# A New Method for Magnetic Position and Orientation Tracking

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Abstract-The method is based on two-axis generation of a quasi-static rotating magnetic field and three-axis sensing. Two mutually orthogonal coils fed with phase-quadrature currents comprise the excitation source, which is equal to a mechanically rotating magnetic dipole. The resulting excitation field rotates elliptically at any position in the near-field region. The ac part of the squared field magnitude is a sinusoidal wave at twice the excitation frequency. The following set of parameters uniquely characterize the excitation at the sensor's position: the phase of the squared field waveform, relative to the excitation currents, the minimum field value, the ratio of the field extremes, and the orientation of the excitation field plane. Simple and explicit analytical expressions are given which relate the first three parameters to the azimuth, elevation, and distance from the source to the sensor, respectively. The orientation of the sensor axes, relative to the plane of the excitation, can easily be determined by comparing the phase and amplitude of the measured signals against the phase and amplitude of the excitation field at the sensor's position. Apart from simplicity, the proposed method increases the speed of tracking; a single period of excitation is in principle sufficient to obtain all of the information needed to determine both the sensor's position and orientation. A continuous sinusoidal excitation mode allows an efficient phase-locking and accurate detection of the sensor output. It also improves the electromagnetic compatibility of the method.

Index Terms—Elliptically rotating excitation field, magnetic position and orientation tracking, magnetic position measurement, magnetic sensing, magnetic tracking system, rotating magnetic dipole field.

# I. INTRODUCTION

OST magnetic tracking techniques are based on accurate mapping of a 3D magnetic field around generating coils and computing from the field mapped the sensor position and orientation relative to the source. Usually, a number of orthogonal coils are used for generating and sensing magnetic fields [1]–[5]. Reference [6] suggests using only two excitation coils to increase the speed and reduce the complexity of the measuring system. According to [6], a sequence of two different field patterns is generated to provide enough information for measuring the sensor's coordinates and orientation. (It is assumed that the sensor resides in a certain quadrant of the source coordinate system.) Since [6] considers a two state excitation,

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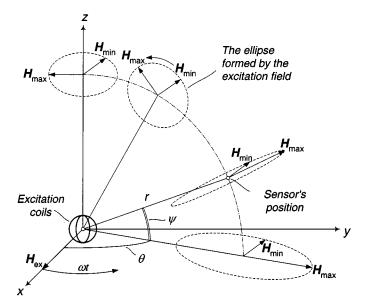


Fig. 1. The excitation field pattern. Magnetic field generated by the two excitation coils that are in space and phase quadrature rotates elliptically at any position in the near-field region.

the period of measurements is limited by at least two periods of the excitation frequency. The computing algorithm suggested in [6] is a rather complicated one.

The aims of the present work are to further increase the speed of magnetic tracking and simplify the computing algorithm. For this purpose, we suggest making the magnetic field rotate continuously by using a pair of excitation coils that are in space and phase quadrature. It is found that the resulting excitation field rotates elliptically at any position in the near-field region (see Fig. 1). Thus, the sensor sees a rotating field as though a properly oriented equivalent pair of excitation coils situated just around the sensor's center produced it. At any position, the "excitation ellipse" (see Fig. 1) has a unique set of parameters: the aspect ratio, size, phase, and orientation. These parameters can be related in a simple manner to the excitation field at the origin. As a result, information about the sensor's position and orientation can easily be extracted from the sensor's output. The period of measurements is limited in this case by a single period of the excitation frequency, during which the excitation field turns by one complete revolution.

#### II. SOURCE-SENSOR COUPLING

Let us assume (see Fig. 2) that the sensor is located within a certain quadrant of the source coordinate system xyz. The

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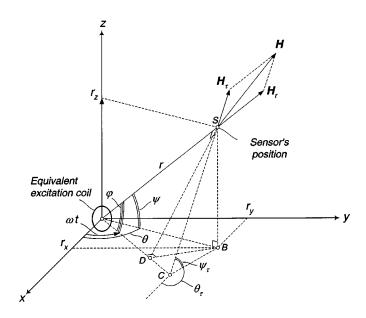


Fig. 2. Description of the excitation field at an arbitrary position,  $S(r, \psi, \theta)$ , in the near-field space. The two excitation coils are replaced by the single equivalent coil, the axis of which makes the angle  $\omega t$  with the x-axis.

TABLE I
DESCRIPTION OF THE QUASI-STATIC MAGNETIC DIPOLE FIELD

| М   | $H_r$                           | $H_{\tau}$                      |
|-----|---------------------------------|---------------------------------|
| NIA | $\frac{M}{2\pi r^3}\cos\varphi$ | $\frac{M}{4\pi r^3}\sin\varphi$ |

sensor's position is given by the distance, r, to the source, the azimuth,  $\theta$ , and the elevation,  $\psi$ .

