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Compton Scattering Lab Report

Abstract

The Compton Effect is the shift in wavelength/energy caused by a photon interacting with an electron. This experiment exposes this phenomenon and uses Compton's formula for energy shifting to solve for the mass/energy of the electron to <2% difference precision. In addition to the Compton Effect, the differential cross section for incident Cs-137 is calculated and is fit with the Klein-Nishina formula to excellent precision.

Introduction and Theory

In short, Compton Scattering is the inelastic scattering of a photon by a charged particle (usually an electron). The inelastic scattering results in a decrease in energy, and an increase in wavelength of the photon (such as a gamma ray photon). This is evidence of light interaction with matter.

In simpler terms, a photon collides with an electron at rest; after the collision, the electron travels in some direction at some angle, and the photon's wavelength is altered as it also travels in some direction, as seen in figure 1.

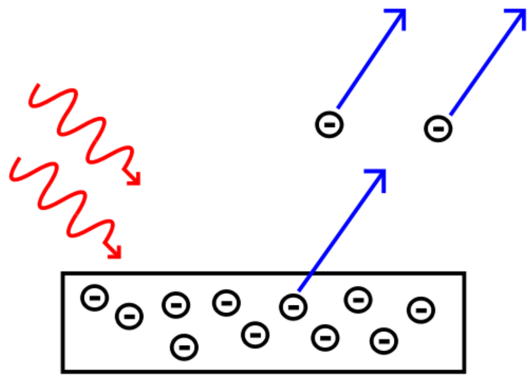

Figure 1: Diagram of the inelastic scattering described in the intro section. Source: Wikipedia

The change in wavelength/energy was modelled by Compton himself in 1923:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\frac{E'}{E} = \frac{1}{1 + (E/mc^2)(1 - \cos\theta)}$$

The differential cross section describes the relationship between the incoming particle and the scattering geometry. It is described below in figure 2:

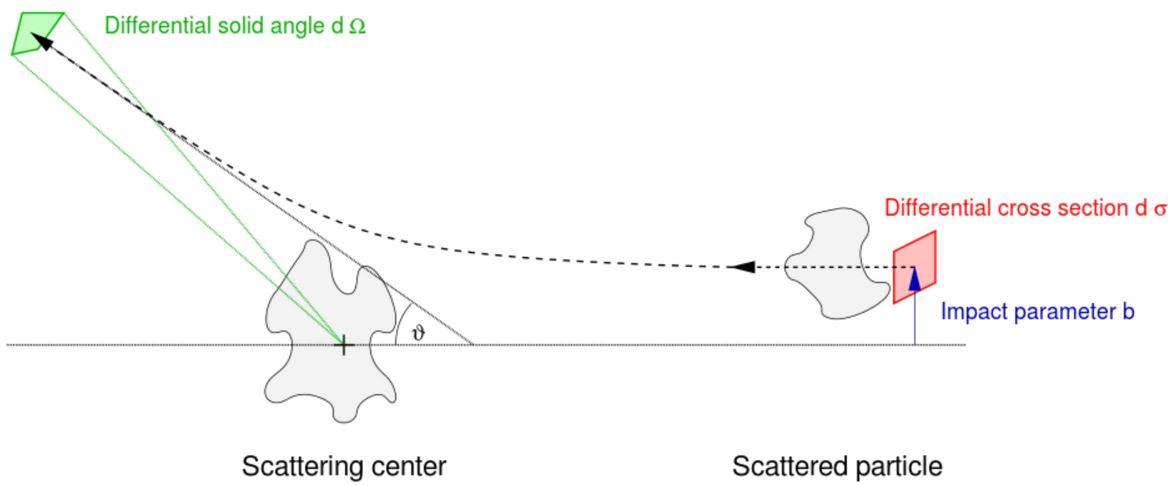

Figure 2: Classical differential cross section visualization. Source: Wikipedia

In a typical scattering experiment, a target is uniformly illuminated by a parallel incident beamed and it produces a scattered flux that varies with angle.[1] Figure 3 is a visualization of this. The target is not marked in figure 3, but it will be placed in the center of the circle.

[1] This is a word-for-word quote from the laboratory instruction sheet that I felt filled in the introduction perfectly.

For the remainder of this report, the quantity $\frac{d\sigma}{d\Omega}$ will be referred to as the differential cross section.

At this point, it can be seen that for every change of θ in figure 3, there will be a corresponding differential cross section.

