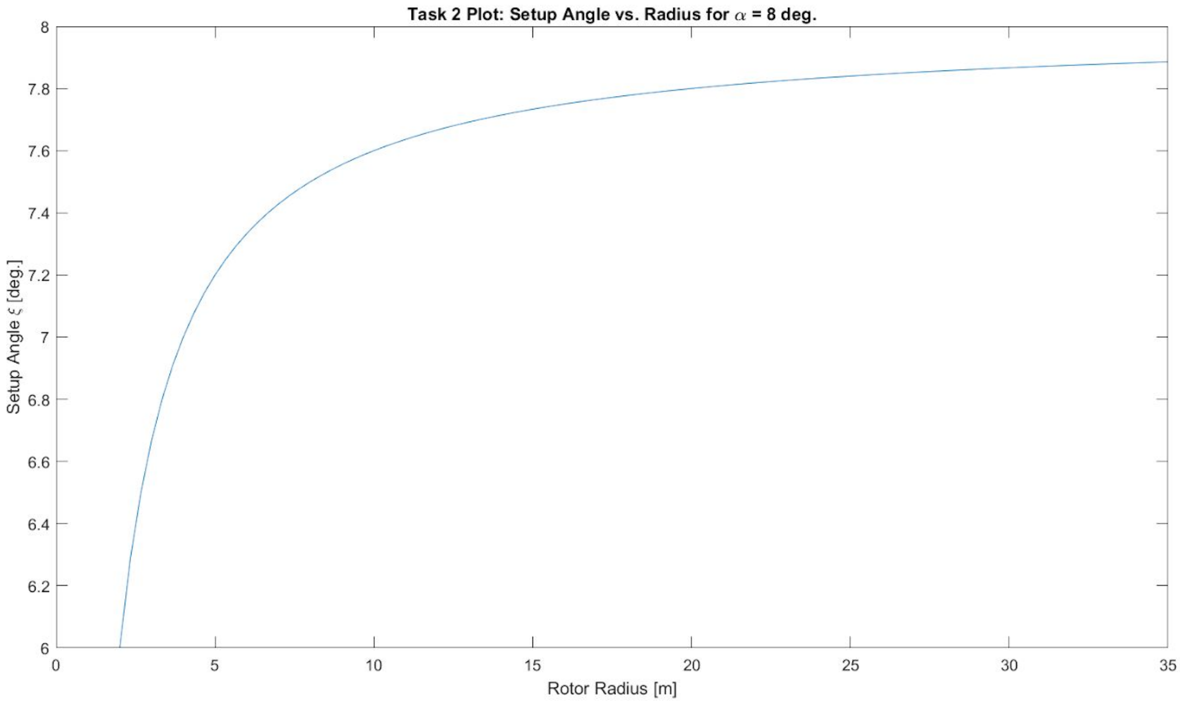
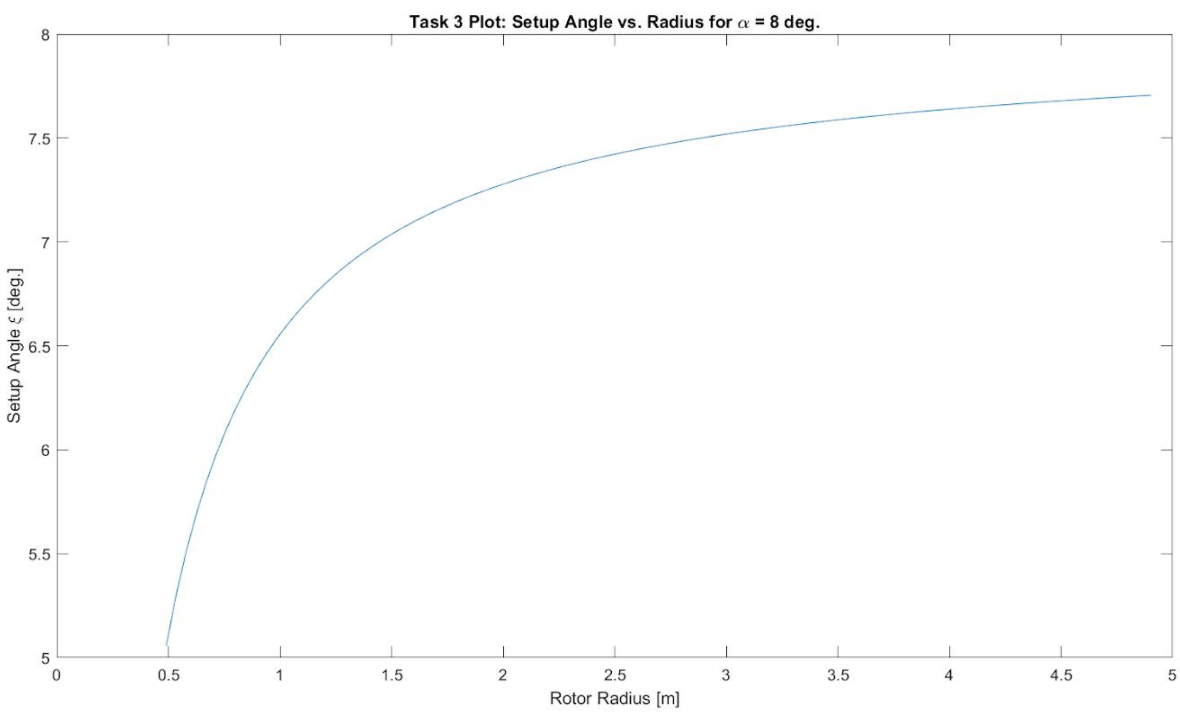
Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Task 1: See Appendix B; Task 2 Plot: See below



