Graphs and Networks

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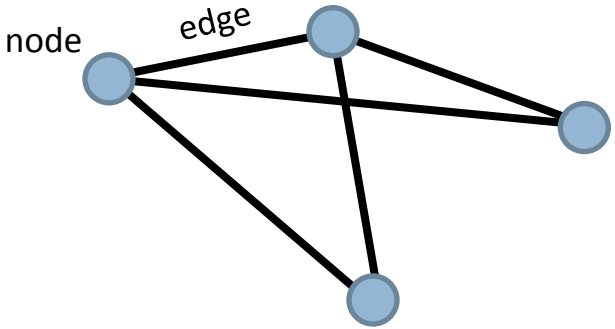
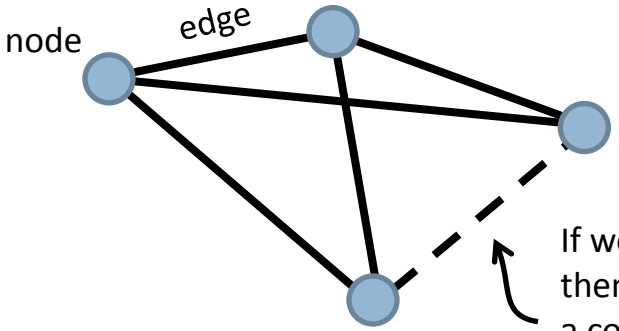
Graphs

