Лабораторная 5
Вариант 3

N6

$u_t = a^2 u_{xx} + xe^t$

$u_x(0,t) + hu(0,t) = 0$

$u_x(l,t) - hu(l,t) = t$

$u(x,0) = x$

$u = v + w$

$w = A(t)x^2 + B(t)x + C(t)$

$w_x = 2A(t)x + B(t)$

$0 = (u_x + hu)\big|_{x=0} - (w_x + hw)\big|_{x=0} = 0 - (B(t) + hC(t)) = 0$

$0 = (u_x - hu)\big|_{x=l} - (w_x - hw)\big|_{x=l} = t - (2A(t)l + B(t)$
$- h(A(t)l^2 + B(t)l + C(t))) = 0$

$A(t) = 0 \qquad B(t) = \dfrac{t}{2 - lh} \qquad C(t) = \dfrac{t}{h(2-lh)}$

$w = x\dfrac{t}{2-lh} + \dfrac{t}{h(2-lh)}$

$$\upsilon_t = a^2 \upsilon_{xx} + xe^t - \left(\frac{x}{2-lh} + \frac{1}{h(2-lh)}\right) = a^2 \upsilon_{xx} +$$

$$+ xe^t + \frac{(x+1)}{lh-2}$$

$$\upsilon|_{t=0} = x$$

при $\lambda \neq 0$

при $\lambda = 0$

$C_1 = C_2 = 0$

не $\exists$ нетрив реш

$$X = A\cos\lambda x + B\sin\lambda x$$

$$X' = -\lambda A\sin\lambda x + \lambda B\cos\lambda x$$

$$X'(0) + hX(0) = \lambda B + hA = 0$$

$$X'(l) - hX(l) = -\lambda A\sin\lambda l + \lambda B\cos\lambda l - hA\cos\lambda l - hB\sin\lambda l$$

$$B = -\frac{hA}{\lambda} \qquad A = -\lambda \quad B = h$$

$$-\lambda A\sin\lambda l - hA\cos\lambda l - hA\cos\lambda l + \frac{h^2A}{\lambda}\sin l\lambda = 0$$

$$A\left(\left(-\lambda + \frac{h^2}{\lambda}\right)\sin\lambda l - 2\cos\lambda l\right) = 0$$

$$\operatorname{tg}\lambda_n l = \frac{2\lambda_n}{h^2 - \lambda_n^2}$$

$$X_n = -\lambda_n\cos\lambda_n x + h\sin\lambda_n x$$

$$\|X_n\|^2 = \int_0^l (-\lambda_n\cos\lambda_n x + h\sin\lambda_n x)^2 dx =$$

$$= \lambda_n^2\left(\frac{\sin 2\lambda_n l}{4\lambda_n} + \frac{l}{2}\right) + h^2\left(\frac{l}{2} + \frac{\sin 2\lambda_n l}{4\lambda_n}\right) +$$

$$+ h(\cos 2\lambda_n l - 1)$$

$$v = v_1^{,\text{неодн}} + v_2^{,\text{одн}}$$

• v_1

$$\sum T_n'(t) X_n(x) = -a^2 \sum \lambda_n^2 T_n X^{n} +$$

$$+ \sum \left(\left(e^t + \frac{1}{(\ell h - 2)} \right) f_n + \left(\frac{1}{\ell h - 2} \right) g_n \right) X_n(x)$$

$$f_n = \frac{1}{\|X_n\|^2} \int_0^\ell x(-\lambda_n \cos \lambda_n x + h \sin \lambda_n x)\, dx$$

$$g_n = \frac{1}{\|X_n\|^2} \int_0^\ell (-\lambda_n \cos \lambda_n x + h \sin \lambda_n x)\, dx$$

$$f_n = \frac{1}{\|X_n\|^2} \left(\frac{h(\sin \lambda_n \ell - \lambda_n \ell \cos \lambda_n \ell)}{\lambda_n^2} - \frac{(\lambda_n \ell \sin \lambda_n \ell + \cos \lambda_n \ell) + 1}{\lambda_n} \right)$$

$$g_n = \frac{1}{\|X_n\|^2} \left(-\sin \lambda_n \ell - \frac{h}{\lambda_n} \cos \lambda_n \ell + \frac{h}{\lambda_n} \right)$$

$$\begin{cases} T_n' + a^2 \lambda_n^2 T_n = \left(e^t + \frac{1}{\ell h - 2} \right) f_n + \frac{1}{\ell h - 2} g_n \\ T_n(0) = 0 \end{cases}$$

$$T_n = \tilde{A}_n e^{-a^2 \lambda_n^2 t} + \frac{(f_n + g_n)}{(\ell h - 2) a^2 \lambda_n^2} + \frac{f_n e^t}{(a^2 \lambda_n^2 + 1)}$$

$$\tilde{A}_n = -\frac{(f_n + g_n)}{(\ell h - 2) a^2 \lambda_n^2} - \frac{f_n}{a^2 \lambda_n^2 + 1}$$

