

Field Line Resonance in Two and a Half Dimensions

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⁵ Acknowledgements

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¹³ Dedication

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Abstract

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¹¹⁶ **Chapter 1**

¹¹⁷ **Introduction**

¹¹⁸ 1859 was a pivotal year in human history. The United States moved steadily toward
¹¹⁹ the American Civil War, which would abolish slavery and consolidate the power of
¹²⁰ the federal government. A slew of conflicts in Southern Europe set the stage for the
¹²¹ unification of Italy. The Taiping Civil War — one of the bloodiest conflicts of all time
¹²² — is considered by many to mark the beginning of modern Chinese history. Origin of
¹²³ Species was published. The first transatlantic telegraph cable was laid.

¹²⁴ Meanwhile, ambivalent to humanity, the Sun belched an intense burst of charged par-
¹²⁵ ticles and magnetic energy directly toward Earth. The resulting geomagnetic storm¹
¹²⁶ caused telegraph systems to fail across the Western hemisphere, electrocuting operators
¹²⁷ and starting fires[35, 94]. Displays of the northern lights were visible as far south as
¹²⁸ Cuba.

¹²⁹ The Solar Storm of 1859 was perhaps the most powerful in recorded history, but by no
¹³⁰ means was it a one-time event. The Sun discharges hundreds of coronal mass ejections
¹³¹ (CMEs) per year, of all sizes, in all directions. In fact, a comparably large CME narrowly
¹³² missed Earth in 2012[70]. Had it not, it's estimated it would have caused widespread,
¹³³ long-term electrical outages, with a damage toll on the order of 10^{12} dollars[66].

¹The Solar Storm of 1859 is also called the Carrington Event, after English astronomer Richard Carrington. He drew a connection between the storm's geomagnetic activity and the sunspots he had observed the day before.

134 The Sun’s extreme — and temperamental — effect on Earth’s magnetic environment
135 makes a compelling case for the ongoing study of space weather. Such research has
136 evolved over the past century from sunspot counts and compass readings to multi-
137 satellite missions and supercomputer simulations. Modern methods have dramatically
138 increased humanity’s understanding of the relationship between the Sun and the Earth;
139 however, significant uncertainty continues to surround geomagnetic storms, substorms,
140 and the various energy transport mechanisms that make them up.

141 The present work focuses in particular on the phenomenon of field line resonance: Alfvén
142 waves bouncing between the northern and southern hemispheres. Such waves play an
143 important part in the energization of magnetospheric particles, the transport of energy
144 from high to low altitude, the precipitation of particles into the atmosphere, and the
145 driving of currents at the top of the atmosphere. It is these currents which give rise to
146 potentially-catastrophic magnetic disturbances at Earth’s surface.

147 The study of resonance in the near-Earth environment is furthermore valuable as a proxy
148 for other (less-accessible) plasma environments. Similar waves occur in astrophysical
149 plasmas, observation of which is limited by distance. Field line resonance is also analo-
150 gous to the so-called “fishbone instability” in fusion reactors. As a plasma laboratory,
151 the magnetosphere is unique in that it is both close enough to measure directly, and
152 also large enough that measurements can be performed without disrupting its behavior.

153 1.1 Structure of the Present Work

154 The present work is laid out as follows.

155 Chapter 2 surveys the near-Earth environment. Prominent features of the magneto-
156 sphere are defined. The behavior of the magnetosphere during geomagnetic storms and
157 substorms is summarized.

158 Chapter 3 introduces the field line resonance phenomenon, in terms of both the under-
159 lying physics and notable work on the topic. Jargon is introduced to clarify important
160 elements of wave structure. Several open questions about field line resonances (FLRs)
161 are offered as motivations for the present work.

162 Chapter 4 lays the groundwork for a numerical model by exploring the fundamental
163 equations of waves in a cold, resistive plasma — such as Earth’s magnetosphere. Char-
164 acteristic scales are gleaned from the resulting dispersion relations.

165 Chapter 5 presents Tuna, a new two and a half dimensional simulation designed specif-
166 ically for the realistic modeling of FLRs. Tuna’s non-orthogonal geometry, height-
167 resolved ionosphere, novel driving mechanism, and coupling to the atmosphere are jus-
168 tified and explained.

169 Chapter 6 considers the addition of electron inertial effects to Tuna, which are neglected
170 in the core model presented in Chapter 5. These effects allow the computation of parallel
171 currents and electric fields, which have not previously been included in global Alfvén
172 models. The effects are shown to be instability-prone and computationally expensive,
173 but some results are gleaned nonetheless.

174 Chapter 7 showcases the core numerical results of the present work, unifying several of
175 the questions posed in Chapter 3. The interplay between compressional propagation,
176 poloidal-to-toroidal rotation, and Joule dissipation is considered from several angles.

177 Chapter 8 puts the numerical results in context through the analysis of data from the
178 Van Allen Probes mission. FLR occurrence rates are considered in terms of location,
179 harmonic, and polarization, parameters which have been only partially addressed in
180 past FLR surveys.

181 Chapter 9 briefly summarizes the results shown in the above chapters — code devel-
182 opment, analysis of numerical results, and satellite observation — and suggests further
183 directions.

¹⁸⁴ **Chapter 2**

¹⁸⁵ **The Near-Earth Environment**

¹⁸⁶ From Earth’s surface to a height of a hundred kilometers or so, the atmosphere is a
¹⁸⁷ well-behaved fluid: a collisional ensemble of neutral atoms. Higher up, its behavior
¹⁸⁸ changes dramatically. As altitude increases, solar ultraviolet radiation becomes more
¹⁸⁹ intense, which ionizes atmospheric atoms and molecules. Density also decreases, slow-
¹⁹⁰ ing collisional recombination. Whereas the neutral atmosphere is held against Earth’s
¹⁹¹ surface by gravity, the motion of charged particles is dominated by Earth’s geomagnetic
¹⁹² field, as well as the electromagnetic disturbances created as that field is hammered by
¹⁹³ the solar wind.

