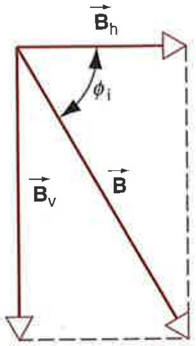

FIGURE 35-17. Sample Problem 35-6. The horizontal and vertical components of the Earth's magnetic field near Tucson, Arizona. The angle ϕ_i is the dip angle.

angle at Tucson was measured to be 59°. Find the magnitude of the field and its vertical component at that location.

Solution As Fig. 35-17 shows, the magnitude of the field can be found from

$$B = \frac{B_h}{\cos \phi_i} = \frac{26\ \mu\text{T}}{\cos 59°} = 50\ \mu\text{T}.$$

The vertical component is given by

$$B_v = B_h \tan \phi_i = (26\ \mu\text{T})(\tan 59°) = 43\ \mu\text{T}.$$

As expected for a dipole field (see Fig. 35-13), measured values of the dip angle range from 0° near the equator (actually, the *magnetic* equator) to 90° near the magnetic poles.

35-7 GAUSS' LAW FOR MAGNETISM

Figure 35-18*a* shows the electric field associated with an insulating rod having equal quantities of positive and negative charge placed on opposite ends. This is an example of an electric dipole. Figure 35-18*b* shows the analogous case of the magnetic dipole, such as the familiar bar magnet, with a north pole at one end and a south pole at the other end. At this level the electric and magnetic cases look quite similar. (Compare Fig. 26-14*b* with Fig. 32-6 to see another illustration of this similarity.) In fact, we might be led to postulate the existence of individual magnetic poles analogous to electric charges; such poles, if they existed, would produce magnetic fields (similar to electric fields produced by charges) proportional to the strength of the pole and inversely proportional to the square of the distance from the pole. As we shall see, this hypothesis is not consistent with experiment.

Let us cut the objects of Fig. 35-18 in half and separate the two pieces. Figure 35-19 shows that the electric and magnetic cases are no longer similar. In the electric case, we have two objects that, if separated by a sufficiently large distance, could be regarded as point charges of opposite polarities, each producing a field characteristic of a point charge. In the magnetic case, however, we obtain not isolated north and south poles but instead a pair of magnets, each with its own north and south poles.

This appears to be an important difference between electric and magnetic dipoles: an electric dipole can be separated into its constituent single charges (or "poles"), but a magnetic dipole cannot. Each time we try to divide a magnetic dipole into separate north and south poles, we create a new pair of poles. This process occurs microscopically, down to the level of individual atoms. Each atom behaves like a magnetic dipole having a north and a south pole, and as far as we yet know the dipole, rather than the single isolated pole, appears to be the smallest fundamental unit of magnetic structure.

This difference between electric and magnetic fields has a mathematical expression in the form of Gauss' law. In Fig. 35-18*a*, the flux of the electric field through the different Gaussian surfaces depends on the net charge enclosed

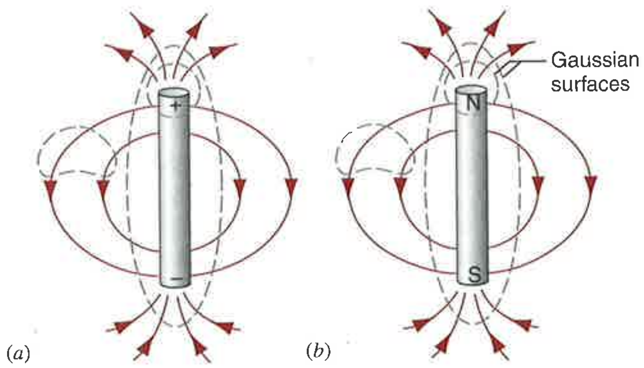

FIGURE 35-18. (*a*) An electric dipole, consisting of an insulating rod with a positive charge at one end and a negative charge at the other. Several Gaussian surfaces are shown. (*b*) A magnetic dipole, consisting of a bar magnet with a north pole at one end and a south pole at the other.

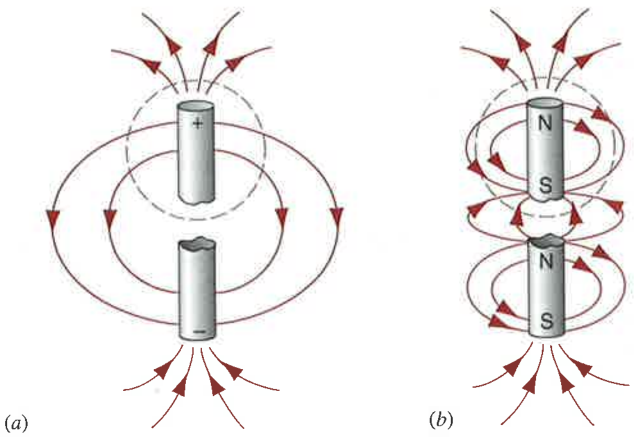

FIGURE 35-19. (*a*) When the electric dipole of Fig. 37-18*a* is cut in half, the positive charge is isolated on one piece and the negative charge on the other. (*b*) When the magnetic dipole of Fig. 35-18*b* is cut in half, a new pair of north and south poles appears. Note the difference in the field patterns.

