Executive Summary

1. *Facts and Assumptions Of Our Model*

The construction of our model is based on the following facts and assumptions:

1. Each employee's hourly wage is 105 dollars and each employee's hourly gross profit is 210 dollars, which gives us 105 dollars of net profit per employee per hour.
2. It is assumed that all staff would be 100% billable, equaling to 1870 alliable a year or 156 hours a month.
3. Each employee's hourly wage is 105 dollars and each employee's hourly gross profit is 210 dollars, which gives us 105 dollars of net profit per employee per hour.
4. A new employee requires one month's worth of training prior to being fully billable.
5. Once an employee is hired, he cannot be laid off, even though the demand is low.
6. We currently have 35 fully billable employees on our staff team.
7. There will be a certain penalty for not finishing the project in time each month.
8. The amount of time needed for each phase is given in Exhibit 2 and the total demand is given in Exhibit 3.
9. We use the following calculation for the monthly demand (in hours) for each month: (Phase1_time _needed) * (Phase1_demand) + (Phase2_time _needed) * (Phase2_demand) + (Phase3_time _needed) * (Phase3_demand), the calculation yields the following table which summarizes the total amount of time needed (in hours) for each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Hours Needed	5801	7272	7877	7042	7254	7360	7572	10064	12556	16121	16651	17181

The implementation of our model is given as follows:

1. The total amount of time needed (in hours) for each month is represented by:
$$T_i \; such \; that \; i = 1, 2, 3 \dots 12$$
The exact number of T_i can be found in the above table
2. Our objective variable is the number of new workers we decide to hire for each month, represented by:
$$X_i \; such \; that \; i = 1, 2, 3 \dots 12$$
3. The number of total employees is represented by:
$$B_i \text{ such that } B_i = B_{i-1} + X_i$$
and we initialize the value of $B_1 = 35$, which means the total number of employees in our company is the number of employees from last month plus the new hires from this month
4. The total number of hours billed each month is represented by:

$$A_i \text{ such that } A_i = Min(T_i\ ,\ B_i * 156)$$

If $B_i * 156 > T_i$, that means our working capacity exceeds the demand, thus $A_i = T_i$. Otherwise, if our working capacity is smaller than the demand $A_i = B_i * 156$

5. The number of hours that falls short each is represented by:

$$C_i,\ such\ that\ C_i = Max(0\ ,\ T_i - B_i * 156)$$

If $T_i - B_i * 156 \geq 0$ that means we have some hours that's been leftover, which makes $C_i = T_i - B_i * 156$. If $T_i - B_i * 156 \leq 0$, that means we don't have any hours left hour, so $C_i = 0$.

6. The penalty factor for not completing the required work each month is represented by:

$$P,\ such\ that\ 0 \leq P \leq 1$$

7. The objective function is defined as:

$$Max \sum_{i=1}^{12} A_i * 210 - B_i * 105 * 156 - 210 * C_i * P$$

Subject to: X_i being positive whole numbers

A brief interpretation of our model: $A_i * 210$ means the total number of hours billed each month times hourly income which is our total gross profit. $B_i * 105 * 156$ means the wage we have to pay to all of our employees. $210 * C_i * P$ is the penalty fee we have to pay each month due to leftover hours. This is covered in significant depth in the Technical Analysis.

2. Proactive VS Reactive Hiring & Tradeoffs

Proactive hiring is defined as "the process of acquiring talent based on future goals and expectations, giving managers more time to discover and train employees for specific roles" such procedure will bring the following benefits:

1. Can reserve suitable talents for emergencies: hiring proactively can ensure the human-resource team has adequate time to advertise the job opening and receive more applicants, thus giving us the leisure to select the best applicants from the pool.
2. Will prevent penalty for not finishing the work in time: hiring proactively ensures that we have the adequate labor capacity to meet our demands, in case of a high penalty score P, we should hire proactively, trying to avoid the penalty as much as possible.
3. Will boost the stability and growth of start-up companies: for a start company, if we are not able to provide the customers their desired service in time, we are subject to low growth. Using proactive employment thus will ensure that we can always meet our demands thus greatly boosting growth.