¹⁹⁴ Before discussing specific interactions, it’s appropriate to introduce the so-called “frozen-
¹⁹⁵ in condition.” In a collisionless plasma, magnetic field lines are equipotential contours.
¹⁹⁶ Charged particles move freely along the contours, but cannot move across them. Com-
¹⁹⁷ pression of the magnetic field is synonymous with compression of the ambient plasma,
¹⁹⁸ as any magnetic field lines that thread a moving plasma are dragged along with it. This
¹⁹⁹ assumption is valid throughout most of the magnetosphere — that is, the region of
²⁰⁰ space primarily governed by Earth’s magnetic field — and provides an invaluable tool
²⁰¹ for understanding the large-scale motions of plasmas and fields.

2.1 The Outer Magnetosphere

Plasma behavior within Earth's magnetosphere is ultimately driven by the solar wind: a hot (~ 100 eV), fast-moving (~ 100 km/s) plasma threaded by the interplanetary magnetic field (~ 10 nT)¹. The density of the solar wind is on the order of 10^3 /cm³; in a laboratory setting, this would constitute an ultra-high vacuum (atmospheric density at sea level is $\sim 10^{19}$ /cm³), but compared to much of the magnetopause it's quite dense.

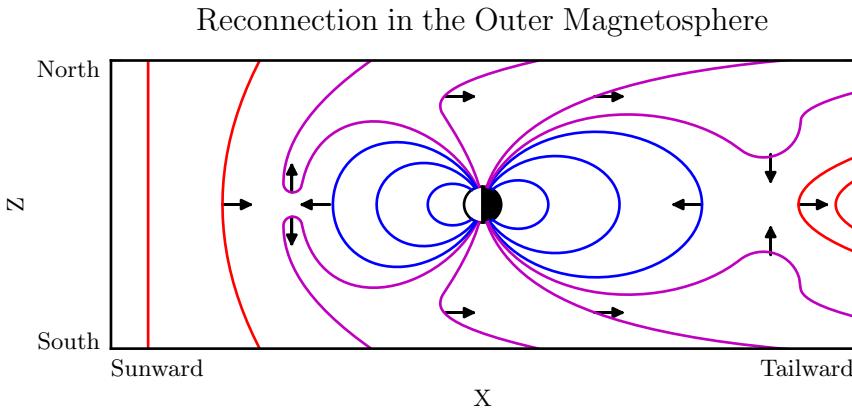


Figure 2.1: When the solar wind magnetic field (red) points southward, reconnection can occur between it and Earth's (northward) closed magnetic field lines (blue). The resulting open field lines (magenta) convect nightward over the poles, ultimately arriving in the magnetotail. There, the open field lines reconnect again. Newly closed field lines move Earthward, carrying flux across the flanks and back to the dayside. The rest are completely decoupled from Earth, and are lost to the solar wind.

- The magnetosphere's outer boundary represents a balance between the solar wind dynamic pressure and the magnetic pressure of Earth's dipole field. On the dayside, the dipole is compressed, pushing this boundary to within about $10 R_E$ of Earth². The nightside magnetosphere is stretched into a long tail which may exceed $50 R_E$ in width and $100 R_E$ in length.
- When the interplanetary magnetic field opposes the geomagnetic field at the nose of the magnetosphere, magnetic reconnection occurs. Closed magnetospheric field lines

¹Listed values correspond to the solar wind at Earth's orbit.

²Distances in the magnetosphere are typically measured in units of Earth radii: $1 R_E \equiv 6378$ km.

215 “break,” opening up to the interplanetary magnetic field³. They then move tailward
216 across the poles, dragging their frozen-in plasma with them. Reconnection in the tail
217 allows magnetic field lines to convect back to the day side, across the flanks. This
218 process is called the Dungey cycle[21].

Viewed from the north, the Dungey cycle involves a clockwise flow of flux and plasma on
the dusk side and a counterclockwise flow on the dawn side. The motion is accompanied
by a convection electric field, per Ohm’s law in an ideal plasma:

$$\underline{E} + \underline{U} \times \underline{B} = 0 \quad (2.1)$$

219 Where \underline{B} , \underline{E} , and \underline{U} are the magnetic field, electric field, and plasma velocity vectors
220 respectively.

221 Consistent with Ampère’s law, the interplanetary magnetic field is separated from the
222 magnetosphere by a current sheet: the magnetopause. On the dayside, the magne-
223 topause current flows duskward; on the nightside, it flows downward around the mag-
224 netotail.

225 Earth’s dipole is significantly deformed in the magnetotail; field lines in the northern
226 lobe of the tail points more or less Earthward, and vice versa. Plasma within the lobes
227 is cool (~ 100 eV) and rarefied ($\sim 10^{-2}$ /cm³). The two lobes are divided by the plasma
228 sheet, which is comparably hot ($\sim 10^3$ eV) and dense (~ 1 /cm³). The plasma sheet
229 carries a duskward current which connects to the magnetopause current.

³Closed field lines are more or less dipolar; one end connects to the north pole of Earth’s magnetic core, and the other end to the south pole. Open field lines are tethered to Earth at one end. In principle, the other end eventually doubles back to Earth, but for practical purposes it is lost to the solar wind. The distinction is important because charged particle motion is guided by magnetic field lines, as discussed in Chapter 3.

2.2 The Inner Magnetosphere

Within $L \sim 8$ (where L is the McIlwain parameter⁴), the dipole magnetic field is not appreciably deformed by the solar wind. As a result, the structures in the inner magnetosphere follow closely from the motion of charged particles in an ideal dipole field.