There is a classical and a quantum model for the differential cross section. The classical model is Thomson scattering and it is accurate so long as the photon energy is much less than the mass energy of the particle:

$$v \ll mc^2/h$$

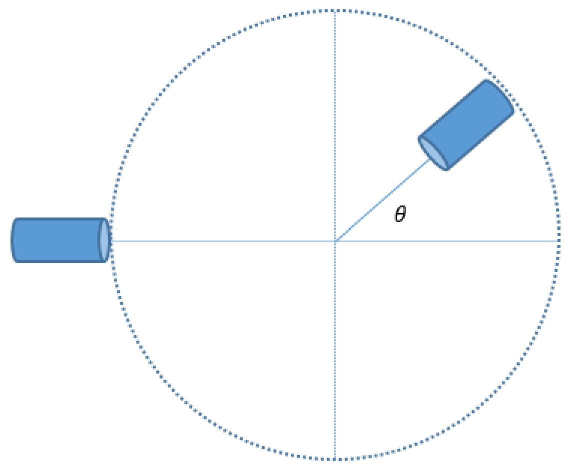

Figure 3: Topological view of the detector (left cylinder) and the collimator where gamma ray particles are being emitted from (right cylinder)

The quantum model is described by the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 r_c^2 P(E_\gamma, \theta)^2 \left[P(E_\gamma, \theta) + P(E_\gamma, \theta)^{-1} - 1 + \cos^2\theta\right]}{2}$$

The variable α is the fine structure constant ($\approx 1/137.04$), θ is the scattering angle (seen in figure 3), $r_c = \hbar/m_e c$ is the “reduced” Compton wave length ($\approx 0.38616 \times 10^{-12}$m), $m_e = \sim .511\text{MeV}/c^2$ is the mass of the electron, and $P(E_\gamma, \theta) = E'/E$ from the beginning of this section:

$$P(E_\gamma, \theta) = \frac{E'}{E} = \frac{1}{1 + (E/mc^2)(1 - \cos\theta)}$$

If $E_\gamma \ll m_e c^2$, then $P(E_\gamma, \theta) \to 1$, and the Klein-Nishina formula reduces to the classical Thomson expression.

Experiment

A Caesium-137 source is attached to the collimator in figure 3. This source emits γ-rays at 0.6617 MeV and it has a half-life of 30 years. Therefore the initial energy $E = 0.6617$.

The collection of data looks similar to that pictured in figure 4. For a set time of 720 seconds, a detector receives incoming particles and amplifies the signal by 1200V. The data is read by a Quantum MCA and is serialized. Once this occurs, the data is readable by a computer.

The Quantum MCA reads 2048 channels. The energy being read by each channel is not representative of the actual energy of the particles coming into the detector, so in order to resolve this, an offset is found by collecting a set of data and then measuring the Gaussian peaks of a set of three known radiative elements. This is the energy calibration process and it is done every time the setup is altered.

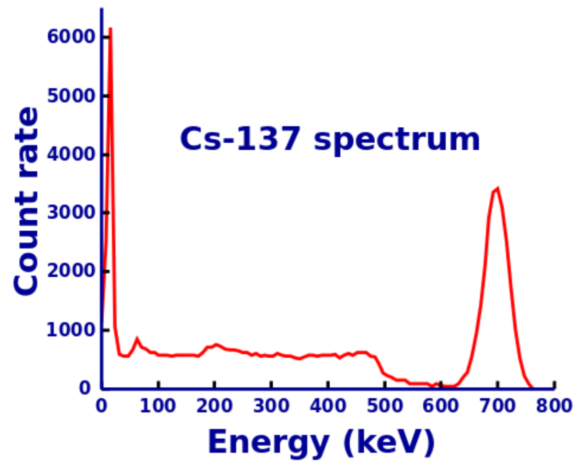

Figure 4: This is a plot of the Cs-137 spectrum, likely with an angle $\theta = 0$ degrees.

The main photon peak of Ba-137m is .662 MeV, which is the value we will be using – the effect we are observing is that of an emitted photon.

Source: Wikipedia article for Caesium-137

In figure 3, the angle θ is the independent variable and its value is incremented by 10, from 20 degrees to 100 degrees. For each increment, there are two readings taken: One with the target in the center, and one with the target out. The target is the incident scattering center in figure 2. Data must be taken with the target out in order to filter out extraneous noise.

This is the entire experiment. The remainder of the work is analytic work, and will be described in the next section.

