Project 4.
Due: Bring deliverables to the final exam on Wednesday December 12, 2018, at 3:00 PM.

You may team up with a partner for this project. Do not share information or results with other groups.

This project will focus on evaluating the operating characteristics of a horizontal axis wind turbine in a power generation system. The turbine envisioned is shown schematically in Fig. 1.

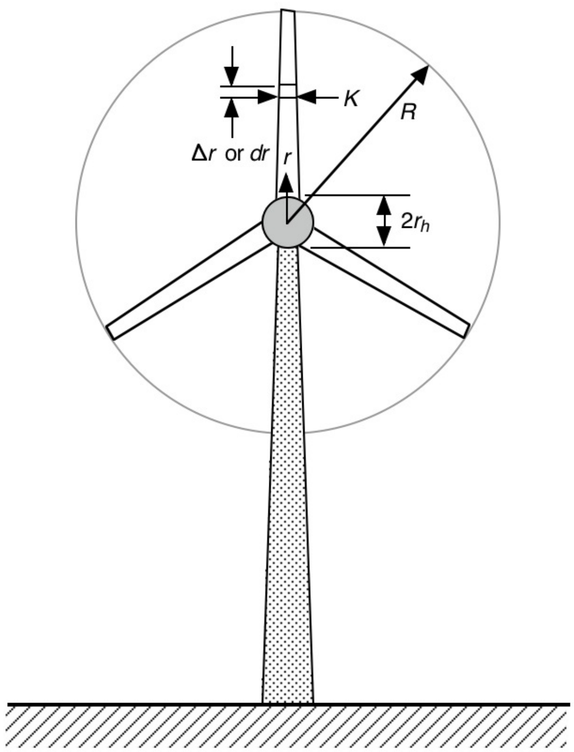

Figure 1.

Note that the angle of attack α, setup angle ξ, and the angle between the rotation plane and the relative air velocity ψ are related as shown in Figure 2.

Task 1

As shown in class, for the *i*th segment of the blade of a horizontal axis turbine rotor with *n* blades, if the lift coefficient is large compared to the drag coefficient, the power contribution of the segment is given by

$$\dot{W}_i = \frac{1}{3} n \rho w_i v_1 C_{L,i} K_i r_i \omega \Delta r \tag{1}$$

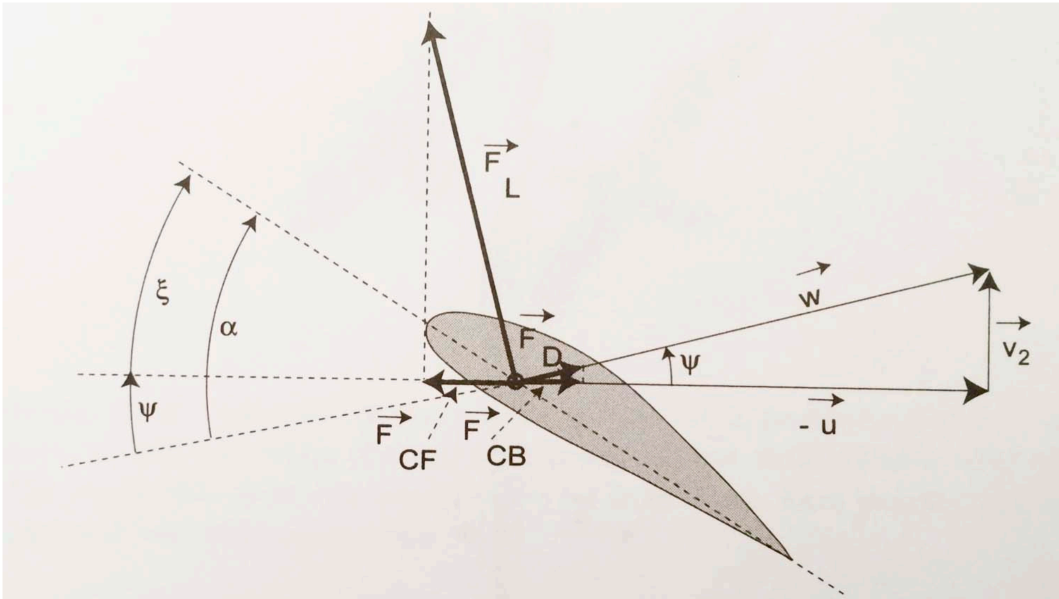

Figure 2. (Figure 15.19 in text)

where

ρ = air density ~1.2 kg/m^3
w_i = magnitude of the air relative velocity
v_1 = far field wind speed
n = the number of blades
$C_{L,i}$ = lift coefficient of segment i
K_i = chord of segment i
r_i = radius of segment i
ω = angular rotation speed (radians/second)
Δr = length of segment in radial direction

