Design and Analysis of Algorithms I

Graph Primitives

Dijkstra's Algorithm: The Basics

Single-Source Shortest Paths

Input: directed graph G=(V, E). (m=|E|, n=|V|)
- each edge has non negative length l_e
- source vertex s

Length of path
= sum of edge lengths

Output: for each $v \in V$, compute
L(v) := length of a shortest s-v path in G

Path length = 6

Assumption:

If w/o this assumption, we can use dfs or bfs to eliminate the irrelevant part of the graph

1. [for convenience] $\forall v \in V, \exists s \Rightarrow v$ path
2. [important] $le \geq 0 \;\; \forall e \in E$ There are algos that deal with edge length negative

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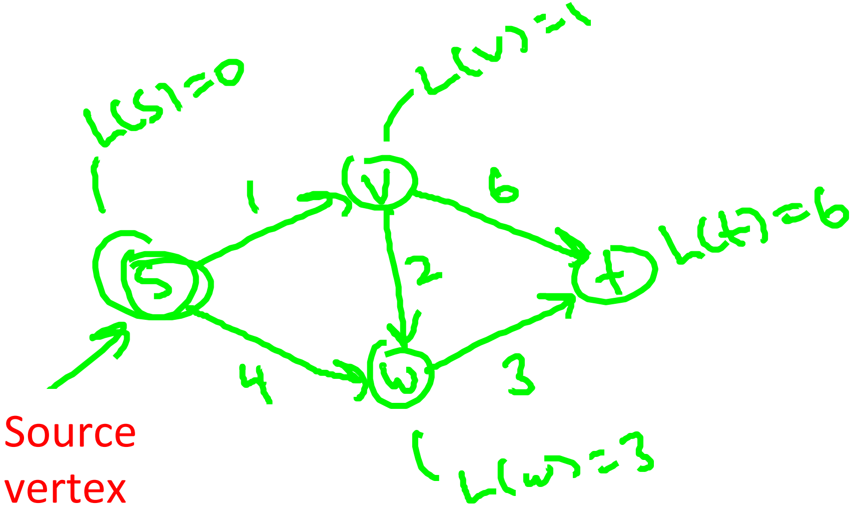
One of the following is the list of shortest-path distances for the nodes s,v,w,t, respectively. Which is it?

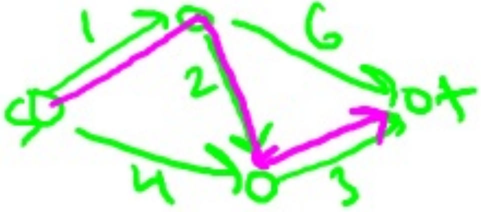
- 0,1,2,3
- 0,1,4,7
- 0,1,4,6
- 0,1,3,6



Why Another Shortest-Path Algorithm?

Question: doesn't BFS already compute shortest paths in linear time?

Answer: yes, IF $l_e = 1$ for every edge e.



Question: why not just replace each edge e by directed path of l_e unit length edges:

Answer: blows up graph too much

Too wasteful

Solution: Dijkstra's shortest path algorithm.

Dijkstra's Algorithm

This array only to help explanation!

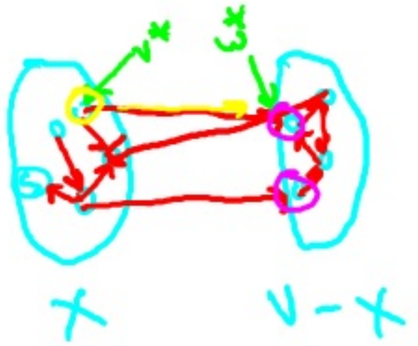
Initialize:

- X = [s] [vertices processed so far]
- A[s] = 0 [computed shortest path distances]
- B[s] = empty path [computed shortest paths]

Main Loop

- while X≠V:

-need to grow x by one node



Main Loop cont'd: remember: compute all that qualifie

- among all edges $(v,w) \in E$ with $v \in X, w \notin X$, pick the one that minimizes $A[v] + l_{vw}$