Analysis

There are three main goals to be accomplished:

- Find a difference spectra and then find a Gaussian fit for each of the regions of interest.
- Make a linearized fit of the energy vs. angle and use the slope of that line to solve for the mass of the electron.
- Determine the differential scattering cross section and plot the results with the Klein Nishina formula and the Thomson formula.

The primary method of all three of these was accomplished using a python script. From nothing more than the raw Quantum MCA ascii data, all three of the above goals are accomplished.

Regions of Interest

The difference spectra was obtained by subtracting the target out data from the target in data (the energies are calibrated as well).

Looking at counts of energy between 200-625 KeV, the python script attempts to find a maximum value. Once it finds it, it takes a set of 50 data points in each direction and determines the average number of counts over that data set. If the average number of counts is below a threshold (28), then a lower/upper limit guess is made for the Gaussian peak (region of interest).

Once the python script has a rough estimate of the region of interest, it then performs a best-Gaussian fit over that data.

When developing the script, the results of the Gaussian fits were *always* accurate, so the upper/lower limits were then changed to match the lower/upper limits of the Gaussian fit instead of keeping the upper/lower limits established by the max-value guesstimate.

At this point, a region of interest is established, and the Gaussian distribution is integrated to get the number of counts for the θ value. The midpoint of the Gaussian distribution is the value of E' for each θ.

At this point, the first goal has been accomplished, and the results can be seen in the appendix page 1 (the set of illustrated plots).

Solve for Electron Mass

Plotting $1/E'$ vs. $1 - \cos\theta$ from the above data gives a linear function with two very pleasant results. The slope of the line is the mass of the electron, and the y-intercept is the energy of the electron (because the angle is at 0 degrees).

