

# CHAPTER 1

## TUNED CIRCUITS

### LEARNING OBJECTIVES

Learning objectives are stated at the beginning of each chapter. These learning objectives serve as a preview of the information you are expected to learn in the chapter. The comprehensive check questions are based on the objectives. By successfully completing the OCC/ECC, you indicate that you have met the objectives and have learned the information. The learning objectives are listed below.

Upon completion of this chapter, you will be able to:

1. State the applications of a resonant circuit.
2. Identify the conditions that exist in a resonant circuit.
3. State and apply the formula for resonant frequency of an a.c. circuit.
4. State the effect of changes in inductance (L) and capacitance (C) on resonant frequency ( $f_r$ ).
5. Identify the characteristics peculiar to a series resonant circuit.
6. Identify the characteristics peculiar to a parallel resonant circuit.
7. State and apply the formula for Q.
8. State what is meant by the bandwidth of a resonant circuit and compute the bandwidth for a given circuit.
9. Identify the four general types of filters.
10. Identify how the series- and parallel-resonant circuit can be used as a bandpass or a band-reject filter.

### INTRODUCTION TO TUNED CIRCUITS

When your radio or television set is turned on, many events take place within the "receiver" before you hear the sound or see the picture being sent by the transmitting station.

Many different signals reach the antenna of a radio receiver at the same time. To select a station, the listener adjusts the tuning dial on the radio receiver until the desired station is heard. Within the radio or TV receiver, the actual "selecting" of the desired signal and the rejecting of the unwanted signals are accomplished by what is called a TUNED CIRCUIT. A tuned circuit consists of a coil and a capacitor connected in series or parallel. Later in this chapter you will see the application and advantages of both series- and parallel-tuned circuits. Whenever the characteristics of inductance and capacitance are found in a tuned circuit, the phenomenon as RESONANCE takes place.

You learned earlier in the *Navy Electricity and Electronics Training Series, Module 2*, chapter 4, that inductive reactance ( $X_L$ ) and capacitive reactance ( $X_C$ ) have opposite effects on circuit impedance (Z).

You also learned that if the frequency applied to an LCR circuit causes  $X_L$  and  $X_C$  to be equal, the circuit is RESONANT.

If you realize that  $X_L$  and  $X_C$  can be equal ONLY at ONE FREQUENCY (the resonant frequency), then you will have learned the most important single fact about resonant circuits. This fact is the principle that enables tuned circuits in the radio receiver to select one particular frequency and reject all others. This is the reason why so much emphasis is placed on  $X_L$  and  $X_C$  in the discussions that follow.

Examine figure 1-1. Notice that a basic tuned circuit consists of a coil and a capacitor, connected either in series, view (A), or in parallel, view (B). The resistance ( $R$ ) in the circuit is usually limited to the inherent resistance of the components (particularly the resistance of the coil). For our purposes we are going to disregard this small resistance in future diagrams and explanations.

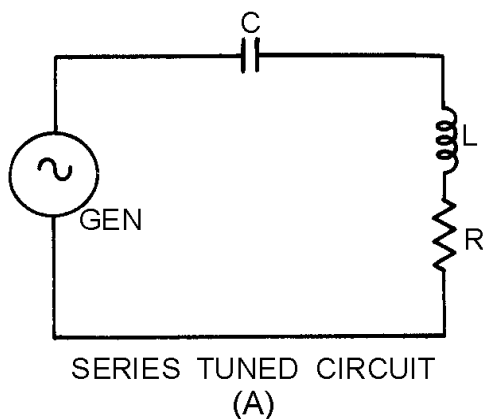


Figure 1-1A.—Basic tuned circuits. **SERIES TUNED CIRCUIT**

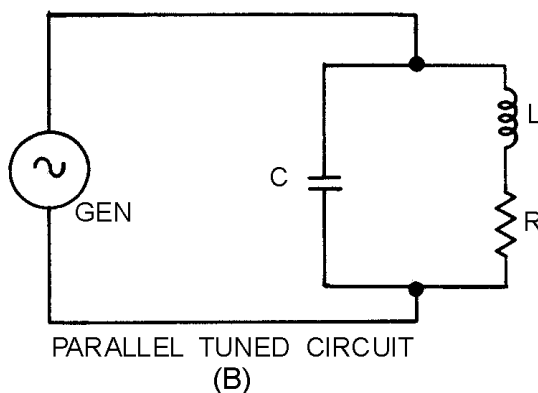


Figure 1-1B.—Basic tuned circuits. **PARALLEL TUNED CIRCUIT**

You have already learned how a coil and a capacitor in an a.c. circuit perform. This action will be the basis of the following discussion about tuned circuits.

Why should you study tuned circuits? Because the tuned circuit that has been described above is used in just about every electronic device, from remote-controlled model airplanes to the most sophisticated space satellite.

You can assume, if you are going to be involved in electricity or electronics, that you will need to have a good working knowledge of tuned circuits and how they are used in electronic and electrical circuits.

## REVIEW OF SERIES/PARALLEL A.C. CIRCUITS

First we will review the effects of frequency on a circuit which contains resistance, inductance, and capacitance. This review recaps what you previously learned in the Inductive and Capacitive Reactance chapter in *module 2* of the *NEETS*.

### FREQUENCY EFFECTS ON RLC CIRCUITS

Perhaps the most often used control of a radio or television set is the station or channel selector. Of course, the volume, tone, and picture quality controls are adjusted to suit the individual's taste, but very often they are not adjusted when the station is changed. What goes on behind this station selecting? In this chapter, you will learn the basic principles that account for the ability of circuits to "tune" to the desired station.

#### Effect of Frequency on Inductive Reactance

In an a.c. circuit, an inductor produces inductive reactance which causes the current to lag the voltage by 90 degrees. Because the inductor "reacts" to a changing current, it is known as a reactive component. The opposition that an inductor presents to a.c. is called inductive reactance ( $X_L$ ). This opposition is caused by the inductor "reacting" to the changing current of the a.c. source. Both the inductance and the frequency determine the magnitude of this reactance. This relationship is stated by the formula:

$$X_L = 2\pi fL$$

Where:

$X_L$  = the inductive reactance in ohms

$f$  = the frequency in hertz

$L$  = the inductance in henries

$\pi = 3.1416$

As shown in the equation, any increase in frequency, or " $f$ ," will cause a corresponding increase of inductive reactance, or " $X_L$ ." Therefore, the INDUCTIVE REACTANCE VARIES DIRECTLY WITH THE FREQUENCY. As you can see, the higher the frequency, the greater the inductive reactance; the lower the frequency, the less the inductive reactance for a given inductor. This relationship is illustrated in figure 1-2. Increasing values of  $X_L$  are plotted in terms of increasing frequency. Starting at the lower left corner with zero frequency, the inductive reactance is zero. As the frequency is increased (reading to the right), the inductive reactance is shown to increase in direct proportion.

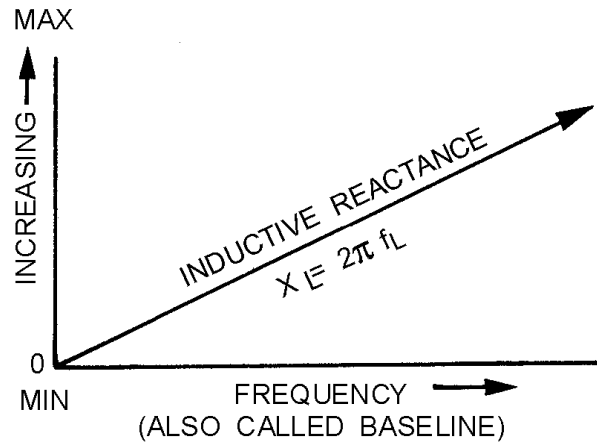


Figure 1-2.—Effect of frequency on inductive reactance.

### Effect of Frequency on Capacitive Reactance

In an a.c. circuit, a capacitor produces a reactance which causes the current to lead the voltage by 90 degrees. Because the capacitor "reacts" to a changing voltage, it is known as a reactive component. The opposition a capacitor presents to a.c. is called capacitive reactance ( $X_C$ ). The opposition is caused by the capacitor "reacting" to the changing voltage of the a.c. source. The formula for capacitive reactance is:

$$X_C = \frac{1}{2\pi fC}$$

Where:

$X_C$  = the capacitive reactance in ohms

$f$  = the frequency in hertz

$C$  = the capacitance in farads

$\pi = 3.1416$

In contrast to the inductive reactance, this equation indicates that the CAPACITIVE REACTANCE VARIES INVERSELY WITH THE FREQUENCY. When  $f = 0$ ,  $X_C$  is infinite ( $\infty$ ) and decreases as frequency increases. That is, the lower the frequency, the greater the capacitive reactance; the higher the frequency, the less the reactance for a given capacitor.

As shown in figure 1-3, the effect of capacitance is opposite to that of inductance. Remember, capacitance causes the current to lead the voltage by 90 degrees, while inductance causes the current to lag the voltage by 90 degrees.

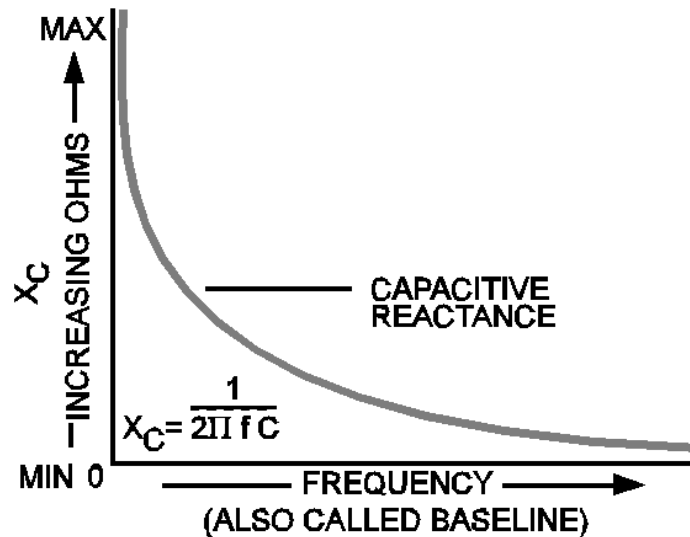


Figure 1-3.—Effect of frequency on capacitive reactance.

### Effect of Frequency on Resistance

In the expression for inductive reactance,  $X_L = 2\pi fL$ , and in the expression for capacitive reactance,

$$X_C = \frac{1}{2\pi fC}$$

both contain "f" (frequency). Any change of frequency changes the reactance of the circuit components as already explained. So far, nothing has been said about the effect of frequency on resistance. In an Ohm's law relationship, such as  $R = E/I$  no "f" is involved. Thus, for all practical purposes, a change of frequency does not affect the resistance of the circuit. If a 60-hertz a.c. voltage causes 20 milliamperes of current in a resistive circuit, then the same voltage at 2000 hertz, for example, would still cause 20 milliamperes to flow.

NOTE: Remember that the total opposition to a.c. is called impedance (Z). Impedance is the combination of inductive reactance ( $X_L$ ), capacitive reactance ( $X_C$ ), and resistance (R). When dealing with a.c. circuits, the impedance is the factor with which you will ultimately be concerned. But, as you have just been shown, the resistance (R) is not affected by frequency. Therefore, the remainder of the discussion of a.c. circuits will only be concerned with the reactance of inductors and capacitors and will ignore resistance.

### A.c. Circuits Containing Both Inductive and Capacitive Reactances

A.c. circuits that contain both an inductor and a capacitor have interesting characteristics because of the opposing effects of L and C.  $X_L$  and  $X_C$  may be treated as reactors which are 180 degrees out of phase. As shown in figure 1-2, the vector for  $X_L$  should be plotted above the baseline; vector for  $X_C$ , figure 1-3, should be plotted below the baseline. In a series circuit, the effective reactance, or what is termed the RESULTANT REACTANCE, is the difference between the individual reactances. As an equation, the resultant reactance is:

$$X = X_L - X_C$$

Suppose an a.c. circuit contains an  $X_L$  of 300 ohms and an  $X_C$  of 250 ohms. The resultant reactance is:

$$X = X_L - X_C = 300 - 250 = 50 \text{ ohms (inductive)}$$

In some cases, the  $X_C$  may be larger than the  $X_L$ . If  $X_L = 1200$  ohms and  $X_C = 4000$  ohms, the difference is:  $X = X_L - X_C = 1200 - 4000 = -2800$  ohms (capacitive). The total carries the sign (+ or -) of the greater number (factor).

*Q-1. What is the relationship between frequency and the values of (a)  $X_L$ , (b)  $X_C$ , and (c)  $R$ ?*

*Q-2. In an a.c. circuit that contains both an inductor and a capacitor, what term is used for the difference between the individual reactances?*

## RESONANCE

For every combination of  $L$  and  $C$ , there is only ONE frequency (in both series and parallel circuits) that causes  $X_L$  to exactly equal  $X_C$ ; this frequency is known as the RESONANT FREQUENCY. When the resonant frequency is fed to a series or parallel circuit,  $X_L$  becomes equal to  $X_C$ , and the circuit is said to be RESONANT to that frequency. The circuit is now called a RESONANT CIRCUIT; resonant circuits are tuned circuits. The circuit condition wherein  $X_L$  becomes equal to  $X_C$  is known as RESONANCE.

Each LCR circuit responds to resonant frequency differently than it does to any other frequency. Because of this, an LCR circuit has the ability to separate frequencies. For example, suppose the TV or radio station you want to see or hear is broadcasting at the resonant frequency. The LC "tuner" in your set can divide the frequencies, picking out the resonant frequency and rejecting the other frequencies. Thus, the tuner selects the station you want and rejects all other stations. If you decide to select another station, you can change the frequency by tuning the resonant circuit to the desired frequency.

## RESONANT FREQUENCY

As stated before, the frequency at which  $X_L$  equals  $X_C$  (in a given circuit) is known as the resonant frequency of that circuit. Based on this, the following formula has been derived to find the exact resonant frequency when the values of circuit components are known:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

There are two important points to remember about this formula. First, the resonant frequency found when using the formula will cause the reactances ( $X_L$  and  $X_C$ ) of the  $L$  and  $C$  components to be equal. Second, any change in the value of either  $L$  or  $C$  will cause a change in the resonant frequency.

An increase in the value of either  $L$  or  $C$ , or both  $L$  and  $C$ , will lower the resonant frequency of a given circuit. A decrease in the value of  $L$  or  $C$ , or both  $L$  and  $C$ , will raise the resonant frequency of a given circuit.

The symbol for resonant frequency used in this text is  $f$ . Different texts and references may use other symbols for resonant frequency, such as  $f_o$ ,  $F_r$ , and  $fR$ . The symbols for many circuit parameters have been standardized while others have been left to the discretion of the writer. When you study, apply the rules given by the writer of the text or reference; by doing so, you should have no trouble with nonstandard symbols and designations.

The resonant frequency formula in this text is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where:

$f_r$  = the resonant frequency in hertz

$L$  = the inductance in henries

$C$  = the capacitance in farads

$\pi$  = 3.1416

By substituting the constant .159 for the quantity

$$\frac{1}{2\pi}$$

the formula can be simplified to the following:

$$f_r = \frac{.159}{\sqrt{LC}}$$

Let's use this formula to figure the resonant frequency ( $f_r$ ). The circuit is shown in the practice tank circuit of figure 1-4.

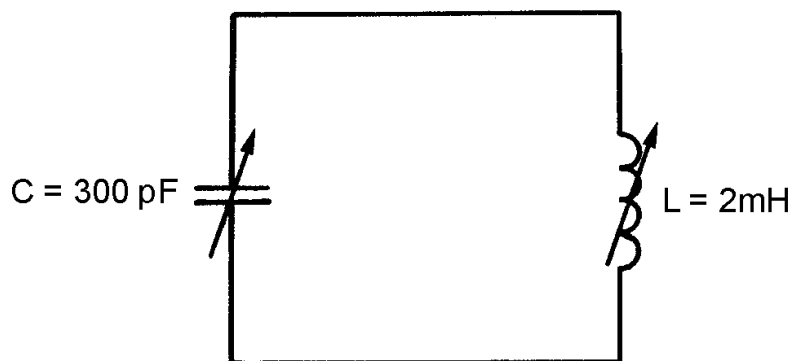


Figure 1-4.—Practice tank circuit.

