Recurrent Neural Network

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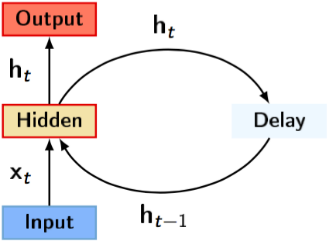
Recurrent Neural Network (RNN)

The RNN has recurrent hidden states whose output at each time is dependent on that of the previous time.
Given a sequence $x(1:n) = (x_1, x_2, ..., x_t, ..., x_n)$ (each x_i is a vector), the RNN updates its recurrent hidden state h(t) by

$$h_t = \begin{cases} 0 & \text{if t=0} \\ f(h_{t-1}, x_t) & \text{otherwise} \end{cases} \tag{1}$$



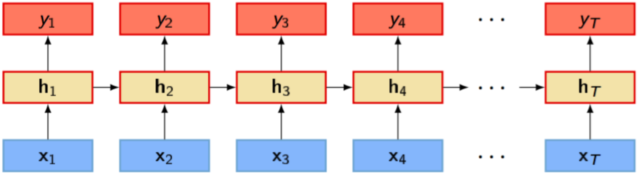
Simple Recurrent Network

- The recurrence function:

$$h_t = f(Uh_{t-1} + Wx_t + b) \tag{2}$$

where f is non-linear function (for instance, *sigmoid* or *tanh*).

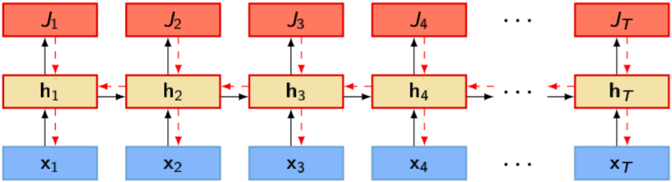
- A typical RNN structure can be illustrated as follows:



Backpropagation Through Time, BPTT

Suppose loss at time t is J_t, then the total loss is $\Sigma_{t=1}^{T} J_t$. The gradient of J is

$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T} \frac{\partial J_t}{\partial U} = \sum_{t=1}^{T} \frac{\partial h_t}{\partial U} \frac{\partial J_t}{\partial h_t} \tag{3}$$



Matrix Calculus

$$\frac{\partial y}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} & \cdots & \frac{\partial y}{\partial x_n} \end{bmatrix}$$

Figure: $y = f(x_1, x_2, ..., x_n)$

$$\frac{\partial \mathbf{Y}}{\partial x} = \begin{bmatrix} \frac{\partial y_{11}}{\partial x} & \frac{\partial y_{12}}{\partial x} & \cdots & \frac{\partial y_{1n}}{\partial x} \\ \frac{\partial y_{21}}{\partial x} & \frac{\partial y_{22}}{\partial x} & \cdots & \frac{\partial y_{2n}}{\partial x} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_{m1}}{\partial x} & \frac{\partial y_{m2}}{\partial x} & \cdots & \frac{\partial y_{mn}}{\partial x} \end{bmatrix}$$

Figure: Y is a matrix.

Matrix Calculus (cont'd)

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

Figure: Y, X are column vectors.

$$\frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{bmatrix} \frac{\partial \mathbf{F}}{\partial X_{1,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial X_{1,n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}}{\partial X_{m,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial X_{m,n}} \end{bmatrix}$$

Figure: F, X are matrices.

Gradient of RNN

$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T} \sum_{k=1}^{t} \frac{\partial h_k}{\partial U} \frac{\partial h_t}{\partial h_k} \frac{\partial y_t}{\partial h_t} \frac{\partial J_t}{\partial y_t} \tag{4}$$

$$\frac{\partial h_t}{\partial h_k} = \prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=k+1}^{t} U^T diag[f'(h_{i-1})] \tag{5}$$

$$\frac{\partial J}{\partial U} = \sum_{t=1}^{T} \sum_{k=1}^{t} \frac{\partial h_k}{\partial U} \left(\prod_{i=k+1}^{t} U^T diag[f'(h_{i-1})] \right) \frac{\partial y_t}{\partial h_t} \frac{\partial J_t}{\partial y_t} \tag{6}$$

Long-Term Dependencies

Define $\gamma = ||U^T diag[f'(h_{i-1})]||$,

- Exploding Gradient Problem: When $\gamma > 1$ and $t - k \to \infty$, $\gamma^{t-k} \to \infty$.
- Vanishing Gradient Problem: When $\gamma < 1$ and $t - k \to \infty$, $\gamma^{t-k} \to 0$.