$\bullet\, v_2$

$T_n' + a^2 \lambda_n^2 T_n = 0$

$T_n(0) = x$

$T_n = A_n e^{-a^2 \lambda_n t}$

$A_n = \frac{1}{\|X_n\|^2} \int_0^l x(-\lambda_n \cos\lambda_n x + h \sin\lambda_n x)\, dx$

$u = v_1 + v_2 + w$

N8

$$u_t = a^2 u_{xx} + 3t$$

$$u_x - hu\big|_{x=0} = 0$$

$$u\big|_{x=l} = t^3$$

$$u\big|_{t=0} = 1$$

$$w = A(t)x^2 + B(t)x + C(t)$$

$$0 = 0 - (B(t) - hC(t)) = 0$$

$$0 = t^3 - A(t)l^2 - B(t)l - C(t)$$

$$A(t) = 0 \qquad C(t) = \frac{t^3}{lh+1} \qquad B(t) = \frac{ht^3}{lh+1}$$

$$w = \frac{ht^3}{lh+1}x + \frac{t^3}{lh+1}$$

$$v_t = a^2 v_{xx} + 3t - \left(\frac{3t^2}{hl+1}(hx+1)\right)$$

$$v\big|_{t=0} = 1$$

$\lambda = 0$ $\quad C_1 = C_2 = 0$ нет нетрив реш.

$\lambda \neq 0$

$X = A\cos\lambda x + B\sin\lambda x$

$\lambda B - hA = 0$

$B = \frac{hA}{\lambda} \quad A = \lambda \quad B = h$

$\lambda + h\,tg\lambda l = 0 \quad tg\lambda_n l = \frac{-\lambda_n}{h}$

$X_n = \lambda_n \cos\lambda_n x + h\sin\lambda_n x$

$v = v_1 + v_2$

- v_1

$\sum v_n' X_n = -a^2 \sum v_n \lambda_n^2 X^n + (3t - \frac{3t}{hl+1})$

$\cdot \sum f_n X_n - \frac{3t^2 h}{hl+1} \sum g_n X_n$

$f_n = \frac{1}{\|X_n\|^2} \int_0^l (\lambda_n \cos\lambda_n x + h\sin\lambda_n x) =$

$= \frac{1}{\|X_n\|^2} (\sin\lambda_n l - \frac{h}{\lambda_n}\cos\lambda_n l + \frac{h}{\lambda_n})$

$g_n = \frac{1}{\|X_n\|^2} \int_0 x(\lambda_n \cos\lambda_n x + h\sin\lambda_n x)\,dx =$

$= \frac{1}{\|X_n\|^2} \left(\frac{h(\sin\lambda_n l - \lambda_n l\cos\lambda_n l)}{\lambda_n^2} + \frac{(\lambda_n l\sin\lambda_n l + \cos\lambda_n l) - 1}{\lambda_n} \right)$

$$T_n' + a^2\lambda^2 T_n = \left(3t - \frac{3t^2}{hl+1}\right) f_n - \frac{3t^2 h}{lh+1} q_n$$

$$T_n(0) = 0$$

$$T_n = \tilde{A}_n e^{-a^2\lambda_n^2 t} - \frac{3f_n t^2}{a^2\lambda_n^2(hl+1)} + \frac{3f_n t}{a^2\lambda_n^2}$$

$$- \frac{3q_n h t^2}{(hl+1)a^2\lambda_n^2} + \frac{6f_n t}{(hl+1)a^4\lambda_n^4} - \frac{3f_n}{a^4\lambda_n^4} + \frac{6hq_n t}{a^4\lambda_n^4(hl+1)}$$

$$- \frac{6f_n}{(hl+1)a^6\lambda_n^6} - \frac{6hq_n}{(hl+1)a^6\lambda_n^6}$$

$$\tilde{A}_n = \frac{3t^2}{a^2\lambda_n^2(hl+1)}(f_n + hq_n) - \frac{6(f_n+hq_n)}{a^4\lambda_n^4(hl+1)} + \frac{6(f_n+hq_n)}{(hl+1)a^6\lambda_n^6}$$

$$+ \frac{8f_n}{a^4\lambda_n^4} - \frac{3f_n t}{a^2\lambda_n^2}$$

- v_2

$$T_n' + a^2\lambda_n^2 T = 0$$

$$T_n = A_n e^{-a^2\lambda_n^2 t}$$

$$A_n = f_n$$

$$u = v_1 + v_2 + w$$

№14

$u_t = a^2 u_{xx} + x\sin t$

$u_x|_{x=0} = t^2$

$u_x|_{x=l} = t$

$u|_{t=0} = 3x+4$

$u = v + w$

$w(x,t) = xt^2 + \frac{x^2}{2l}(t-t^2)$ (как в модели)

$v_t = a^2 v_{xx} + x\sin t - (2xt + \frac{x^2}{2l}(1-2t) - \frac{a^2}{l}(t-t^2))$

$v_{t=0} = 3x+4$

• при $\lambda = 0$

$C_1 = 0 \quad C_2 = \forall$

∃ нетрив. реш.