by each surface. If the surface encloses no charge at all, or no net charge (that is, equal quantities of positive and negative charge, such as the entire dipole), the flux of the electric field vector through the surface is zero. If the surface cuts through the dipole, so that it encloses a net charge q, the flux Φ_E of the electric field is given by Gauss' law:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = q/\epsilon_0. \tag{35-21}$$

We can similarly construct Gaussian surfaces for the magnetic field, as in Fig. 35-18*b*. If the Gaussian surface contains no net "magnetic charge," the flux Φ_B of the magnetic field through the surface is zero. However, as we have seen, even those Gaussian surfaces that cut through the bar magnet enclose no net magnetic charge, because every cut through the magnet gives a piece having both a north and a south pole. The magnetic form of Gauss' law is written

$$\Phi_B = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0. \tag{35-22}$$

The net flux of the magnetic field through any closed surface is zero.

Figure 35-20 shows a more detailed representation of the magnetic fields of a bar magnet and a solenoid, both of which can be considered as magnetic dipoles. Note in Fig. 35-20*a* that lines of $\vec{\mathbf{B}}$ enter the Gaussian surface inside the magnet and leave it outside the magnet. The total inward flux equals the total outward flux, and the net flux Φ_B for the surface is zero. The same is true for the Gaussian surface through the solenoid shown in Fig. 35-20*b*. In neither case is there a single point from which the lines of $\vec{\mathbf{B}}$ originate or to which they converge; that is, *there is no isolated magnetic charge.*

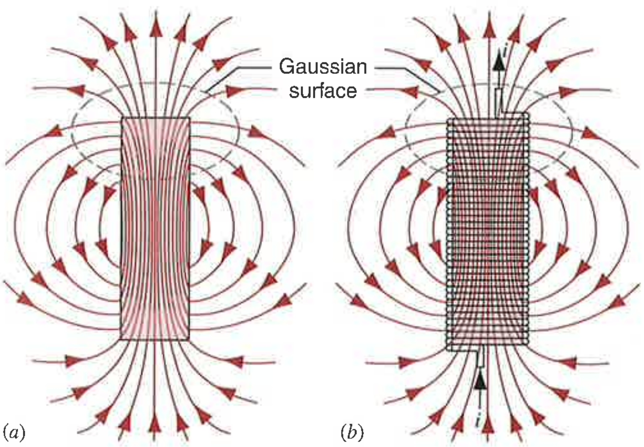

FIGURE 35-20. Lines of $\vec{\mathbf{B}}$ for (*a*) a bar magnet and (*b*) a short solenoid. In each case, the north pole is at the top of the figure. The dashed lines represent Gaussian surfaces.

Magnetic Monopoles

We showed in Chapter 27 that Gauss' law for electric fields is equivalent to Coulomb's law, which is based on the experimental observation of the force between point charges. Gauss' law for magnetism is also based on an experimental observation: the failure to observe isolated magnetic poles, such as a single north pole or south pole.

The existence of isolated magnetic charges was proposed in 1931 by theoretical physicist Paul Dirac on the basis of arguments using quantum mechanics and symmetry. He called those charges *magnetic monopoles* and derived some basic properties expected of them, including the magnitude of the "magnetic charge" (analogous to the electronic charge e). Following Dirac's prediction, searches for magnetic monopoles were made using large particle accelerators as well as by examining samples of terrestrial and extraterrestrial matter. None of these early searches turned up any evidence for the existence of magnetic monopoles.

Recent attempts to unify the laws of physics, bringing together the strong, weak, and electromagnetic forces into a single framework, have reawakened interest in magnetic monopoles. These theories predict the existence of extremely massive magnetic monopoles, roughly 10^{16} times the mass of the proton. This is certainly far too massive to be made in any accelerator on Earth; in fact, the only known conditions under which such monopoles could have been made would have occurred in the hot, dense matter of the early universe. Searches for magnetic monopoles continue to be made, but convincing evidence for their existence has not yet been obtained.* For the present, we assume either that monopoles do not exist, so that Eq. 35-22 is exactly and universally valid, or else that if they do exist they are so exceedingly rare that Eq. 35-22 is a highly accurate approximation. Equation 35-22 then assumes a fundamental role as a description of the behavior of magnetic fields in nature, and it is included as one of the four Maxwell equations of electromagnetism.

*See "Searches for Magnetic Monopoles and Fractional Electric Charges," by Susan B. Felch, *The Physics Teacher,* March 1984, p. 142. See also "Superheavy Magnetic Monopoles," by Richard A. Carrigan, Jr. and W. Peter Trower, *Scientific American,* April 1982, p. 106.