1 Introduction

A 3D seismic device measures the three components of an acceleration vector in 3 specific directions attached to the device. These 3 directions form a trihedron almost orthogonal but not exactly. We denote $v_{u,1}$, $v_{u,2}$, $v_{u,3}$ the unitary vectors of these 3 directions of the seismic device under test (SUT), and $v_{r,1}$, $v_{r,2}$, $v_{r,3}$ those of the reference system (SREF). It follows that the 3 signals on SUT write:

$$\left\{\begin{array}{rcl} x_{u,1}(t) & = & h_{u,1}(t)\star(v_{u,1,1}g_x(t)+v_{u,1,2}g_y(t)+v_{u,1,3}g_z(t)) \\ x_{u,2}(t) & = & h_{u,2}(t)\star(v_{u,2,1}g_x(t)+v_{u,2,2}g_y(t)+v_{u,2,3}g_z(t)) \\ x_{u,3}(t) & = & h_{u,3}(t)\star(v_{u,3,1}g_x(t)+v_{u,3,2}g_y(t)+v_{u,3,3}g_z(t)) \end{array}\right.$$

and similar equations for the SREF. In frequency domain we get:

$$\left\{\begin{array}{rcl} X_{u,1}(f) & = & H_{u,1}(f)(v_{u,1,1}G_x(f)+v_{u,1,2}G_y(f)+v_{u,1,3}G_z(f)) \\ X_{u,2}(f) & = & H_{u,2}(f)(v_{u,2,1}G_x(f)+v_{u,2,2}G_y(f)+v_{u,2,3}G_z(f)) \\ X_{u,3}(f) & = & H_{u,3}(f)(v_{u,3,1}G_x(f)+v_{u,3,2}G_y(f)+v_{u,3,3}G_z(f)) \end{array}\right.$$

More concisely we write

$$X_u(f) \;=\; H_u(f)V_uG(f)$$

where

$$H_u(f) \;=\; \begin{bmatrix} H_{u,1}(f) & 0 & 0 \\ 0 & H_{u,2}(f) & 0 \\ 0 & 0 & H_{u,3}(f) \end{bmatrix}$$

and where V_u is the square matrix whose entries are $v_{u,k}$. Because the columns of V_u are normalized, V_u depends on 6 free parameters. A possible parametrization is:

$$V_u(\phi) \;=\; \begin{bmatrix} \cos(a_1)\cos(e_1) & \sin(a_1)\cos(e_1) & \sin(e_1) \\ \cos(a_2)\cos(e_2) & \sin(a_2)\cos(e_2) & \sin(e_2) \\ \cos(a_3)\cos(e_3) & \sin(a_3)\cos(e_3) & \sin(e_3) \end{bmatrix} \tag{1}$$

In our protocol, the reference system (SREF) is a portable seismic device whose the trihedron is perfectly known. Therefore we have in the absence of noise:

$$\left\{\begin{array}{rcl} X_u(f) & = & H_u(f)V_uG(f) \\ X_r(f) & = & H_r(f)V_rG(f) \end{array}\right. \tag{2}$$

The objective is to estimate the matrix V_u, knowing $H_r(f)$ and observing $X_u(f)$ and $X_r(f)$. More specifically we can estimate the product $H_u(f)V_u$, but in the absence of a priori knowledge it is impossible to solve separately $H_u(f)$ and V_u.

2 Resolution w.r.t. V_u

In this section we assume that we know the response $H_u(f)$ of the SUT. Then solving the second equation of (2) w.r.t. $G(f)$, we get:

$$X_u(f) \;=\; H_u(f)V_uV_r^{-1}H_r^{-1}(f)X_r(f)$$

It is worth to notice that integrating w.r.t. f leads

$$\sum H_u^{-1}(f) X_u(f) X_u^H(f) \quad = \quad V_u V_r^{-1} \sum H_r^{-1}(f) X_r(f) X_u^H(f)$$

We let

$$S_{uu} \quad = \quad \sum H_u^{-1}(f) X_u(f) X_u^H(f)$$

and

$$S_{ru} \quad = \quad \sum H_r^{-1}(f) X_r(f) X_u^H(f)$$

and provides the solution:

$$\hat{\phi} \quad = \quad \arg\min_{\phi} \| S_{uu} - V_u(\phi) V_r^{-1} S_{ru} \|^2$$

which can be solved numerically. Finally, carrying this estimate in (1) provides an estimate of V_u.

3 General resolution

In this section we assume that the response $H_u(f)$ of the SUT depends on a P-dimensional parameter a. Then:

$$[\hat{\phi}, \hat{a}] \quad = \quad \arg\min_{\phi, a} \| S_{uu}(a) - V_u(\phi) V_r^{-1} S_{ru} \|^2$$

where

$$S_{uu}(a) \quad = \quad \sum H_u^{-1}(f; a) X_u(f) X_u^H(f)$$

which can be solved numerically with reasonable computational effort (maybe!)