Given:

$$L = 2\text{mH } (2 \times 10^{-3} \text{ H})$$

$$C = 300\text{pF } (300 \times 10^{-12} \text{ F})$$

Solution:

$$f_r = \frac{.159}{\sqrt{LC}}$$

$$f_r = \frac{.159}{\sqrt{(2 \times 10^{-3} \text{ H}) \times (300 \times 10^{-12} \text{ F})}}$$

$$f_r = \frac{.159}{\sqrt{600 \times 10^{-15}}} \quad \begin{array}{l} \text{(F and H are shown in this} \\ \text{step to show units)} \end{array}$$

$$f_r = \frac{.159}{60 \times 10^{-14}} \quad \begin{array}{l} \text{(Solving for square root} \\ 60 = 7.75 \times 10^{-14} = 10^{-7}) \end{array}$$

$$f_r = \frac{.159}{7.75 \times 10^{-7}}$$

$$f_r = \frac{.159 \times 10^7}{7.75}$$

$$f_r = \frac{.159 \times 10^4}{7.75}$$

$$f_r = 20.5 \times 10^4 \text{ (rounded off)}$$

$$f_r = 205,000 \text{ Hz or } 205 \text{ kHz}$$

The important point here is not the formula nor the mathematics. In fact, you may never have to compute a resonant frequency. The important point is for you to see that any given combination of L and C can be resonant at only one frequency; in this case, 205 kHz.

The universal reactance curves of figures 1-2 and 1-3 are joined in figure 1-5 to show the relative values of  $X_L$  and  $X_C$  at resonance, below resonance, and above resonance.



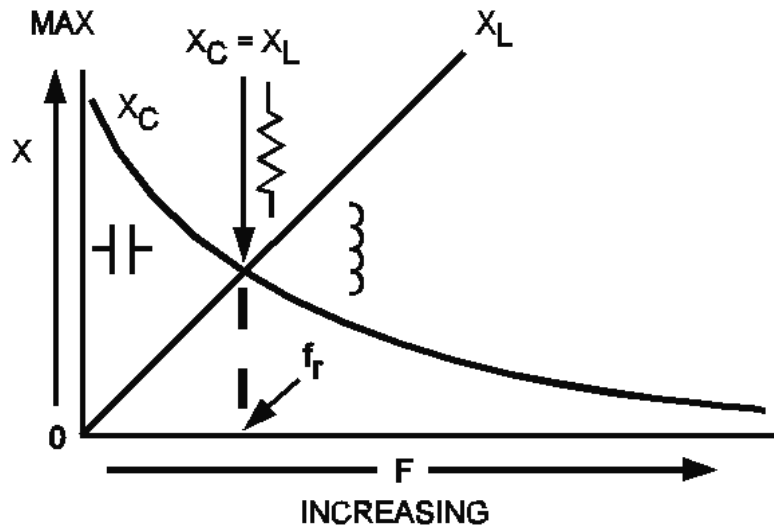


Figure 1-5.—Relationship between  $X_L$  and  $X_C$  as frequency increases.

First, note that  $f_r$ , (the resonant frequency) is that frequency (or point) where the two curves cross. At this point, and ONLY this point,  $X_L$  equals  $X_C$ . Therefore, the frequency indicated by  $f_r$  is the one and only frequency of resonance. Note the resistance symbol which indicates that at resonance all reactance is cancelled and the circuit impedance is effectively purely resistive. Remember, a.c. circuits that are resistive have no phase shift between voltage and current. Therefore, at resonance, phase shift is cancelled. The phase angle is effectively zero.

Second, look at the area of the curves to the left of  $f_r$ . This area shows the relative reactances of the circuit at frequencies BELOW resonance. To these LOWER frequencies,  $X_C$  will always be greater than  $X_L$ . There will always be some capacitive reactance left in the circuit after all inductive reactance has been cancelled. Because the impedance has a reactive component, there will be a phase shift. We can also state that below  $f_r$  the circuit will appear capacitive.

Lastly, look at the area of the curves to the right of  $f_r$ . This area shows the relative reactances of the circuit at frequencies ABOVE resonance. To these HIGHER frequencies,  $X_L$  will always be greater than  $X_C$ . There will always be some inductive reactance left in the circuit after all capacitive reactance has been cancelled. The inductor symbol shows that to these higher frequencies, the circuit will always appear to have some inductance. Because of this, there will be a phase shift.

## RESONANT CIRCUITS

Resonant circuits may be designed as series resonant or parallel resonant. Each has the ability to discriminate between its resonant frequency and all other frequencies. How this is accomplished by both series- and parallel-LC circuits is the subject of the next section.

NOTE: Practical circuits are often more complex and difficult to understand than simplified versions. Simplified versions contain all of the basic features of a practical circuit, but leave out the nonessential features. For this reason, we will first look at the IDEAL SERIES-RESONANT CIRCUIT—a circuit that really doesn't exist except for our purposes here.

## THE IDEAL SERIES-RESONANT CIRCUIT

The ideal series-resonant circuit contains no resistance; it consists of only inductance and capacitance in series with each other and with the source voltage. In this respect, it has the same characteristics of the series circuits you have studied previously. Remember that current is the same in all parts of a series circuit because there is only one path for current.

Each LC circuit responds differently to different input frequencies. In the following paragraphs, we will analyze what happens internally in a series-LC circuit when frequencies at resonance, below resonance, and above resonance are applied. The L and C values in the circuit are those used in the problem just studied under resonant-frequency. The frequencies applied are the three inputs from figure 1-6. Note that the resonant frequency of each of these components is 205 kHz, as figured in the problem.

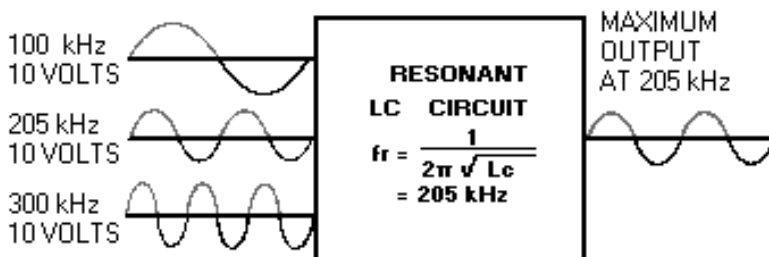


Figure 1-6.—Output of the resonant circuit.

### How the Ideal Series-LC Circuit Responds to the Resonant Frequency (205 kHz)

Given:

$$L = 2 \text{ mH } (2 \times 10^{-3} \text{ H})$$

$$C = 300 \text{ pF } (300 \times 10^{-12} \text{ F})$$

$$f_r = 205 \text{ kHz (rounded off)}$$

$$f_r = \frac{.159}{\sqrt{LC}}$$

$$X_L = 2580 \text{ ohms } (2\pi fL)$$

$$X_C = 2580 \text{ ohms } \left(\frac{1}{2\pi fC}\right)$$

$$E_s = 10 \text{ volts at a frequency } 205 \text{ kHz}$$

Note: You are given the values of  $X_L$ ,  $X_C$ , and  $f_r$  but you can apply the formulas to figure them. The values given are rounded off to make it easier to analyze the circuit.

First, note that  $X_L$  and  $X_C$  are equal. This shows that the circuit is resonant to the applied frequency of 205 kHz.  $X_L$  and  $X_C$  are opposite in effect; therefore, they subtract to zero. (2580 ohms – 2580 ohms = zero.) At resonance, then,  $X = \text{zero}$ . In our theoretically perfect circuit with zero resistance and zero reactance, the total opposition to current ( $Z$ ) must also be zero.

Now, apply Ohm's law for a.c. circuits:

$$I = \frac{E}{Z}$$

$$I = \frac{10 \text{ volts}}{0 \text{ ohms}}$$

$$I = \text{INFINITELY HIGH}$$

Don't be confused by this high value of current. Our perfect, but impossible, circuit has no opposition to current. Therefore, current flow will be extremely high. The important points here are that AT RESONANCE, impedance is VERY LOW, and the resulting current will be comparatively HIGH.

If we apply Ohm's law to the individual reactances, we can figure relative values of voltage across each reactance.

$$E_L = I \times X_L$$

$$E_C = I \times X_C$$

These are reactive voltages that you have studied previously. The voltage across each reactance will be comparatively high. A comparatively high current times 2580 ohms yields a high voltage. At any given instant, this voltage will be of opposite polarity because the reactances are opposite in effect.  $E_L + E_C =$  zero volts

### WARNING

**THE INDIVIDUAL VOLTAGES MAY REACH QUITE HIGH VALUES.  
ALTHOUGH LITTLE POWER IS PRESENT, THE VOLTAGE IS REAL AND  
CARE SHOULD BE TAKEN IN WORKING WITH IT.**

Let's summarize our findings so far. In a series-LC circuit with a resonant-frequency voltage applied, the following conditions exist:

- $X_L$  and  $X_C$  are equal and subtract to zero.
- Resultant reactance is zero ohms.
- Impedance ( $Z$ ) is reduced to a MINIMUM value.
- With minimum  $Z$ , current is MAXIMUM for a given voltage.
- Maximum current causes maximum voltage drops across the individual reactances.

All of the above follow in sequence from the fact that  $X_L = X_C$  at the resonant frequency.

### How the Ideal Series-LC Circuit Respond to a Frequency Below Resonance (100 kHz)

Given:

$$L = 2 \text{ mH } (2 \times 10^{-3} \text{ H})$$

$$C = 300 \text{ pF } (300 \times 10^{-12} \text{ F})$$

$$f_r = 205 \text{ kHz (at resonant frequency)}$$

$$f_r = \frac{.159}{\sqrt{LC}}$$

$$X_L = 1260 \text{ ohms (rounded off) (at 100 kHz)}$$

$$X_C = 5300 \text{ ohms (rounded off) (at 100 kHz)}$$

$$E_s = 10 \text{ volts (at 100 kHz)}$$

(As in the previous analysis, you are given values that are possible for you to compute. If you do the computations, remember that most values are rounded off.)

First, note that  $X_L$  and  $X_C$  are no longer equal.  $X_C$  is larger than it was at resonance;  $X_L$  is smaller. By applying the formulas you have learned, you know that a lower frequency produces a higher capacitive reactance and a lower inductive reactance. The reactances subtract but do not cancel ( $X_L - X_C = 1260 - 5300 = 4040$  ohms (capacitive)). At an input frequency of 100 kHz, the circuit (still resonant to 205 kHz) has a net reactance of 4040 ohms. In our theoretically perfect circuit, the total opposition ( $Z$ ) is equal to  $X$ , or 4040 ohms.

As before, let's apply Ohm's law to the new conditions.

$$I = \frac{E}{Z}$$

$$I = \frac{10 \text{ volts}}{4040 \text{ ohms}}$$

$$I = .00248 \text{ ampere} \\ \text{(approximately 2.5 mA)}$$

The voltage drops across the reactances are as follows:

$$E_L = I \times X_L$$

$$E_L = .0025 \text{ A} \times 1260 \Omega$$

$$E_L = 3 \text{ volts (approximately)}$$

$$E_C = I \times X_C$$

$$E_C = .0025 \text{ A} \times 5300 \Omega$$

$$E_C = 13 \text{ volts (approximately)}$$

In summary, in a series-LC circuit with a source voltage that is below the resonant frequency (100 kHz in the example), the resultant reactance ( $X$ ), and therefore impedance, is higher than at resonance. In addition current is lower, and the voltage drops across the reactances are lower. All of the above follow in sequence due to the fact that  $X_C$  is greater than  $X_L$  at any frequency lower than the resonant frequency.

### **How the Ideal Series-LC Circuit Responds to a Frequency Above Resonance (300 kHz)**

Given:

$$L = 2 \text{ mH } (2 \times 10^{-3} \text{ H})$$

$$C = 300 \text{ pF } (300 \times 10^{-12} \text{ F})$$

$$f_r = 205 \text{ kHz (at resonant frequency)}$$

$$X_L = 3770 \text{ ohms (rounded off) (at 300 kHz)}$$

$$X_C = 1770 \text{ ohms (rounded off) (at 300 kHz)}$$

$$E_s = 10 \text{ volts (at 300 kHz)}$$

Again,  $X_L$  and  $X_C$  are not equal. This time,  $X_L$  is larger than  $X_C$ . (If you don't know why, apply the formulas and review the past several pages.) The resultant reactance is 2000 ohms ( $X_L - X_C = 3770 - 1770 = 2000$  ohms.) Therefore, the resultant reactance ( $X$ ), or the impedance of our perfect circuit at 300 kHz, is 2000 ohms.

By applying Ohm's law as before:

$$I = 5 \text{ milliamperes}$$

$$E_L = 19 \text{ volts (rounded off)}$$

$$E_C = 9 \text{ volts (rounded off)}$$

In summary, in a series-LC circuit with a source voltage that is above the resonant frequency (300 kHz in this example), impedance is higher than at resonance, current is lower, and the voltage drops across the reactances are lower. All of the above follow in sequence from the fact that  $X_L$  is greater than  $X_C$  at any frequency higher than the resonant frequency.

### **Summary of the Response of the Ideal Series-LC Circuit to Frequencies Above, Below, and at Resonance**

The ideal series-resonant circuit has zero impedance. The impedance increases for frequencies higher and lower than the resonant frequency. The impedance characteristic of the ideal series-resonant circuit results because resultant reactance is zero ohms at resonance and ONLY at resonance. All other frequencies provide a resultant reactance greater than zero.

Zero impedance at resonance allows maximum current. All other frequencies have a reduced current because of the increased impedance. The voltage across the reactance is greatest at resonance because voltage drop is directly proportional to current. All discrimination between frequencies results from the fact that  $X_L$  and  $X_C$  completely counteract ONLY at the resonant frequency.

## How the Typical Series-LC Circuit Differs From the Ideal

As you learned much earlier in this series, resistance is always present in practical electrical circuits; it is impossible to eliminate. A typical series-LC circuit, then, has  $R$  as well as  $L$  and  $C$ .

If our perfect (ideal) circuit has zero resistance, and a typical circuit has "some" resistance, then a circuit with a very small resistance is closer to being perfect than one that has a large resistance. Let's list what happens in a series-resonant circuit because resistance is present. This is not new to you - just a review of what you have learned previously.

In a series-resonant circuit that is basically  $L$  and  $C$ , but that contains "some"  $R$ , the following statements are true:

- $X_L$ ,  $X_C$ , and  $R$  components are all present and can be shown on a vector diagram, each at right angles with the resistance vector (baseline).
- At resonance, the resultant reactance is zero ohms. Thus, at resonance, The circuit impedance equals only the resistance ( $R$ ). The circuit impedance can never be less than  $R$  because the original resistance will always be present in the circuit.
- At resonance, a practical series-RLC circuit ALWAYS has MINIMUM impedance. The actual value of impedance is that of the resistance present in the circuit ( $Z = R$ ).

Now, if the designers do their very best (and they do) to keep the value of resistance in a practical series-RLC circuit LOW, then we can still get a fairly high current at resonance. The current is NOT "infinitely" high as in our ideal circuit, but is still higher than at any other frequency. The curve and vector relationships for the practical circuit are shown in figure 1-7.