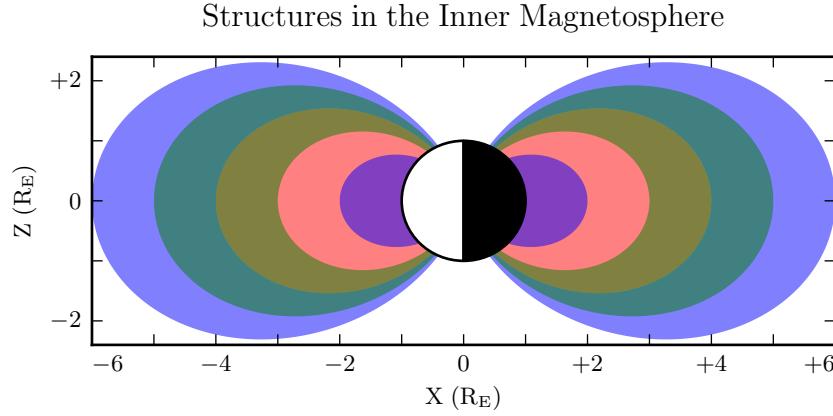


Figure 2.2: The above figure shows typical ranges in L for the plasmasphere (red, $L < 4$), ring current (green, $3 < L < 5$) and radiation belts (blue, $L < 2$ and $4 < L < 6$). These values, particularly the size of the plasmasphere, can vary significantly in response to geomagnetic activity.

The plasmasphere — a cold (~ 1 eV), dense ($10^2 / \text{cm}^3$ to $10^4 / \text{cm}^3$) torus of corotating plasma — is formed by the outward drift of atmospheric ions along magnetic closed field lines. Its outer boundary is thought to represent a balance between the corotation electric field (per the rotation of Earth's magnetic dipole) and the convection electric field (associated with the convection of magnetic flux during the Dungey cycle). Particle density drops sharply at the edge of the plasmasphere; the boundary is called the plasmapause. The plasmapause typically falls around $L = 4$, though during prolonged quiet times it can extend to $L = 6$ or larger.

⁴The McIlwain parameter L is used to index field lines in Earth's dipole geometry: $L \equiv \frac{r}{\sin^2 \theta}$ for colatitude θ and radius r in Earth radii. For example, the $L = 5$ field line passes through the equatorial plane at a geocentric radius of $5 R_E$, then meets the Earth at a colatitude of $\arcsin \sqrt{\frac{1}{5}} \sim 27^\circ$ (equally, a latitude of $\arccos \sqrt{\frac{1}{5}} \sim 63^\circ$).

242 Energetic particles trapped within the inner magnetosphere are divided into two popu-
243 lations.

244 The Van Allen radiation belts are made up of particles with energy above 10^5 eV or
245 so. The inner belt ($L \lesssim 2$) is primarily composed of protons, the decay remnants of
246 neutrons freed from the atmosphere by cosmic rays. The outer belt ($L \gtrsim 4$) is primarily
247 composed of high-energy electrons. The density of radiation belt particles is significantly
248 affected by geomagnetic storms and substorms; a typical value is $10 / \text{cm}^3$.

249 Particles with energies of 10^3 eV to 10^5 eV make up the ring current, which extends
250 from $L \sim 3$ to $L \sim 5$. Gradient-curvature drift carries ions and electrons in opposite
251 directions; the net result is a westward current. During quiet times, the ring current
252 causes magnetic a southward magnetic field on the order of 1 nT at Earth's equator,
253 while during geomagnetically active times (discussed in Section 2.4) the effect may be
254 100 nT or more⁵.

255 2.3 The Ionosphere

256 Earth's neutral atmosphere is an excellent insulator. Collisions are so frequent that
257 charged particles quickly thermalize and recombine. The breakdown of air molecules
258 into a conductive plasma (as happens during a lightning strike, for example) requires
259 electric fields on the order of 10^9 mV/m.

260 Cold particles in the magnetosphere are likewise not conducive to currents. In the
261 absence of collisions, electrons and ions drift alongside one another in response to an
262 electric field, creating no net current perpendicular to the magnetic field⁶. Magnetic
263 field lines can typically be considered as equipotential contours, devoid of field-aligned
264 potential structures.

265 The ionosphere is a sweet spot between the two regimes. Collisions are frequent enough
266 to disrupt the drift of ions, but not frequent enough to immobilize the electrons. The

⁵For comparison, Earth's dipole field points north at the equator with a magnitude over 10^4 nT.

⁶The so-called E -cross- B drift is associated with a velocity of $\underline{U} = \frac{1}{B^2} \underline{E} \times \underline{B}$, independent of a charged particle's mass or sign.

267 result is a finite-valued conductivity tensor. Pedersen currents (which scale with the
268 Pedersen conductivity) flow in the direction of the perpendicular electric field. Hall
269 currents (due to the Hall conductivity) flow in the $\underline{B} \times \underline{E}$. It is these currents — par-
270 ticularly the Hall current — which give rise to magnetic fields at the ground. Collisions
271 in the ionosphere also result in a finite parallel conductivity, allowing for the formation
272 of potential structures along the magnetic field line.

273 The convection electric field (associated with the Dungey cycle, Section 2.1) drives
274 Pedersen currents in the ionosphere. Pedersen currents flow downward on the flanks
275 and duskward across the poles. The currents remain divergence-free by connecting
276 to field-aligned currents at the edges of the polar cap. The field-aligned currents, in
277 turn, connect to the magnetopause current, the cross-tail current, and the (partial) ring
278 current.

