CHAPTER 35

MAGNETIC PROPERTIES OF MATERIALS

Magnetic materials play increasingly important roles in our daily lives. Materials such as iron, which can be permanent magnets at ordinary temperatures, are commonly used in electric motors and generators as well as in certain types of loudspeakers. Other materials can be "magnetized" and "demagnetized" with relative ease; these materials have found wide use for storing information in such applications as magnetic recording tape (used in audio tape recorders and VCRs), computer disks and credit cards. Still other materials are analogous to dielectrics in that they acquire an induced magnetic field in response to an external magnetic field; the induced field vanishes when the external field is removed.

In this chapter we consider the internal structure of materials that is responsible for their magnetic properties. We show that the behavior of different magnetic materials can be understood in terms of the magnetic dipole moments of individual atoms. A complete understanding of magnetic properties requires methods of quantum mechanics that are beyond the level of this text, but a qualitative understanding can be achieved based on principles discussed in this chapter. Finally, we consider a magnetic form of Gauss' law, which takes into account the apparent nonexistence of isolated magnetic poles.

35-1 THE MAGNETIC DIPOLE

For static electric fields, the single isolated charge is the fundamental quantity. Individual charges *produce* an electric field, and in turn the electric field set up by one group of charges can *influence* the behavior of other charges. On the basis of this elementary interaction between electric charges, we can explain many common phenomena: the force exerted by the nucleus on the electrons, which holds the atom together; the force exerted by one atom on another in ionic molecules and solids; elastic and frictional forces; and so forth.

In some electrically neutral molecules, it is useful to regard the fundamental interaction to be based on the electric dipole (which in turn can be analyzed as two point charges). We have seen how the dipole can *produce* an electric field (Section 26-3) and also how a dipole is *influenced* by other electric fields (Section 26-7).

For steady magnetic fields, the fundamental quantity is the moving electric charges in a current element, which can *produce* a magnetic field and can also be *influenced* by the magnetic field due to other current elements. However, in trying to explain the magnetic properties of materials, this explanation in terms of current elements is not as convenient as one based on the *magnetic dipole*. Ultimately the magnetic dipole can be regarded as caused by moving charges, just as the electric dipole can be regarded as two static charges. However, when we discuss the magnetic properties of materials we gain more insight by considering the materials to be a collection of atoms with individual magnetic dipole moments.

Let us begin by considering the magnetic field due to a circular current loop (Section 33-2) at a point on the z axis:

$$B = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}. \qquad (35\text{-}1)$$

If we are far from the loop ($z >> R$), this becomes

$$B = \frac{\mu_0 i R^2}{2z^3} = \frac{\mu_0}{2\pi}\frac{i\pi R^2}{z^3}. \tag{35-2}$$

The quantity $i\pi R^2$ in Eq. 35-2 can be written as iA, where $A = \pi R^2$ is the area of the circular loop. We define this quantity to be the magnitude of the *magnetic dipole moment* μ of the loop:

$$\mu = iA. \tag{35-3}$$

The magnetic dipole moment of a current loop is the product of the current and the area of the loop. Although we derived Eq. 35-3 for a circular loop, it holds for loops of any shape. If the loop has N turns, then $\mu = NiA$.

Equation 35-3 suggests that the units for μ are $A \cdot m^2$ (ampere-meter^2). Later in this section we will find that equivalent units are J/T (joules per tesla).

Like the electric dipole moment, the magnetic dipole moment is a vector quantity. The direction of $\vec{\mu}$ is perpendicular to the plane of the current loop, determined by the right-hand rule: if the fingers of your right hand are in the direction of the current, then your thumb indicates the direction of $\vec{\mu}$ (Fig. 35-1). With this definition, we can write Eq. 35-2 as a vector equation:

$$\vec{\mathbf{B}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}. \tag{35-4}$$

Note that $\vec{\mathbf{B}}$ and $\vec{\mu}$ are vectors in the same direction, as Fig. 35-1 shows. In Eq. 35-4, $\vec{\mathbf{B}}$ is the magnetic field *produced by* the magnetic moment $\vec{\mu}$.