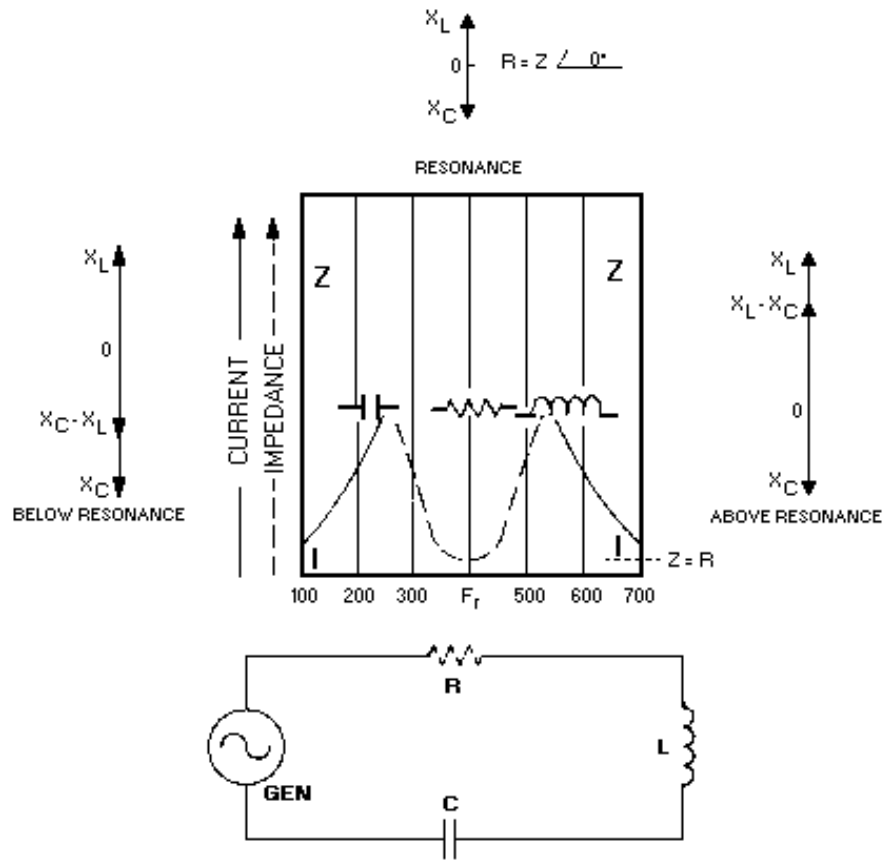


Figure 1-7.—Curves of impedance and current in an RLC series resonant circuit.

Note that the impedance curve does not reach zero at its minimum point. The vectors above and below resonance show that the phase shift of the circuit at these frequencies is less than 90 degrees because of the resistance.

The horizontal width of the curve is a measure of how well the circuit will pick out (discriminate) the one desired frequency. The width is called **BANDWIDTH**, and the ability to discriminate between frequencies is known as **SELECTIVITY**. Both of these characteristics are affected by resistance. Lower resistance allows narrower bandwidth, which is the same as saying the circuit has better selectivity. Resistance, then, is an unwanted quantity that cannot be eliminated but can be kept to a minimum by the circuit designers.

More on bandwidth, selectivity, and measuring the effects of resistance in resonant circuits will follow the discussion of parallel resonance.

*Q-3. State the formula for resonant frequency.*

*Q-4. If the inductor and capacitor values are increased, what happens to the resonant frequency?*

*Q-5. In an "ideal" resonant circuit, what is the relationship between impedance and current?*

*Q-6. In a series-RLC circuit, what is the condition of the circuit if there is high impedance, low current, and low reactance voltages?*

## How the Parallel-LC Circuit Stores Energy

A parallel-LC circuit is often called a TANK CIRCUIT because it can store energy much as a tank stores liquid. It has the ability to take energy fed to it from a power source, store this energy alternately in the inductor and capacitor, and produce an output which is a continuous a.c. wave. You can understand how this is accomplished by carefully studying the sequence of events shown in figure 1-8. You must thoroughly understand the capacitor and inductor action in this figure before you proceed further in the study of parallel-resonant circuits.

In each view of figure 1-8, the waveform is of the charging and discharging CAPACITOR VOLTAGE. In view (A), the switch has been moved to position C. The d.c. voltage is applied across the capacitor, and the capacitor charges to the potential of the battery.

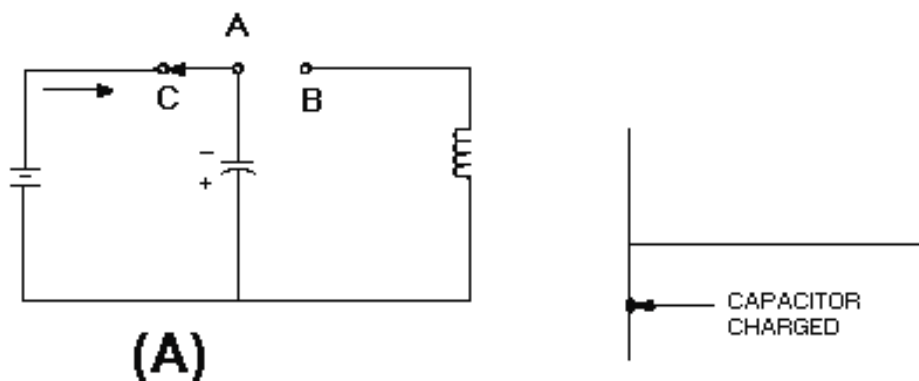


Figure 1-8A.—Capacitor and inductor action in a tank circuit.

In view (B), moving the switch to the right completes the circuit from the capacitor to the inductor and places the inductor in series with the capacitor. This furnishes a path for the excess electrons on the upper plate of the capacitor to flow to the lower plate, and thus starts neutralizing the capacitor charge. As these electrons flow through the coil, a magnetic field is built up around the coil. The energy which was first stored by the electrostatic field of the capacitor is now stored in the electromagnetic field of the inductor.

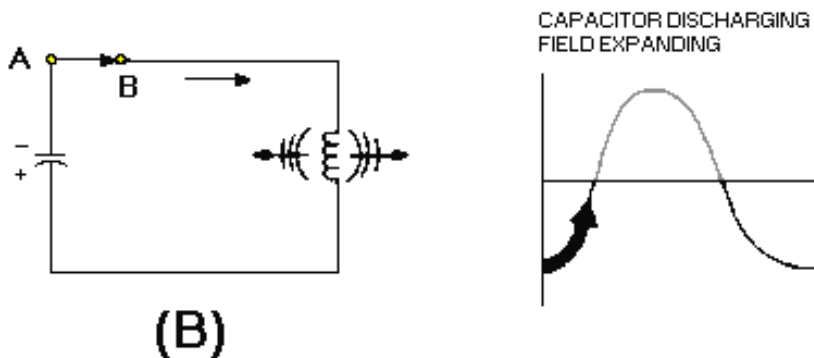


Figure 1-8B.—Capacitor and inductor action in a tank circuit.



View (C) shows the capacitor discharged and a maximum magnetic field around the coil. The energy originally stored in the capacitor is now stored entirely in the magnetic field of the coil.

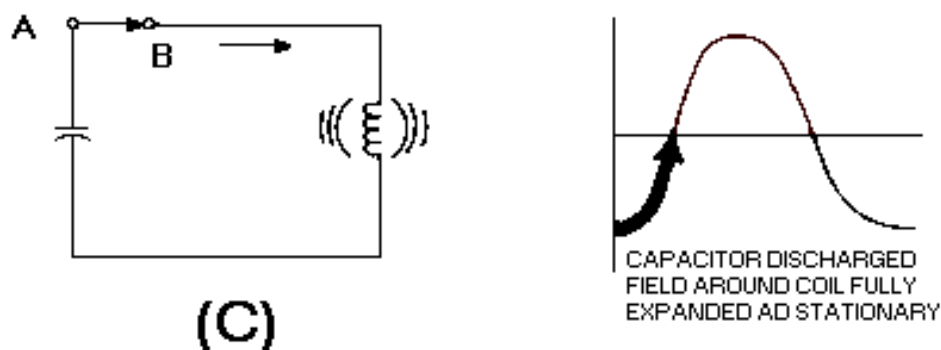


Figure 1-8C.—Capacitor and inductor action in a tank circuit.

Since the capacitor is now completely discharged, the magnetic field surrounding the coil starts to collapse. This induces a voltage in the coil which causes the current to continue flowing in the same direction and charges the capacitor again. This time the capacitor charges to the opposite polarity, view (D).

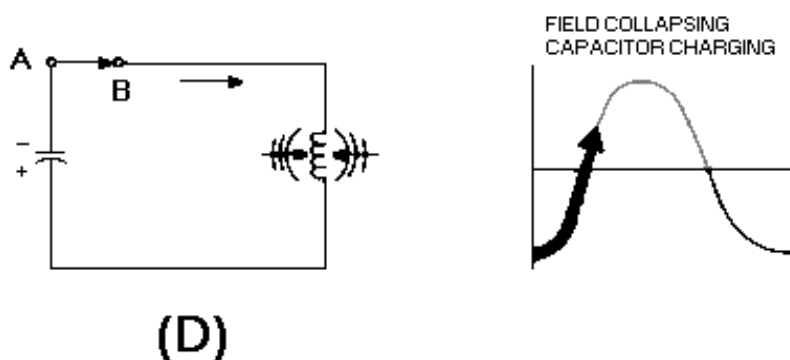


Figure 1-8D.—Capacitor and inductor action in a tank circuit.

In view (E), the magnetic field has completely collapsed, and the capacitor has become charged with the opposite polarity. All of the energy is again stored in the capacitor.

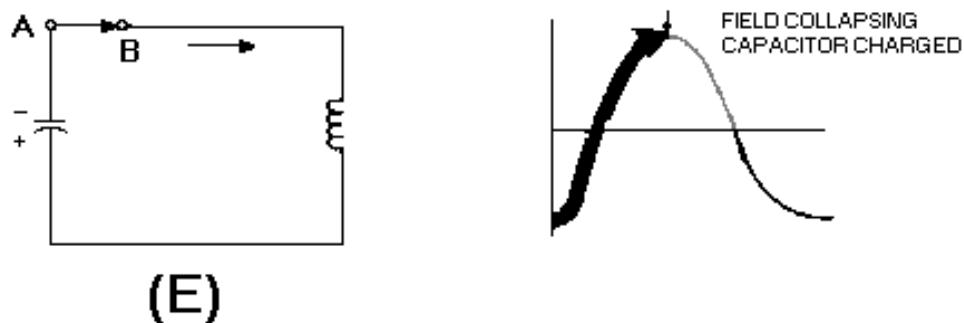


Figure 1-8E.—Capacitor and inductor action in a tank circuit.

In view (F), the capacitor now discharges back through the coil. This discharge current causes the magnetic field to build up again around the coil.

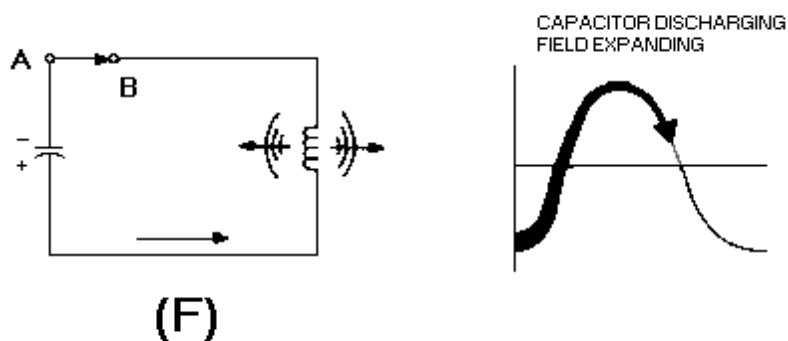


Figure 1-8F.—Capacitor and inductor action in a tank circuit.

In view (G), the capacitor is completely discharged. The magnetic field is again at maximum.

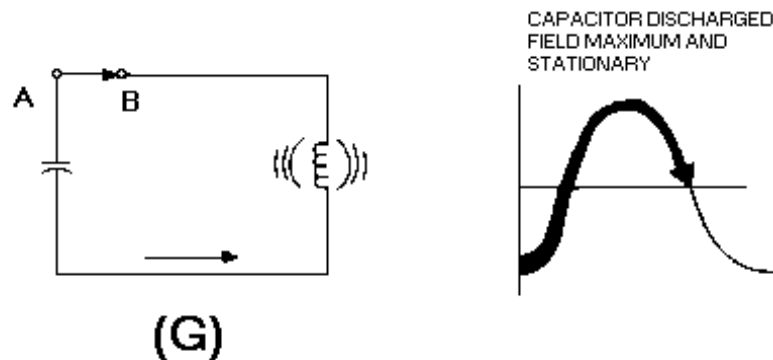
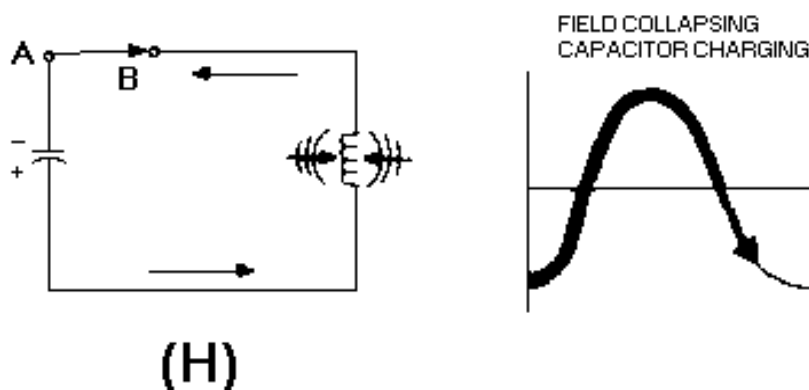


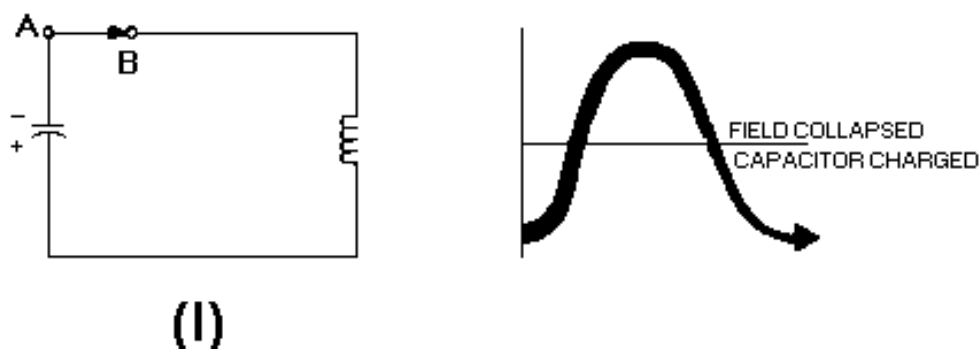
Figure 1-8G.—Capacitor and inductor action in a tank circuit.

In view (H), with the capacitor completely discharged, the magnetic field again starts collapsing. The induced voltage from the coil maintains current flowing toward the upper plate of the capacitor.



**Figure 1-8H.—Capacitor and inductor action in a tank circuit.**

In view (I), by the time the magnetic field has completely collapsed, the capacitor is again charged with the same polarity as it had in view (A). The energy is again stored in the capacitor, and the cycle is ready to start again.

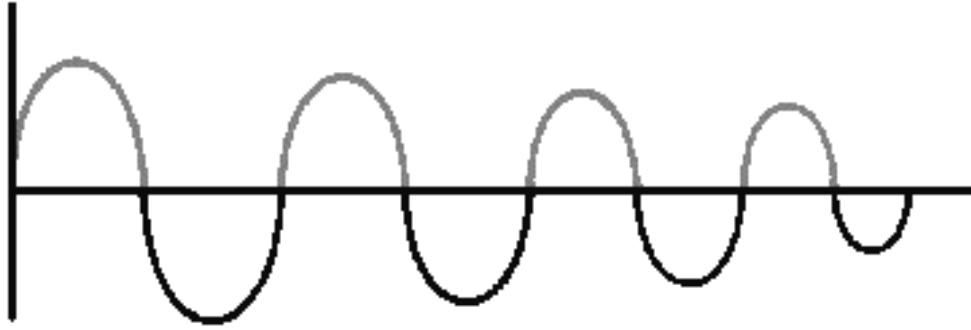


**Figure 1-8I.—Capacitor and inductor action in a tank circuit.**

The number of times per second that these events in figure 1-8 take place is called **NATURAL FREQUENCY** or **RESONANT FREQUENCY** of the circuit. Such a circuit is said to oscillate at its resonant frequency.