However, hiring proactively does come with a certain degree of cost: As described in the case study: "due to family-based company culture, the executive team does not want to lay off staff during times of low demand". If the company is not able to achieve the projected demand as shown in Exhibit 3, then we will still pay wages for the staff that's not contributing any profits.

And in cases where the penalty score P is very low, then we would rather pay the penalty fees rather than waste money on employee's wages.

3. Optimal Staffing Strategy Recommendation

Our Optimal Staffing Strategy can be variable depending on what the firm desires. If optimality means maximizing profit, our Optimal Strategy would be:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Hire	1	11	0	0	1	0	0	17	15	24	2	5

Table 1

This would result in a profit of $12,882,729.3. However, it would lead to a high amount of delay every month, which is covered further in our technical analysis.

The optimal strategy is not necessarily the one that the firm would like to pick. As of now, the firm has not provided any information on how important it is to them to be on time with their projects. The value that the firm gets from being on time can be quantified, and it will potentially be able to help us identify a more optimal strategy. We will study this further in our technical analysis, however, since it is possible to quantify this desire to be on time, we can define a penalty function which better helps our decision-making process.

4. *Risk Profile*

- Once an employee is hired, he cannot be laid off, even though the demand is low
    - Risk: Even if we are unable to reach our projected demand, we still will have to pay wages to all staff (even those who are not contributing to any project)
    - Effectiveness of Controls:
        - Hire only based on expected demand per month
    - Importance / Priority:
        - Depends on P (see below)
- There will be a penalty for not finishing the project in time each month
    - Risk: We incur a penalty each time we do not finish a project in a month, resulting in a loss in our overall revenue
    - Effectiveness of Controls:
        - Hiring Proactively allows us to avoid a penalty since we will then have an adequate supply of labor to meet our expected demand for the projects. The value of P greatly affects our control strategy in mitigating this risk.
    - Importance / Priority: Depends on P (see below)

The importance of these two 2 risks is somewhat dependent on each other. In the end, the priority that we assign to each of these risks is dependent on the coefficient of P. This is because, depending on the penalty, we will develop a staffing solution accordingly that will be optimal.

Say for example that our penalty = 0.9, this is a significant penalty that would greatly diminish our overall revenue. Therefore, in this case, we must hire proactively to ensure that we always have an adequate supply of workers to meet project demands since any penalty would result in a 90% proportion loss for that project. However, if our penalty was much less significant like 0.01 or 0.1, then the risk of facing a penalty for an incomplete project would not be given that much importance since our overall net revenue on the project can still be quite high. It would on the other hand be much worse to hire proactively in this situation since if we have too many workers, many of them will still be getting paid full wages even if they are not having to complete any project. This is because one of the key facts in our model is that we do not fire any employees once they have been hired at the firm. Our optimal staffing solution, based only on expected demand per month, is obtained using only a penalty value of p=0.01. This is because minimizing penalties leads to maximizing revenue as we can see from the tests using different coefficients for P. This, however, is not a good hiring strategy long term for the firm. But, since we are only interested in maximizing revenue in 2017, we can maximize our objective function by using p = 0.01, giving us our optimal objective value of $12,882,725.10. It also means that we can deem penalties as a low-risk priority risk.

Example:

- P = 0.01
    - Hiring Proactively
            - Risk Priority: High
        - Facing Penalty at end of month
        - Risk Priority: Low
- P = 0.9
    - Hiring Proactively
        - Risk Priority: Low
    - Facing Penalty at end of month
        - Risk Priority: High

5. *Flexibility in our Hiring Strategy*

The objective of our model is to maximize the number of workers we hire per month. With the addition of each new employee, we are able to increase our objective function, since each employee is netting $105 profit per hour. However, this statement ONLY holds if each of our employees is occupied with a project. In the case where we have an excess of workers, we will still have to pay those employees regardless of whether or not we are gaining anything from them (due to our pendent on the expected number of hours required across all upcoming and current projects. policy of not firing any employees). Depending on how significant our penalty is, the effect of this could cost us even more money than hiring that worker in the first place.