Letting the segment length pass to the limit of differential size ($\dot{W}_i \rightarrow d\dot{W}, \Delta r \rightarrow dr$), this relation becomes

$$d\dot{W} = \frac{1}{3} n\rho w(r) v_1 C_L(r) K(r) \omega r dr \tag{2}$$

where the magnitude of the relative velocity is a function of r:

$$w(r) = \sqrt{(2v_1/3)^2 + \omega^2 r^2} \tag{3}$$

and the lift coefficient $C_L(r)$ and chord $K(r)$ may be functions of r. If $w(r)$, $C_L(r)$ and $K(r)$ are known functions, the differential relation (2) can be integrated from the hub radius r_h to the tip radius R to determine the total power output of the rotor.

For the purposes of this project, take the angle of attack and lift coefficient to be constant at

$$C_L = 1.27 \tag{4}$$

$$\alpha = 8° \tag{5}$$

The blades are to be tapered linearly so that the chord varies according to the following relation

$$K(r) = K_h \left[1 - \sigma \left(\frac{r - r_h}{R - r_h} \right) \right] \quad (6)$$

where K_h is the chord at the hub ($r = r_h$) and σ is a constant between zero and 1 that dictates the amount of taper. Note that the tip chord will be equal to $K_h(1-\sigma)$ at $r = R$.

For this task, substitute relations (4) and (6) into Eq. (2) and integrate Eqn. (2) from $r = r_h$ to $r = R$ and use one or more of the relations in the appendix to derive a closed-form relation for the power output of the entire rotor $\dot{W}$. **Show all work in this derivation.** Reorganize the resulting relation for so that it is written in the form

$$\dot{W} = f(n, \rho, v_1, R, \hat{r}_h, C_L, K_h, \sigma, \lambda) \quad (7)$$

where

$\hat{r}_h = r_h / R$
$\lambda = \omega R / v_1$

Also, combine the resulting equation for $\dot{W}$ obtained from the integration with the formula for the Betz power output $\dot{W}_{Betz}$ for the rotor to obtain an equation for the Betz efficiency $\eta^* = \dot{W} / \dot{W}_{Betz}$. Cast this relation in the form:

$$\eta^* = f(n, \hat{r}_h, C_L, K_h / R, \sigma, \lambda) \quad (8)$$

Note: the formulas for $\dot{W}$ and η^* derived in this task are used in later portions of the project, so it is wise to derive these relations carefully.

<u>Task 2</u>
(a) Program the relations obtained in Task 1 into a computer program so that the power output of the turbine and its Betz efficiency can be computed for a specific set of input parameters.
(b) Compute the Betz efficiency and power output for a turbine with the following design parameters:

ρ = air density = 1.18 kg/m^3
v_1 = 12 m/s
$\alpha = 8°$
C_L = 1.27
n = 3
K_h = 2.7 m
σ = 0.40
r_h = 2.0 m
R = 35.0 m
ω = 2.0 radian/s

For this large-scale design, determine and plot how the setup angle ξ must vary between $r = r_h$ and $r = R$ to keep the angle of attack at 8° all along the blade.

Task 3
Wind speed and air density data for Berkeley, CA are:

mean air density = 1.18 kg/m^3
mean wind speed (v) = 6.5 m/s

For this task you are to use your model to design a horizontal axis wind turbine for the top of the Berkeley hills that will provide 1.5 kW of power at the mean wind speed indicted above. For your design analysis, use the parameter values indicated below:

ρ = air density - 1.18 kg/m^3
v_1 = 6.5 m/s
$\alpha = 8°$
C_L = 1.26
n = 3
K_h = 0.085R
r_h = 0.1R
ω = 3.0 radian/s

Based on your design analysis, recommend a rotor radius R and a taper constant σ that will meet the design power requirement while maximizing efficiency subject to the parameter specifications. For your design, determine and plot how the setup angle ξ must vary between $r = r_h$ and $r = R$ to keep the angle of attack at 8° all along the blade.