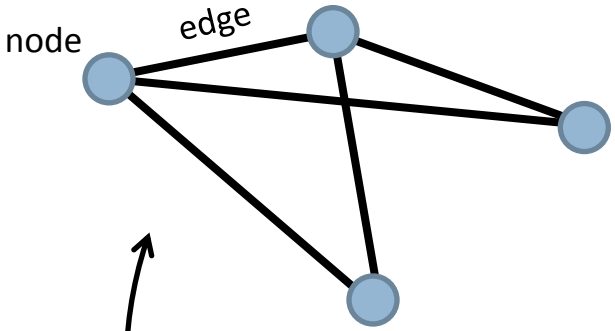


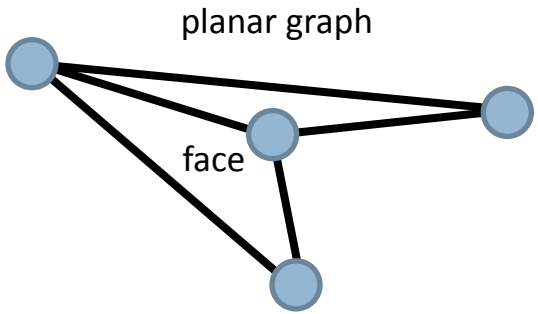
Graphs



If we add this edge, then this would be a complete graph, also called a *clique*

Graphs

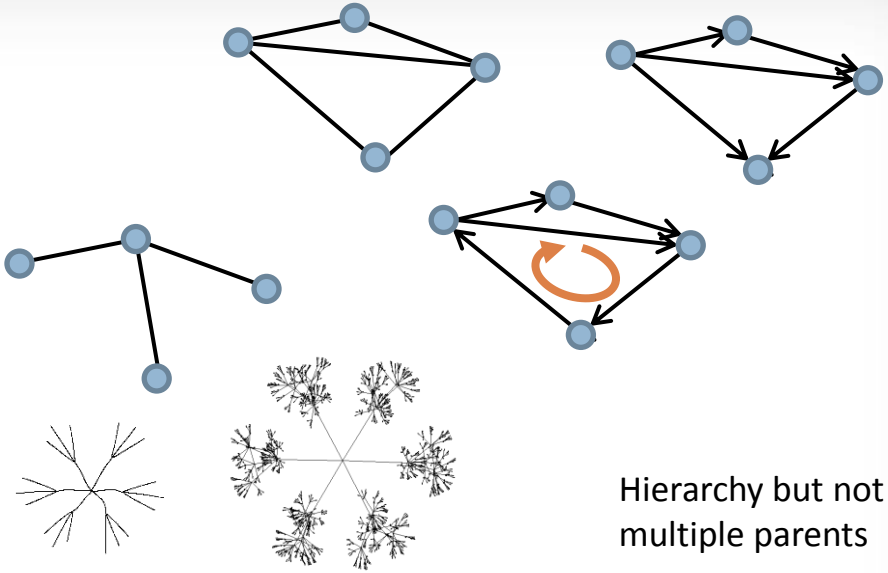




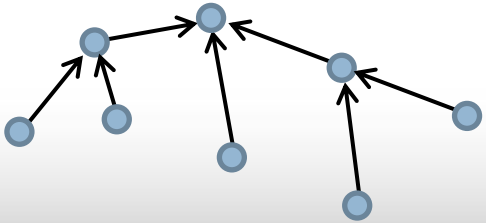
The left graph and the right graph are *isomorphic*, two different *embeddings* of the same relationship. Together, then they form a single *disconnected* graph with two *connected components*

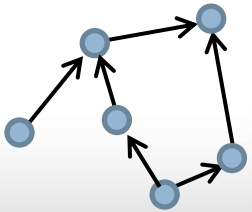
Directed Graphs and Hierarchies

- Directed v. undirected
- Cyclic v. acyclic
- Tree
  - minimally connected
  - n nodes, n-1 edges
  - Single parent node can have multiple child nodes (siblings)
- Hierarchy
  - acyclic directed graph
  - root node



Hierarchy but not a tree, multiple parents

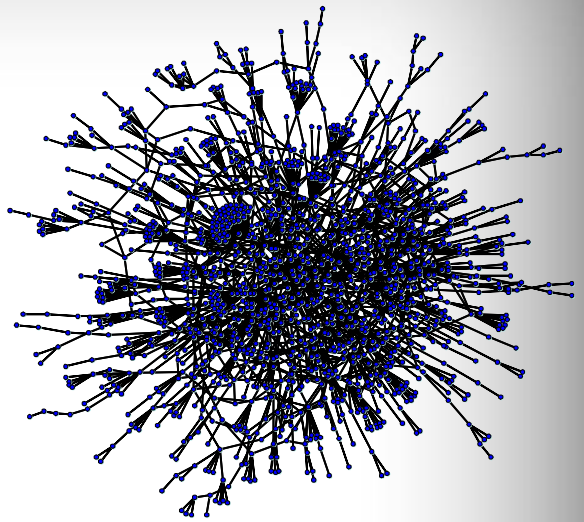




Node Degree

1,458 yeast proteins
1,948 interactions

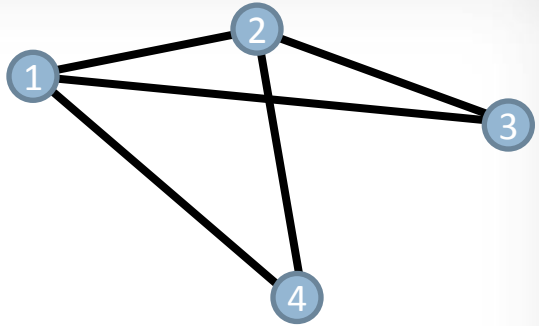
- Degree of a node = number of edges
  - Directed graph nodes have an in-degree and an out-degree
- Social networks
  - Many low degree nodes and fewer high degree nodes
  - Also called logarithmic, power-law or scale-free graphs



Adjacency Matrix

- Matrix representation of a graph
- Square matrix

  # of rows & columns = # of nodes
- Row i column j set to one if edge connects node i to node j, otherwise zero
- Symmetric, unless directed graph
- Can also use weighted edges
- Diagonal can represent edge between a node and itself



$$A=\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

One if this node is connected to…

…this node