An Improved 6DOF Electromagnetic Tracking Algorithm with Anisotropic System Parameters

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Abstract. An improved 6DOF electromagnetic tracking algorithm is proposed. It utilizes the scattering parameters, which are about the transmitter coils, the receiver coils and the involved circuits, to determine the pose and orientation information of the target. Compared with the traditional algorithm which takes the system parameters as same in all the three directions, it is more accordant to the nature of the current tracking device. And it also simplifies the system hardware design. The experimental results prove the algorithm is valid and can improve the accuracy of the system.

1 Introduction

Magnetic Tracking System is one of the most widely used tracking devices in virtual reality, augmented reality and other applications. It is usually composed of three parts : a source which generates a magnetic field, a sensor that detects the field and a processor. The system discussed here, both the source and the sensor, uses 3 coils oriented orthogonal to each other called transmitter and receiver to generate and detect magnetic field, along with the related electronic circuitry. The system determines the position and orientation depending on the sensor outputs and the system parameters. The parameters, including the area and turn number of the coils, the amplitude and frequency of the exciting current, and the system gain etc., affect the accuracy of the system seriously [1]. Among these parameters, due to large number of turns and irregular coil winding, the actual turn number and area of the coils are not only difficult to get, but different from each other largely.

Currently, Polhemus and Ascension are the two most famous manufacture of magnetic tracking device. But we have not detailed and published system designing resources about their products. The other traditional magnetic tracking algorithms are all based on the assumption, which is that the parameters are approximately the same value in three directions of the coil [2, 3]. To make the parameter same, more extra work must be done during system hardware design. This increases the design complexity. But in most case large difference between the parameters still exits. Our experiment results providing in back part of this paper will show this. So the parameters themselves are not accurate. As a result, the tracking system will have errors inevitably. At present, various calibration

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methods are proposed to calibrate the errors caused by these system errors and the other noise [4, 5]. But these on the other hand increase the system hardware design complexity and the system delay.

In order to overcome these adverse effects, we have developed an algorithm which utilizes the scattering parameters to determine the pose and orientation of the target. It allows the coils on different directions to have different turn number and area. The circuit also need not amplify the different channel signals with same gain. So it simplifies the system hardware design. Compared with the traditional algorithm, it is more accordant to the nature of the current tracking devices. The experimental results indicate that the algorithm is valid and can improve the accuracy of the system.

2 The Definition of System Parameter and Variable

2.1 Definition of System Parameter

Magnetic tracking system determines the position and orientation depending on the sensor outputs and the system parameters. Because the parameters are only decided by the system hardware. We define the anisotropic parameters as

$$\omega C_x K_x, \omega C_x K_y, \omega C_x K_z, \omega C_y K_x, \omega C_y K_y, \omega C_y K_z, \omega C_z K_x, \omega C_z K_y, \omega C_z K_z$$

where

- ω is the radian frequency of the source excitation;
- $C_i = \mu_0 N_i I_i S_i / 4\pi (i = x, y, z)$ is the transmitter parameter on i direction. μ_0 is the permittivity of the free space; N_i and S_i are the turn number and the area of the coil on i direction; I_i is the amplitude of the excitation current.
- $k_i = g_i n_i s_i (i = x, y, z)$ is the receiver parameter on i direction. n_i and s_i are the turn number and the area of the coil on i direction; g_i is system gain for the signal feeding from the receiver coil on i direction.

2.2 System Variable and Representation

Suppose now the coil on $i (i = x, y, z)$ direction is excited with a current $i(t) = I_i \sin(\omega t + \phi)$. Base on the formula given in [6], the magnetic field produced at $P(x, y, z)$ will be

$$\boldsymbol{B}_i = C_i \sin(\omega t + \phi)[\frac{3xz}{r^5}, -\frac{3yz}{r^5}, (\frac{3z^2}{r^5} - \frac{1}{r^3})] \tag{1}$$

So we define the excitation signal as:

$$f_0 = [X_t, Y_t, Z_t] = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{bmatrix}$$

If the receiver is placed on point P with random orientation, from (1), the magnitude of the magnetic field on receiver coil on $j (j = x, y, z)$ direction can be expressed as

$$B'_{ij} = C_i \sin(\omega t + \phi) f = \frac{\mu_0 N_i I_i S_i}{4\pi} \sin(\omega t + \phi) f$$

where f is a function only decided by the position and orientation of the receiver. When the two is invariant, f is a constant.