Task 3 Plot: $R = 4.9042$ m, $\sigma = 0$



Task 4 Results:

a) Delivered current: I_L = 125 A
b) V_{cell} = 1.48V (recharge, V_{in}); V_L = 0.9765V (discharge, V_{out}); V_{in}/V_{out} = 0.658.

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Appendix A: Matlab Code

Task2.m

```
rau = 1.18
v1 = 12
alpha = 8
CL = 1.27
n = 3
K_h = 2.7
const = 0.4
r_h = 2
R = 35
w = 2
lamda = w*R/v1
r_head = r_h/R
x = const/(1-r_head)
power1 = (1/3)*n*rau*v1^3*CL*lamda^2*K_h*R
a =
(((1/3)+(const/(1-r_head))*((r_head/3)-1/4))*(1+(4/(9*lamda^2)))^1.5)-(((1/3)+(const/
(1-r_head))*((r_head/12)))*(r_head+(4/(9*lamda^2)))^1.5)
b =
x*1/(18*lamda^2)*((sqrt(1+(4/(9*lamda^2)))-(r_head*sqrt(r_head^2+(4/(9*lamda^2))))))
c =
x*2/(81*lamda^4)*log((1+sqrt(1+(4/(9*lamda^2))))/(r_head+sqrt(r_head^2+(4/(9*lamda^2)
))))
ans = power1*(a-b-c)
Wbetz = 8*pi/27*rau*v1^3*R^2*(1-r_head^2);

eff = (9*n*CL*lamda^2*K_h)*(a-b-c)/(8*pi*R*(1-r_head^2))
```

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Task2Plot.m

```
%% Task 2 Problem Statement

%% Constants/Initializations

p = 1.18; %[kg/m3]
v1 = 12; %[m/s]
a = 8; %[deg]
CL = 1.27; %unitless, coeff. of lift
n = 3; %unitless, 3 blades
Kh = 2.7; %[m], chord at hub r = rh
o = 0.40; %unitless, taper ratio
rh = 2.0; %[m], radius at hub
R = 35.0; %[m], radius at tip
w = 2.0; %[rad/s], angular rotation speed

lambda = w*R/v1;
rhat = rh/R;

%% Plot

r = linspace(rh,R,100); %[m], linspace
w_deg = rad2deg(w);
zeta = a - atand(2*R*v1./(3*w_deg*R.*r)); %[deg]
plot(r,zeta);
xlabel("Rotor Radius [m]");
ylabel("Setup Angle \xi [deg.]");
title("Task 2 Plot: Setup Angle vs. Radius for \alpha = 8 deg.");
```