There are various ways to solve Long-Term Dependency problem.

Gradient Explosion & Vanishing

- Gradient norm clipping (Mikolov thesis 2012; Pascanu, Mikolov, Bengio, ICML 2013)

$$\hat{g} \leftarrow \frac{\partial \mathcal{J}}{\partial \theta}$$

IF

$$||\hat{g}|| > \mathit{Threshold}$$

THEN

$$\hat{g} \leftarrow \frac{\mathit{Threshold}}{||\hat{g}||} * \hat{g}$$

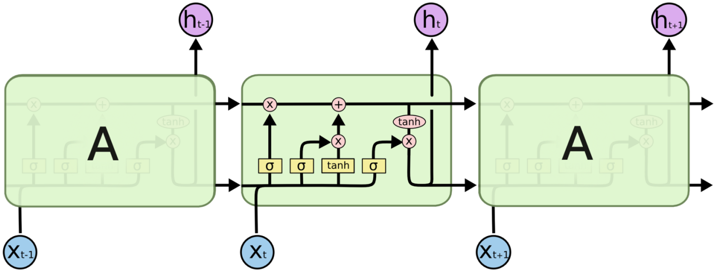
- Gradient propagation regularizer (avoid vanishing gradient)
- LSTM self-loops (avoid vanishing gradient)

Long Short-Term Memory (LSTM)

Key difference to RNN: a memory cell c which is controlled by three gates:

- input gate i:
$$i_t = \sigma(W_i x_t + U_i h_{t-1} + b_i) \tag{7}$$
- output gate o:
$$o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o) \tag{8}$$
- forget gate f:
$$f_t = \sigma(W_f x_t + U_f h_{t-1} + b_f) \tag{9}$$
- update of memory cell and hidden state:
$$\widetilde{C_t} = tanh(W_c x_t + U_c h_{t-1} + b_C) \tag{10}$$
$$C_t = f_t \otimes C_{t-1} + i_t \otimes \widetilde{C_t} \tag{11}$$
$$h_t = o_t \otimes tanh(C_t) \tag{12}$$

LSTM Architecture



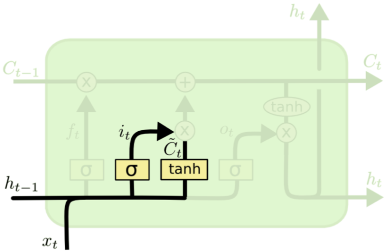
$$\widetilde{C_t} = tanh(W_c x_t + U_c h_{t-1} + b_C)$$

$$C_t = f_t \otimes C_{t-1} + i_t \otimes \widetilde{C_t}$$

$$h_t = o_t \otimes tanh(C_t)$$

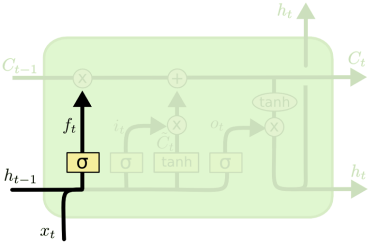
The picture and following 4 pictures are from http://colah.github.io/posts/2015-08-Understanding-LSTMs/.