It might seem that these oscillations could go on forever. You know better, however, if you apply what you have already learned about electric circuits.

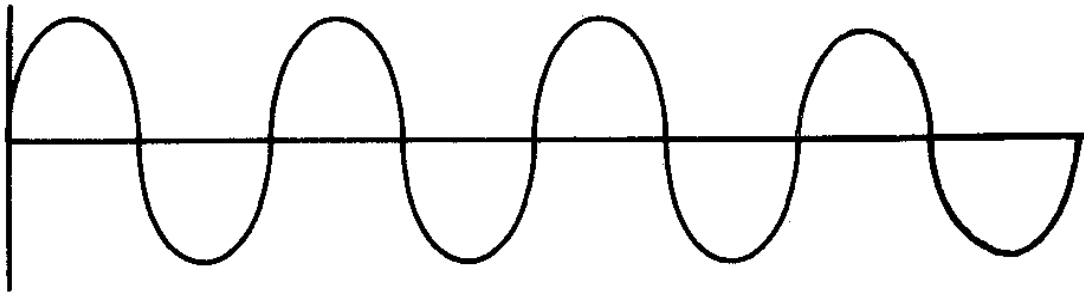
This circuit, as all others, has some resistance. Even the relatively small resistance of the coil and the connecting wires cause energy to be dissipated in the form of heat ( $I^2R$  loss). The heat loss in the circuit resistance causes the charge on the capacitor to be less for each subsequent cycle. The result is a **DAMPED WAVE**, as shown in figure 1-9. The charging and discharging action will continue until all of the energy has been radiated or dissipated as heat.



**Figure 1-9.—Damped wave.**

If it were possible to have a circuit with absolutely no resistance, there would be no heat loss, and the oscillations would tend to continue indefinitely. You have already learned that tuned circuits are designed to have very little resistance. Reducing  $I^2R$  losses is still another reason for having low resistance.

A "perfect" tuned circuit would produce the continuous sine wave shown in figure 1-10. Its frequency would be that of the circuit.



**Figure 1-10.—Sine wave-resonant frequency.**

Because we don't have perfection, another way of causing a circuit to oscillate indefinitely would be to apply a continuous a.c. or pulsing source to the circuit. If the source is at the resonant frequency of the circuit, the circuit will oscillate as long as the source is applied.

The reasons why the circuit in figure 1-8 oscillates at the resonant frequency have to do with the characteristics of resonant circuits. The discussion of parallel resonance will not be as detailed as that for series resonance because the idea of resonance is the same for both circuits. Certain characteristics differ as a result of L and C being in parallel rather than in series. These differences will be emphasized.

*Q-7. When the capacitor is completely discharged, where is the energy of the tank circuit stored?*

*Q-8. When the magnetic field of the inductor is completely collapsed, where is the energy of the tank circuit stored?*

## **PARALLEL RESONANCE**

Much of what you have learned about resonance and series-LC circuits can be applied directly to parallel-LC circuits. The purpose of the two circuits is the same — to select a specific frequency and reject all others.  $X_L$  still equals  $X_C$  at resonance. Because the inductor and capacitor are in parallel, however, the circuit has the basic characteristics of an a.c. parallel circuit. The parallel hookup causes

frequency selection to be accomplished in a different manner. It gives the circuit different characteristics. The first of these characteristics is the ability to store energy.

### The Characteristics of a Typical Parallel-Resonant Circuit

Look at figure 1-11. In this circuit, as in other parallel circuits, the voltage is the same across the inductor and capacitor. The currents through the components vary inversely with their reactances in accordance with Ohm's law. The total current drawn by the circuit is the vector sum of the two individual component currents. Finally, these two currents,  $I_L$  and  $I_C$ , are 180 degrees out of phase because the effects of L and C are opposite. There is not a single fact new to you in the above. It is all based on what you have learned previously about parallel a.c. circuits that contain L and C.

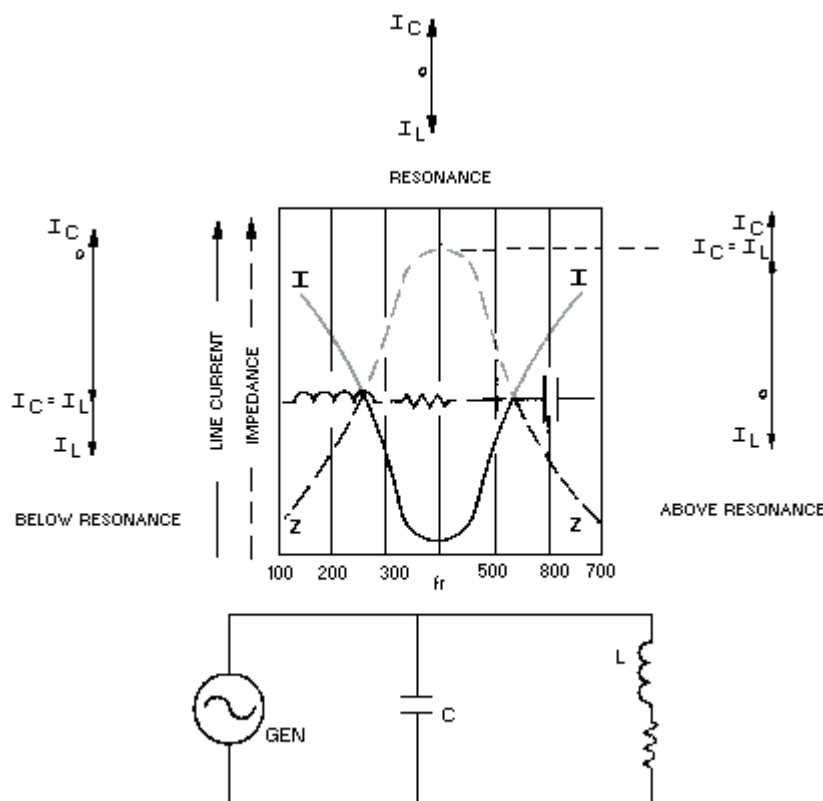


Figure 1-11.—Curves of impedance and current in an RLC parallel-resonant circuit.

Now, at resonance,  $X_L$  is still equal to  $X_C$ . Therefore,  $I_L$  must equal  $I_C$ . Remember, the voltage is the same; the reactances are equal; therefore, according to Ohm's law, the currents must be equal. But, don't forget, even though the currents are equal, they are still opposites. That is, if the current is flowing "up" in the capacitor, it is flowing "down" in the coil, and vice versa. In effect, while the one component draws current, the other returns it to the source. The net effect of this "give and take action" is that zero current is drawn from the source at resonance. The two currents yield a total current of zero amperes because they are exactly equal and opposite at resonance.

A circuit that is completed and has a voltage applied, but has zero current, must have an INFINITE IMPEDANCE (apply Ohm's law — any voltage divided by zero yields infinity).

By now you know that we have just ignored our old friend resistance from previous discussions. In an actual circuit, at resonance, the currents will not quite counteract each other because each component will have different resistance. This resistance is kept extremely low, but it is still there. The result is that a relatively small current flows from the source at resonance instead of zero current. Therefore, a basic characteristic of a practical parallel-LC circuit is that, at resonance, the circuit has MAXIMUM impedance which results in MINIMUM current from the source. This current is often called line current. This is shown by the peak of the waveform for impedance and the valley for the line current, both occurring at  $f_r$ , the frequency of resonance in figure 1-11.

There is little difference between the circuit pulsed by the battery in figure 1-8 that oscillated at its resonant (or natural) frequency, and the circuit we have just discussed. The equal and opposite currents in the two components are the same as the currents that charged and discharged the capacitor through the coil.

For a given source voltage, the current oscillating between the reactive parts will be stronger at the resonant frequency of the circuit than at any other frequency. At frequencies below resonance, capacitive current will decrease; above the resonant frequency, inductive current will decrease. Therefore, the oscillating current (or circulating current, as it is sometimes called), being the lesser of the two reactive currents, will be maximum at resonance.

If you remember, the basic resonant circuit produced a "damped" wave. A steady amplitude wave was produced by giving the circuit energy that would keep it going. To do this, the energy had to be at the same frequency as the resonant frequency of the circuit.

So, if the resonant frequency is "timed" right, then all other frequencies are "out of time" and produce waves that tend to buck each other. Such frequencies cannot produce strong oscillating currents.

In our typical parallel-resonant (LC) circuit, the line current is minimum (because the impedance is maximum). At the same time, the internal oscillating current in the tank is maximum. Oscillating current may be several hundred times as great as line current at resonance.

In any case, this circuit reacts differently to the resonant frequency than it does to all other frequencies. This makes it an effective frequency selector.

### **Summary of Resonance**

Both series- and parallel-LC circuits discriminate between the resonant frequency and all other frequencies by balancing an inductive reactance against an equal capacitive reactance.

In series, these reactances create a very low impedance. In parallel, they create a very high impedance. These characteristics govern how and where designers use resonant circuits. A low-impedance requirement would require a series-resonant circuit. A high-impedance requirement would require the designer to use a parallel-resonant circuit.

### **Tuning a Band of Frequencies**

Our resonant circuits so far have been tuned to a single frequency - the resonant frequency. This is fine if only one frequency is required. However, there are hundreds of stations on many different frequencies.

Therefore, if we go back to our original application, that of tuning to different radio stations, our resonant circuits are not practical. The reason is because a tuner for each frequency would be required and this is not practical.

What is a practical solution to this problem? The answer is simple. Make either the capacitor or the inductor variable. Remember, changing either L or C changes the resonant frequency.

Now you know what has been happening all of these years when you "pushed" the button or "turned" the dial. You have been changing the L or C in the tuned circuits by the amount necessary to adjust the tuner to resonate at the desired frequency. No matter how complex a unit, if it has LC tuners, the tuners obey these basic laws.

*Q-9. What is the term for the number of times per second that tank circuit energy is either stored in the inductor or capacitor?*

*Q-10. In a parallel-resonant circuit, what is the relationship between impedance and current?*

*Q-11. When is line current minimum in a parallel-LC circuit?*

## **RESONANT CIRCUITS AS FILTER CIRCUITS**

The principle of series- or parallel-resonant circuits have many applications in radio, television, communications, and the various other electronic fields throughout the Navy. As you have seen, by making the capacitance or inductance variable, the frequency at which a circuit will resonate can be controlled.

In addition to station selecting or tuning, resonant circuits can separate currents of certain frequencies from those of other frequencies.

Circuits in which resonant circuits are used to do this are called FILTER CIRCUITS.

If we can select the proper values of resistors, inductors, or capacitors, a FILTER NETWORK, or "frequency selector," can be produced which offers little opposition to one frequency, while BLOCKING or ATTENUATING other frequencies. A filter network can also be designed that will "pass" a band of frequencies and "reject" all other frequencies.

Most electronic circuits require the use of filters in one form or another. You have already studied several in modules 6, 7, and 8 of the NEETS.

One example of a filter being applied is in a rectifier circuit. As you know, an alternating voltage is changed by the rectifier to a direct current. However, the d.c. voltage is not pure; it is still pulsating and fluctuating. In other words, the signal still has an a.c. component in addition to the d.c. voltage. By feeding the signal through simple filter networks, the a.c. component is reduced. The remaining d.c. is as pure as the designers require.

Bypass capacitors, which you have already studied, are part of filter networks that, in effect, bypass, or shunt, unwanted a.c. components to ground.

## **THE IDEA OF "Q"**

Several times in this chapter, we have discussed "ideal" or theoretically perfect circuits. In each case, you found that resistance kept our circuits from being perfect. You also found that low resistance in tuners was better than high resistance. Now you will learn about a factor that, in effect, measures just how close to perfect a tuner or tuner component can be. This same factor affects BANDWIDTH and SELECTIVITY. It can be used in figuring voltage across a coil or capacitor in a series-resonant circuit and the amount of circulating (tank) current in a parallel-resonant circuit. This factor is very important

and useful to designers. Technicians should have some knowledge of the factor because it affects so many things. The factor is known as Q. Some say it stands for quality (or merit). The higher the Q, the better the circuit; the lower the losses ( $I^2R$ ), the closer the circuit is to being perfect.

Having studied the first part of this chapter, you should not be surprised to learn that resistance (R) has a great effect on this figure of merit or quality.

### Q Is a Ratio

Q is really very simple to understand if you think back to the tuned-circuit principles just covered. Inductance and capacitance are in all tuners. Resistance is an impurity that causes losses. Therefore, components that provide the reactance with a minimum of resistance are "purer" (more perfect) than those with higher resistance. The actual measure of this purity, merit, or quality must include the two basic quantities, X and R.

The ratio

$$\frac{X}{R}$$

does the job for us. Let's take a look at it and see just why it measures quality.

First, if a perfect circuit has zero resistance, then our ratio should give a very high value of Q to reflect the high quality of the circuit. Does it?

Assume any value for X and a zero value for R.

Then:

$$Q = \frac{X}{R} = \frac{\text{Some Value}}{0} = \text{Infinity}$$

Remember, any value divided by zero equals infinity. Thus, our ratio is infinitely high for a theoretically perfect circuit.

With components of higher resistance, the Q is reduced. Dividing by a larger number always yields a smaller quantity. Thus, lower quality components produce a lower Q. Q, then, is a direct and accurate measure of the quality of an LC circuit.

Q is just a ratio. It is always just a number — no units. The higher the number, the "better" the circuit. Later as you get into more practical circuits, you may find that low Q may be desirable to provide certain characteristics. For now, consider that higher is better.

Because capacitors have much, much less resistance in them than inductors, the Q of a circuit is very often expressed as the Q of the coil or:

$$Q = \frac{X_L}{R}$$

The answer you get from using this formula is very near correct for most purposes. Basically, the Q of a capacitor is so high that it does not limit the Q of the circuit in any practical way. For that reason, the technician may ignore it.



## The Q of a Coil

Q is a feature that is designed into a coil. When the coil is used within the frequency range for which it is designed, Q is relatively constant. In this sense, it is a physical characteristic.

Inductance is a result of the physical makeup of a coil - number of turns, core, type of winding, etc. Inductance governs reactance at a given frequency. Resistance is inherent in the length, size, and material of the wire. Therefore, the Q of a coil is mostly dependent on physical characteristics.

Values of Q that are in the hundreds are very practical and often found in typical equipment.

## Application of Q

For the most part, Q is the concern of designers, not technicians. Therefore, the chances of you having to figure the Q of a coil are remote. However, it is important for you to know some circuit relationships that are affected by Q.

## Q Relationships in Series Circuits

Q can be used to determine the "gain" of series-resonant circuits. Gain refers to the fact that at resonance, the voltage drop across the reactances are greater than the applied voltage. Remember, when we applied Ohm's law in a series-resonant circuit, it gave us the following characteristics:

- Low impedance, high current.
- High current; high voltage across the comparatively high reactances.

This high voltage is usable where little power is required, such as in driving the grid of a vacuum tube or the gate of a field effect transistor (F.E.T.). The gain of a properly designed series-resonant circuit may be as great or greater than the amplification within the amplifier itself. The gain is a function of Q, as shown in the following example:

$E$  = the input voltage to the tuned circuit

$E_L$  = the voltage drop across the coil at  
resonance Q.

$Q$  = the Q of the coil

Then:

$$E_L = EQ$$

If the Q of the coil were 100, then the gain would be 100; that is, the voltage of the coil would be 100 times that of the input voltage to the series circuit.

Resistance affects the resonance curve of a series circuit in two ways — the lower the resistance, the higher the current; also, the lower the resistance, the sharper the curve. Because low resistance causes high Q, these two facts are usually expressed as functions of Q. That is, the higher the Q, the higher and sharper the curve and the more selective the circuit.