Let us now consider the effect of a magnetic field on a magnetic dipole. Figure 35-2 shows a current loop in a uniform magnetic field $\vec{\mathbf{B}}$. (This field $\vec{\mathbf{B}}$ is produced by some external agent not shown in the figure, perhaps a large solenoid.) In Section 32-6 we considered a similar problem (see Fig. 32-26) and concluded that in a uniform field the loop experiences no net force but does experience a net torque given by $\vec{\tau} = iA\hat{\mathbf{n}} \times \vec{\mathbf{B}}$ (Eq. 32-35), where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the loop in a direction determined by the right-hand rule. Because we defined the directions of $\vec{\mu}$ and $\hat{\mathbf{n}}$ in exactly the same way, we can write $\vec{\mu} = iA\hat{\mathbf{n}}$, and so Eq. 32-35 becomes

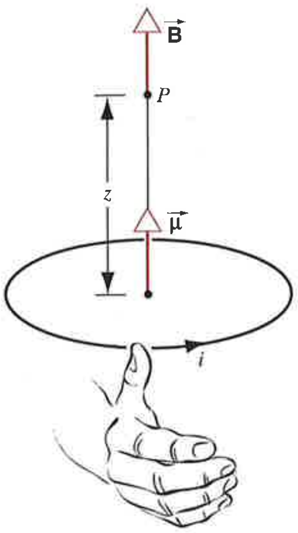

FIGURE 35-1. The magnetic dipole moment of a current loop and the magnetic field at point P a distance z from the loop on its axis.

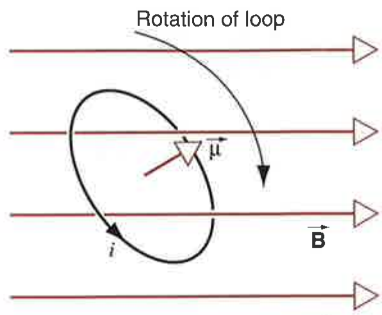

FIGURE 35-2. In an external magnetic field, a magnetic dipole moment experiences a torque that rotates $\vec{\mu}$ into alignment with $\vec{\mathbf{B}}$.

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}. \tag{35-5}$$

That is, the torque tends to rotate the loop so that $\vec{\mu}$ lines up with $\vec{\mathbf{B}}$. Note the similarity of Eq. 35-5 with the corresponding result for the torque that rotates an electric dipole in an electric field: $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ (Eq. 26-27). Equation 35-5 is valid regardless of the shape of the loop or its orientation relative to the magnetic field.

Equations 35-4 and 35-5 satisfy our two goals: Eq. 35-4 indicates how a magnetic field is *produced* by a magnetic dipole, and Eq. 35-5 shows how a magnetic dipole is *influenced by* an applied magnetic field. Keeping these two concepts in mind will help us understand the magnetic behavior of materials.

We can continue the analogy between electric and magnetic fields by considering the work done to change the orientation of a magnetic dipole in a magnetic field and relating that work to the potential energy of a magnetic dipole in a magnetic field. We can write the potential energy as

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{\mathbf{B}}, \tag{35-6}$$

for a magnetic dipole whose moment $\vec{\mu}$ makes an angle θ with $\vec{\mathbf{B}}$. This equation is similar to the corresponding expression for an electric dipole, $U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$ (Eq. 26-32). In Eq. 35-6, $U = 0$ when $\theta = 90°$ ($\vec{\mu}$ is perpendicular to $\vec{\mathbf{B}}$ or equivalently $\vec{\mathbf{B}}$ is parallel to the plane of the loop). U has its smallest value ($= -\mu B$) when $\vec{\mu}$ and $\vec{\mathbf{B}}$ are parallel, and U is largest $= +\mu B$) when $\vec{\mu}$ and $\vec{\mathbf{B}}$ are antiparallel.