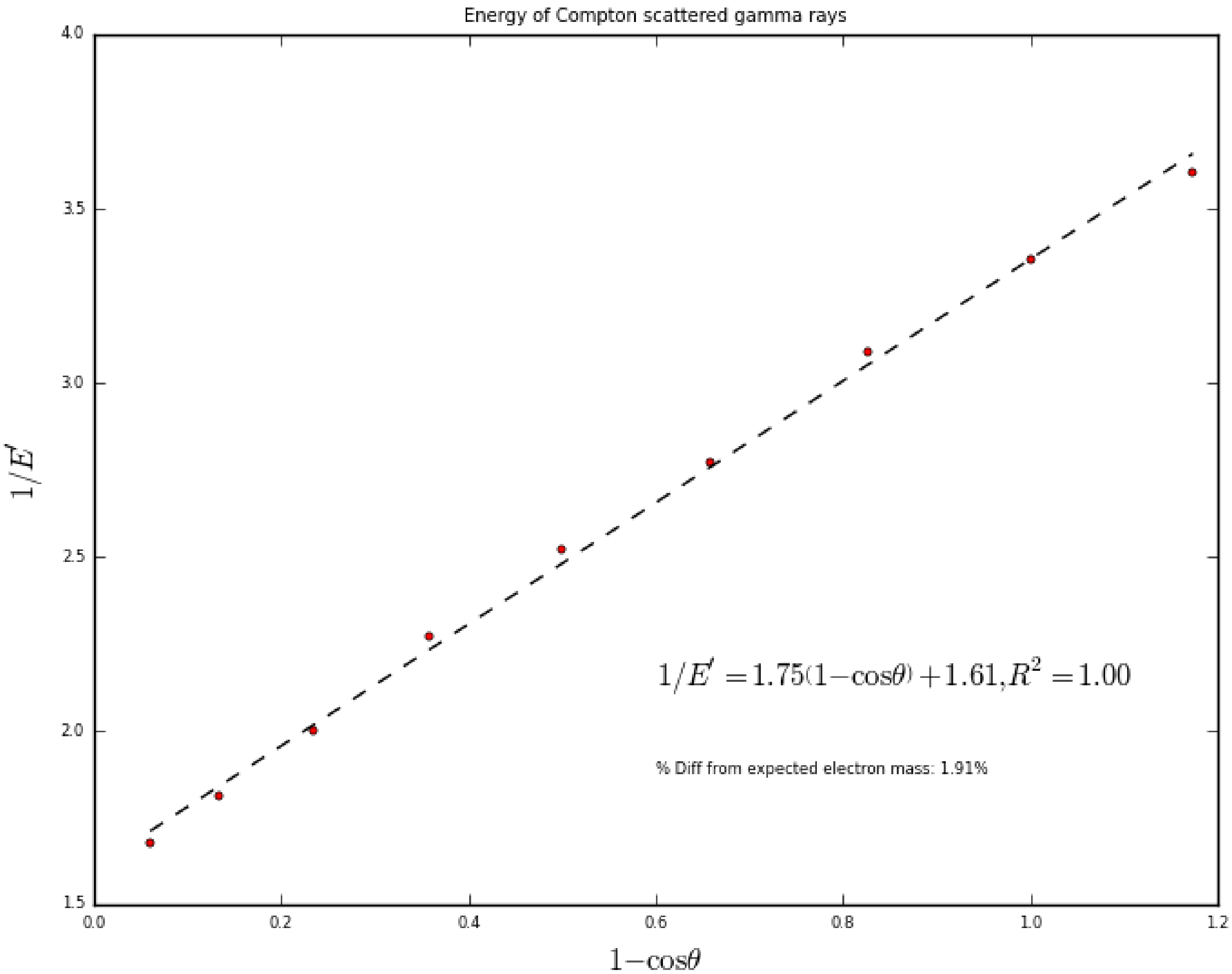

Figure 5: Plot of the inverse-energy shift in KeV^{-1} versus 1 minus the angle (unitless).

Figure 5 predicts the mass of the electron to 1.91% difference (expected 1.78), and the energy of the electron to 0.5% difference (expected 1.602). What a result!

Klein-Nishina, Thomson, Experimental Fit

The data at this point is not ready to be plotted as $d\sigma/d\Omega$. Some geometry must be analyzed. This formula is presented from Dr Robert Marzke:

$$\frac{d\sigma}{d\Omega} = \frac{\text{yield}}{d\Omega_{\text{det}} \times N \times I_0}$$

(continued next page)

Each quantity in the denominator is carefully calculated.

I_0 represents the incident γ intensity at the target. The collimator is emitting gamma rays in a specific geometry. $I_0 = I/A$ is the number of γ rays disintegrated per area-second – also called the flux of the incident particles. Since the Caesium-137 is from 1995 (20 yrs), the γ emission per second is given by the formula

$$I = R_0 \exp(-20/43.28) = 5.46 \times 0.6299 = 3.439\text{mCi} = 12.72 \times 10^7 \frac{\text{disintegrations}}{\text{s}}$$

The area over which the γ rays disintegrate is $4\pi(400) = 50 \times 10^2 \text{cm}^2$. The particles coming from the collimator are scattered into a full spherical 4π solid angle, and the radius is of the sphere is 20cm. Therefore

$$I_0 = \frac{12.72 \times 10^7}{50 \times 10^2} = 2.5448 \times 10^4/\text{cm}^2 s$$

N represents the number of electrons in the 1.9cm long by 1.9cm diameter in the aluminum target cylinder.

$$N = \#\text{ moles in target} \times 6.023 \times 10^{23} \times 13$$

$$N = (\text{target volume} \times \rho_{Al}) \times 6.023 \times 10^{23} \times 13/27$$

The quantity 13 represents the number of electrons in the aluminum atom, and $13/27$ is g/mole.

The target volume is given by: (units cm^3)

$$\text{target volume} = \pi r^2 h = \frac{\pi (1.9)^2 \, (1.9)}{4} = 5.387$$

Thus N becomes

$$N = 0.5387 \times 2.7 \times 6.023 \times 10^{23} \times \frac{13}{27} = 4.218 \times 10^{24} \text{electrons}$$

The last piece of the formula is $d\Omega_{det}$. This is the detector solid angle as seen from the target. The distance from the center of the target to the detector's front face is approximately 41cm:

$$d\Omega_{\text{det}} = \frac{\pi (2.3)^2}{41^2} \approx 0.010 \text{ sr}$$

Now, the differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{\text{yield}}{d\Omega_{\text{det}} \times N \times I_0} = \frac{\text{yield}}{1.0734 \times 10^{27}} \text{cm}^2$$

At this point, yield is nothing more than counts/sec. All of the counts can be seen in appendix 1. As previously mentioned, the number of seconds the data was collected over was 720.

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A formula for the differential cross section that depends on only the number of counts at our regions of interest allows us to plot the Klein Nishina formula, the Thomson formula, and the experimental data all together.