The lower the Q (because of higher resistance), the lower the current curve; therefore, the broader the curve, the less selective the circuit. A summary of the major characteristics of series RLC-circuits at resonance is given in table 1-1.

**Table 1-1.—Major Characteristics of Series RLC Circuits at Resonance**

| QUANTITY   | SERIES CIRCUIT                    |
|--|-----------------------------------|
| At resonance:<br>Reactance ( $X_L - X_C$ )       | Zero, because $X_L = X_C$         |
| Resonant frequency                               | $f_r = \frac{1}{2\pi\sqrt{LC}}$   |
| Impedance  | Minimum: $Z = R$                  |
| $I_{LINE}$                                       | Maximum value                     |
| $I_L$  | $I_{LINE}$                        |
| $I_C$  | $I_{LINE}$                        |
| $E_L$  | $Q \cdot E_{LINE}$                |
| $E_C$  | $Q \cdot E_{LINE}$                |
| Phase angle between<br>$E_{LINE}$ and $I_{LINE}$ | $0^\circ$                         |
| Angle between $E_L$ & $E_C$                      | $180^\circ$                       |
| Angle between $I_L$ & $I_C$                      | $0^\circ$                         |
| Desired value of Q                               | 10 or more                        |
| Desired value of R                               | Low                               |
| Highest selectivity                              | High Q, low R, high $\frac{L}{C}$ |
| When f is greater than $f_r$<br>Reactance        | Inductive                         |
| Phase angle between<br>$I_{LINE}$ and $E_{LINE}$ | Lagging current                   |
| When f is less than $f_r$<br>Reactance           | Capacitive                        |
| Phase angle between<br>$I_{LINE}$ and $E_{LINE}$ | Leading current                   |

### Q Relationships in a Parallel-Resonant Circuit

There is no voltage gain in a parallel-resonant circuit because voltage is the same across all parts of a parallel circuit. However, Q helps give us a measure of the current that circulates in the tank.

Given:

$I_{\text{LINE}}$  = current drawn from the source

$I_L$  = current through the coil (or  
circulating current)

$Q$  = the  $Q$  of the coil

Then:

$$I_L = I_{\text{LINE}} Q$$

Again, if the  $Q$  were 100, the circulating current would be 100 times the value of the line current. This may help explain why some of the wire sizes are very large in high-power amplifying circuits.

The impedance curve of a parallel-resonant circuit is also affected by the  $Q$  of the circuit in a manner similar to the current curve of a series circuit. The  $Q$  of the circuit determines how much the impedance is increased across the parallel-LC circuit. ( $Z = Q \times X_L$ )

The higher the  $Q$ , the greater the impedance at resonance and the sharper the curve. The lower the  $Q$ , the lower impedance at resonance; therefore, the broader the curve, the less selective the circuit. The major characteristics of parallel-RLC circuits at resonance are given in table 1-2.

**Table 1-2.—Major Characteristics of Parallel RLC Circuits at**

| QUANTITY   | PARALLEL CIRCUIT                           |
|--|--|
| At resonance :<br>Reactance ( $X_L - X_C$ )      | Zero; because nonenergy currents are equal |
| Resonant frequency                               | $f_r = \frac{1}{2\pi\sqrt{LC}}$            |
| Impedance  | Maximum: $Z = \frac{L}{CR}$                |
| $I_{LINE}$                                       | Minimum value                              |
| $I_L$  | $Q \cdot I_{LINE}$                         |
| $I_C$  | $Q \cdot I_{LINE}$                         |
| $E_L$  | $E_{LINE}$                                 |
| $E_C$  | $E_{LINE}$                                 |
| Phase angle between<br>$E_{LINE}$ and $I_{LINE}$ | $0^\circ$                                  |
| Angle between $E_L$ & $E_C$                      | $0^\circ$                                  |
| Angle between $I_L$ & $I_C$                      | $180^\circ$                                |
| Desired value of $Q$                             | 10 or more                                 |
| Desired value of $R$                             | Low  |
| Highest selectivity                              | High $Q$ , low $R$ , $\frac{L}{C}$         |
| When $f$ is greater than $f_r$<br>Reactance      | Capacitive                                 |
| Phase angle between<br>$I_{LINE}$ and $E_{LINE}$ | Leading current                            |
| When $f$ is less than $f_r$<br>Reactance         | Inductive                                  |
| Phase angle between<br>$I_{LINE}$ and $E_{LINE}$ | Lagging current                            |

#### Resonance

#### Summary of Q

The ratio that is called Q is a measure of the quality of resonant circuits and circuit components. Basically, the value of Q is an inverse function of electrical power dissipated through circuit resistance. Q is the ratio of the power stored in the reactive components to the power dissipated in the resistance. That is, high power loss is low Q; low power loss is high Q.

Circuit designers provide the proper Q. As a technician, you should know what can change Q and what quantities in a circuit are affected by such a change.

## BANDWIDTH

If circuit  $Q$  is low, the gain of the circuit at resonance is relatively small. The circuit does not discriminate sharply (reject the unwanted frequencies) between the resonant frequency and the frequencies on either side of resonance, as shown by the curve in figure 1-12, view (A). The range of frequencies included between the two frequencies (426.4 kHz and 483.6 kHz in this example) at which the current drops to 70 percent of its maximum value at resonance is called the BANDWIDTH of the circuit.

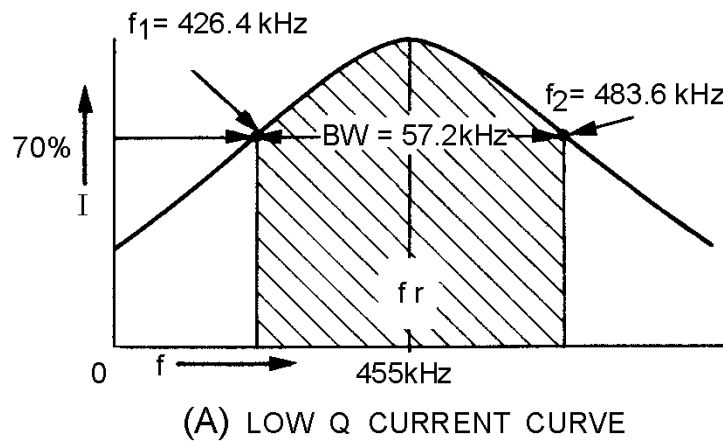


Figure 1-12A.—Bandwidth for high- and low- $Q$  series circuit. LOW  $Q$  CURRENT CURVE.

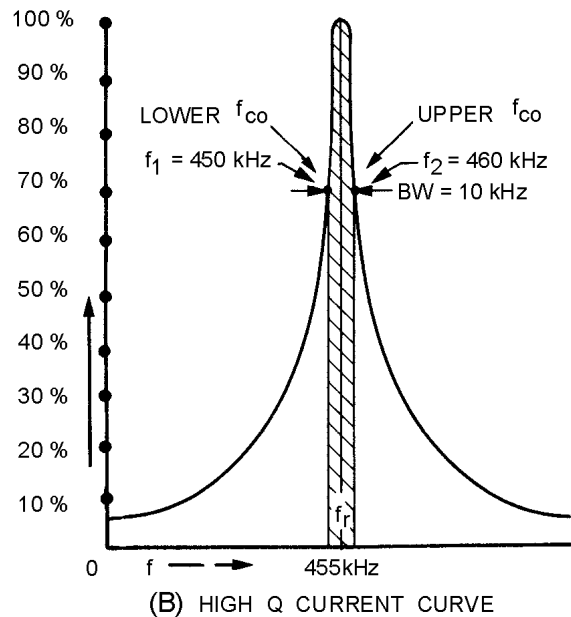


Figure 1-12B.—Bandwidth for high- and low- $Q$  series circuit. HIGH  $Q$  CURRENT CURVE.

It is often necessary to state the band of frequencies that a circuit will pass. The following standard has been set up: the limiting frequencies are those at either side of resonance at which the curve falls to a point of .707 (approximately 70 percent) of the maximum value. This point is called the HALF-POWER point. Note that in figure 1-12, the series-resonant circuit has two half-power points, one above and one

below the resonant frequency point. The two points are designated upper frequency cutoff ( $f_{co}$ ) and lower frequency cutoff ( $f_{co}$ ) or simply  $f_1$  and  $f_2$ . The range of frequencies between these two points comprises the bandwidth. Views (A) and (B) of figure 1-12 illustrate the bandwidths for low- and high-Q resonant circuits. The bandwidth may be determined by use of the following formulas:

$$BW = \frac{f_r}{Q}$$

or

$$BW = f_2 - f_1$$

Where:

BW = bandwidth of a circuit  
in units of frequency

$f_r$  = resonant frequency

$f_2$  = the upper cutoff frequency

$f_1$  = the lower cutoff frequency

For example, by applying the formula we can determine the bandwidth for the curve shown in figure 1-12, view (A).

Solution:

$$BW = f_2 - f_1$$

$$BW = 483.6 \text{ kHz} - 426.4 \text{ kHz}$$

$$BW = 57.2 \text{ kHz}$$

If the Q of the circuit represented by the curve in figure 1-12, view (B), is 45.5, what would be the bandwidth?

Solution:

$$BW = \frac{f_r}{Q}$$

$$BW = \frac{455 \text{ kHz}}{45.5}$$

$$BW = 10 \text{ kHz}$$

If Q equals 7.95 for the low-Q circuit as in view (A) of figure 1-12, we can check our original calculation of the bandwidth.

Solution:

$$BW = \frac{f_r}{Q}$$

$$BW = \frac{455 \text{ kHz}}{7.95}$$

$$BW = 57.2 \text{ kHz}$$

The Q of the circuit can be determined by transposing the formula for bandwidth to:

$$Q = \frac{f_r}{BW}$$

To find the Q of the circuit using the information found in the last example problem:

Given:

$$f_r = 455 \text{ kHz}$$

$$BW = 57.2 \text{ kHz}$$

Solution:

$$Q = \frac{f_r}{BW}$$

$$Q = \frac{455 \text{ kHz}}{57.2 \text{ kHz}}$$

$$Q = 7.95$$

*Q-12. What is the relationship of the coil to the resistance of a circuit with high "Q"?*

*Q-13. What is the band of frequencies called that is included between the two points at which current falls to 70 percent of its maximum value in a resonant circuit?*

## **FILTERS**

In many practical applications of complex circuits, various combinations of direct, low-frequency, audio-frequency, and radio-frequency currents may exist. It is frequently necessary to have a means for separating these component currents at any desired point. An electrical device for accomplishing this separation is called a FILTER.

A filter circuit consists of inductance, capacitance, and resistance used singularly or in combination, depending upon the purpose. It may be designed so that it will separate alternating current from direct current, or so that it will separate alternating current of one frequency (or a band of frequencies) from other alternating currents of different frequencies.

The use of resistance by itself in filter circuits does not provide any filtering action, because it opposes the flow of any current regardless of its frequency. What it does, when connected in series or parallel with an inductor or capacitor, is to decrease the "sharpness," or selectivity, of the filter. Hence, in some particular application, resistance might be used in conjunction with inductance or capacitance to provide filtering action over a wider band of frequencies.

Filter circuits may be divided into four general types: LOW-PASS, HIGH-PASS, BANDPASS, AND BAND-REJECT filters.

Electronic circuits often have currents of different frequencies. The reason is that a source produces current with the same frequency as the applied voltage. As an example, the a.c. signal input to an audio amplifier can have high- and low-audio frequencies; the input to an rf amplifier can have a wide range of radio frequencies.

In such applications where the current has different frequency components, it is usually necessary for the filter either to accept or reject one frequency or a group of frequencies. The electronic filter that can pass on the higher-frequency components to a load or to the next circuit is known as a HIGH-PASS filter. A LOW-PASS filter can be used to pass on lower-frequency components.

Before discussing filters further, we will review and apply some basic principles of the frequency-response characteristics of the capacitor and the inductor. Recall the basic formula for capacitive reactance and inductive reactance:

$$X_C = \frac{1}{2\pi fC}$$

and

$$X_L = 2\pi fL$$

Assume any given value of L and C. If we increase the applied frequency,  $X_C$  decreases and  $X_L$  increases. If we increase the frequency enough, the capacitor acts as a short and the inductor acts as an open. Of course, the opposite is also true. Decreasing frequency causes  $X_C$  to increase and  $X_L$  to decrease. Here again, if we make a large enough change,  $X_C$  acts as an open and  $X_L$  acts as a short. Figure 1-13 gives a pictorial representation of these two basic components and how they respond to low and high frequencies.

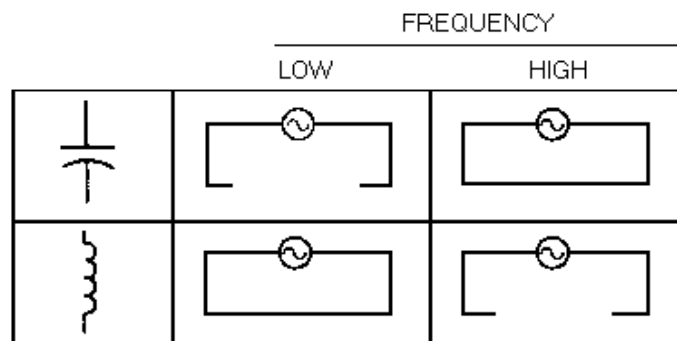
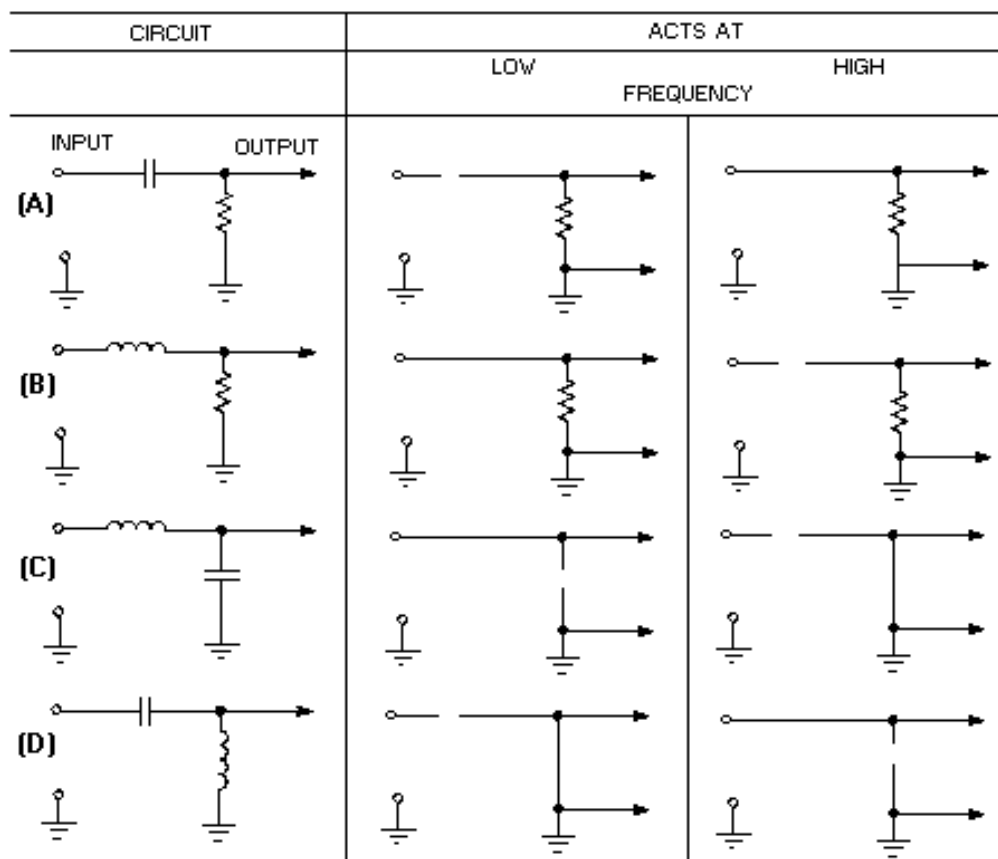


Figure 1-13.—Effect of frequency on capacitive and inductive reactance.