279 When electron density is low, thermal velocities may be unable to carry enough current
280 to satisfy $\nabla \cdot \underline{J} = 0$. This leads to the formation of potential structures along geomagnetic
281 field lines in the ionosphere. Such structures accelerate particles along magnetic field
282 lines, leading to the precipitation of energetic particles into the atmosphere. As the
283 particles thermalize, they excite atmospheric atoms. The resulting spontaneous emission
284 is often in the visible spectrum, giving rise to the aurora.

285 2.4 Geomagnetic Storms and Substorms

286 The quiet geomagnetic behavior described above is periodically disturbed by transient
287 solar phenomena such as corotating interaction regions (CIRs) and coronal mass ejec-
288 tions (CMEs). CMEs, such as the one that caused the Solar Storm of 1859 mentioned
289 in Chapter 1, are bursts of unusually dense solar wind which are ejected from regions of
290 high magnetic activity on the Sun; they are most common at the height of the eleven-
291 year solar cycle. CIRs, on the other hand, occur when a relatively fast region of the
292 solar wind catches up to an earlier and slower-moving pocket of solar wind, resulting in
293 a pair of shockwaves.

294 During a storm, increased solar wind intensity results in enhanced magnetic reconnection
295 on the dayside. As the newly-opened field lines are swept tailward, the convection
296 electric field is strengthened. The plasmasphere — the outer boundary of which is set by
297 a balance between the convection electric field and the (more or less constant) corotation
298 electric field — sheds its outer layers[33]. A large number of energetic particles are also
299 injected into the ring current[67].

300 The strength of the storm is gauged by the size of the magnetic perturbation created
301 by the ring current⁷. A small storm has a magnitude of 50 nT to 100 nT. Large storms
302 may reach 250 nT to 500 nT. The Carrington Event, mentioned in Chapter 1, is thought
303 to have exceeded 1700 nT[94].

304 The main phase of a storm typically lasts for several hours. Storm recovery — the
305 gradual return of the storm index to zero, and the refilling of the plasmasphere — lasts
306 several days. Geomagnetic storms occur tens of times per year at the height of the solar
307 cycle, and just a few times per year otherwise.

308 Whereas storms are prompted by large solar wind events on the dayside, geomagnetic
309 substorms are primarily a nightside occurrence. As flux accumulates in the tail, mag-
310 netic tension builds in the stretched field lines. A substorm is an impulsive release of
311 that tension.

312 At substorm onset, a burst of reconnection occurs in the tail. A jet of plasma is launched
313 Earthward from the reconnection site (and another is launched tailward, and lost to the
314 solar wind). The Earthward plasma injection injects particles into the ring current.
315 The outer radiation belt is depleted, then repopulated. Energetic particles precipitate
316 into the atmosphere, giving rise to a distinctive sequence of auroral signatures over the
317 course of about an hour.

318 Concurrent with substorm onset, impulsive Alfvén waves are observed with periods of
319 a minute or two. The precise ordering of events — whether reconnection causes the
320 waves, or vice versa, or if they share a common cause — remains controversial.

⁷The most commonly used storm index is DST, which is computed hourly using measurements from several ground magnetometers near the equator. The Sym-H index, used in Section 5.3, is computed once per minute.

- 321 Each substorm lasts several hours, including the time it takes for the ring current to
322 return to pre-substorm levels. Several substorms may occur per day during quiet times.
323 During a storm, substorms become far more frequent; by the time one has ended,
324 another may have already begun.

³²⁵ **Chapter 3**

³²⁶ **Field Line Resonance**

³²⁷ The motion of a charged particle in a dipole field can be described in terms of three
³²⁸ fundamental motions.

³²⁹ The first is cyclotron motion. Given a uniform magnetic field line, a particle follows a
³³⁰ helical path. It moves in a circular path in a plane normal to the magnetic field line,
³³¹ and keeps a constant velocity along the direction of the field. Close to Earth, where the
³³² magnetic field is strongest, the proton (electron) cyclotron timescale is on the order of
³³³ 10^{-3} s (10^{-6} s); at $L \sim 5$, a typical value is closer to 10 s (10^4 s).

³³⁴ The second fundamental motion is bounce motion. As it moves along the magnetic field
³³⁵ line like a bead on a wire, the particle experiences a change in magnetic field magnitude.

³³⁶ In order to conserve its magnetic moment (also called the first adiabatic invariant), the
³³⁷ particle's perpendicular kinetic energy increases in proportion with the magnetic field.

³³⁸ When the perpendicular kinetic energy can no longer increase — that is, when all of
³³⁹ the particle's kinetic energy is perpendicular — the particle bounces back. Particles
³⁴⁰ undergoing bounce motion continuously move back and forth between the northern and
³⁴¹ southern hemispheres, with timescales of a few seconds to a few minutes.

³⁴² Particles with more parallel kinetic energy (compared to their perpendicular kinetic
³⁴³ energy) bounce at lower altitudes. If the particle's motion is sufficiently field-aligned, the

344 bounce altitude drops into the atmosphere, and the particle is collisionally thermalized.

345 This process is called precipitation.

346 The third fundamental motion is drift motion. Over the course of a particle's cyclotron
347 motion, the Earthward half of the orbit experiences a slightly stronger magnetic field
348 (and thus a slightly smaller orbit radius). The net effect, called the gradient-curvature
349 drift, is an azimuthal motion around Earth on timescales of $\sim 10^3$ s.

350 Wave-particle resonance arises when a particle's periodic motion matches with the fre-
351 quency of a coincident electromagnetic wave[23, 63, 72, 83]. In the particle's rest frame,
352 the wave then appears as a net electric field. This allows a net movement of energy
353 between the wave and the particle. The interaction is analogous to a surfer moving
354 along with — and being accelerated by — a wave in the ocean. Such resonance can
355 arise for any of the three fundamental motions, or even for a combination of them.