6. Innovative Strategies to Implement Recommendations

We recommend that the firm not only use our model but identify their willingness to delay a project. This could either be through a number that they quantify as the penalty themselves, or by considering the maximum delay that they are willing to have in a year, or per month. Since we introduced the idea of quantifying the firm's desire to delay, we can take this one step further and set a range for the amount of delay the firm is willing to have on a project every month. Additionally, we can also set a range on what the minimum amount of profit is desired by the firm. Since we have new constraints, instead of trying random penalty functions that quantify the willingness to delay, we can use the two constraints to pick the optimal value for the firm's willingness to delay, given more information about minimum profit, and maximum delay from them.

This is a creative way to ensure that the firm not only reaches the solution that maximizes its profit but also the solution that considers its reputation amongst its clients. Since it may be possible for a firm to lose a client due to excessive delay, by considering not just the profit, but also the delay, which determines its reputation, we make sure that the firm maximizes this year's profit, without losing clients for the future. This will be better in the long run for the firm.

Another aspect the firm can improve on is to utilize multiple market demand forecasting and take the average of all the forecasts. Since the entirety of our model is dependent on the forecasting data, the accuracy of the forecasting is detrimental to the result of our model. A more accurate model will ensure our staffing model ultimately benefits the growth of the company in real-life market demands. And finally, the firm should negotiate with their clients to strive for a lower penalty value in their contract. As a result, due to decreased penalty score, the firm can thus hire more reactive, and avoid unnecessary wage payouts even in low market demands.

Technical Analysis

Key Variables

For this model, we begin by defining our Decision Variables. We want to identify the number of employees that should be hired each month, which we define as the variable $X_i, \forall i \in [1, 12]$, over 12 months in 2017. We will also define a term $H_i, \forall i \in [1, 12]$, which is not a variable but is a term that represents the total number of project hours that we have for a month. This term for a month i is the sum of the hours needed for all Phase 1, Phase 2, and Phase 3 projects for a month.

A factor that we must consider in our model is the fact that since an employee who is hired in a month, cannot begin working for a month, due to the fact that they must undergo a 4-week training period. So, we define a variable, B_i, where B_i, is the total number of employees at the company, i.e. number of new employees, plus a number of employees already at the company. For any month, since a newly hired employee must train first, the actual number of employees available to work is equal to the total number of employees at the end of the previous month.

For month i, B_{i-1} is the number of employees available to work. This is a safe statement to make since we don't fire employees at the firm, so anyone hired will continue to be in the employee count, $B_i, \forall i \in [0, 12]$. This range starts at 0 since we already have 35 employees at the company.

We start off with 35 employees, so $B_0 = 35$

For every other month, $B_i = B_{i-1} + X_i$

Now, we look at our objective. Our target for this project is to maximize profit. In order to do so, we need to identify the total number of hours spent working on contracts by the firm. To track this bit of information, we set a variable, A_i.

$$A_i = min(H_i, B_{i-1} * 156)$$

The way the variable works is that it picks the number of hours actually worked on projects in a given month. If we are meeting the total number of hours, then it implies we have at least enough "working" staff hours to cover it, so $B_{i-1} * 156$ in this situation will be greater than equal to hours needed. However, since we cannot bill a client for more hours than the project takes, in this scenario, the variable will be equal to the number of hours needed for all projects in the month.

Now conversely, it is possible that in a month we do not have enough staff hours to meet our project requirements, so in this case, the minimum would be B_{i-1}, which is the total number of hours that we can bill the client for, and as a result, is the number of hours we make revenue in.