So according to the Falady's magnetic-electric inductive law, the amplitude of the amplified output voltage signal of the receiver coil on j direction will be

$$S_{ij} = g_j n_j s_j \frac{\mu_0 N_i \omega I_i S_i}{4\pi} f = \omega K_j C_i f = \omega K_j B_{ij} \tag{2}$$

where B_{ij} is the amplitude of B'_{ij}.

Define the final output of the sensor system as

$$f_s = [X_s, Y_s, Z_s] = \begin{bmatrix} s_{xx} & s_{yx} & s_{zx} \\ s_{xy} & s_{yy} & s_{zy} \\ s_{xz} & s_{yz} & s_{zz} \end{bmatrix}$$

where $s_{ij} (i, j = x, y, z)$ is the amplitude of the amplified induced voltage signal of the receiver coil on j direction. s_{ij} corresponds to the sending of the transmitter coil on i direction. According to (2), we get

$$f_s = K \bullet B = \begin{bmatrix} \omega k_x & & \\ & \omega k_y & \\ & & \omega k_z \end{bmatrix} \begin{bmatrix} B_{xx} & B_{yx} & B_{zx} \\ B_{xy} & B_{yy} & B_{zy} \\ B_{xz} & B_{yz} & B_{zz} \end{bmatrix} \tag{3}$$

where

$$K = \begin{bmatrix} \omega k_x & & \\ & \omega k_y & \\ & & \omega k_z \end{bmatrix}$$

B_{ij} $(i, j = x, y, z)$ is the amplitude of the magnetic field on the receiver coil on j direction. It corresponds to the sending of the transmitter coil on i direction.

3 Electromagnetic Tracking Algorithm

As shown in Fig.1, the source frame $X_0Y_0Z_0$ defined by the axes of the transmitter is used as source reference frame. $X_sY_sZ_s$ defined by the axes of the receiver is used as the sensor frame. Suppose now the receiver is located at $P(\alpha, \beta, r)$ in $X_0Y_0Z_0$. In order to get the receiver output f_s, considering first the coupling between $X_0Y_0Z_0$ and the source tracking frame $X_1Y_1Z_1$, which is defined by azimuth rotation $T(\alpha)$ and elevation rotation $T(\beta)$ from $X_0Y_0Z_0$. The X_1 axis of $X_1Y_1Z_1$ is aligned with the line connecting the transmitter and the receiver. So the coupled signal f_1 from f_0 is

$$f_1 = T(\beta)T(\alpha)f_0 \tag{4}$$

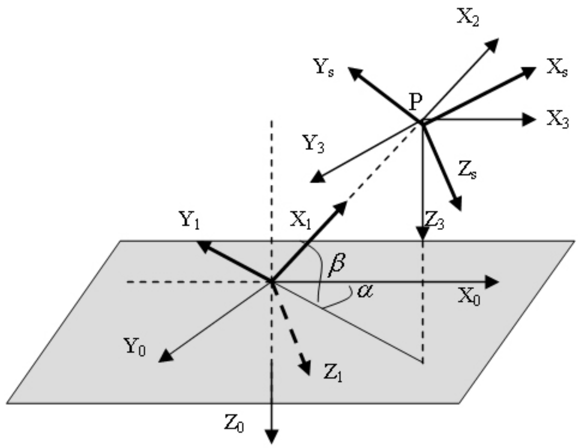

Fig. 1. Position and orientation

where

$$T(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, T(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

Then coupling f_1 from $X_1Y_1Z_1$ to the sensor tracking frame $X_2Y_2Z_2$, which locates at P and has the same orientation as $X_1Y_1Z_1$ (Fig.1). According to (1), the coupled signal in $X_2Y_2Z_2$ is

$$f_2 = \frac{2}{r^3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{bmatrix} f_1 \tag{5}$$