356 In the present work, the waves under consideration are field line resonances (FLRs). An
357 FLR is a standing harmonic on a geomagnetic field line. It can also be envisioned as
358 a superposition of traveling waves, reflecting back and forth between its northern and
359 southern foot points at the conducting ionosphere.

These waves travel at the Alfvén speed, v_A , defined per

$$v_A^2 \equiv \frac{B^2}{\mu_0 \rho} \quad \text{or, equally,} \quad v_A^2 \equiv \frac{1}{\mu_0 \epsilon_{\perp}} \quad (3.1)$$

360 Where B is the magnetic field magnitude, ρ is the mass density, and μ_0 is the magnetic
361 constant. The perpendicular electric constant ϵ_{\perp} is analogous to the electric constant
362 ϵ_0 , and arises in cases (such as the magnetosphere) where a dielectric medium exhibits
363 a preferred direction. In the magnetosphere, mass density and magnetic field strength
364 depend strongly on position. As a result, the Alfvén speed varies by several orders
365 of magnitude over the length of a field line. The fundamental equations of field line
366 resonance were presented by Dungey in 1954[20]. Since then, they have remained a
367 topic of active study.

368 So-called ultra low frequency waves — of which FLRs are a subset — are categorized
 369 by morphology. Continuous (long-lived) pulsations are termed Pc, while irregular pulsations
 370 are called Pi. Within each are a number of frequency bands; see Table 3.1[44].
 371 In practice, frequency demarcations are not strict, but rather serve as a heuristic for
 372 grouping phenomenologically similar waves[41].

Table 3.1: IAGA Magnetic Pulsation Frequency Bands

	Pc1	Pc2	Pc3	Pc4	Pc5	Pi1	Pi2
Period (s)	0.2–5	5–10	10–45	45–150	150–600	1–40	40–150
Frequency (mHz)	200–5000	100–200	22–100	7–22	2–7	25–1000	7–25

373 FLRs fall into the Pc3, Pc4, and Pc5 ranges. The present work focuses specifically
 374 on the Pc4 band: waves with periods of a minute or two. Frequencies in the Pc4
 375 range typically coincide with Alfvén bounce times¹ near the plasmapause: $L \sim 4$ to
 376 $L \sim 6$ [2, 16, 24, 56]². In fact, the large radial gradients in the Alfvén speed near the
 377 plasmapause act as an effective potential well, trapping FLRs[15, 49, 53, 54, 62, 86].

378 In addition to being radially localized, FLRs in the Pc4 frequency band (also called Pc4
 379 pulsations, or just Pc4s) are localized in magnetic local time (MLT³). They have also
 380 been shown to occur preferentially on the dayside, during storms or storm recovery[2,
 381 16, 24, 52, 56, 95].

382 In the inner magnetosphere, the Alfvén frequency — and thus the frequency of FLRs
 383 — often coincides with integer or half-integer⁴ multiples of particle drift frequencies[17].
 384 The resulting wave-particle interactions can give rise to significant energization and ra-
 385 dial diffusion of the particles. In some cases, the waves also include an electric field
 386 parallel to the background magnetic field, breaking the assumption that magnetic field

¹The Alfvén frequency is the inverse of the Alfvén bounce time: $\frac{2\pi}{\omega_A} \equiv \oint \frac{dz}{v_A}$.

²Not coincidentally, these are the same L -shells where the Van Allen Probes spend most of their time; see Chapter 8.

³Noon points toward the Sun and midnight away from it, with 06:00 and 18:00 at the respective dawn and dusk flanks.

⁴See Section 3.1.

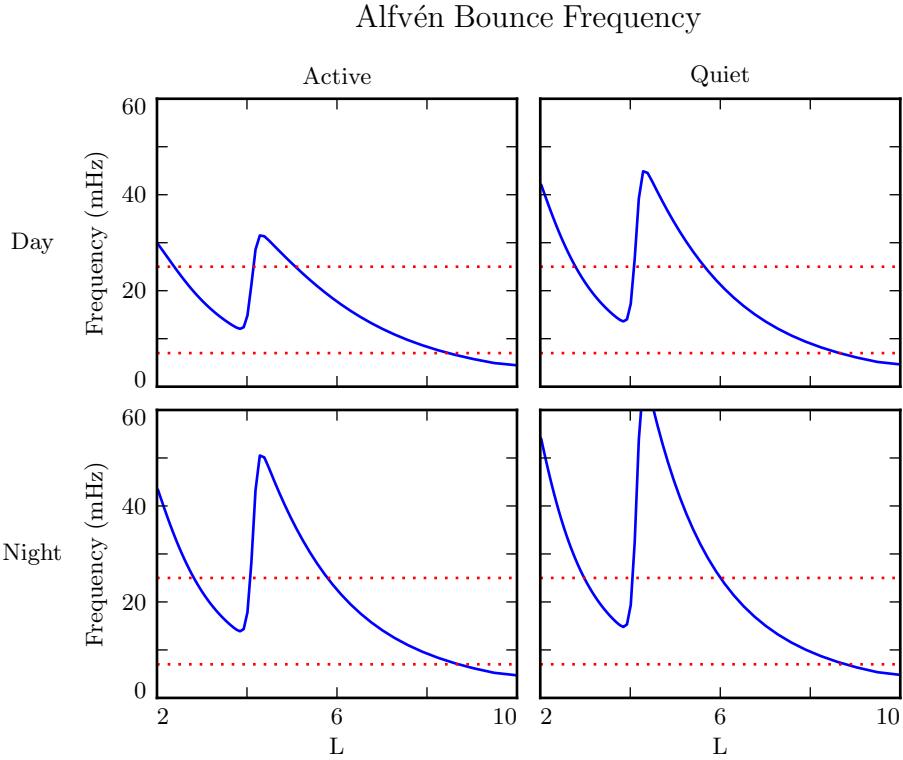


Figure 3.1: The above are bounce frequencies for an Alfvén wave moving back and forth along a geomagnetic field line. Alfvén speeds are computed based on profiles from Kelley[47], as discussed in Section 5.2. Notably, the bounce frequency depends significantly on the position of the plasmasphere, taken here to be at $L = 4$. Dotted lines indicate the P_{c4} frequency range: 7 mHz to 25 mHz.