The magnetic force, like all forces that depend on velocity, is in general *not* conservative and therefore cannot generally be represented by a potential energy. In this special case, in which the torque on a dipole depends on its posi-

TABLE 35-1 Selected Values of Magnetic Dipole Moments

System	μ (J/T)
Nucleus of nitrogen atom	2.04×10^{-28}
Proton	1.41×10^{-26}
Neutron	9.65×10^{-27}
Electron	9.28×10^{-24}
Nitrogen atom	2.8×10^{-23}
Typical small coil[a]	5.4×10^{-6}
Small bar magnet	5
Superconducting coil	400
The Earth	8.0×10^{22}

[a] That of Sample Problem 35-1, for instance.

tion relative to the field, it *is* possible to define a potential energy for the *system* consisting of the dipole in the field. Note that the potential energy is not characteristic of the field alone, but of the dipole *in* the field. In general, we cannot define a scalar "magnetic potential energy" of a point charge or "magnetic potential" of the field itself such as we did for electric fields in Chapter 28.

Many physical systems have magnetic dipole moments: the Earth, bar magnets, current loops, atoms, nuclei, and elementary particles. Table 35-1 gives some typical values.

Note that Eq. 35-6 suggests units for μ of energy divided by magnetic field, or J/T. Equation 35-3 suggests units of current times area, or $A \cdot m^2$. You can show that these two units are equivalent, and the choice between them is one of convenience.

As indicated by the examples of the proton and the nitrogen atom in Table 35-1, nuclear magnetic dipole moments are typically three to six orders of magnitude smaller than atomic magnetic dipole moments. Several conclusions follow immediately from this observation. (1) Electrons cannot be constituents of the nucleus; otherwise nuclear magnetic dipole moments would typically have magnitudes about the same as that of the electron. (2) Ordinary magnetic effects in materials are determined by *atomic* magnetism, rather than the much weaker *nuclear* magnetism. (3) To exert a particular torque necessary to align nuclear dipoles requires a magnetic field about three to six orders of magnitude larger than that necessary to align atomic dipoles.

SAMPLE PROBLEM 35-1. (*a*) A 250-turn rectangular coil of length 2.10 cm and width 1.25 cm carries a current of 85 μA. What is the magnetic dipole moment of this coil? (*b*) The magnetic dipole moment of the coil is lined up with an external magnetic field whose strength is 0.85 T. How much work would be done by an external agent to rotate the coil through 180°?

Solution (*a*) The magnitude of the magnetic dipole moment of the coil, whose area A is $(0.0210\text{ m})(0.0125\text{ m}) = 2.52 \times 10^{-4}\text{ m}^2$, is

$$\mu = NiA = (250)(85 \times 10^{-6}\text{ A})(2.52 \times 10^{-4}\text{ m}^2)$$
$$= 5.36 \times 10^{-6}\text{ A} \cdot \text{m}^2 = 5.36 \times 10^{-6}\text{ J/T}.$$

(*b*) The external work is equal to the increase in potential energy of the system, which is

$$W = \Delta U = -\mu B \cos 180° - (-\mu B \cos 0°) = 2\mu B$$
$$= 2(5.36 \times 10^{-6}\text{ J/T})(0.85\text{ T}) = 9.1 \times 10^{-6}\text{ J} = 9.1\ \mu\text{J}.$$

This is about equal to the work needed to lift an aspirin tablet through a vertical height of about 3 mm.

The Field of a Dipole

So far we have discussed the field of a magnetic dipole (a current loop) only at points on the axis. Now we consider the complete magnetic dipole field. In the case of the electric dipole, a complete pattern of field lines was shown in Fig. 26-12. A few field lines for an electric dipole are shown in Fig. 35-3*a* and can be compared with the field lines for a current loop shown in Fig. 35-3*b*. You can see a great similarity between the pattern of field lines outside the loop. Another similarity between the electric and magnetic dipole fields is that both vary as r^{-3} when we are far from the dipole. A significant difference between electric and magnetic field lines is that electric field lines start on positive charges and end on negative charges, whereas magnetic field lines always form closed loops.