Task 4
Storing power generated by wind turbines efficiently is a major challenge. One possible approach is to run a hydrogen-oxygen fuel cell backwards to generate H_2 and O_2 gas that can be stored until the energy is needed. It can be retrieved by running it through the fuel cell to produce power.

As discussed in class, activation losses, Ohmic losses and mass transport losses generally cause the output voltage to differ from the reversible potential for a fuel cell. We specifically consider here a H_2-O_2 fuel cell having an operating curve that is well approximated over the range of interest by the following equation

$$V_L = 1.12 - 0.041 I_L$$

where V_L is in volts, and I_L is in amperes. Note that the constant 1.11 is below the reversible cell voltage of 1.23 V, which is likely the result of activation and reaction rate loss effects. The term -0.041 I_L suggests a reduction in output voltage due to Ohmic electrical resistance. The above relation can be written in the form

$$V_L = V_{rev} - 0.11 - 0.041 I_L, \text{ where } V_{rev} = 1.23 \text{ V}$$

For the fuel cell driven in reverse, a plausible estimate is to assume that the loss effects are symmetric. In reverse operation, extra applied voltage is required to overcome activation/reaction and Ohmic losses. This suggests that for production of hydrogen, the following operating curve would link the applied voltage to the current driven through the cell as it produces hydrogen:

$$V_{cell} = V_{rev} + 0.11 + 0.041 I_{cell}$$

where again, V_{rev} = 1.23 V.

(a) For your wind turbine design determined in Task 3, if the turbine generator nominally produces 12.0 V DC current, determine the delivered current if the turbine runs at the design wind speed of 6.5 m/s.

(b) Energy from a wind turbine is first stored in a H_2-O_2 fuel cell that is part of a storage system by inputting current I_{cell} = 3.5 A for 1 hr. Then energy is extracted at the same current flow rate for 1 hour. What fraction of the energy input in the first process is retrieved in the extraction process?

(c) <u>ME246 students only</u>: For the processes described in part (b) above, is the answer to the question different if the input/output current is increased by 25%? If so, by how much? Briefly explain the physical reasons for your answer.

<u>Tasks to be divided between coworkers:</u>

Tasks 1-4
Write-up of results

<u>Deliverables (Due Wednesday 12/12/16 at the end of the final exam):</u>

The following must be handed in at the final exam:

(1) A one page summary of your results for Tasks 1-4.
(2) The plots specified in Tasks 2 & 3. Make sure these are at least a half page in size with clear labels.
(3) A copy of your program should be attached to the report as an appendix.

A separate copy of the materials (1)-(3) must be handed-in by each team member.

Note: The final exam will include one question that relates to results you generate in this miniproject. Your score on that question will be based on what you hand in for this miniproject and the work you do answering that question on the exam.

Appendix

Indefinite integral formulas:

$$\int \sqrt{x^2+a^2}\,dx = \left(\tfrac{1}{2}\right)\left[x\sqrt{x^2+a^2} + a^2 \ln(x+\sqrt{x^2+a^2})\right]$$

$$\int x\sqrt{x^2+a^2}\,dx = \left(\tfrac{1}{3}\right)\left[x^2+a^2\right]^{3/2}$$

$$\int x^2\sqrt{x^2+a^2}\,dx = \left(\tfrac{x}{4}\right)\sqrt{(x^2+a^2)^3} - \left(\tfrac{a^2 x}{8}\right)\sqrt{x^2+a^2} - \tfrac{a^4}{8}\ln(x+\sqrt{x^2+a^2})$$

$$\int x^3\sqrt{x^2+a^2}\,dx = \left(\tfrac{1}{5}x^2 - \tfrac{2}{15}a^2\right)\sqrt{(x^2+a^2)^3}$$