Suppose now that one of the excitation coils, which is orthogonal to the x-axis, is fed with a current  $i_x = I\cos(\omega t)$ , and another excitation coil, which is orthogonal to the y-axis, is fed with a current  $i_y = I\sin(\omega t)$ . For the sake of simplicity, one can replace the two excitation coils by a single, imaginary coil that is rotating mechanically with an angular speed  $\omega t$ . The near field, H, (see Fig. 2) produced by such imaginary coil can be described in terms of radial,  $H_r$ , and tangential,  $H_\tau$ , components, which are given in Table I. M in Table I is the magnetic moment, A and N are the area and number of turns of the coil, and  $\varphi$  is the angle between the axis of the coil and direction toward the sensor (see Fig. 2).

To describe the total excitation field,  $\boldsymbol{H}$ , at any position in the near-field region, we involve angles  $\psi_{\tau}$  and  $\theta_{\tau}$  (see Fig. 2) that define direction of the  $\boldsymbol{H}_{\tau}$  field. Table II gives simple formulas describing the  $\boldsymbol{H}_r$  and  $\boldsymbol{H}_{\tau}$  components at any arbitrary position  $S(r,\theta,\psi)$ . These formulas are obtained by using the geometric constructions of Fig. 2 and elementary trigonometric transformations. The formulas of Tables I and II also allow one to write explicit equations describing the total excitation field components,  $H_x$ ,  $H_y$ ,  $H_z$ , for any sensor's position.

Our current aim is to investigate the behavior of the total excitation field  $\mathbf{H} = f(\omega t, r, \theta, \psi)$ . For the sake of simplicity, we

TABLE II
DESCRIPTION OF THE ROTATING MAGNETIC DIPOLE FIELD

| $r = \sqrt{r_x^2 + r_y^2 + r_z^2}$  | $\overline{BD}$                             | arphi  |
|---|---|--|
| $r_x = r \cos \psi \cos \theta$ $r_y = r \cos \psi \sin \theta$ $r_z = r \sin \psi$ | $r_y \cos \omega t - r_x \sin \omega t$     | $\arcsin \frac{\sqrt{\overline{BD}^2 + r_z^2}}{r}$                       |
| $\Psi_t$  | $\overline{BC}$                             | $\theta_t$   |
| $\arcsin \frac{r_z}{r \tan \varphi}$  | $r 	an arphi \cos arphi_{	au}$              | $\frac{\pi}{2} + \varpi t + \arccos \frac{\overline{BD}}{\overline{BC}}$ |
| $H_x$   | $H_{y}$                                     | $H_z$  |
| $H_r \cos \psi \cos \theta$   | $H_r \cos \psi \sin \theta$                 | $H_r \sin \psi$  |
| $+H_{\tau}\cos\psi_{\tau}\cos\theta_{\tau}$   | $+H_{\tau}\cos\psi_{\tau}\sin\theta_{\tau}$ | $+H_{\tau}\sin\psi_{\tau}$   |

TABLE III
DETERMINATION OF THE SENSOR'S POSITION

| $H^2$   | $H_{\min} = H \Big _{\varphi = \pi/2}$ | $H_{\max} = H \big _{\varphi = \psi}$                            |
|---|--|--|
| $H_r^2 + H_\tau^2$  | $\frac{M}{4\pi r^3}$                   | $H_{\min}\sqrt{3\cos^2\psi+1}$                                   |
| Azimuth, $\theta$   | Distance, r                            | Elevation, $\psi$  |
| is equal to the phase of $dH_{ac}^2/dt$ with reference to $i_y$ | $\sqrt[3]{\frac{M}{4\pi H_{\min}}}$    | $\arccos\sqrt{\frac{H_{\text{max}}^2}{3H_{\text{min}}^2}} - 1/3$ |

took  $\theta$  equal to zero, r equal to a constant and then expressed the  $H_x$ ,  $H_y$ ,  $H_z$  components in terms of  $\psi$  and  $\omega t$ . Eventually, the equations derived were reduced to the following form:

$$\begin{cases} H_x = H_{\min}(3\cos^2\psi - 1)\cos\varpi t \\ H_y = H_{\min}\sin\varpi t \\ H_z = H_{\min}1.5\sin2\psi\cos\varpi t \end{cases}$$
 (1)

where  $H_{\rm min}=M/4\pi r^3$  and  $\psi$  are fixed parameters.

It can easily be seen that equations (1) describe an ellipse with the center at point  $S(r,\theta,\psi)$  (see Fig. 2). Thus, it proves that the excitation field does rotate elliptically at any position in the near-field region, as shown in Fig. 1.

It is also worth noting that, for any  $\theta$ , the excitation field reaches its minima,  $H_{\min}$ , when  $\varphi=\pi/2$  and  $\omega t=\theta\pm\pi/2$ , and its maxima,  $H_{\max}$ , when  $\varphi=\psi$  and  $\omega t=\theta$  or  $\theta\pm\pi$ .

# III. Position Determination

Determination of the sensor's position is based on measuring the phase,  $\theta$ , and extremes,  $H_{\min}$ ,  $H_{\max}$ , of the ac part of the squared total excitation field,  $H_{ac}^2$ . Table III gives simple formulas relating the sensor's coordinates r,  $\theta$ , and  $\psi$  to the above parameters.