Figure 35-3*c* shows the field lines of a bar magnet. It shows the same pattern of field lines as the current loop, so a bar magnet can also be considered to be a magnetic dipole. It is convenient to label the two ends of a bar magnet as the north (N) and south (S) poles, with field lines leaving the N pole and converging on the S pole. Superficially the poles may seem to behave like the positive and negative charges of an electric dipole. However, close inspection of Fig. 35-3*c* shows that the field lines do not start and end on the poles but instead continue through the interior of the magnet, again forming closed loops. The N and S poles do not behave like the charges in an electric dipole, and as we discuss in Section 35-7, isolated magnetic poles do not appear to exist in nature.

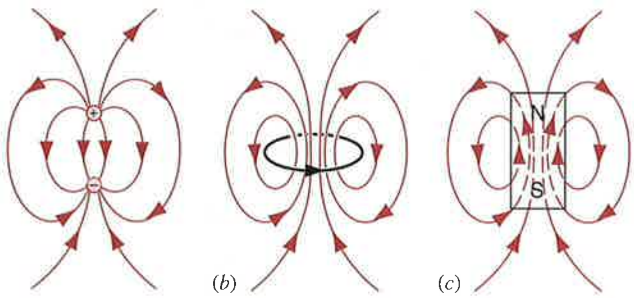

FIGURE 35-3. (*a*) The electric field of an electric dipole. (*b*) The dipole magnetic field of a current loop. (*c*) The dipole magnetic field of a bar magnet. The dashed lines show the field lines inside the magnet.

35-2 THE FORCE ON A DIPOLE IN A NONUNIFORM FIELD

In a uniform electric field, the forces on the two charges of an electric dipole are of equal magnitude and in opposite directions (Fig. 26-19). If the field is nonuniform, the forces are of different magnitudes and so a net force can act on the dipole. The same conclusion holds for magnetic dipoles: in a uniform magnetic field, there may be a net torque on the dipole, but there is no net force. For a net force to be exerted on the dipole, the magnetic field must be nonuniform.

Consider the pair of current loops shown in Fig. 35-4. The loops lie along a common axis, and both loops carry current in the counterclockwise direction as viewed from above. Loop 1 sets up a magnetic field $\vec{\mathbf{B}}_1$, which then interacts with loop 2. (We assume that loop 2 has already been rotated by the torque due to the field of loop 1, which lines up the dipole moment of loop 2 with the field of loop 1.) At points C and D, which are on opposite ends of a diameter of loop 2, the forces $d\vec{\mathbf{F}} = i_2\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}_1$ on the elements $d\vec{\mathbf{s}}$ have downward and radially outward components. When we add the forces on all such pairs of elements, we find that the radial components cancel and the downward components sum to give a net downward force on the current loop.

We can also analyze this force in terms of magnetic poles. Each of the current loops can be represented as a magnet with north and south poles oriented as shown in Fig. 35-4. The attraction of loop 1 for loop 2 can be described in terms of the force between magnetic poles: the N pole of the magnet representing loop 1 attracts the S pole of the magnet representing loop 2. In Fig. 35-4, there is also a repulsion between the two N poles and the two S poles, but the N–S attraction is the stronger force because the poles are closer together.

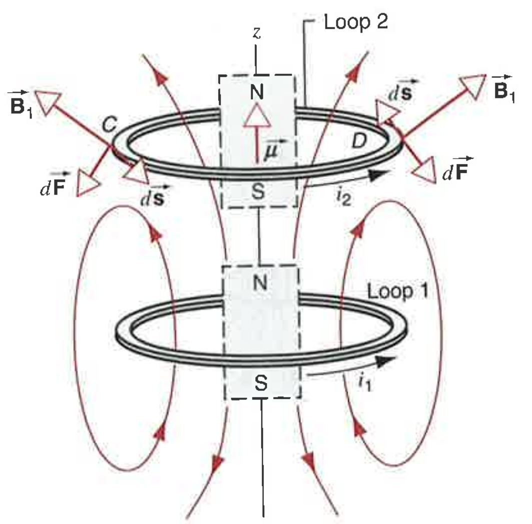

FIGURE 35-4. The magnetic field $\vec{\mathbf{B}}_1$ due to loop 1 causes a net downward force on loop 2.