If we apply these same principles to simple circuits, such as the ones in figure 1-14, they affect input signals as shown. For example, in view (A) of the figure, a low frequency is blocked by the capacitor which acts as an open and at a high frequency the capacitor acts as a short. By studying the figure, it is easy to see how the various components will react in different configurations with a change in frequency.



**Figure 1-14.—Reaction to circuit by change in frequency.**

As mentioned before, high-pass and low-pass filters pass the specific frequencies for which circuits are designed.

There can be a great deal of confusion when talking about high-pass, low-pass, discrimination, attenuation, and frequency cutoff, unless the terms are clearly understood. Since these terms are used widely throughout electronics texts and references, you should have a clear understanding before proceeding further.

- **HIGH-PASS FILTER.** A high-pass filter passes on a majority of the high frequencies to the next circuit and rejects or attenuates the lower frequencies. Sometimes it is called a low-frequency discriminator or low-frequency attenuator.
- **LOW-PASS FILTER.** A low-pass filter passes on a majority of the low frequencies to the next circuit and rejects or attenuates the higher frequencies. Sometimes it is called a high-frequency discriminator or high-frequency attenuator.

- **DISCRIMINATION.** The ability of the filter circuit to distinguish between high and low frequencies and to eliminate or reject the unwanted frequencies.
- **ATTENUATION.** The ability of the filter circuit to reduce the amplitude of the unwanted frequencies below the level of the desired output frequency.
- **FREQUENCY CUTOFF ( $f_{co}$ ).** The frequency at which the filter circuit changes from the point of rejecting the unwanted frequencies to the point of passing the desired frequency; OR the point at which the filter circuit changes from the point of passing the desired frequency to the point of rejecting the undesired frequencies.

## LOW-PASS FILTER

A low-pass filter passes all currents having a frequency below a specified frequency, while opposing all currents having a frequency above this specified frequency. This action is illustrated in its ideal form in view (A) of figure 1-15. At frequency cutoff, known as  $f_c$  the current decreases from maximum to zero. At all frequencies above  $f_c$  the filter presents infinite opposition and there is no current. However, this sharp division between no opposition and full opposition is impossible to attain. A more practical graph of the current is shown in view (B), where the filter gradually builds up opposition as the cutoff frequency ( $f$ ) is approached. Notice that the filter cannot completely block current above the cutoff frequency.

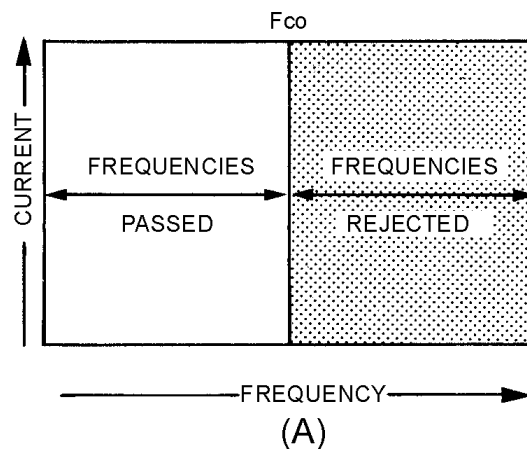


Figure 1-15A.—Low-pass filter.

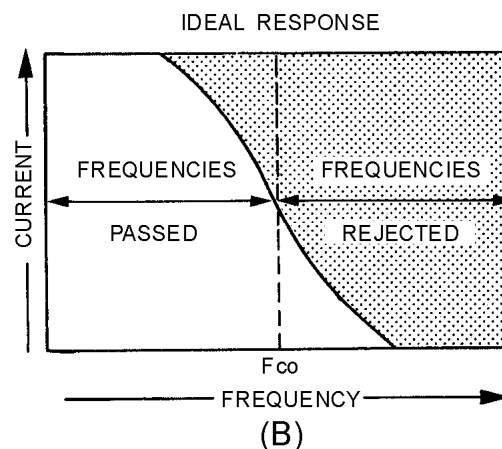


Figure 1-15B.—Low-pass filter.

View (A) of figure 1-16 shows the electrical construction of a low-pass filter with an inductor inserted in series with one side of a line carrying both low and high frequencies. The opposition offered by the reactance will be small at the lower frequencies and great at the higher frequencies. In order to divert the undesired high frequencies back to the source, a capacitor must be added across the line to bypass the higher frequencies around the load, as shown in view (B).

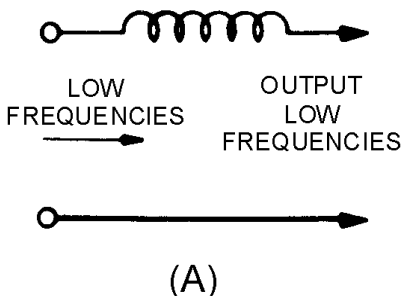


Figure 1-16A.—Components of a simple low-pass filter.

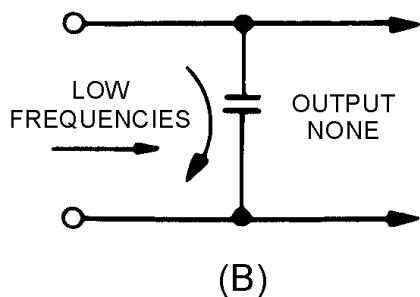


Figure 1-16B.—Components of a simple low-pass filter.

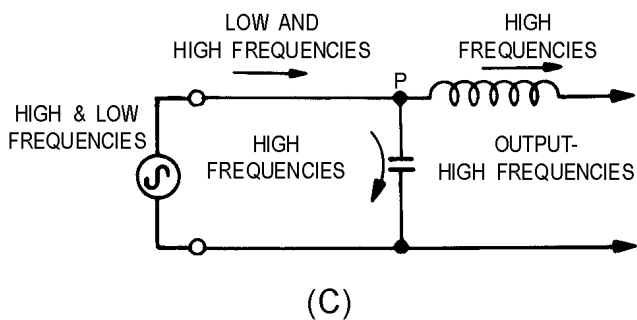


Figure 1-16C.—Components of a simple low-pass filter.

The capacitance of the capacitor must be such that its reactance will offer little opposition to frequencies above a definite value, and great opposition to frequencies below this value. By combining the series inductance and bypass capacitance, as shown in view (C), the simplest type of low-pass filter is obtained. At point P, a much higher opposition is offered to the low frequencies by the capacitor than by the inductor, and most of the low-frequency current takes the path of least opposition. On the other hand,

the least amount of opposition is offered to the high frequencies by the capacitor, and most of the high-frequency energy returns to the source through the capacitor.

## HIGH-PASS FILTER

A high-pass filter circuit passes all currents having a frequency higher than a specified frequency, while opposing all currents having a frequency lower than its specified frequency. This is illustrated in figure 1-17. A capacitor that is used in series with the source of both high and low frequencies, as shown in view (A) of figure 1-18, will respond differently to high-frequency, low-frequency, and direct currents. It will offer little opposition to the passage of high-frequency currents, great opposition to the passage of low-frequency currents, and completely block direct currents. The value of the capacitor must be chosen so that it allows the passage of all currents having frequencies above the desired value, and opposes those having frequencies below the desired value. Then, in order to shunt the undesired low-frequency currents back to the source, an inductor is used, as shown in view (B). This inductor must have a value that will allow it to pass currents having frequencies below the frequency cutoff point, and reject currents having frequencies above the frequency cutoff point, thus forcing them to pass through the capacitor. By combining inductance and capacitance, as shown in view (C), you obtain the simplest type of high-pass filter. At point P most of the high-frequency energy is passed on to the load by the capacitor, and most of the low-frequency energy is shunted back to the source through the inductor.

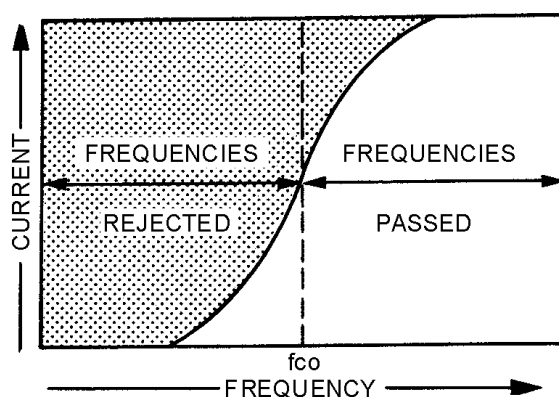


Figure 1-17.—High-pass filter response curve.

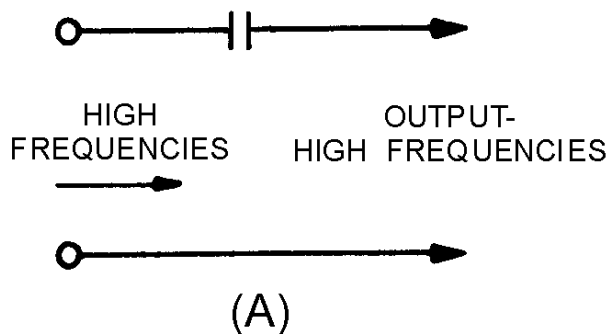


Figure 1-18A.—Components of a simple high-pass filter.

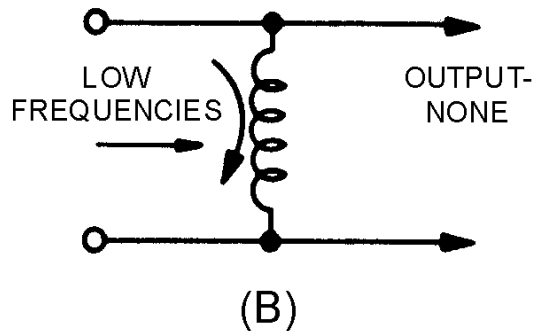


Figure 1-18B.—Components of a simple high-pass filter.

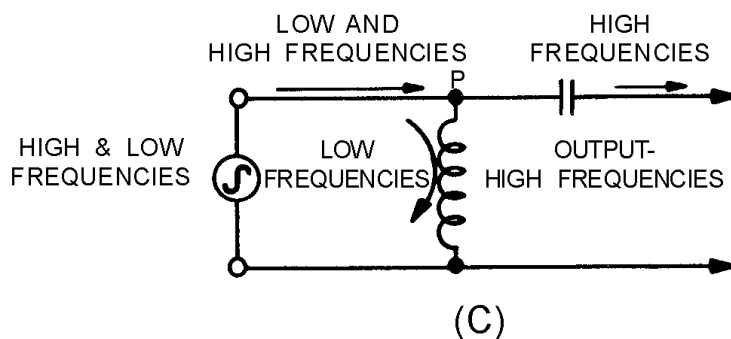


Figure 1-18C.—Components of a simple high-pass filter.

## RESONANT CIRCUITS AS FILTERS

Resonant circuits can be made to serve as filters in a manner similar to the action of individual capacitors and inductors. As you know, the series-LC circuit offers minimum opposition to currents that have frequencies at or near the resonant frequency, and maximum opposition to currents of all other frequencies.

You also know that a parallel-LC circuit offers a very high impedance to currents that have frequencies at or near the resonant frequency, and a relatively low impedance to currents of all other frequencies.

If you use these two basic concepts, the BANDPASS and BAND-REJECT filters can be constructed. The bandpass filter and the band-reject filter are two common types of filters that use resonant circuits.

### Bandpass Filter

A bandpass filter passes a narrow band of frequencies through a circuit and attenuates all other frequencies that are higher or lower than the desired band of frequencies. This is shown in figure 1-19 where the greatest current exists at the center frequency ( $f_r$ ). Frequencies below resonance ( $f_1$ ) and frequencies above resonance ( $f_2$ ) drop off rapidly and are rejected.

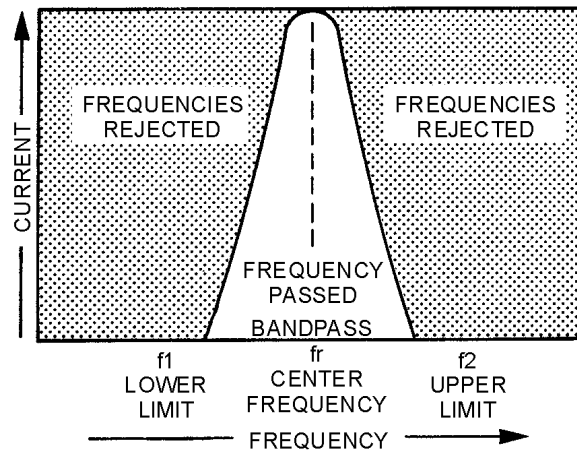


Figure 1-19.—Bandpass filter response curve.

In the circuit of figure 1-20, view (A), the series-LC circuit replaces the inductor of figure 1-16, view (A), and acts as a BANDPASS filter. It passes currents having frequencies at or near its resonant frequency, and opposes the passage of all currents having frequencies outside this band.

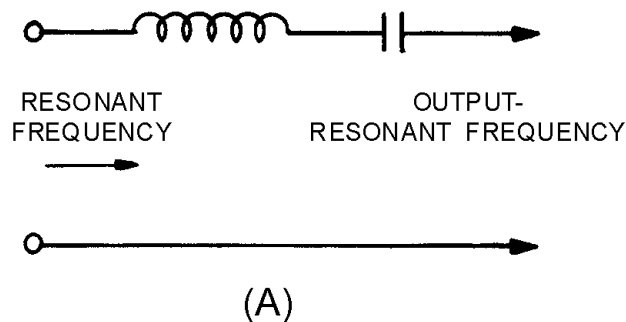


Figure 1-20A.—Components of a simple bandpass filter.

Thus, in the circuit of figure 1-20, view (B), the parallel-LC circuit replaces the capacitor of figure 1-16, view (B). If this circuit is tuned to the same frequency as the series-LC circuit, it will provide a path for all currents having frequencies outside the limits of the frequency band passed by the series-resonant circuit. The simplest type of bandpass filter is formed by connecting the two LC circuits as shown in figure 1-20, view (C). The upper and lower frequency limits of the filter action are filter cutoff points.

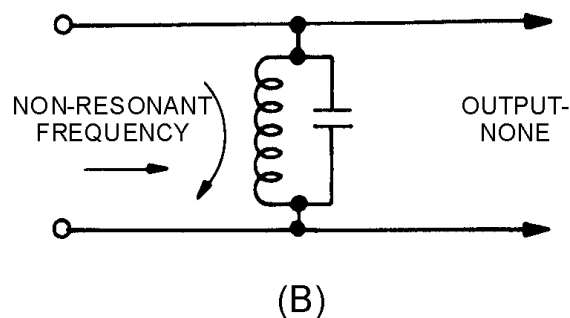


Figure 1-20B.—Components of a simple bandpass filter.

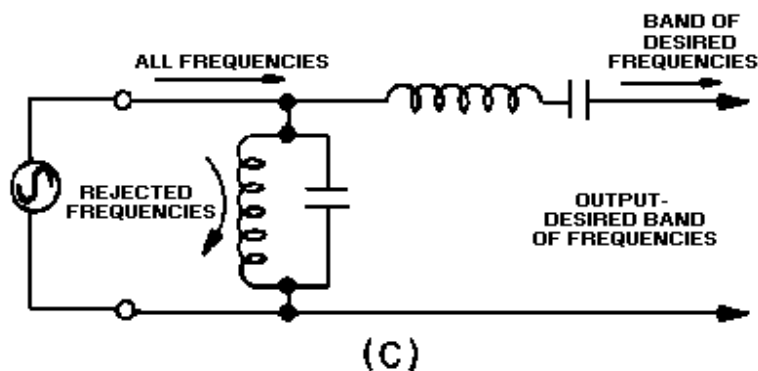


Figure 1-20C.—Components of a simple bandpass filter.