Define the sensor reference frame $X_3Y_3Z_3$ which locates at P and has same orientation as $X_0Y_0Z_0$. Consider the coupling output on $X_3Y_3Z_3$. From its definition, the output can be got by inverse rotation $T(-\alpha)$ and $T(-\beta)$, thus

$$f_3 = T(-\alpha)T(-\beta)f_2 \tag{6}$$

Suppose the receiver located at P with arbitrary orientation (φ, ξ, θ) which is Euler angle. The magnetic field on the sensor frame can be got by azimuth, elevation, and roll rotation, which is

$$B = T(\varphi)T(\xi)T(\theta)f_3$$

According to (3), the final output of the sensor can be written as

$$f_s = KB = \frac{2}{r^3} KT(\varphi)T(\xi)T(\theta)T(-\alpha)T(-\beta)ST(\beta)T(\alpha)f_0 \tag{7}$$

where

$$T(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\varphi) \end{bmatrix} T(\xi) = \begin{bmatrix} \cos(\xi) & 0 & -\sin(\xi) \\ 0 & 1 & 0 \\ \sin(\xi) & 0 & \cos(\xi) \end{bmatrix}$$

$$T(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{bmatrix}$$

$$f_0 = \begin{bmatrix} C_x & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C_z \end{bmatrix}, \qquad f_s = \begin{bmatrix} s_{xx} & s_{yx} & s_{zx} \\ s_{xy} & s_{yy} & s_{zy} \\ s_{xz} & s_{yz} & s_{zz} \end{bmatrix}$$

3.1 Determining the Value of r

From (4) and (5), f_2 can be written as

$$f_2 = \frac{2}{r^3} \begin{bmatrix} C_x \cos\alpha \cos\beta & C_y \sin\alpha \cos\beta & -C_z \sin\beta \\ \frac{1}{2} C_x \sin\alpha & -\frac{1}{2} C_y \cos\alpha & 0 \\ -\frac{1}{2} C_x \cos\alpha \sin\beta & -\frac{1}{2} C_y \sin\alpha \sin\beta & -\frac{1}{2} C_z \cos\beta \end{bmatrix} \tag{8}$$

According to (8) when the transmitter coil on Z direction is excited, the square amplitude of the magnetic field coupling on $X_2Y_2Z_2$ will be

$$P_2(Z) = \frac{4C_z^2}{r^6}(\sin^2\beta + \frac{1}{4}\cos^2\beta)$$

From Fig.1, we can see

$$\sin^2\beta = \frac{z^2}{r^2}, \cos^2\beta = \frac{x^2 + y^2}{r^2}$$

So $P_2(Z)$ can be rewritten as

$$P_2(Z) = \frac{4C_z^2}{r^6}(\frac{1}{4}x^2 + \frac{1}{4}y^2 + z^2)$$

Because the coils on three axes of the transmitter are equivalent, we can get the similar expression as $P_2(Z)$.

$$P_2(X) = \frac{4C_x^2}{r^6}(x^2 + \frac{1}{4}y^2 + \frac{1}{4}z^2), P_2(Y) = \frac{4C_y^2}{r^6}(\frac{1}{4}x^2 + y^2 + \frac{1}{4}z^2)$$

Adding $P_2(X)$,$P_2(Y)$ and $P_2(Z)$ together yields

$$\frac{P_2(X)}{C_x^2} + \frac{P_2(Y)}{C_y^2} + \frac{P_2(Z)}{C_z^2} = \frac{6}{r^6}(x^2 + y^2 + z^2)$$

So we can get

$$\frac{1}{r^4} = \frac{1}{6}[\frac{P_2(X)}{C_x^2} + \frac{P_2(Y)}{C_y^2} + \frac{P_2(Z)}{C_z^2}] \tag{9}$$

