1 Introduction

A 3D seismic sensor consists of 3 direction measurements. Each direction is characterized by an unitary vector u_k with $k = 1$, 2 and 3. The signal depends of the dot product between u_k and the DOA of the source, assumed to be faraway in such a way the wave can be considered as plane. Usually, by construction, the 3 vectors u_k are orthogonal.

If we considered 2 seismic sensors it is not sure that the 3 arm directions coincide. If not the transformation that brings one sensor to the other is a 3 D rotation.

We consider a 3D sensor denoted SUT and a 3D sensor denoted SREF. We denote $x_u(t)$ the 3D time series of the SUT and $x_r(t)$ the 3D time series of the SREF. Therefore we can write in the frequency domain:

$$\left\{\begin{array}{lcl} X_u(f) & = & R(\theta)H_u(f)S(f) \\ X_r(f) & = & H_r(f)S(f) \end{array}\right.$$

where R is a rotation matrix, which in the most general case depends the 3D parameter θ, where $H_u(f)$ and $H_r(f)$ are the 3D frequency responses of each arm of the respective sensors. It follows that

$$X_u(f) = R(\theta)H_u(f)(H_r^T(f)H_r(f))^{-1}H_r^T(f)X(f)$$

Then

$$X_u(f)X_u^T(f) = R(\theta)H_u(f)(H_r^T(f)H_r(f))^{-1}H_r^T(f)X_r(f)X_u^T(f)$$

and

$$X_u(f)X_u^T(f) = R(\theta)H_u(f)(H_r^T(f)H_r(f))^{-1}H_r^T(f)X_r(f)X_u^T(f)$$

$$X_u(f)X_u^T(f) = R(\theta)a(f)H_u(f)X_u^T(f)$$

where $a(f) = (H_r^T(f)H_r(f))^{-1}H_r^T(f)X_r(f)$ is a scalar. Let $S(f) = X_u(f)X_u^T(f)$ and $Q(f) = H_u(f)X_u^T(f)$. We have

$$S(f) = R(\theta)a(f)Q(f)$$

By integration on f, we have:

$$S = R(\theta)P$$

Usually S and P are full rank, therefore R can be derived from S and P but with the constraint that R is unitary. A way could be to numerically minimize

$$J(\theta) = \|S - R(\theta)P\|_F^2$$

where $R(\theta) = R_x(\theta_x)R_y(\theta_y)R_z(\theta_z)$ and

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix} \quad R_y(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix}$$

$$R_z(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$