387 lines are equipotential contours, and contributing to the precipitation of energetic par-
 388 ticles into the neutral atmosphere[31, 32, 92, 101].

389 The present chapter introduces the structural characteristics of FLRs, how those charac-
 390 teristics affect wave behavior, and several unresolved questions related to that behavior.

₃₉₁ **3.1 Harmonic Structure**

₃₉₂ Wave structure along a geomagnetic field line is indicated by harmonic number. The
₃₉₃ first (or fundamental) harmonic has a wavelength twice as long as the field line. The
₃₉₄ electric field perturbation is zero at the ionospheric foot points of the field line, due to
₃₉₅ the conductivity of the ionosphere. For the first harmonic, this puts an electric field
₃₉₆ antinode at the equator, along with a node in the perpendicular⁵ perturbation to the
₃₉₇ magnetic field. For the second harmonic, the electric field has a third node at the
₃₉₈ equator, in addition to the two at the ionospheric foot points, which is accompanied by
₃₉₉ an antinode in the perpendicular wave magnetic field. Figure 3.2 shows a qualitative
₄₀₀ sketch of the first and second harmonics: a series of snapshots in time, in the rest frame
₄₀₁ of the wave. Perpendicular electric and magnetic field perturbations are shown in blue
₄₀₂ and red respectively.

₄₀₃ A first-harmonic FLR that is periodic or localized in the azimuthal direction is conducive
₄₀₄ to drift-resonant wave-particle interactions[17, 73]. The particle is like a child on a swing:
₄₀₅ whenever the path of the particle (or child) gets close to the wave (parent), it gets a
₄₀₆ push, and always in the same direction. The wave fields spend half its time pointing
₄₀₇ the other direction, just as the parent must shift their weight backward to get ready for
₄₀₈ the next push, but at that point the particle (child) is far away.

₄₀₉ Second-harmonic FLRs interact with particles through the drift-bounce resonance, which
₄₁₀ is slightly more complicated. As the particle drifts azimuthally, it also zig-zags north-
₄₁₁ south. The combination of those two periodic motions must align with the phase of
₄₁₂ the wave electric field. An example path is shown by the purple line in Figure 3.2:
₄₁₃ the particle's drift and bounce motions together ensure that it experiences a rightward
₄₁₄ electric field throughout the wave's oscillation.

The drift and drift-bounce resonance conditions is written, respectively[87]:

$$\omega - m\omega_D = 0 \quad \text{and} \quad \omega - m\omega_D = \omega_B \quad (3.2)$$

⁵The parallel, or compressional, wave magnetic field exhibits the same nodes and antinodes as the perpendicular electric field[76].

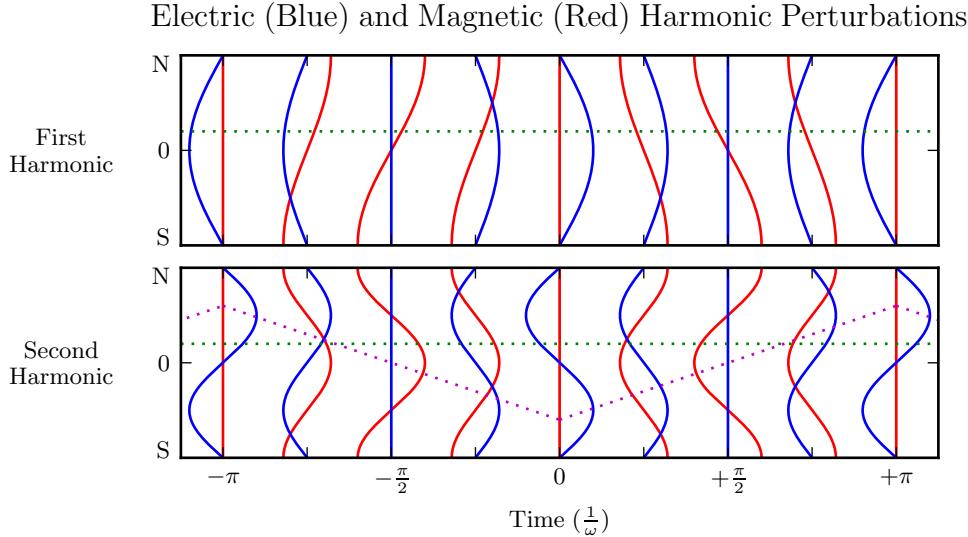


Figure 3.2: The first (or fundamental) harmonic has an antinode in its perpendicular electric field (blue) at the equator, along with a node in its perpendicular magnetic field perturbation (red). A single satellite can identify the first harmonic by the relative phase of the electric and magnetic field perturbations: an observer north of the equator (green) will see the magnetic field perturbation lead the electric field by $\pm 90^\circ$. The second harmonic is the opposite: it has an electric field node at the equator, and an observer north of the equator will see the magnetic field perturbation lag the electric field by $\mp 90^\circ$. Top and bottom signs correspond to the poloidal (shown) and toroidal polarizations respectively. The purple line sketches the path of a particle in drift-bounce resonance; in the particle’s rest frame, the electric field is always to the right.