In order to get the value of $P_2(X)$, $P_2(Y)$ and $P_2(Z)$ from the receiver output f_s, some transformation must be made. According to (3), the magnetic field couples on $X_sY_sZ_z$ is

$$B = K^{-1}f_s = \begin{bmatrix} \frac{s_{xx}}{\omega k_x} & \frac{s_{yx}}{\omega k_x} & \frac{s_{zx}}{\omega k_x} \\ \frac{s_{xy}}{\omega k_y} & \frac{s_{yy}}{\omega k_y} & \frac{s_{zy}}{\omega k_y} \\ \frac{s_{xz}}{\omega k_z} & \frac{s_{yz}}{\omega k_z} & \frac{s_{zz}}{\omega k_z} \end{bmatrix}$$

Because at one point the square of amplitude of a vector is unique, $P_2(X)$,$P_2(Y)$ and $P_2(Z)$ can be written as

$$P_2(X) = (\frac{s_{xx}}{\omega k_x})^2 + (\frac{s_{xy}}{\omega k_y})^2 + (\frac{s_{xz}}{\omega k_z})^2$$

$$P_2(Y) = (\frac{s_{yx}}{\omega k_x})^2 + (\frac{s_{yy}}{\omega k_y})^2 + (\frac{s_{yz}}{\omega k_z})^2$$

$$P_2(Z) = (\frac{s_{zx}}{\omega k_x})^2 + (\frac{s_{zy}}{\omega k_y})^2 + (\frac{s_{zz}}{\omega k_z})^2$$

Combining these with (9) yields

$$\frac{1}{r^4} = \frac{1}{6}[(\frac{s_{xx}}{\omega C_x k_x})^2 + (\frac{s_{xy}}{\omega C_x k_y})^2 + (\frac{s_{xz}}{\omega C_x k_z})^2 + (\frac{s_{yx}}{\omega C_y k_x})^2 + (\frac{s_{yy}}{\omega C_y k_y})^2$$

$$+(\frac{s_{yz}}{\omega C_y k_z})^2 + (\frac{s_{zx}}{\omega C_z k_x})^2 + (\frac{s_{zy}}{\omega C_z k_y})^2 + (\frac{s_{zz}}{\omega C_z k_z})^2]$$

Because all variable are known in the expression, the distance information from transmitter to receiver can be got.

3.2 Determining the Value of α and β

Rewriting (7) as

$$K^{-1}f_s = \frac{2}{r^3}T(\varphi)T(\xi)T(\theta)T(-\alpha)T(-\beta)ST(\beta)T(\alpha)f_0 \tag{10}$$

Setting

$$X = \frac{2}{r^3}T(\varphi)T(\xi)T(\theta)T(-\alpha)T(-\beta)ST(\beta)T(\alpha)$$

$$W = X^TX = \frac{4}{r^6}T^{-1}(\alpha)T^{-1}(\beta)S^TT(\beta)T(\alpha)$$

From the definition of $T(\alpha)$ and $T(\beta)$, W can be written out as

$$W = X^TX = \frac{1}{r^6}F$$

where

$$F = \begin{bmatrix} 1+3\cos^2\alpha\cos^2\beta & 3\sin\alpha\cos\alpha\cos^2\beta & -3\cos\alpha\sin\beta\cos\beta \\ 3\sin\alpha\cos\alpha\cos^2\beta & 1+3\sin^2\alpha\cos^2\alpha & -3\sin\alpha\sin\beta\cos\beta \\ -3\cos\alpha\sin\beta\cos\beta & -3\sin\alpha\sin\beta\cos\beta & 1+3\sin^2\beta \end{bmatrix} \tag{11}$$

Applying these to (10) yields

$$(K^{-1}f_s)^T K^{-1} f_s = f_0^T X^T X f_0 = \frac{1}{r^6} f_0^T F f_0$$

thus

$$\begin{aligned} F = [F_1, F_2, F_3] &= r^6 (f_0^T)^{-1} (K^{-1} f_s)^T K^{-1} f_s f_0^{-1} \\ &= r^6 (f_0^T)^{-1} f_s^T K^{-1} K^{-1} f_s f_0^{-1} \end{aligned} \tag{12}$$

