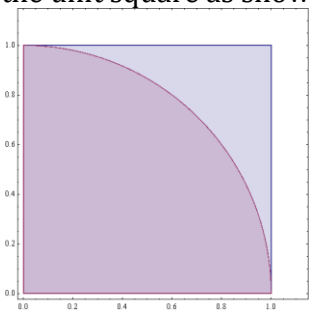
EE 511 Simulation Methods for Stochastic Systems
Project #5: MC & MCMC
[Due 2-Dec-2016]

Problem 1 [Monte Carlo]

Generate n=100 samples of i.i.d 2-dimensional uniform random variables in the unit-square. Count how many of these samples fall within the quarter unit-circle centered at the origin. This quarter circle inscribes the unit square as shown below:



i.) Use these random samples to estimate the area of the inscribed quarter circle. Use this area estimate to estimate the value of pi. Do k=50 runs of these pi-estimations. Plot the histogram of the 50 pi-estimates.

ii.) Repeat the experiment with different numbers of uniform samples, n (using k=50 for all these runs). Plot the sample variance of the pi-estimates for these different values of n. What is the relationship the estimate variance and Monte Carlo sample size?

iii.) Adapt your Monte Carlo solution to provide integral and error estimates for the function:

$$g(x,y) = |4x-2| \times |4y-2| \qquad \text{x, y in } [0,1]$$

Problem 2 [Variance Reduction Methods for Monte Carlo]

Use a total sample budget of n=1000 to obtain Monte Carlo estimates and sample MC estimate variances for the definite integrals in 2 dimensions (x_1, x_2):

(a) $\exp\left(\sum_{i=1}^{2} 5|x_i - 0.5|\right)$ for $x_i \; in \; [0,1]$

(b) $\cos\left(\pi + \sum_{i=1}^{2} 5x_i\right)$ for $x_i \; in \; [-1,1]$

(c) $|4x-2| \times |4y-2|$ for $x, y \; in \; [0,1]$

Implement *stratification* and *importance sampling* (separately) in the Monte Carlo estimation procedures using the same sample budget n=1000. Compare the 3 different Monte Carlo integral estimates and their sample variances. Discuss the quality of the Monte Carlo estimates from each method.

Problem 3 [MCMC for Optimization]

The n-dimensional Scwefel function

$$f(\vec{x}) = 418.9829n - \sum_{i=1}^{n} x_i \sin\sqrt{|x_i|}$$
$$x_i \; in \; [-500, 500]$$

is a very bumpy surface with many local critical points and one global maximum. We will explore the surface for the case n=2 dimensions.

i) Plot a contour plot of the surface for the 2-D surface

ii) Implement a simulated annealing procedure to find the global minimum of this surface

iii) Explore the behavior of the procedure starting from the origin with an exponential, a polynomial, and a logarithmic cooling schedule. Run the procedure for t={20, 50, 100, 1000} iterations for k=100 runs each. Plot a histogram of the function minima your procedure converges to.

iv) Choose your best run and overlay your 2-D sample path on the contour plot of the Schwefel function to visualize the locations your optimization routine explored.

Turn in:

- Analyses & discussions of your findings
- A summary of your experiments including histograms of your data, sample path plots, and other relevant numerical results
- A print out of your code.