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Task3.m

```
%% Task 3 Problem Statement
% Provide 1.5 kW of power. This means Wdot = 1.5e3.
% Maximize efficiency, Wdot/Wbetz

p = 1.18;
v1 = 6.5;
alpha = 8;
CL = 1.26;
n = 3;
w = 3.0;

Wdot_des = 1500;
ospace = linspace(0,1,101); % Deliverable
Rspace = linspace(2,190,17001); % Deliverable

% power_results = zeros(31,11);
% eff_results = zeros(31,11);
results = [];
for i = 1:length(ospace)
    for j = 1:length(Rspace)
        o = ospace(i);
        R = 4.904;%Rspace(j);
        rh = 0.1*R;
        Kh = 0.085*R;
        lambda = w*R/v1;
        rhat = rh/R;

        x = o/(1-rhat);
        power1 = (1/3)*n*p*v1^3*CL*lambda^2*Kh*R;
        a =
(((1/3)+(o/(1-rhat))*((rhat/3)-1/4))*(1+(4/(9*lambda^2)))^1.5)-(((1/3)+(o/(1-rhat))*((rhat/12)))*(rhat+(4/(9*
lambda^2)))^1.5);
        b = x*1/(18*lambda^2)*((sqrt(1+(4/(9*lambda^2)))-(rhat*sqrt(rhat^2+(4/(9*lambda^2))))));
        c = x*2/(81*lambda^4)*log((1+sqrt(1+(4/(9*lambda^2))))/(rhat+sqrt(rhat^2+(4/(9*lambda^2)))));
        Wdot = power1*(a-b-c);
        Wbetz = 8*pi/27*p*v1^3*R^2*(1-rhat^2);

        eff = (9*n*CL*lambda^2*Kh)*(a-b-c)/(8*pi*R*(1-rhat^2));

        error = abs(Wdot-Wdot_des);
        if(error < 0.025*Wdot_des) && (eff<1)% && (eff>0.3)
            col = [o; R; Wdot; eff];
            results = [results col];
        end
        power_results(i,j) = Wdot;
        eff_results(i,j) = eff;
    end
end

power_results = [Rspace; power_results];
eff_results = [Rspace; eff_results];
```

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Task3Plot.m

```
%% Task 3 Problem Statement
% Plot setup angle to keep alpha = 8

%% Constants/Initializations

p = 1.18;
v1 = 6.5;
a = 8;
CL = 1.26;
n = 3;
w = 3.0;

% Deliverables
o = 0;
R = 4.9042; %[m]

rh = 0.1*R;
Kh = 0.085*R;
lambda = w*R/v1;
rhat = rh/R;

%% Plot

r = linspace(rh,R,100); %[m], linspace
w_deg = rad2deg(w);
zeta = a - atand(2*R*v1./(3*w_deg*R.*r)); %[deg]
plot(r,zeta);
xlabel("Rotor Radius [m]");
ylabel("Setup Angle \xi [deg.]");
title("Task 3 Plot: Setup Angle vs. Radius for \alpha = 8 deg.");
```

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Appendix B: Task 1 Derivation

Task 1: Derivation

1

$$d\dot{W} = \frac{1}{3} n \rho W(r) V_1 C_L(r) K(r) w r\, dr$$

$$\int d\dot{W} = \int_{r_h}^{R} \frac{1}{3} n \rho \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot V_1 \cdot C_L \cdot w r\, dr \cdot k_h \left(1 - 6\left(\frac{r - r_h}{R - r_h}\right)\right) dr$$

$$\dot{W} = \frac{1}{3} n \rho V_1 C_L \cdot w \cdot k_h \left[\underbrace{\int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot \left(1 + \frac{6 r_h}{R - r_h}\right) \cdot r\, dr}_{A} - \underbrace{\int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot \left(\frac{6}{R - r_h}\right) r^2\, dr}_{B} \right]$$

(A)

$$\int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot \left(1 + \frac{6 r_h}{R - r_h}\right) \cdot r\, dr$$

$$= \left(1 + \frac{6 r_h}{R - r_h}\right) \int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot r\, dr$$

$$= w\left(1 + \frac{6 r_h}{R - r_h}\right) \int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3w}\right)^2 + r^2} \cdot r\, dr = w\left(1 + \frac{6 r_h}{R - r_h}\right) \left[\frac{1}{3}\left(r^2 + \left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}}\right]_{r_h}^{R}$$

$$= \frac{w}{3}\left(1 + \frac{6 r_h}{R - r_h}\right)\left(\left(R^2 + \left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}} - \left(r_h^2 + \left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}}\right)$$

$$= \boxed{\frac{w}{3}\left(1 + \frac{6 r_h}{R - r_h}\right)\left(\left(R^2 + \left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}} - \left(r_h^2 + \left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}}\right)}$$