Using Eq. 35-6 for the potential energy of the magnetic dipole moment of loop 2 in the magnetic field caused by loop 1 ($U = -\vec{\boldsymbol{\mu}}_2 \cdot \vec{\mathbf{B}}_1$), we obtain $U = -\mu_{2z}B_{1z}$, because the magnetic moment of loop 2 has only a z component. The z component of the force $\vec{\mathbf{F}}_{21}$ exerted on loop 2 by loop 1 is related to the potential energy by $F_z = -dU/dz$, so

$$F_{21z} = -\frac{dU}{dz} = -\frac{d}{dz}(-\mu_{2z}B_{1z}) = \mu_{2z}\frac{dB_{1z}}{dz}. \quad (35\text{-}7)$$

In Fig. 35-4, taking the z axis as positive upward, we have $\mu_{2z} > 0$ and $dB_{1z}/dz < 0$ (since the z component of the field decreases as we go upward), and so $F_{21z} < 0$. The force on loop 2 due to loop 1 is downward, as we have already determined.

Similar considerations show that the force on loop 1 due to loop 2 is upward, so the two loops attract each other.

Induced Magnetic Dipole Moments

In some materials in which the molecules do not have a permanent electric dipole moment, as we discussed in Section 29-6, an applied electric field can induce a dipole moment by causing a separation of the positive and negative charges in the molecule. A similar effect occurs for magnetic fields: in materials that lack permanent magnetic dipole moments, an applied magnetic field can induce a dipole moment.

Figure 35-5 shows how this might occur. Consider a double loop, consisting of two single loops carrying identical currents in opposite directions, in a nonuniform field that might be produced by a permanent magnet. The net magnetic moment of the double loop is zero, because the two single loops have magnetic moments of equal magni-

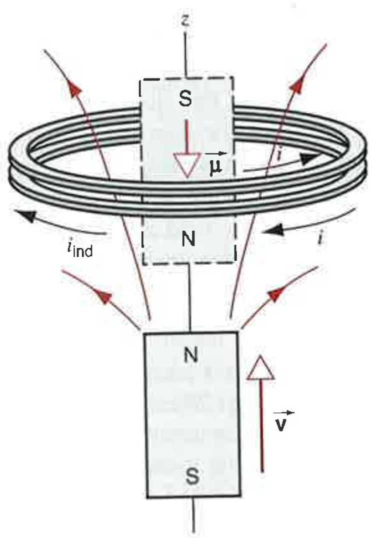

FIGURE 35-5. The double loop has no permanent magnetic dipole moment, but it acquires an induced dipole moment when the magnet approaches the loop. The loop is repelled by the force on the induced moment.

tudes but opposite directions. As the magnet is brought closer to the double loop, the flux through the loops increases, causing an induced current that, according to Lenz' law, must be directed clockwise (viewed from above). This induced current, which adds to the currents in the two loops, gives a net current $i - i_{ind}$ in the upper loop and $i + i_{ind}$ in the lower loop. The result is a net induced magnetic moment directed downward. The N and S poles of the equivalent magnet are shown, from which it can be seen that the force on the double loop due to the magnet is repulsive (upward).

Using Eq. 35-7, we see that (again taking the z axis as positive upward), $\mu_z < 0$ and $dB_z/dz < 0$, so that $F_z > 0$, corresponding to an upward force, in agreement with the previous conclusion.

In summary, in a nonuniform magnetic field, permanent dipoles are rotated into alignment with the field and attracted to the source of the field, but induced dipoles are repelled from the source of the field.