Here is the result:

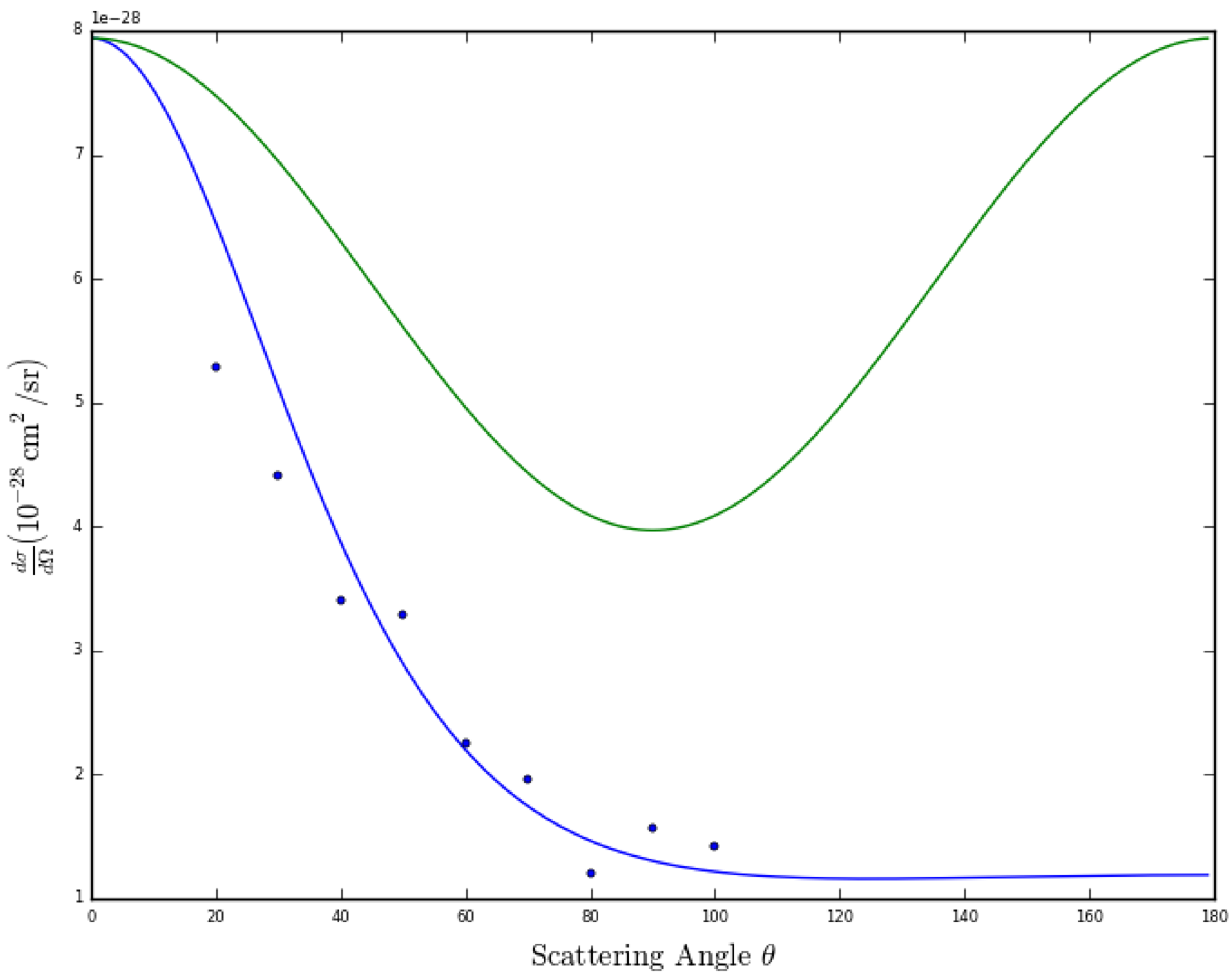

Figure 6: Klein Nishina formula (blue), Thomson formula (green), experimental data (scatter).

The experimental data points are tightly knit with the Klein Nishina formula. The lower the incident photon energy, the more it would converge to the Thomson formula, i.e. $E_\gamma \ll m_e c^2$. However since $E_\gamma \approx m_e c^2$ (approximate within the order of magnitude), the result is that which converges to the Klein-Nishina formula.

Conclusion

For every incident angle θ, the regions of interest were found automatically using an algorithm in Python, and the mass/energy of the electron was found within 2% accuracy of expected results. The differential cross section for the incident Cs-137 gamma photons was calculated and the fit with the Klein-Nishina formula is precise.

(appendix begins next page)