• $\lambda \neq 0$

$X = A\cos\lambda x + B\sin\lambda x$

$\lambda_n = \frac{\pi n}{l}$

$X_n = \cos\lambda_n x$

$\|X_n\|^2 = l/2$

$v = v_1 + v_2$

• v_1

$\sum T_n' X_n = -a^2 \sum \lambda_n^2 T_n X_n - \frac{(1-2t)}{2l} \sum f_n X_n$

$+ (\sin t - 2t) \sum g_n X_n + \frac{a^2}{l}(t-t^2) \sum h_n \cos\lambda_n x$

$$f_n = \frac{2}{l}\int_0^l x^2 \cos\lambda_n x\, dx = \frac{2}{l}\cdot\frac{2\cdot l(-1)^n}{\lambda_n^2} = \frac{4(-1)^n}{\lambda_n^2}$$

$$g_n = 2/l \int_0^l x\cos\lambda_n x\, dx = \frac{2((-1)^n-1)}{l\lambda_n^2}$$

$$h_n = 0$$

$$T_n = \tilde{A}_n e^{-a^2\lambda_n^2 t} + \frac{a^2\lambda_n^2 f_n t}{l(a^4\lambda_n^4+1)} - \frac{a^2\lambda_n^2 f_n}{2l(a^4\lambda_n^4+1)}$$

$$-\frac{2a^2\lambda_n^2 g_n t}{a^4\lambda_n^4+1} + \frac{a^2\lambda_n^4 g_n \sin t}{a^4\lambda_n^4+1} - \frac{f_n}{(a^4\lambda_n^4+1)l} + \frac{2g_n}{(a^4\lambda_n^4+1)} +$$

$$+\frac{f_n t}{l(a^6\lambda_n^6+a^2\lambda_n^2)} - \frac{f_n}{2l(a^6\lambda_n^6+a^2\lambda_n^2)} - \frac{2g_n t}{a^6\lambda_n^6+a^2\lambda_n^2} - \frac{f_n}{l(a^6\lambda_n^6+a^4\lambda_n^4)} + \frac{2g_n}{a^2\lambda_n^2+a^4\lambda_n^4}$$

$$\tilde{A}_n = v_n(0)$$

- v_2

$$T_n' + a^2\lambda_n^2 T_n = 0$$

$$v_n = A_n e^{-a^2\lambda_n^2 t}$$

$$A_n = 2/l \int_0^l (3x+4)\cos\lambda_n x\, dx = \frac{2}{l}\left(\frac{3(-1)^n+1}{\lambda_n^2}\right)$$

$$u = v_1 + v_2 + w$$

N15

$U_t = a^2 U_{xx} + e^{xt}$

$U_x|_{x=0} = 1 \qquad U|_{x=l} = 2$

$U_{t=0} = 3$

$U = V + W$

$W = 2 + (x - l)$

$V_t = a^2 V_{xx} + e^{xt}$

$V_{t=0} = 1 - x + l$

при $\lambda \neq 0$ нет нетрив реш.

$\lambda \neq 0$

$X = A\cos\lambda x + B\sin\lambda x$

$X'(0) = \lambda B = 0 \qquad B = 0$

$X(l) = A\cos\lambda l = 0 \qquad \lambda_n = \frac{\pi(2n+1)}{2l}$

$X_n = \cos\lambda_n x$

$\|X_n\|^2 = l/2$

$v = v_1 + v_2$

• v_1

$\sum T' X = -a^2 \sum \lambda^2 T_n X^n + \sum f_n \cos\lambda_n x$

$$f_n = \frac{2}{l}\int_0^l e^{xt}\cos\lambda_n x\,dx = \frac{2}{l}\left(\frac{e^{tl}(\lambda_n(-1)^n - t)}{t^2+\lambda_n^2}\right)$$

$T_n' + a^2\lambda_n^2 T_n = f_n$

$T_n = \tilde{A}_n e^{-a^2\lambda_n^2 t} + T_{ч.н}$

$\tilde{A}_n = -T_{ч.н.}(0)$ из условие $T_n(0) = 0$

• v_2

$T_n' + a^2\lambda_n^2 T_n = 0$

$T_n(0) = 1 - x + l$

$T_n = A_n e^{-a^2\lambda_n t}$ $\quad A_n = 2/l\int_0^l (1-x+l)\cos\lambda_n x\,dx =$

$= -\frac{2}{l\lambda_n^2}(\lambda_n(-1)^n - 1)$

$u = v_1 + v_2 + w$