Building on the idea that it is possible to not meet the client project requirements, we need to take into account the number of leftover hours every month. Since all products need to be completed at some point, if a project is not completed in a month, the missed hours will be moved over to the next month. We define another variable, C_i, which accounts for the number of hours that we fall short of our project needs. $C_i \forall i \in [0, 12]$.

$$C_i = max(0, H_i - B_{i-1} * 156)$$

This variable builds off the previous variable in some sense. For a given month, if we meet all project hour requirements, it implies that $H_i \leq B_{i-1} * 156$, that is, we have more than enough working hours, to satisfy all our project requirements. So, the value of C_i for such a month is 0, because we have no leftover hours to send to next month. Now, if we had fewer hours to work than what is needed, $H_i > B_{i-1} * 156$, so $C_i = H_i - B_{i-1} * 156$ in this case, which would be the number of hours that we haven't met.

Objective Function

Now that we have our decision variables, we can define our objective function.

$$max \sum_{k=1}^{12} 210 * A_i - 16380 * B_i - C_i * P$$

1. $210 * A_i$ is the revenue earned by the firm. Since A_i is the total number of hours spent working on contracts in a month i, and each hour gets billed at 210, our revenue is $210 * A_i$.
2. We know B_i is the total number of employees at the firm in a month i, including all existing employees and new hires. Since each employee gets paid 105 and works 156 hours a month, for any given month, every employee works $105 * 156 = 16380$ hours. This is our cost function for every month, and we define it as $16380 * B_i$.
3. The third term is very interesting. Since we have C_i hours every month that may get rolled over to the next month, we would like to try and reduce the number of hours that we may unnecessarily push over to the next month. While in theory there are times where pushing hours works well for maximizing profit, it is not great from a company perspective, and clients may get upset if there is a delay in project completion. As a result, we define a new term P, which penalizes the number of hours that we push over. This can be any number, which we define in our model computation, and we can play around with it to look at different hiring strategies.

Constraints

Now that we have defined our variables and objective function, we would like to look at our constraints as well.

1. The most obvious constraint is that of the nonnegativity of our decision variable, X_i. We cannot hire a negative number of people in a given month. This can be written as: $X_i \geq 0, \forall i \in [1, 12]$
2. Similarly, we cannot hire a partial person. All our hires need to be whole numbers, so we
3. can set a constraint that $X_i \in \mathbf{Z}$. We don't need to specify the positive integers since our previous constraint already enforces positivity.
4. You may have observed that our current definition of the variables A_i and C_i is not linear, which clearly does not work since we are looking for Linear Solutions to our problems. We must convert our max and min into linear equations, through constraints:
    a. We can write $A_i = min(H_i, B_{i-1} * 156) \ \ \forall i \in [1, 12]$ as:
        i. $A_i \leq H_i$
        ii. $A_i \leq B_{i-1} * 156$
        iii. $A_i \geq H_i - M_1(1 - y_1)$
        iv. $A_i \geq (B_{i-1} * 156) - M_1 y_1$
    b. Similarly, $C_i = max(0, H_i - B_{i-1} * 156) \ \ \forall i \in [1, 12]$ becomes:
        i. $C_i \geq 0$
        ii. $C_i \geq H_i - B_{i-1} * 156$
        iii. $C_i \leq 0 + M_2(1 - y_2)$
        iv. $C_i \leq H_i - B_{i-1} * 156 + M_2 y_2$

    This is similar to the manner in which we have attempted to convert maximum and minimum functions into constraints in the course, however, as we iterated through our solution, we discovered that simply using constraints a.i), a.ii), b.i, and b.ii) is not enough. When inputted in this manner to AMPL, it results in solutions that may not be optimal. These constraints on their own signify a range, not an exact value, however, we desire an exact value that satisfies the max and min functions. Out of the terms that we take the maximum and minimum for, we would like either one or the other of the terms input, however, the first two constraints don't always do that. As a result, we introduced two more constraints for each function.