(B) $$\int_{r_h}^{R} \sqrt{\left(\frac{2V_1}{3}\right)^2 + w^2 r^2} \cdot \left(\frac{6}{R-r_h}\right) r^2 \, dr$$

$$= \left(\frac{6}{R-r_h}\right)\int_{r_h}^{R}\sqrt{\left(\frac{2V_1}{3w}\right)^2+r^2}\cdot w \cdot r^2 \, dr$$

$$= \left(\frac{w6}{R-r_h}\right)\int_{r_h}^{R}\sqrt{\left(\frac{2V_1}{3w}\right)^2+r^2}\cdot r^2 \, dr$$

$$= \left(\frac{w6}{R-r_h}\right)\left[\left(\frac{r}{4}\right)\sqrt{\left(r^2+\left(\frac{2V_1}{3w}\right)^2\right)^3} - \frac{\left(\frac{2V_1}{3w}\right)^2 r}{8}\sqrt{r^2+\left(\frac{2V_1}{3w}\right)^2} - \frac{\left(\frac{2V_1}{3w}\right)^4}{8}\ln\left(r+\sqrt{r^2+\left(\frac{2V_1}{3w}\right)^2}\right)\right]_{r=r_h}^{r=R}$$

$$\dot{W} = \frac{1}{3} n\rho V_1 L_L \cdot w \cdot K_n \left[\frac{w}{3}\left(1+\frac{6r_h}{R-r_h}\right)\left(\left(R^2+\left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}} - \left(r_h^2+\left(\frac{2V_1}{3w}\right)^2\right)^{\frac{3}{2}}\right)\right.$$

$$+\frac{w6}{R-r_h}\left(\left(\frac{R}{4}\right)\sqrt{\left(R^2+\left(\frac{2V_1}{3w}\right)^2\right)^3} - \left(\frac{r_h}{4}\right)\sqrt{\left(r_h^2+\left(\frac{2V_1}{3w}\right)^2\right)^3}\right.$$

$$+\frac{\left(\frac{2V_1}{3w}\right)^2 R}{8}\sqrt{R^2+\left(\frac{2V_1}{3w}\right)^2} - \frac{\left(\frac{2V_1}{3w}\right)^2 r_h}{8}\sqrt{r_h^2+\left(\frac{2V_1}{3w}\right)^2}$$

$$\left.\left.+\frac{\left(\frac{2V_1}{3w}\right)^4}{8}\ln\left(R+\sqrt{R^2+\left(\frac{2V_1}{3w}\right)^2}\right) - \frac{\left(\frac{2V_1}{3w}\right)^4}{8}\ln\left(r_h+\sqrt{r_h^2+\left(\frac{2V_1}{3w}\right)^2}\right)\right)\right]$$

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

3

$$\dot{W} = \frac{1}{3}\eta \rho V_1 C_L \frac{\lambda^2 V_1^2}{R^2} K_n R^3 \Bigg[\left(\frac{1}{3} + \frac{6}{1-\hat{r}_n}\left(\frac{\hat{r}_n}{3} - \frac{1}{4}\right)\right) \left(1+\frac{4}{9\lambda^2}\right)^{\frac{3}{2}}$$

$$- \left(\frac{1}{3} + \frac{6}{1-\hat{r}_n}\left(\frac{10\hat{r}_n}{12}\right)\right) \left(\hat{r}_n + \frac{4}{9\lambda^2}\right)^{\frac{3}{2}}\Bigg)$$

$$- \frac{6}{(1-\hat{r}_n)18\lambda^2}\left(\sqrt{1+\frac{4}{9\lambda^2}} - \hat{r}_n\sqrt{\hat{r}_n^2 + \frac{4}{9\lambda^2}}\right)$$

$$= \frac{2 6}{(1-\hat{r}_n)81\lambda^4} \ln \frac{1+\sqrt{1+\frac{4}{9\lambda^2}}}{\hat{r}_n + \sqrt{\hat{r}_n^2 + \frac{4}{9\lambda^2}}}\Bigg]$$

