

Analyzing Actuary Data to Predict Outcomes for Insurance Policies
Applied Discrete and Continuous Probability

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Project Overview

Insurance companies are faced with decisions to determine how much to charge for premiums for policies, given the estimate that a probability that an individual will collect on that policy.

The Social Security Administration (SSA) within the United States collects and determines the probability of life expectancy at a given age for males and females, with the most recent in 2019, as used in the 2022 Trustees Report (TR).

Terminology

Continuous Probability describes the probabilities of possible values of a variable. The variable is random and has a set of possible values, known as the range that can be infinite.

Discrete Distribution describes the probability of an occurrence of each value of a variable. The variable is random and has values that can be counted.

The Dataset

This data will utilize the 2015 US Period Life Tables to explore the various probabilities and predictions that can be made associated with death, insurance rates, and payouts that a life insurance company should take into account to maintain profitability.

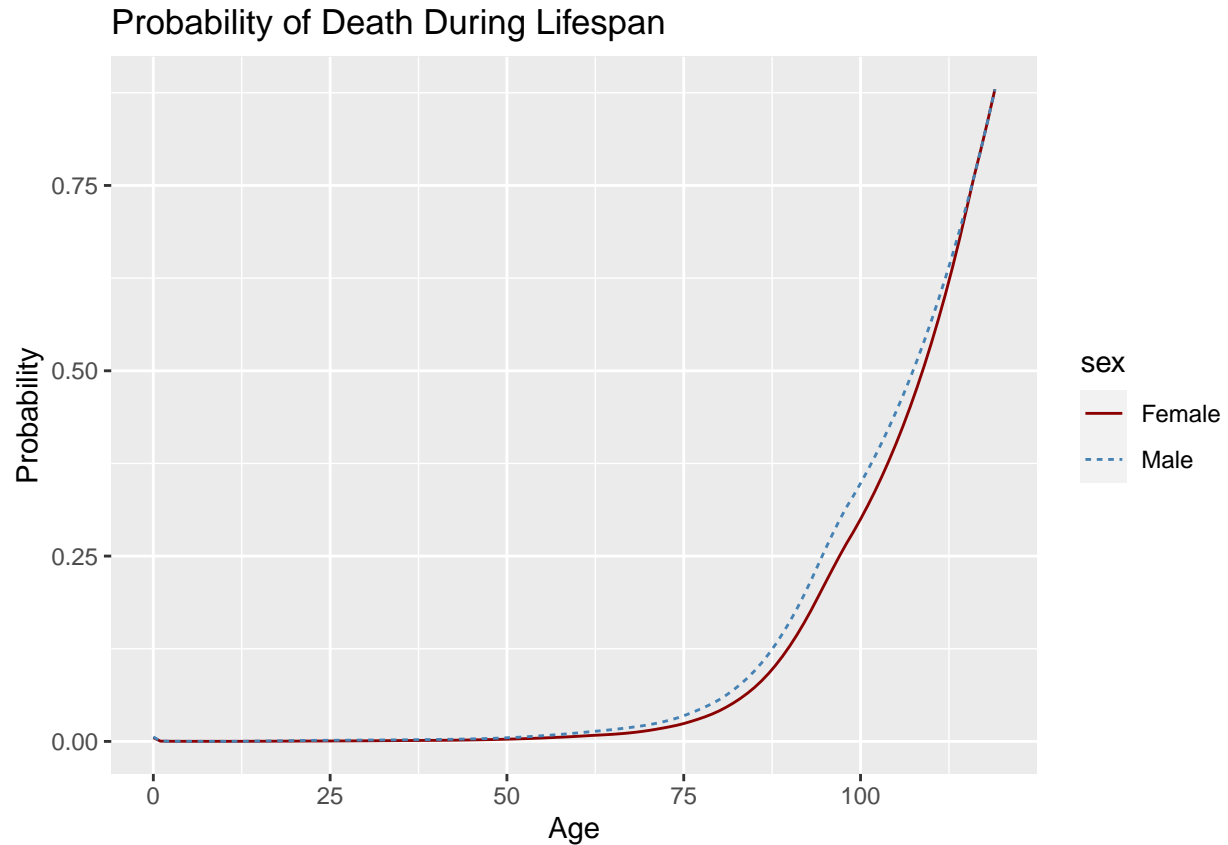
Load the data and explore data frame:

```
### Analyzing Actuary Data to Predict Outcomes for Insurance Policies
options(digits = 3)
library(tidyverse)
# library(dslabs)
# str(death_prob)
# summary(death_prob)
# write.csv(death_prob, "data/old/death_prob2015.csv")
death_prob <- read.csv("data/2019_ActuaryData.csv")
```