LSTM: Input and Forget Gates



$$i_t = \sigma(W_i x_t + U_i h_{t-1} + b_i)$$

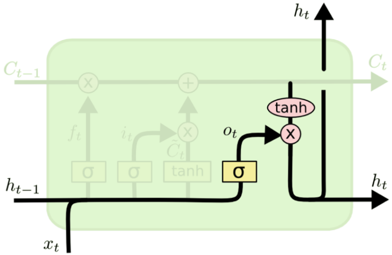
$$\widetilde{C}_t = tanh(W_c x_t + U_c h_{t-1} + b_C)$$



$$f_t = \sigma(W_f x_t + U_f h_{t-1} + b_f)$$

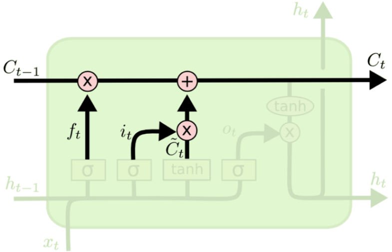
The picture is from http://colah.github.io/posts/2015-08-Understanding-LSTMs/.

LSTM:Output Gates and State Update



$$o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o)$$

$$h_t = o_t \otimes tanh(C_t)$$



$$C_t = f_t \otimes C_{t-1} + i_t \otimes \widetilde{C_t}$$

The picture is from
http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Gated Recurrent Unit, GRU

Instead of using three gates, only two gates is employed:

- update gate z:

$$z_t = \sigma(W_z x_t + U_z h_{t-1}) \tag{13}$$

- reset gate r:

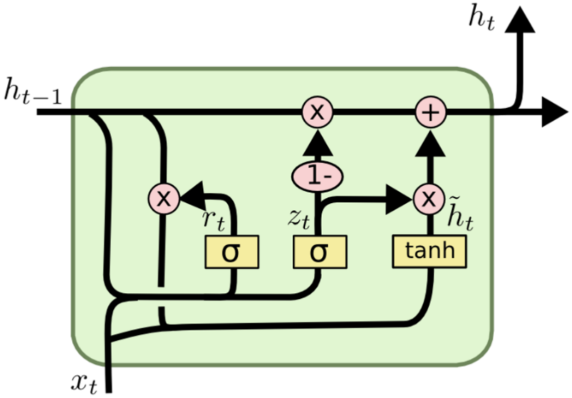
$$r_t = \sigma(W_r x_t + U_r h_{t-1}) \tag{14}$$

- state update:

$$\widetilde{h_t} = tanh(W_c x_t + U(r_t \otimes h_{t-1})) \tag{15}$$

$$h_t = (1 - z_t) \otimes h_{t-1} + z_t \otimes \widetilde{h_t} \tag{16}$$

GRU Architecture



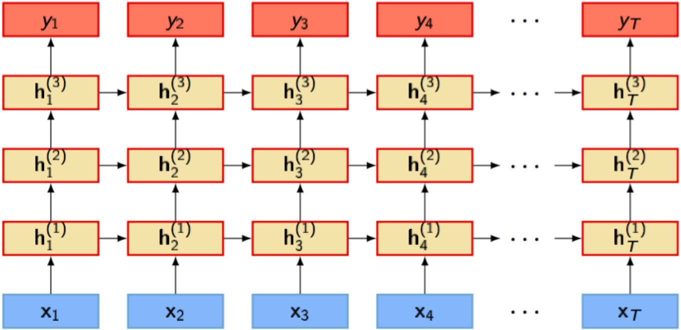
The picture is from
http://colah.github.io/posts/2015-08-Understanding-LSTMs/.

More Exploring to LSTM...

- How does LSTM deal with the explosion and vanishing gradient problems?
- refer to: Hochreiter S, Schmidhuber J. Long short-term memory[J]. Neural computation, 1997, 9(8): 1735-1780.

Stacked RNN/LSTM

- Stack multiple RNN/LSTM in vertical direction



- More common is seen that raw input (x_i) is connected to all layers as input.