$$\dot{W} = \frac{1}{3}\eta \rho V_1^3 C_L \cdot \lambda^2 \cdot K_n R^3 \Bigg[\left(\frac{1}{3} + \frac{6}{1-\hat{r}_n}\left(\frac{\hat{r}_n}{3} - \frac{1}{4}\right)\right) \left(1+\frac{4}{9\lambda^2}\right)^{\frac{3}{2}}$$

$$- \left(\frac{1}{3} + \frac{6}{1-\hat{r}_n}\left(\frac{10\hat{r}_n}{12}\right)\right) \left(\hat{r}_n + \frac{4}{9\lambda^2}\right)^{\frac{3}{2}}\Bigg)$$

$$- \frac{6}{(1-\hat{r}_n)18\lambda^2}\left(\sqrt{1+\frac{4}{9\lambda^2}} - \hat{r}_n\sqrt{\hat{r}_n^2 + \frac{4}{9\lambda^2}}\right)$$

$$- \frac{2 \cdot 6}{(1-\hat{r}_n)81\lambda^4} \ln \frac{1+\sqrt{1+\frac{4}{9\lambda^2}}}{\hat{r}_n + \sqrt{\hat{r}_n^2 + \frac{4}{9\lambda^2}}}\Bigg]$$

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

4

$$\dot{W}_{Betz} = \frac{16}{27} \cdot \frac{1}{2} \cdot \rho \cdot \pi \cdot (R^2 - r_h^2) \cdot V_1^3$$

$$= \frac{8\pi}{27} \rho V_1^3 R^2 \left(1 - \hat{r}_h^2\right)$$

$$\eta^* = \frac{\dot{W}}{\dot{W}_{betz}} = \frac{\frac{1}{3} \eta \cancel{\rho V_1^3} C_L \cdot X^2 \cdot k_n R [\cdots]}{\frac{8\pi}{27} \cdot \cancel{\rho V_1^3} \cdot R^2 \left(1 - \hat{r}_h^2\right)}$$

$$\boxed{\eta^* = \frac{27}{24\pi} \frac{9 C_L \eta X^2 \cdot k_n \cdot [\cdots]}{8\pi \cdot R \left(1 - \hat{r}_h^2\right)}}$$

$$\eta^* = 9 C_L X^2 \cdot k_n [\,]$$

Project 4 Deliverables: Charles Lin [undergraduate]; Jung Hwan Ha [undergraduate]

Task 1: Results

Task 1

part (A)

$$\boxed{\dot{W} = \frac{1}{3} n \rho V_1^3 C_L \lambda^2 K_n R \left[\left(\frac{1}{3} + \frac{\sigma}{1-\hat{r}_n}\left(\frac{\hat{r}_n}{3} - \frac{1}{4}\right)\right)\left(1+\frac{4}{9\lambda}\right)^{\frac{3}{2}} - \left(\frac{1}{3} + \frac{\sigma}{1-\hat{r}_n}\left(\frac{\hat{r}_n}{12}\right)\right)\left(\hat{r}_n + \frac{4}{9\lambda^2}\right)^{\frac{3}{2}} - \frac{\sigma}{(1-\hat{r}_n)18\lambda^2}\left(\sqrt{1+\frac{4}{9\lambda^2}} - \hat{r}_n\sqrt{\hat{r}_n^2+\frac{4}{9\lambda^2}}\right) - \frac{2\cdot\sigma}{(1-\hat{r}_n)81\lambda^4}\cdot \ln \frac{1+\sqrt{1+\frac{4}{9\lambda^2}}}{\hat{r}_n+\sqrt{\hat{r}_n^2+\frac{4}{9\lambda^2}}} \right]}$$

so, $\dot{W} = f(n, \rho, V_1, R, \hat{r}_n, C_L, K_n, \sigma, \lambda)$

Part (B)

$$\eta^* = \frac{\dot{W}}{\dot{W}_{betz}} = \boxed{\frac{9 C_L n \lambda^2 \cdot \frac{K_n}{R} \cdot [\cdots]}{8\pi(1-\hat{r}_n^2)}}$$

$\eta^* = f(n, \hat{r}_n, C_L, K_n/R, \sigma, \lambda)$