Take notice of the above definitions, F can be further written out

$$F_1 = r^6 \begin{bmatrix} \frac{s_{xx}^2}{(\omega C_x k_x)^2} + \frac{s_{xy}^2}{(\omega C_x k_y)^2} + \frac{s_{xz}^2}{(\omega C_x k_z)^2} \\ \frac{s_{xx}s_{yx}}{(\omega C_x k_x)(\omega C_y k_x)} + \frac{s_{xy}s_{yy}}{(\omega C_x k_y)(\omega C_y k_y)} + \frac{s_{xz}s_{yz}}{(\omega C_x k_z)(\omega C_y k_z)} \\ \frac{s_{xx}s_{zx}}{(\omega C_x k_x)(\omega C_z k_x)} + \frac{s_{xy}s_{zy}}{(\omega C_x k_y)(\omega C_z k_y)} + \frac{s_{xz}s_{zz}}{(\omega C_x k_z)(\omega C_z k_z)} \end{bmatrix}$$

$$F_2 = r^6 \begin{bmatrix} \frac{s_{xx}s_{yx}}{(\omega C_x k_x)(\omega C_y k_x)} + \frac{s_{xy}s_{yy}}{(\omega C_x k_y)(\omega C_y k_y)} + \frac{s_{xz}s_{yz}}{(\omega C_x k_z)(\omega C_y k_z)} \\ \frac{s_{yx}^2}{(\omega C_y k_x)^2} + \frac{s_{yy}^2}{(\omega C_y k_y)^2} + \frac{s_{yz}^2}{(\omega C_y k_z)^2} \\ \frac{s_{yx}s_{zx}}{(\omega C_y k_x)(\omega C_z k_x)} + \frac{s_{yy}s_{zy}}{(\omega C_y k_y)(\omega C_z k_y)} + \frac{s_{yz}s_{zz}}{(\omega C_y k_z)(\omega C_z k_z)} \end{bmatrix}$$

$$F_3 = r^6 \begin{bmatrix} \frac{s_{xx}s_{zx}}{(\omega C_x k_x)(\omega C_z k_x)} + \frac{s_{xy}s_{zy}}{(\omega C_x k_y)(\omega C_z k_y)} + \frac{s_{xz}s_{zz}}{(\omega C_x k_z)(\omega C_z k_z)} \\ \frac{s_{yx}s_{zx}}{(\omega C_y k_x)(\omega C_z k_x)} + \frac{s_{yy}s_{zy}}{(\omega C_y k_y)(\omega C_z k_y)} + \frac{s_{yz}s_{zz}}{(\omega C_y k_z)(\omega C_z k_z)} \\ \frac{s_{zx}^2}{(\omega C_z k_x)^2} + \frac{s_{zy}^2}{(\omega C_z k_y)^2} + \frac{s_{zz}^2}{(\omega C_z k_z)^2} \end{bmatrix}$$

Because (11) and (12) is equal, we get

$$\sin^2\beta = (F_{33} - 1)/3 \tag{13}$$

$$\begin{cases} \cos a = \dfrac{-F_{13}}{3\sin\beta\cos\beta} \\ \\ \sin a = \dfrac{-F_{23}}{3\sin\beta\cos\beta} \end{cases} \tag{14}$$

If the value field of β is $[0, 90°]$, the position information of α and β can be resolved from (13) and (14).

3.3 Determining the Value of θ, ξ, φ

After α, β and r have been known, f_3 can be got from (4) to (6). Then according to (7), we get

$$f_s = KT(\varphi)T(\xi)T(\theta)f_3 = KAf_3$$

where

$$\begin{aligned} A &= K^{-1} f_s f_3^{-1} \\ &= \begin{bmatrix} \cos\varphi\cos\theta & \cos\varphi\sin\theta & -\sin\varphi \\ \sin\xi\sin\varphi\cos\theta - \cos\xi\sin\theta & \sin\varphi\sin\xi\sin\theta + \cos\xi\cos\theta & \sin\xi\cos\varphi \\ \cos\xi\sin\varphi\cos\theta + \sin\xi\sin\theta & \cos\xi\sin\varphi\sin\theta + \sin\xi\cos\theta & \cos\xi\cos\varphi \end{bmatrix} \end{aligned}$$