    M_1 and M_2 are both constants that we define, with the only requirement that $H_i, B_{i-1} * 156 \leq M_1$, and $0, H_i - B_{i-1} * 156 \leq M_2$. Simultaneously, y_1, y_2 are dummy variables, which we will have the model solve for as well. These constraints help ensure that more than just checking to find a solution in a range, we pick either of the two values inputted to the max and min functions. This is a great way to linearize the functions.
5. We have one last constraint that we have already defined earlier, however, for clarity, we will rewrite it. We start off with 35 employees, so $B_0 = 35$.

Further Technical Analysis Behind Some Recommendations

3. Optimal Staffing Strategy

Exhibit 3

Staffing at the Strategic Advising Consulting Group: The "S" Word

2017 Staffing Model

Key Assumptions	
Billable hours capacity per FTE per month	156
Blended hourly revenue	210
Blended hourly cost	105
Current number of delivery staff	35

Modeling by Month	Oct-16	Nov-16	Dec-16	Jan-17	Feb-17	Mar-17	Apr-17	May-17	Jun-17	Jul-17	Aug-17	Sep-17	Oct-17	Nov-17	Dec-17	Total
RAP: Number of new clients	1	2	3	2	2	2	2	2	2	5	5	5	5	5	3	40
% of 2017 new sales per month				5%	5%	5%	5%	5%	5%	12%	12%	12%	12%	12%		90.00%

# of Projects Running at a Time by Phase

Active		Nov-16	Dec-16	Jan-17	Feb-17	Mar-17	Apr-17	May-17	Jun-17	Jul-17	Aug-17	Sep-17	Oct-17	Nov-17	Dec-17
Phase 1 (new + existing)				5	5	4	4	4	4	4	7	10	10	10	10
Phase 2 (new + existing)				1	2	3	2	2	2	2	2	2	5	5	5
Total Phases 1 & 2				6	7	7	6	6	6	6	9	12	15	15	15
Phase 3 (new)					1	3	6	8	10	12	14	16	18	23	28
Phase 3 (existing contracts)				8	10	10	10	10	9	9	9	9	8	8	8
Total Phase 3				8	11	13	16	18	19	21	23	25	26	31	36

From Exhibit 3 above, we can see our expected demand for each month per Phase of our RAPs. Inputting this demand into our model, as our data, we can reach an output for an optimal staffing strategy. Our optimal staffing strategy however depends on the value of P, i.e, our penalty variable. Upon observation, the lower the value of our penalty, the greater our revenue. Hence, from a pure optimality perspective, it makes sense to minimize the penalty for delays. Using a $P = 0.01$, we have an optimal objective value of $12,882,729.3. For this staffing solution, our hiring strategy is:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Hire	1	11	0	0	1	0	0	17	15	24	2	5

Table 1

However, we noticed that simultaneously, we also push work a lot every month. Since we are barely penalizing delay, the model will find a solution with a higher amount of delay, if it results in higher profit. Our delay across the year is:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Delay (Hrs)	185	125	670	380	146	18	102	26	102	0	115	0

Table 2

So, while our solution is great for our profit in 2017, it definitely leaves clients disappointed with the timeline often. So, it is a mathematically good solution, however, it is not great for the firm's reputation. Conversely, if we penalize delay higher, with $P = 0.9$, our new optimal solution has a lower profit, at $12,719,385. Our monthly delay however is much lower. This works because any sort of delay is being penalized heavily, so the model is more skeptical when choosing to delay a project. It however results in a lower penalty.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Delay (Hrs)	29	0	389	0	0	0	0	0	76	0	0	21

Table 3

To conclude, our optimal staffing recommendation if optimality only involves firms' profit would be the one provided in Table 1, with a penalty function of 0.01. It would simultaneously have a higher amount of delay each month.

A more reasonable solution, which considers profit for the firm, without compromising too much on work standards would have a penalty somewhere between the two extremes, i.e. high penalty, and low penalty. Depending on what the firm prefers, the penalty function can be tuned to find the right solution for them.