### Band-Reject Filter

A band-reject filter circuit is used to block the passage of current for a narrow band of frequencies, while allowing current to flow at all frequencies above or below this band. This type of filter is also known as a BAND-SUPPRESSION or BAND-STOP filter. The way it responds is shown by the response curve of figure 1-21. Since the purpose of the band-reject filter is directly opposite to that of a bandpass filter, the relative positions of the resonant circuits in the filter are interchanged. The parallel-LC circuit shown in figure 1-22, view (A), replaces the capacitor of figure 1-18, view (A). It acts as a band-reject filter, blocking the passage of currents having frequencies at or near resonant frequency and passing all currents having frequencies outside this band. The series-LC circuit shown in figure 1-22, view (B), replaces the inductor of figure 1-18, view (B). If this series circuit is tuned, to the same frequency as the parallel circuit, it acts as a bypass for the band of rejected frequencies. Then, the simplest type of band-reject filter is obtained by connecting the two circuits as shown in figure 1-22, view (C).

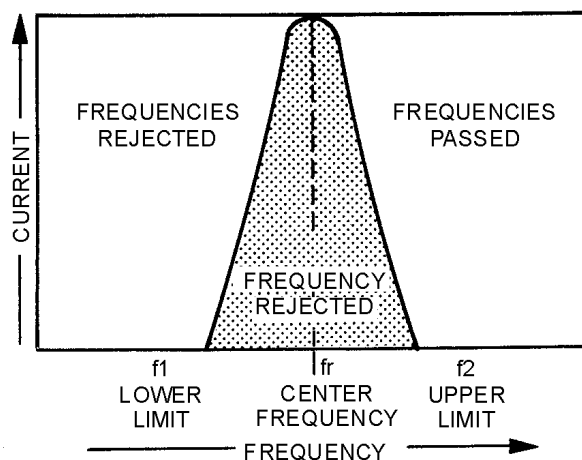


Figure 1-21.—Band-reject filter response curve.

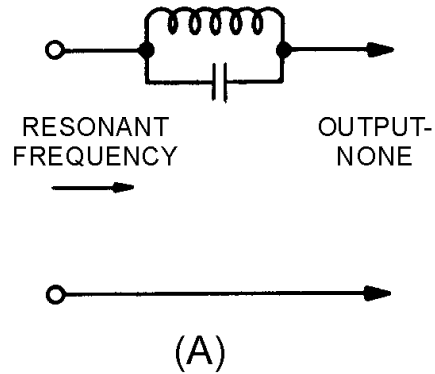


Figure 1-22A.—Components of a simple band-reject filter.

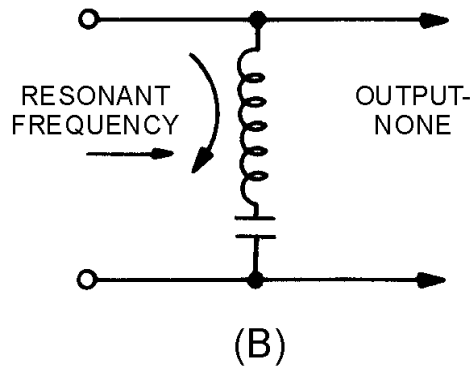


Figure 1-22B.—Components of a simple band-reject filter.

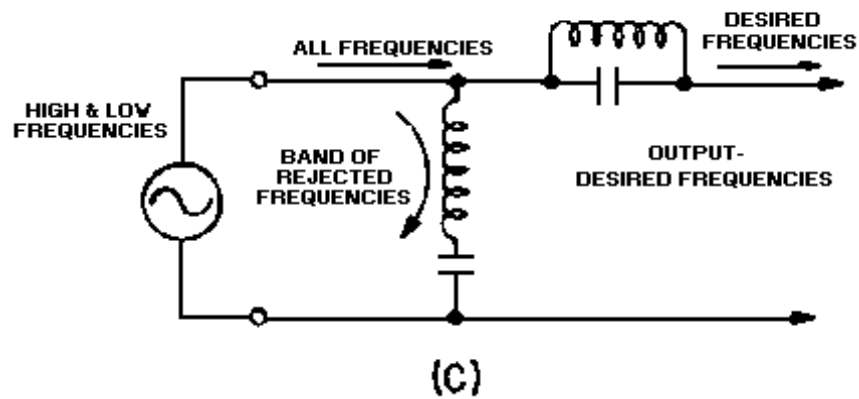


Figure 1-22C.—Components of a simple band-reject filter.

Q-14. What is the device called that will separate alternating current from direct current, or that will separate alternating current of one frequency from other alternating currents of different frequencies?

Q-15. What are the four general types of filters?



- Q-16. What is the filter called in which the low frequencies do not produce a useful voltage?
- Q-17. What is the filter called that passes low frequencies but rejects or attenuates high frequencies?
- Q-18. How does a capacitor and an inductor react to (a) low frequency and (b) high frequency?
- Q-19. What term is used to describe the frequency at which the filter circuit changes from the point of rejecting the unwanted frequencies to the point of passing the desired frequencies?
- Q-20. What type filter is used to allow a narrow band of frequencies to pass through a circuit and attenuate all other frequencies above or below the desired band?
- Q-21. What type filter is used to block the passage of current for a narrow band of frequencies, while allowing current to flow at all frequencies above or below this band?

### MULTISECTION FILTERS

All of the various types of filters we have discussed so far have had only one section. In many cases, the use of such simple filter circuits does not provide sufficiently sharp cutoff points. But by adding a capacitor, an inductor, or a resonant circuit in series or in parallel (depending upon the type of filter action required), the ideal effect is more nearly approached. When such additional units are added to a filter circuit, the form of the resulting circuit will resemble the letter T, or the Greek letter  $\pi$  (pi). They are, therefore, called T- or  $\pi$ -type filters, depending upon which symbol they resemble. Two or more T- or  $\pi$ -type filters may be connected together to produce a still sharper cutoff point.

Figure 1-23, (view A) (view B) and (view C), and figure 1-24, (view A) (view B) and (view C) depict some of the common configurations of the T- and  $\pi$ -type filters. Further discussion about the theory of operation of these circuits is beyond the intended scope of this module. If you are interested in learning more about filters, a good source of information to study is the *Electronics Installation and Maintenance Handbook* (EIMB), section 4 (Electronics Circuits), NAVSEA 0967-LP-000-0120.

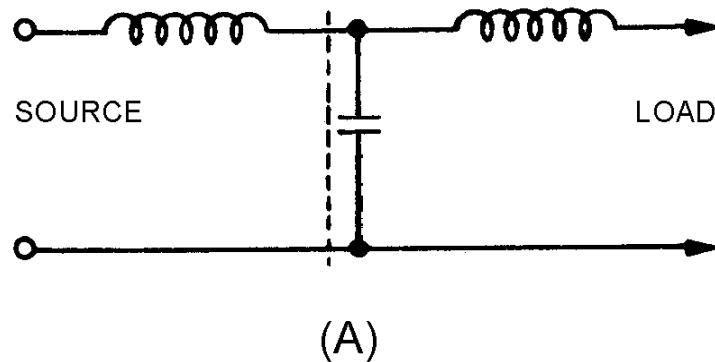


Figure 1-23A.—Formation of a T-type filter.

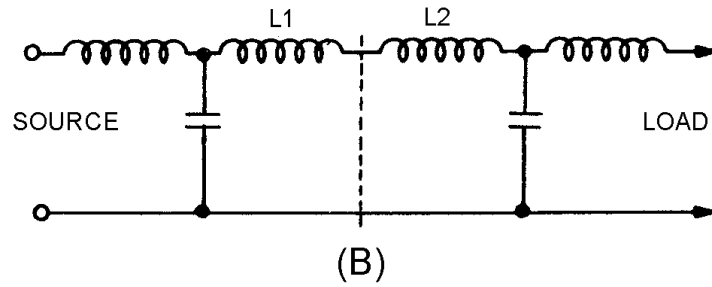


Figure 1-23B.—Formation of a T-type filter.

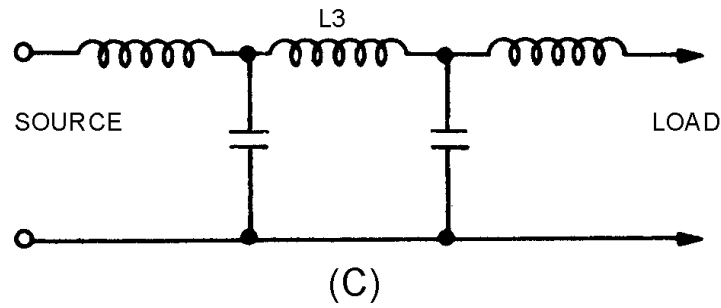


Figure 1-23C.—Formation of a T-type filter.

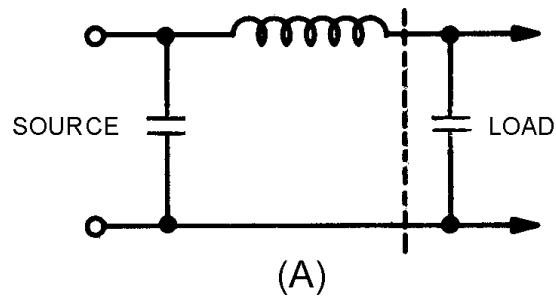


Figure 1-24A.—Formation of a  $\pi$ -type filter.

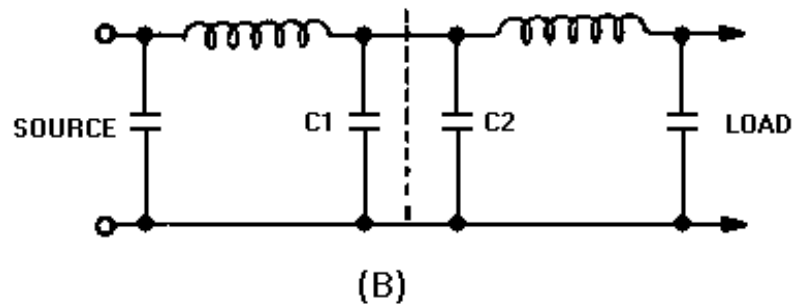


Figure 1-24B.—Formation of a  $\pi$ -type filter.

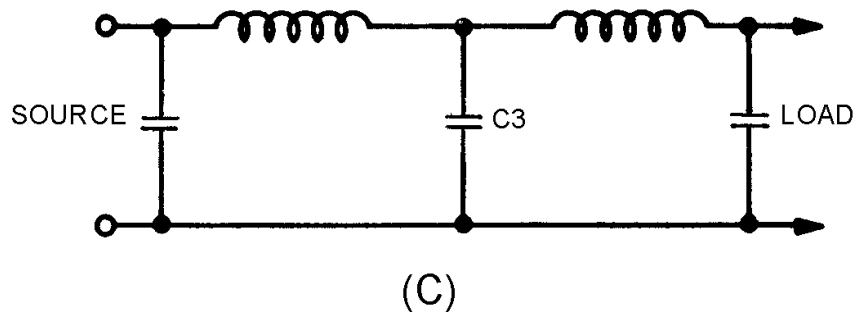


Figure 1-24C.—Formation of a  $\pi$ -type filter.

### SAFETY PRECAUTIONS

When working with resonant circuits, or electrical circuits, you must be aware of the potentially high voltages. Look at figure 1-25. With the series circuit at resonance, the total impedance of the circuit is 5 ohms.

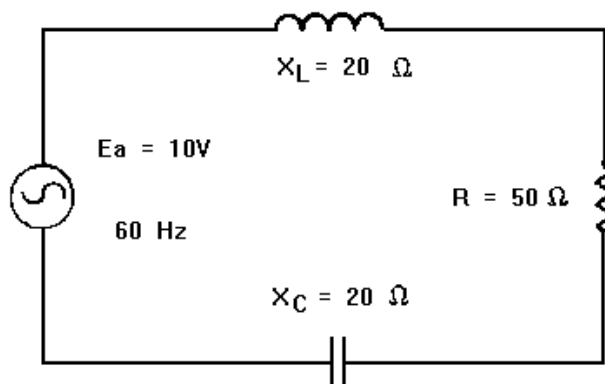


Figure 1-25.—Series RLC circuit at resonance.

Remember, the impedance of a series-RLC circuit at resonance depends on the resistive element. At resonance, the impedance ( $Z$ ) equals the resistance ( $R$ ). Resistance is minimum and current is maximum. Therefore, the current at resonance is:

$$I_T = \frac{E_a}{Z} = \frac{10 \text{ V}}{5\Omega} = 2 \text{ A}$$

The voltage drops around the circuit with 2 amperes of current flow are:

$$E_C = I_T \times X_C$$

$$E_C = 2 \times 20$$

$$E_C = 40 \text{ volts a.c.}$$

$$E_L = I_T \times X_L$$

$$E_L = 2 \times 20$$

$$E_L = 40 \text{ volts a.c.}$$

$$E_R = I_T \times R$$

$$E_R = 2 \times 5$$

$$E_R = 10 \text{ volts a.c.}$$

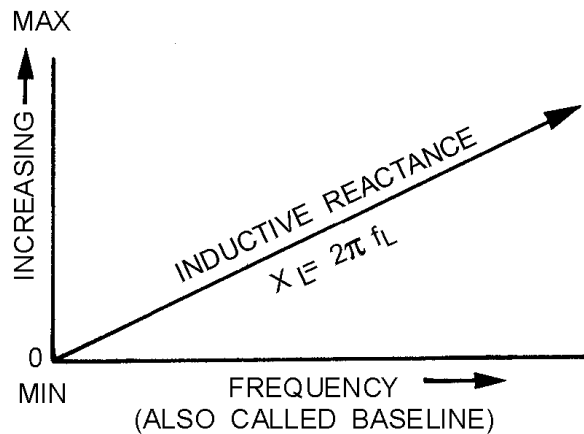
You can see that there is a voltage gain across the reactive components at resonance.

If the frequency was such that  $X_L$  and  $X_C$  were equal to 1000 ohms at the resonant frequency, the reactance voltage across the inductor or capacitor would increase to 2000 volts a.c. with 10 volts a.c. applied. Be aware that potentially high voltage can exist in series-resonant circuits.

### SUMMARY

This chapter introduced you to the principles of tuned circuits. The following is a summary of the major subjects of this chapter.

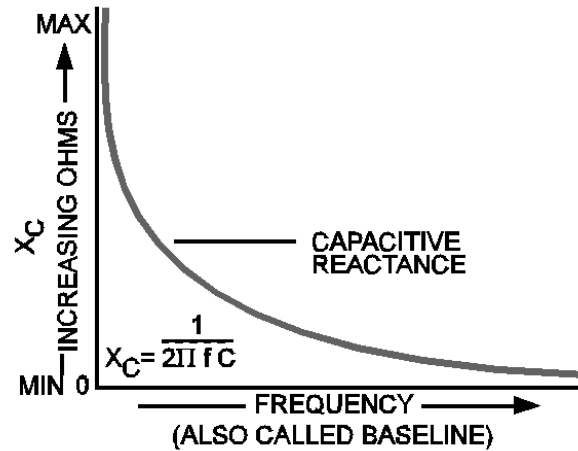
**THE EFFECT OF FREQUENCY** on an **INDUCTOR** is such that an increase in frequency will cause an increase in inductive reactance. Remember that  $X_L = 2\pi fL$ ; therefore,  $X_L$  varies directly with frequency.



**THE EFFECT OF FREQUENCY** on a **CAPACITOR** is such that an increase in frequency will cause a decrease in capacitive reactance. Remember that

$$X_C = \frac{1}{2\pi fC}$$

therefore, the relationship between  $X_C$  and frequency is that  $X_C$  varies inversely with frequency.