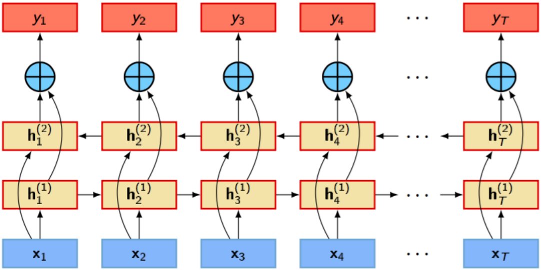
Bidirectional RNN/LSTM

- Modeling the prefix context and suffix context at the same time.

$$h_t^{(1)} = f(U^{(1)} h_{t-1}^{(1)} + W^{(1)} x_t + b^{(1)}) \tag{17}$$

$$h_t^{(2)} = f(U^{(2)} h_{t+1}^{(2)} + W^{(2)} x_t + b^{(2)}) \tag{18}$$

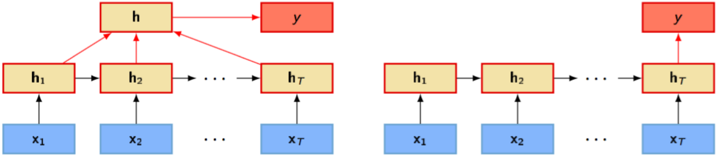
$$h_t = h_t^{(1)} \oplus h_t^{(2)} \tag{19}$$



Application of RNN: Classification

Applicable to many classification problems.

- Text Classification
- Sentiment Classification

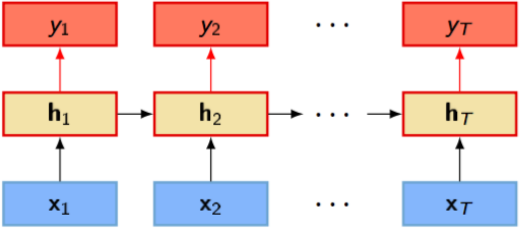


(c) Mean (d) Last

The left: can be enhanced with attention mechanisms.

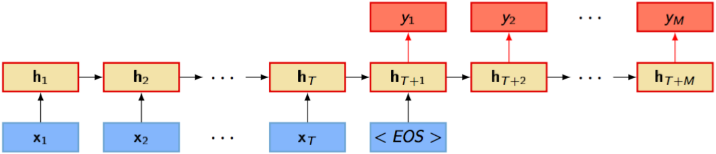
Application of RNN: Sequence Labeling

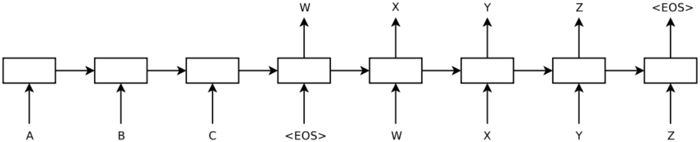
- Sequence Labeling, such as Chinese word segmentation, part-of-speech tagging, semantic role labeling



Application of RNN: Sequence to Sequence Learning

- Machine Translation
- Sequence to sequence generation (for dialogue, question answering, etc.)
- Image captioning: from image to text

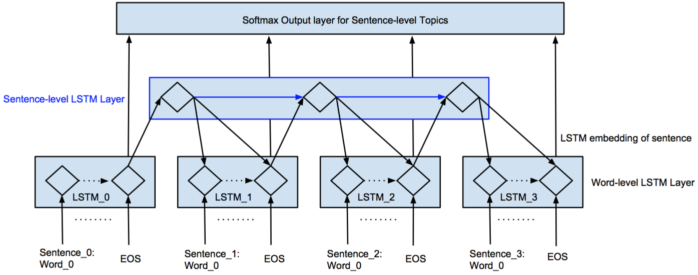




Sutskever I, Vinyals O, Le Q V. Sequence to sequence learning with neural networks[C]//NIPS. 2014: 3104-3112.

Hierarchical LSTM

Hierarchical LSTM (low-level LSTMs are taken as input to the high-level LSTM)



Ghosh S, Vinyals O, Strope B, et al. Contextual LSTM (CLSTM) models for Large scale NLP tasks[J]. arXiv preprint arXiv:1602.06291, 2016.

Summary

- RNN is very good at modeling text/language
- Basic RNN models: RNN, GRU, LSTM
  - Stacked RNN
  - Bi-directional RNN
  - Hierarchical RNN
- Typical Learning Problems with RNN
  - Classification
  - Sequential Labeling
  - Sequence to Sequence translation: MT, Dialogue Generation