35-3 ATOMIC AND NUCLEAR MAGNETISM

The bulk electrical properties of a dielectric substance such as water depend on the individual electric dipole moments of its molecules. Each molecule has a positive side and a negative side and behaves like an electric dipole. If we break apart the molecule we can separate the positive and negative parts.

The magnetic properties of materials similarly depend on the magnetic dipole moments of individual atoms, and we can consider magnetic materials to be composed of a collection of atomic dipoles, which might align when an external magnetic field is applied (Fig. 35-6). However, unlike the electric dipole, we cannot break apart the atoms into separate N and S magnetic poles. Instead, we consider

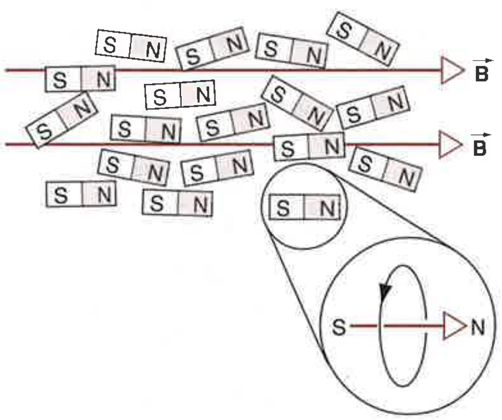

FIGURE 35-6. A magnetic material can be regarded as a collection of magnetic dipole moments, each with a north and a south pole. Microscopically, each dipole is actually a current loop that cannot be split into individual poles.

the magnetic moments to be tiny current loops, caused for example by the circulation of electrons in orbits in the atom. In this section we discuss the magnetic dipole moment associated with a circulating electron.

We consider a simple model of an atom in which an electron moves in a circular orbit of radius r and speed v about the nucleus. This circulating electron can be considered as a current loop, in which the current is the magnitude of the charge of the electron divided by the period T for one orbit:

$$i = \frac{e}{T} = \frac{e}{2\pi r/v}. \qquad (35\text{-}8)$$

The magnetic dipole moment of the loop can be found using Eq. 35-3:

$$\mu = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{erv}{2}. \qquad (35\text{-}9)$$

The magnetic dipole moment we have computed here for atoms is known as the *orbital* magnetic dipole moment, because it is due to the orbital motion of electrons about the nucleus.

In analyzing the properties of atoms, it is convenient to rewrite Eq. 35-9 as

$$\mu_l = \frac{erv}{2} = \frac{e}{2m} mvr = \frac{e}{2m} l, \qquad (35\text{-}10)$$

where m is the mass of an electron. The quantity mvr is the angular momentum l of the electron moving in a circular orbit about the nucleus of the atom. We have labeled the orbital magnetic dipole moment as μ_l to indicate that it arises from the orbital angular momentum l. In the quantum theory of atoms, which is discussed in Chapters 47 and 48, the angular momentum is measured in units of $h/2\pi$, where h is the Planck constant. Substituting this fundamental unit of angular momentum into Eq. 35-10, we obtain a basic unit of the magnetic dipole moment called the *Bohr magneton* μ_B:

$$\mu_B = \frac{e}{2m}\frac{h}{2\pi} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T}, \qquad (35\text{-}11)$$

where the numerical value is obtained by inserting the numerical values of e, h, and m into Eq. 35-11. Atomic magnetic moments are usually measured in units of μ_B and are typically on the order of 1 μ_B in magnitude, as you can see from the example of the nitrogen atom in Table 35-1.

The magnetic dipole moments of atoms can be measured by passing a beam of atoms through a region in which there is a nonuniform magnetic field. As we showed in the previous section, there is a net force on a magnetic dipole in a nonuniform field, so the atoms are deflected from their original paths when they pass through the field region. In the 1920s, experiments of this type showed that atoms with no orbital magnetic dipole moments were still deflected by a magnetic field. This result suggests the presence of another contribution to the magnetic dipole moments of atoms, in this case coming from the magnetic dipole moments of the