4. *Different Penalty Values Analysis:*

Metrics Used:

- Absmipgap - Absolute Mixed-Integer Optimality Gap Tolerance (difference between current best integer solution and the optimal value of LP relaxation)
- Relmipgap - Relative MIP Gap Tolerance (difference between best integer objective and the objective of best node remaining)

Different Penalties:

- Penalty = 0.01
  - Objective Func = 12882729.3
  - Absmipgap = 1287.3
  - Relmipgap = 9.99245e-05
- Penalty = 0.1
  - Objective Func = 12853827
  - Absmipgap = 1176
  - Relmipgap = 9.14903e-05
- Penalty = 0.2
  - Objective Func = 12827934
  - Absmipgap = 1225.32
  - Relmipgap = 9.55198e-05
- Penalty = 0.3
  - Objective Func = 12802041
  - Absmipgap = 1.86265e-09
  - Relmipgap = 1.45496e-16
- Penalty = 0.4
  - Objective Func = 12781440
  - Absmipgap = 1234.48
  - Relmipgap = 9.65839e-05
- Penalty = 0.5
  - Objective Func = 12768000
  - Absmipgap = 617.404
  - Relmipgap = 4.83556e-05
- Penalty = 0.6
  - Objective Func = 12755610

- Absmipgap = 1134
  - Relmipgap = 8.89021e-05
- Penalty = 0.7
  - Objective Func = 12742968
  - Absmipgap = 693
  - Relmipgap = 5.43829e-05
- Penalty = 0.8
  - Objective Func = 12730452
  - Absmipgap = 1134
  - Relmipgap = 8.90777e-05
- Penalty = 0.9
  - Objective Func = 12719385
  - Absmipgap = 1238.88
  - Relmipgap = 9.74008e-05

5. *Sensitivity Analysis (Optimal Solution):*

- Variables
- initialc (number of hours we fall short of project requirements)
- Shadow Price (dual variables): 186.9
- RHS constraint can increase to 21 while maintaining optimality of current basis (_con.up)
- initialb (number of hours billed to client(s))
- Shadow Price (dual variables): 0
- RHS constraint can increase to 35 while maintaining optimality of current basis (_con.up)

```
ampl: display _conname, _con, _con.slack, _con.up, _con.current, _con.down;
:        _conname      _con  _con.slack    _con.up  _con.current   _con.down
 :=
1     initialc        186.9          0         21            0           0
2     initialb          0            0         35           35          35
```

6. Innovative Strategies to Implement Recommendations

Our recommendation is for the firm to identify different penalties and how they impact the objective function, i.e. the profit, as well as how they impact the monthly project delay. A potential way to implement this would be for the firm to identify and assess a threshold, or a range, that they think is suitable. Essentially, this range would determine the maximum delay the firm is willing to have in any month, while also assessing a minimum amount of profit that they would like to reach. This would allow us to attain the best possible solution for the firm while considering what matters most to them.

Hypothetically speaking, if we would like Profit to be at least M, with a maximum delay of D, we could include these variables into our constraint as well. Our objective function would continue to stay the same, looking something like this:

$$maxZ = \sum_{k=1}^{12} 210 * A_i - 16380 * B_i - C_i * P$$

We would however include some new constraints. They would be:

1. $Z \geq M$
2. $C_i \leq D, \forall i \in [0, 12]$

3. $P > 0$
4. $P < 1$

The first constraint ensures our profit, Z is at least equal to the M that we want it to be. The second constraint tracks C_i, which is our delay per month. It sets a maximum delay per month. We could alternatively set a maximum delay for the year, m, and make sure the sum of all delays, $\sum_{i=1} C_i \leq m$. Finally, we would instead of submitting a preset P, penalty, submit P as a variable into our model instead of a predetermined constant. This would pick the best value of P that suits all our needs.

This method should work well to identify the best strategy for the firm while considering everything that matters to them. It would involve the firm having to make a bigger decision of how much of a delay is ideal, and how much minimum profit that they want.

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