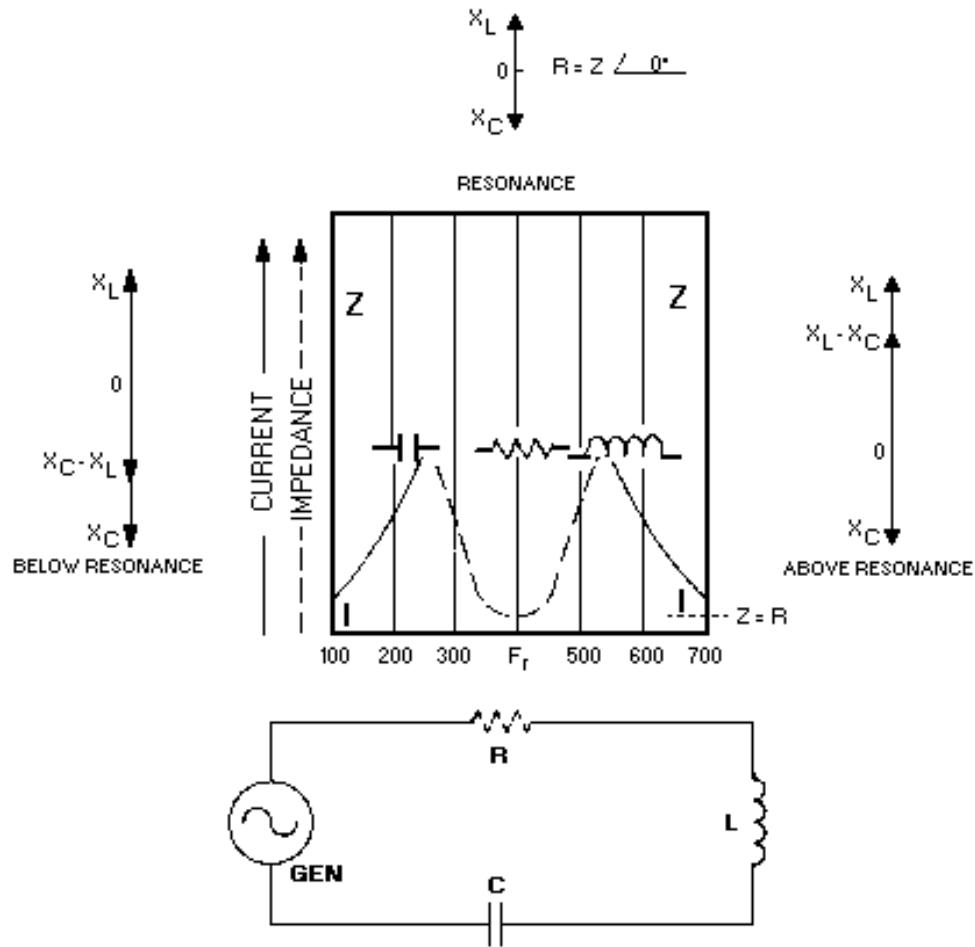


**RESULTANT REACTANCE**  $X = (X_L - X_C)$  or  $X = (X_C - X_L)$ .  $X_L$  is usually plotted above the reference line and  $X_C$  below the reference line. Inductance and capacitance have opposite effects on the current in respect to the voltage in a.c. circuits. Below resonance,  $X_C$  is larger than  $X_L$ , and the series circuit appears capacitive. Above resonance,  $X_L$  is larger than  $X_C$ , and the series circuit appears inductive. At resonance,  $X_L = X_C$ , and the total impedance of the circuit is resistive.

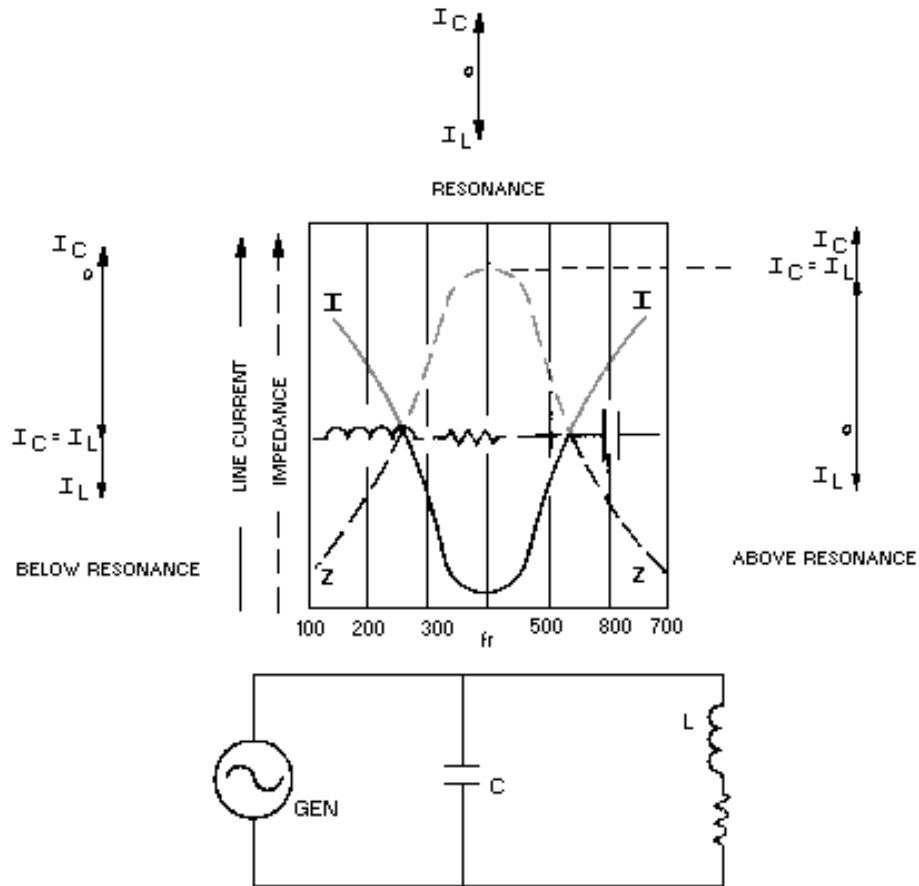
A **RESONANT CIRCUIT** is often called a **TANK CIRCUIT**. It has the ability to take energy fed from a power source, store the energy alternately in the inductor and capacitor, and produce an output which is a continuous a.c. wave. The number of times this set of events occurs per second is called the resonant frequency of the circuit. The actual frequency at which a tank circuit will oscillate is determined by the formula:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

IN A **SERIES-LC CIRCUIT** impedance is minimum and current is maximum. Voltage is the variable, and voltage across the inductor and capacitor will be equal but of opposite phases at resonance. Above resonance it acts inductively, and below resonance it acts capacitively.



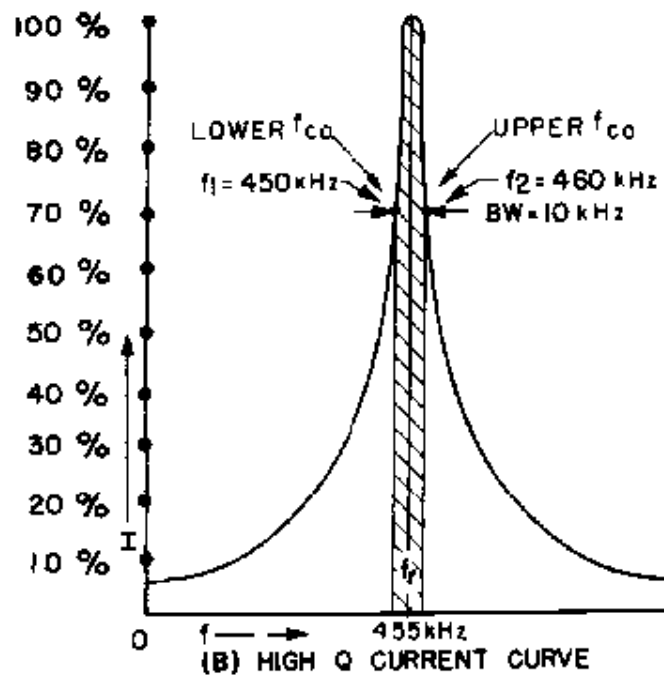
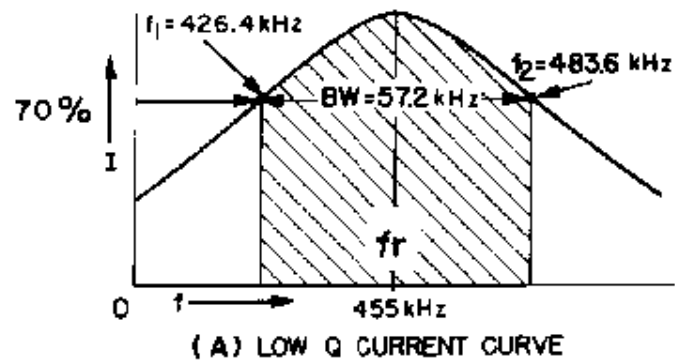
IN A **PARALLEL-LC CIRCUIT** impedance is maximum and current is minimum. Current is the variable and at resonance the two currents are 180 degrees out of phase with each other. Above resonance the current acts capacitively, and below resonance the current acts inductively.



THE "**Q**" OR **FIGURE OF MERIT** of a circuit is the ratio of  $X_L$  to  $R$ . Since the capacitor has negligible losses, the circuit  $Q$  becomes equivalent to the  $Q$  of the coil.

$$(Q = \frac{X_L}{R})$$

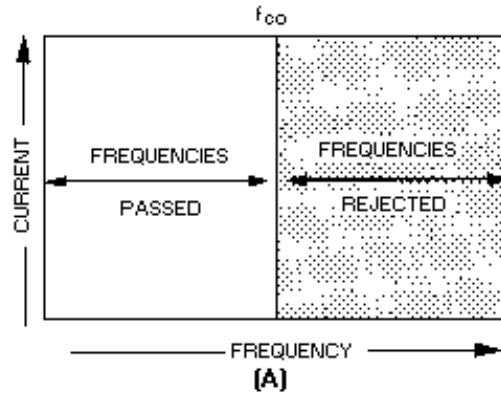
THE **BANDWIDTH** of a circuit is the range of frequencies between the half-power points. The limiting frequencies are those at either side of resonance at which the curve falls to .707 of the maximum value. If circuit  $Q$  is low, you will have a wide bandpass. If circuit  $Q$  is high, you will have a narrow bandpass.



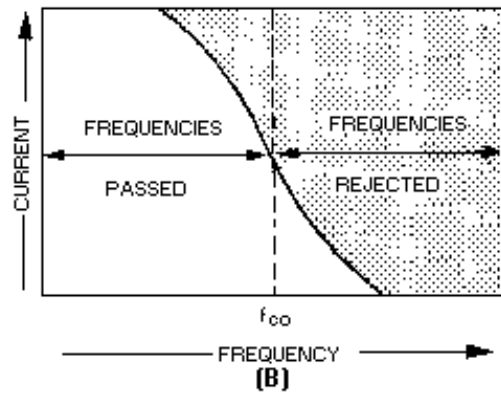
A **FILTER CIRCUIT** consists of a combination of capacitors, inductors, and resistors connected so that the filter will either permit or prevent passage of a certain band of frequencies.

A **LOW-PASS FILTER** passes low frequencies and attenuates high frequencies.

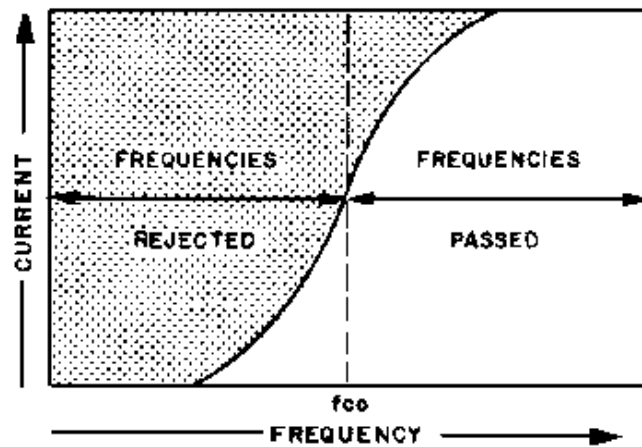




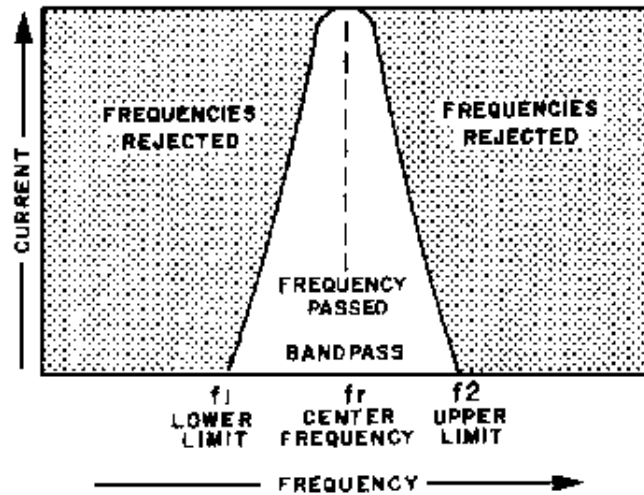
### IDEAL RESPONSE



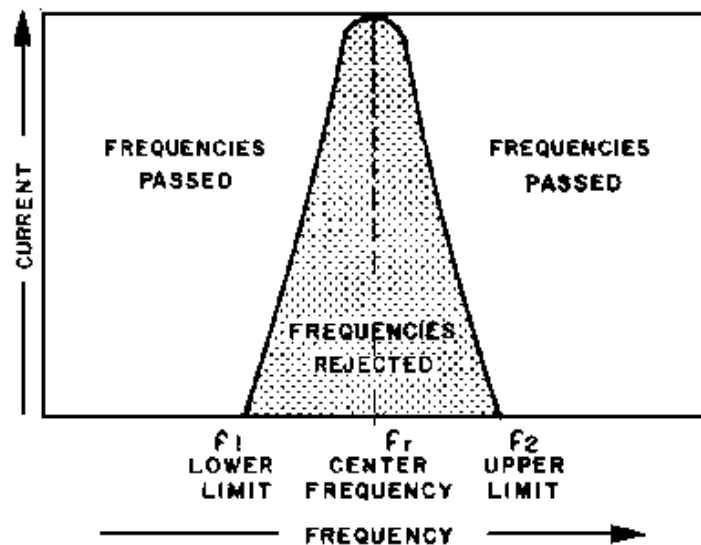
A **HIGH-PASS FILTER** passes high frequencies and attenuates low frequencies.



A **BANDPASS FILTER** will permit a certain band of frequencies to be passed.



A **BAND-REJECT FILTER** will reject a certain band of frequencies and pass all others.



A **SAFETY PRECAUTION** concerning series resonance: Very high reactive voltage can appear across L and C. Care must be taken against possible shock hazard.

**ANSWERS TO QUESTIONS Q1. THROUGH Q21.**

A-1.

- a.  $X_L$  varies directly with frequency.

$$X_L = 2\pi fL$$

- b.  $X_C$  varies inversely with frequency.

$$X_C = \frac{1}{2\pi fC}$$

- c. Frequency has no affect on resistance.

A-2. Resultant reactance.

A-3.

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad f_r = \frac{.159}{\sqrt{LC}}$$

A-4. Decreases.

A-5. Impedance low Current high.

A-6. Nonresonant (circuit is either above or below resonance).

A-7. Inductor magnetic field.

A-8. Capacitor.

A-9. Natural frequency or resonant frequency ( $f_r$ ).

A-10. Maximum impedance, minimum current.

A-11. At the resonant frequency.

A-12.

$$(Q = \frac{X_L}{R}) \text{ (high } X_L, \text{ low } R)$$

A-13. Bandwidth of the circuit.

A-14. A filter.

*A-15.*

- a. Low-pass.*
- b. High-pass*
- c. Bandpass.*
- d. Band-reject.*

*A-16. High-pass filter, low-frequency discriminator, or low-frequency attenuator.*

*A-17. Low-pass filter, high-frequency discriminator or high-frequency attenuator.*

*A-18. At low-frequency, a capacitor acts as an open and an inductor acts as a short. At high-frequency, a capacitor acts as a short and an inductor acts as an open.*

*A-19. Frequency cutoff ( $f_{co}$ ).*

*A-20. Bandpass.*

*A-21. Band-reject.*