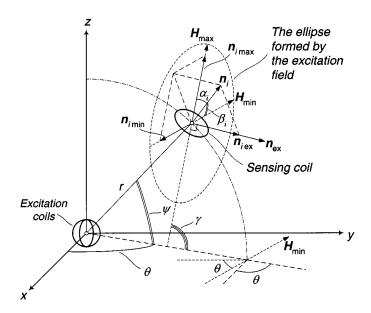


Fig. 3. Description of the sensor orientation. The orientation of the "excitation ellipse" relatively to the source axes system, (x, y, z), is uniquely determined by the angles  $\theta$  and  $\gamma$ . The orientation of the sensor axes system,  $\mathbf{n}_i$ ,  $\mathbf{n}_j$ ,  $\mathbf{n}_k$ , (only  $\mathbf{n}_i$  is shown) relatively to the semiaxes of the "excitation ellipse,"  $\mathbf{H}_{\min}$  and  $\mathbf{H}_{\max}$ , is uniquely determined by the angels  $\alpha_i$  and  $\beta_i$ .

#### IV. ORIENTATION DETERMINATION

The sensor's orientation can be described through a pair from the three unit vectors,  $n_i$ ,  $n_j$ ,  $n_k$ , representing the sensor's axes system. Fig. 3 shows only the vector  $n_i$  that is orthogonal to one of the three mutually orthogonal sensing coils. Fig. 3 also shows the "excitation ellipse" seen by the sensor and the vector  $n_{ex}$  that is orthogonal to the "excitation ellipse" plane.

To determine orientation of the vector  $n_i$  in the source coordinate system, three additional angles are introduced in Fig. 3: the angles  $\alpha_i$  and  $\beta_i$  that the vector  $n_i$  makes with the vectors  $H_{\max}$  and  $H_{\min}$ , and the angle  $\gamma$  that the vector  $H_{\max}$  makes with the projection of the radius vector r onto the x-y plane.

The sensing coil in Fig. 3 sees two sinusoidal fields,  $H_{\max}\cos(\omega t - \theta)$  and  $H_{\min}\sin(\omega t - \theta)$ , and generates the output voltage,  $v_i$ , that is proportional to the derivative of the sum of the projections of these fields onto the  $n_i$ -axis. Therefore,  $v_i \propto H_{i\max}\cos(\omega t - \theta - \delta_i)$ , where  $H_{i\max} = \sqrt{H_{\max}^2\cos^2\alpha_i + H_{\min}^2 \cdot \cos^2\beta_i}$  and  $\theta + \delta_i$  is the phase lag with reference to the excitation current  $i_x$ . Based on this relationship, coordinates of the vector  $n_i(n_{ix}, n_{iy}, n_{iz})$  in the source's axis system can easily be found (see Table IV).

TABLE IV DETERMINATION OF THE SENSOR'S ORIENTATION

| $\alpha_i$  | $oldsymbol{eta}_t$  | γ   |
|---|---|---|
| $\arccos\left(\frac{H_{i\max}}{H_{\max}}\sin\delta_i\right)$                                      | $\arccos\left(\frac{H_{i\max}}{H_{\max}}\cos\delta_i\right)$                                  | $\theta$ + arctan $\frac{H_{\min}}{H_{\max}}$           |
| $n_{i 	ext{max}}$   | $n_{i  m min}$  | $n_{iex}$   |
| $1 \cdot \cos \alpha_i = \frac{H_{i\max}}{H_{\max}} \cos \delta_i$                                | $1 \cdot \cos \beta_i = \frac{H_{i\max}}{H_{\min}} \sin \delta_i$                             | $\sqrt{1-\left(n_{imax}^2+n_{imin}^2\right)}$           |
| $n_{ix}$  | $n_{iy}$  | $n_{iz}$  |
| $n_{i\max} \cos \gamma \cos \theta$ $+ n_{i\min} \sin \theta$ $+ n_{iex} \sin \gamma \cos \theta$ | $n_{i\max} \cos \gamma \sin \theta + n_{i\min} \cos \theta + n_{iex} \sin \gamma \sin \theta$ | $n_{i\max} \sin \gamma \\ + n_{i\text{ex}} \cos \gamma$ |

 $H_{i \max}$ ,  $\theta$  (see also Table III), and  $\delta_i$  in Table IV can accurately be measured by using phase-locking technique.

#### V. CONCLUSIONS

The theoretical background for a new, simple, fast, and accurate magnetic position and orientation tracking method has been developed. The proposed continuous excitation mode allows an effective employment of phase-locking technique for an accurate detecting of the sensor's output. The continuous excitation also reduces the electromagnetic interference.

Preliminary experimental verifications made with the help of a standard lock-in amplifier have shown the resolution of about 1 mm in a 3.6 m range (for  $\psi < 70$  deg).

A computerized magnetic tracking system that is based on the proposed method is currently under development.

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