Already computed in earlier iteration

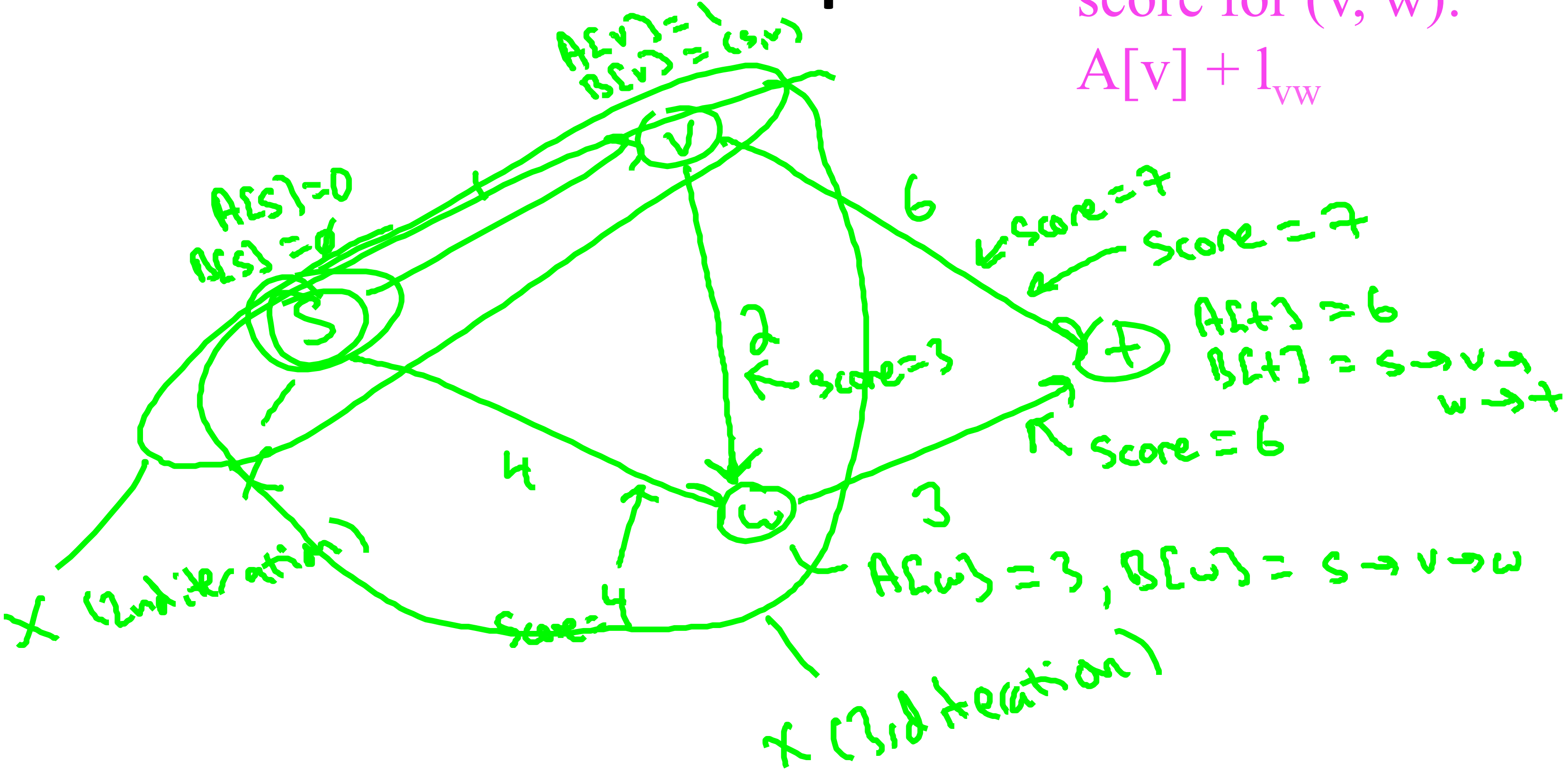
[call it (v*, w*)]

Dijkstra's greedy criterion

- add w* to X
- set $A[w^*] := A[v^*] + l_{v^*w^*}$
- set $B[w^*] := B[v^*]u(v^*, w^*)$

Example

Dijkstra's greedy score for (v, w): $A[v] + l_{vw}$



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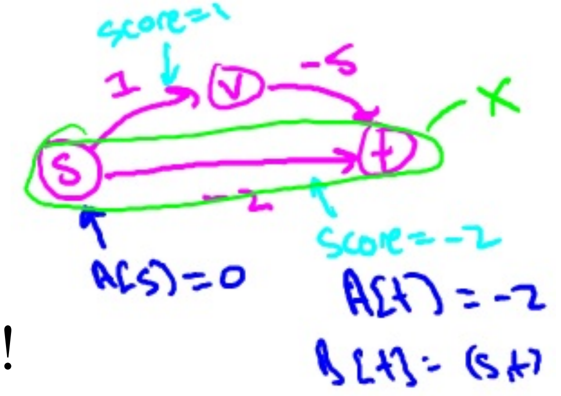
Non-Example

Question: why not reduce computing shortest paths with negative edge lengths to the same problem with non negative lengths? (by adding large constant to edge lengths)

If we add 5 to the right graph, the s->v->t path is 6, which is no longer the shortest path in the graph



Problem: doesn’t preserve shortest paths !

Also: Dijkstra’s algorithm incorrect on this graph !
(computes shortest s-t distance to be -2 rather than -4)