_n x =$

$= \sum (f_n t^2 + g_n (t^3 - 2) + h_n) \sin \lambda_n x$

$$f_n = \frac{2}{l}\left(\frac{(-1)^n(-\lambda_n^4 l^4 + 12\lambda_n^2 l_2 - 24) + 25}{\lambda_n^5}\right)$$

$$g_n = \frac{2}{l}\left(\frac{(-1)^{n+1} + 1}{\lambda_n}\right)$$

$$h_n = -\frac{2}{l}\left(\frac{e^l \lambda_n (-1)^n \lambda_n}{\lambda_n^2 + 1}\right)$$

$T_n^{4} + \lambda_n^2 T_n = f_n t^2 + g_n (t^3 - 2) + h_n$

$$T_n(t) = A_n \cos \lambda_n t + B_n \sin \lambda_n t + \frac{f_n t^2}{\lambda_n} + \frac{g_n t^3}{\lambda_n} - \frac{2 g_n}{\lambda_n} + \frac{h_n}{\lambda_n} - \frac{2 f_n}{\lambda_n^2} - \frac{6 g_n t}{\lambda_n^2}$$

$$T_n(0) = A_n - \frac{2g_n}{\lambda_n} + \frac{h_n}{\lambda_n} - \frac{2f_n}{\lambda_n^2} = \frac{2}{l}\int_0^l (x^2 \sin x - x)\cdot$$

$$\cdot \sin \lambda_n x \, dx$$

$$A_n = \int_0^l \frac{2}{l}(x^2 \sin x - x)\sin \lambda_n x \, dx + \frac{2g_n}{\lambda_n} - \frac{h_n}{\lambda_n} + \frac{2f_n}{\lambda_n^2}$$

$$T_n'(0) = \lambda_n B - \frac{6g_n}{\lambda_n^2} = \frac{2}{l}\int_0^l \cos x \sin \lambda_n x \, dx$$

$$B_n = \left(\frac{6g_n}{\lambda_n^2} + \frac{2}{l}\int_0^l \cos x \sin \lambda_n x \, dx\right)\frac{1}{\lambda_n}$$

N2

$u_{tt} = u_{xx} + x(2+t) + x^2(t^2+t+e^t)$

$u_{t=0} = x \qquad u_t|_{t=0} = 0$

$(u_x - hu)|_{x=l} = t \qquad u|_{x=0} = 1-t$

$u = v + w \qquad w = (1-t) + x\frac{t+h(1-t)}{1-lh}$

$v_{tt} = v_{xx} + x(2+t) + x^2(t^2+t+e^t)$

$v|_{t=0} = x - 1 + \frac{h}{1-lh}x \qquad v_t|_{t=0} = 0$

$X'' + \lambda^2 X = 0$

- $\lambda = 0 \quad X = C_1 X + C_2 \quad C_1 = 0 = C_2$ нет нетр. реш
- $\lambda \neq 0 \quad X = A\cos\lambda x + B\sin\lambda x$

$X(0) = A = 0 \qquad (\lambda B\cos\lambda x - hB\sin\lambda x)|_{x=l} = 0$

$tg\lambda_n l = \frac{\lambda_n}{h} \qquad X_n = \sin\lambda_n x$

$\|X_n\|^2 = \int_0^l \sin^2\lambda_n x\, dx = l/2 - \sin\frac{2\lambda_n l}{4\lambda_n}$

$\sum T_n''(t)\sin\lambda_n x + \sum \lambda_n^2 T_n \sin\lambda_n x =$

$= \sum (f_n(2+t) + g_n(t^2+t+e^t))\sin\lambda_n x$

$$f_n = \frac{(-1)^{n+1} l}{\lambda_n \|X_n\|^2}$$

$$g_n = \frac{1}{\|X_n\|^2} \left(\frac{(2-\lambda_n^2 l^2)(-1)^n - 2}{\lambda_n^3} \right)$$

$$T_n = A_n \cos \lambda_n t + B_n \sin \lambda_n t + \frac{g_n e^t}{(\lambda_n^2+1)} + \frac{f_n t}{\lambda_n+1} + \frac{g_n t^2}{\lambda_n^2} + \frac{2 f_n}{\lambda_n^2} + \frac{g_n t}{\lambda_n^2} - \frac{2 g_n}{\lambda_n^2}$$

$$T_n(0) = A_n + \frac{g_n}{\lambda_n^2+1} + \frac{2 f_n}{\lambda_n^2} - \frac{2 g_n}{\lambda_n^2}$$

$$A_n = g_n\left(\frac{2}{\lambda_n^2} - \frac{1}{\lambda_n^2+1}\right) - f_n\left(1 + \frac{h}{1-l_h} - \frac{2}{\lambda_n^2}\right) - 1$$

$$T_n'(0) = 0 = \lambda_n B_n + \frac{g_n}{\lambda_n^2+1} + \frac{3 f_n}{\lambda_n^2} + \frac{g_n}{\lambda_n^2}$$

$$B_n = -\frac{1}{\lambda_n}\left(g_n\left(\frac{1}{\lambda_n^2+1} + \frac{1}{\lambda_n^2}\right) + 3/\lambda_n^2 f_n\right)$$

N5

$u_{tt} = u_{xx} + (x+t)^4$

$u|_{t=0} = x^2+1$; $u_t|_{t=0} = x$; $u|_{x=0} = 0$ $u_x|_{x=l} = 0$

$w = 1+t+x-2tx$

$v_{tt} = v_{xx} + (x+t)^4$

$v_{x=0} = 0$ $v_x|_{x=l} = 0$ $v|_{t=0} = x^2 - x$ $v_t|_{t=0} = x$

• $\lambda = 0$ $C_1 = C_2 = 0$ ⇒ нет нетрив реш

• $\lambda \neq 0$

$X = A\cos\lambda_n x + B\sin\lambda_n x$

$X_n = \sin\lambda_n x$ $\lambda_n = \frac{\pi(1+2n)}{2l}$

$\|X_n\|^2 = l/2$

$\sum T_n''(t)\sin\lambda_n x + \sum \lambda_n^2 T_n(t)\sin\lambda_n x +$

$+ \sum (f_n + 4t\, g_n + 6t^2 h_n + 4t^3 z_n + t^4 r_n)\sin\lambda_n x$

$$T_n(0) = \frac{2}{l}\int_0^l (x^2+1)\sin\lambda_n x\,dx = K_1$$

$$T_n'(0) = \frac{2}{l}\int_0^l x\sin\lambda_n x\,dx = K_2$$

$$T_n'' + \lambda_n^2 T_n = f_n + 4t g_n + 6t^2 h_n + 4t^3 z_n + t^4 r_n$$

$$T_n = A_n\cos\lambda_n t + B_n\sin\lambda_n t +$$

$$+ \frac{1}{\lambda_n}\left(f_n + 4g_n t + 6h_n t^2 + r_n t^4 + 4t^3 z_n\right)$$

$$- \frac{1}{\lambda_n^2}\left(12h_n + 12r_n t^2 + 24 t z_n - \frac{24 r_n}{\lambda_n}\right)$$

$$A_n = K_1 - \frac{f_n}{\lambda_n} + \frac{12h_n}{\lambda_n^2} + \frac{24 r_n}{\lambda_n^3}$$

$$B_n = -\frac{4g_n}{\lambda_n^2} + \frac{24 z_n}{\lambda_n^3} + K_2$$