- 415 Where ω is the frequency of the wave, ω_D and ω_B are the particle’s drift and bounce
- 416 frequencies respectively, and m is the wave’s azimuthal modenumber, as discussed in
- 417 Section 3.2.
- 418 In principle, the first and second harmonics can be distinguished by their frequencies,
- 419 even from a single-point observation[2, 14, 34]. In practice, however, this is not a reliable
- 420 approach[88]. Significant uncertainties surround the mass density profile — and thus
- 421 the Alfvén speed profile — along a geomagnetic field line.
- 422 Harmonic structure can also be deduced by noting the phase offset between the wave
- 423 magnetic field and its electric field (or the plasma velocity)[16, 91]. In Figure 3.2, the
- 424 green line indicates an observer just north of the magnetic equator. For a wave polarized

425 in the poloidal direction (see Section 3.3), the observer sees the electric field waveform
426 offset from the magnetic field by a phase of $\pm 90^\circ$, where the top sign is for odd modes
427 and the bottom sign is for even modes. The signs are flipped for toroidally-polarized
428 waves, and again for waves observed south of the equator.

429 In addition to a wave’s parity, the phase indicates how energy is divided between stand-
430 ing and traveling waves. Standing waves (phase of $\pm 90^\circ$) have a purely imaginary
431 Poynting flux. Traveling waves (phase of 0° or 180°), on the other hand, have real
432 Poynting flux, indicating a net movement of energy. Wave lifetimes can be estimated
433 by comparing the energy density to the rate at which that energy is carried away by
434 Poynting flux, as is done in Chapter 8.

435 Notably, the measurement of wave phase has only become viable with the advent of
436 satellites carrying both electric and magnetic field instrumentation, such as THEMIS in
437 2007[3] and the Van Allen Probes (formerly RBSP, for Radiation Belt Storm Probes)
438 in 2012[84].

439 Strictly speaking, the the phase offset of the electric and magnetic fields does not provide
440 the harmonic number — only its parity. It’s reasonably safe to assume that an even mode
441 is the second harmonic; the second harmonic is by far the most commonly observed[43,
442 81, 89], due in part to its excitement by the antisymmetric balloon instability[9, 11,
443 12, 83]. However, the distinction between the first and third harmonics is not always
444 clear[13, 34]; this issue is discussed further in Chapter 8. Higher harmonics than that
445 are not expected in the Pc4 frequency band.

446 3.2 Azimuthal Modenumber

447 The wavelength of an FLR in the azimuthal direction is indicated by its azimuthal
448 modenumber. A wave with modenumber m has an azimuthal wavelength that spans $\frac{24}{m}$
449 hours in MLT.

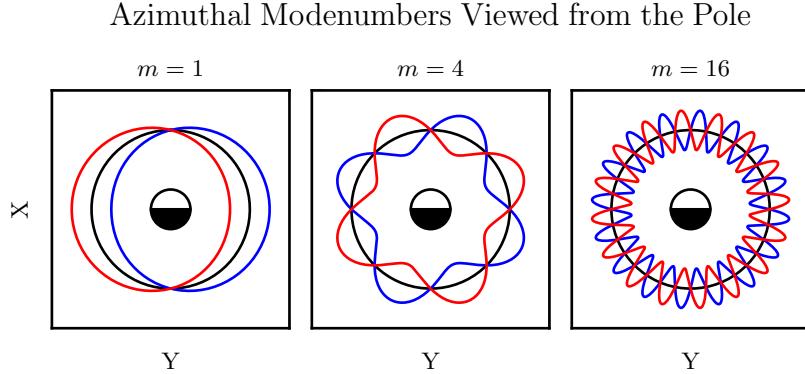


Figure 3.3: Above are qualitative sketches of waves with azimuthal modenumbers 1, 4, and 16, projected into the ecliptic plane. Black circles show unperturbed fields, while the blue and red curves show perturbations. At $m = 1$, the wave is more or less a uniform displacement, while at $m = 16$ azimuthal variations take place on spatial scales small compared to Earth's radius.

- 450 Waves with small azimuthal modenumbers ($0 < m < 10$) are typically driven by broad-
 451 band energy sources at the magnetosphere's boundary, such as variations in the so-
 452 lar wind pressure[18, 38, 48, 104, 105], sporadic magnetic reconnection[41], or Kelvin-
 453 Helmholtz waves on the magnetopause[10, 57, 82]. In the low- m regime, the shear and
 454 compressional Alfvén waves are coupled, which allows energy to move across field lines
 455 until the driving frequency lines up with the local Alfvén frequency[59]. Because of their
 456 broadband energy source, low- m FLRs often have a mishmash of frequencies present in
 457 their spectra[16], though the spectra are coherent in terms of harmonic[25].
- 458 When the azimuthal modenumber is large (or, equally, when the azimuthal wavelength
 459 is small), compressional propagation of Alfvén waves becomes evanescent, so the move-
 460 ment of energy is guided by magnetic field lines[14, 76]⁶. As a result, FLRs must be
 461 driven from within the magnetosphere. Proposed energy sources include phase space
 462 gradients near the plasmapause[17], particularly as the plasmasphere refills after a storm
 463 or substorm[24, 55].

⁶Equally, the strength of a wave's parallel component indicates its modenumber, a point which is revisited in Chapters 7 and 8.

464 The atmosphere is known to attenuate waves with small spatial extent in the perpendic-
465 ular direction[42, 99, 103]. As a result, FLRs may create no signature on the ground if
466 their azimuthal modenumber is large. For example, a recent paper by Takahashi shows
467 a strong (2 nT at $L \sim 10$), clear resonance with $|m| \gtrsim 70$ and no corresponding ground
468 signature[88].