Because f_s and f_3 are known, the orientation information can be got

$$\xi = arctg\left(\frac{A_{23}}{A_{33}}\right), \theta = arctg\left(\frac{A_{12}}{A_{11}}\right), \varphi = arctg\left(\frac{-A_{13}\cos\theta}{A_{11}}\right)$$

4 Experiment

In order to validate the algorithm, a AC magnetic tracking device is developed. It is composed of three parts : a source, a sensor and a processor. The source mainly includes a sine wave generator, a filter and amplifier module, a PWM modulation circuit and a transmitter with three coils. The sine wave generated by the generator is filtered and amplified. The PWM modulate circuit modulate the amplified signal which make the coils be excited sequently. The sensor includes three channels. Each channel is composed of one coil, one filter and amplify module, and one demodulation circuit. The signals of the three channels are digitized by a 12-bit AD. Finally the digitized signals are sent to the processor.

The scattering parameters are got by some calculation on the data which are got by measuring the magnetic field on three different points. They are shown in Table 1. The results show that big difference exits between the parameters. The main reason for this is the actual area and turn number are hard to keep same with large number of turns. In addition, if the inner coil is enclosed wholly by the outer coil, the magnetic field will be attenuated according to the frequency of the signal and this also results in the scattering of the parameter. In addition, the inconsistent circuit parameter also contributes to this phenomenon.

The algorithm using scattering parameter is applied to track the receiver. To compare with the traditional algorithm, an experiment is also done under

Table 1. System Parameters

$\omega C_x K_x$	31445	$\omega C_x K_y$	28183	$\omega C_x K_z$	33075
$\omega C_y K_x$	23992	$\omega C_y K_y$	19411	$\omega C_y K_z$	32976
$\omega C_z K_x$	26958	$\omega C_z K_y$	21291	$\omega C_z K_z$	27321

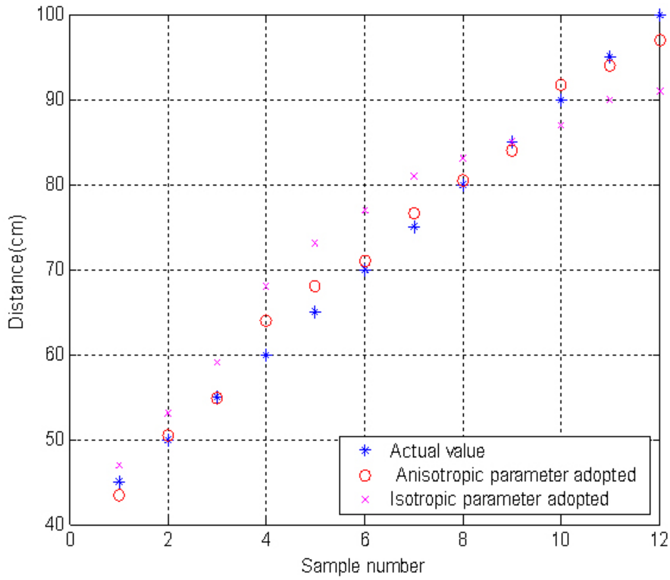

Fig. 2. Output Data Graph

the same condition, which takes an isotropic parameter whose value is 33075. It is an approximate value getting from the hardware design data. Fig.2 is the experiment result. The data is about the distance from transmitter to receiver. It is got by sampling the system output when the receiver moved around the transmitter in the range of 0.5-1m. The experiment results indicates the algorithm adopting scatter parameter improve the accuracy of the system. It also indicates the algorithm is valid.

5 Conclusion

An improved 6 DOF electromagnetic tracking algorithm is proposed. It utilizes the scattering parameters to calculate the pose and orientation information of the target, which are about the transmitter coils, the receiver coils and the involved circuits. Compared with the traditional algorithm which takes the system parameter as same in all the three directions, it is more accordant to the nature of the current tracking device. And it also simplifies the system hardware design. The experimental results prove the algorithm is valid and can improve the accuracy of the system.

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