Visualization - Probability of Death During Lifespan

This time-series visualization shows the probability of death for both men and women over time. Although these values appear to be very close, they begin to have an impact to insurance premiums over time.

```
library(ggplot2)
ggplot(death_prob, aes(age, prob)) +
  geom_line(aes(color = sex, linetype = sex)) +
  scale_color_manual(values = c("darkred", "steelblue")) +
  xlab("Age") +
  ylab("Probability") +
  ggtitle("Probability of Death During Lifespan")
```



The Break-even point for Life Insurance

Further analysis of how death probability is accounted for life insurance is best explored with case study examples. If it is assumed that the loss in the event of the policy holder's death is \$200,000 for a 60 year old male, we want to determine what the break-even point is for the company to earn a profit.

```
a <- -200000
b <- 2317 # adjust this value to test the break-even point
p <- death_prob %>%
  filter(sex == "Male" & age == "60") %>%
  pull(prob)
p # death probability at age 60 (male)
```

```
## [1] 0.0115
```

```
S <- a*p + b*(1-p)
S # using value b, we determine the closest value to zero, which is the annual life insurance r
```

```
## [1] 0.0657
```

If we consider the values and probability of death for this individual, and annual amount of \$2,317 would allow our insurance company to break even for insuring this risk.

Setting an Annual Goal for Insurance Rates to Remain Profitable

Now consider managing 1,000 policies for 30-year-old females, with \$150,000 life insurance policies. How much should the insurance company charge annually to maintain an annual profit of \$100,000?

```
p_female <- death_prob %>%
  filter(sex == "Female" & age == "30") %>%
  pull(prob)
p <- p_female
mu_sum <- 100000
n <- 1000
a <- -150000

b <- (mu_sum/n-a*p)/(1-p)
b
```

```
## [1] 222
```

By charging an annual rate of \$222, 30-year-old females would have annual insurance policies of \$150,000.

Central Limit Theorem

If we use the central limit theorem, it is possible to determine the probability of losing money from the previously mentioned case study as well.

```
options(digits = 10)
n <- 1000
a <- -150000
b <- 222
p <- death_prob %>%
  filter(sex == "Female" & age == "30") %>%
  pull(prob)
p
```

```
## [1] 0.00081
```

```
p <- p_female
mu_sum <- 700000
n <- 1000
a <- -150000
b <- (mu_sum/n-a*p)/(1-p)
b
```

```
## [1] 822.1659544
```

```
sigma_sum <- sqrt(n)*abs(b-a)*sqrt(p*(1-p))  
sigma_sum
```

```
## [1] 135684.9635
```

```
pnorm(0, mu_sum, sigma_sum)
```

```
## [1] 1.241299355e-07
```

There is a 0.000000124% chance that we will lose profit by charging the prescribed amount of \$222 for 1,000 life insurance policies to 30 year old women.

References and Works Cited

1. “Actuarial Life Table: Period Life Table, 2019, as used in the 2022 Trustees Report”, Social Security Administration (SSA). (online) Available: <https://www.ssa.gov/oact/STATS/table4c6.html>

Keywords: Analytics, baseball, data science, multiple linear regression, sports, statistics.

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