Southwood[83] and Glassmeier[29] show that a low frequency wave passing through the ionosphere on its way to Earth will be attenuated per

$$\frac{B_E}{B_I} = \frac{\Sigma_H}{\Sigma_P} \exp\left(-m \frac{R_I - R_E}{R_E \sin \theta}\right) \quad (3.3)$$

469 Where B_E and B_I are the magnetic field strengths at R_E (Earth's surface, 6783 km
470 geocentric) and R_I (the ionosphere, \sim 6900 km geocentric) respectively. The integrated
471 ionospheric Pedersen and Hall conductivities, Σ_P and Σ_H , are typically within a factor
472 of two of one another. Field lines near the plasmapause can be traced to Earth at
473 $\sin \theta \sim 0.4$. That is, by the time it reaches the ground, the magnetic field from an FLR
474 with $m = 10$ is weaker by a factor of two; at $m = 100$, the factor is closer to 100.

475 3.3 Poloidal and Toroidal Polarizations

476 Based on polarization, each FLR can be classified as either poloidal or toroidal. The
477 poloidal mode is a field line displacement in the meridional plane, as shown in Figure 3.4,
478 with an accompanying electric field in the azimuthal direction. The toroidal mode's
479 magnetic displacement is azimuthal, per Figure 3.5; the associated electric field is in the
480 meridional plane.

481 Both poloidal and toroidal waves are noted for their ability to contribute to the energiza-
482 tion and radial diffusion of trapped particles. The poloidal mode interacts more strongly,
483 since its electric field is aligned with the trapped particles' drift motion. Poloidally-
484 polarized waves are also more prone to creating magnetic signatures on the ground, due
485 to ducting in the ionosphere[27, 36].

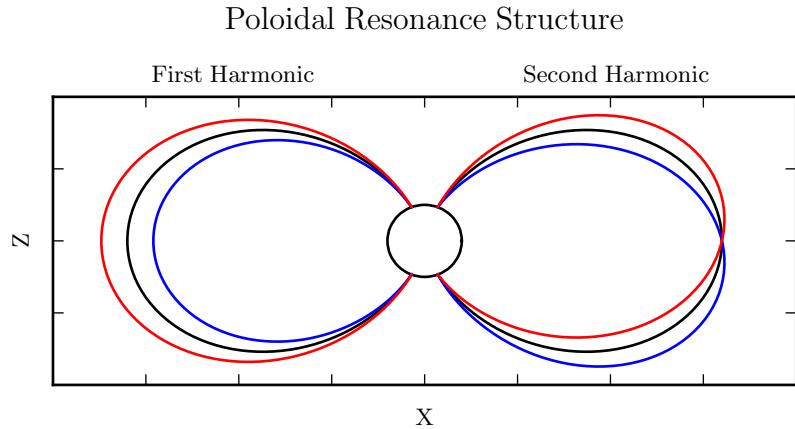


Figure 3.4: The poloidal resonance is a magnetic field in the meridional plane. The displacement is radial near the equator, and can be accompanied by enhancement in the parallel magnetic field near the ionosphere. The first harmonic is shown on the left, and the second harmonic on the right. Lines are colored red and blue only to contrast with the unperturbed black field line.

- 486 Toroidal modes have been shown to outnumber poloidal modes[2]. Perhaps not co-
- 487 incidentally, poloidally-polarized waves rotate asymptotically to the toroidal mode[64,
- 488 65, 76]. Poloidal waves with low azimuthal modenumber — such as those driven by
- 489 broadband sources at the magnetopause — rotate on timescales comparable to their
- 490 oscillation periods. The two modes are also coupled directly by the ionospheric Hall
- 491 conductivity[46].

- 492 The eigenfrequencies for poloidal and toroidal FLRs are similar, though not identical[34].
- 493 It has furthermore been noted that toroidally-polarized waves exhibit a strong relation-
- 494 ship between frequency and L -shell (or latitude), while poloidal waves at fixed frequency
- 495 are spread more broadly in L [25].

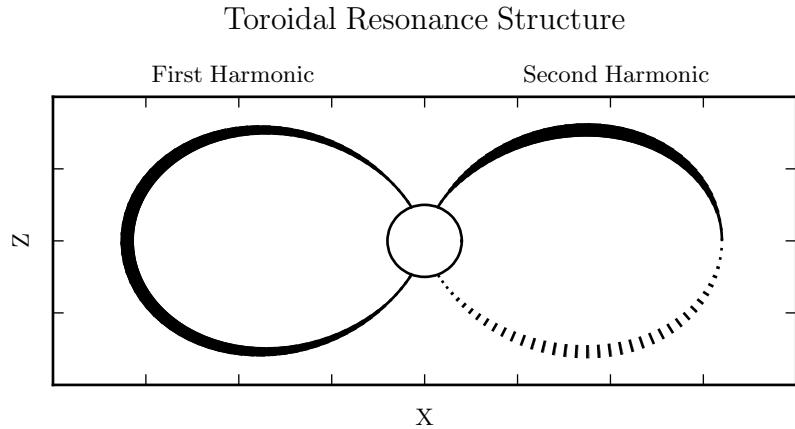


Figure 3.5: A toroidally-polarized FLR has a magnetic perturbation in the azimuthal direction, as shown. Bold lines are taken to be above the page, and dotted lines below the page, with the displacement indicated by the line’s width. The first harmonic — where the entire field line oscillates in and out of the page — is shown on the left. The second harmonic is on the right, with northern and southern halves perturbed in opposite directions.

496 3.4 Giant Pulsations

497 The study of geomagnetic pulsations long predates satellites, sounding rockets, or even
498 the word “magnetohydrodynamics”⁷. Large, regular oscillations in the magnetic field
499 were noted as early as 1901[5]. Eventually, the term “giant pulsation,” or Pg, arose to
500 describe such pulsations.

501 On the ground, Pgs are notable for their strikingly sinusoidal waveforms, their eastward
502 drifts, and their specific occurrence pattern: Pgs are typically seen pre-dawn, at latitudes
503 of 60° to 70° . Pgs generally fall into the Pc4 frequency band⁸. Their harmonic structure
504 was a source of controversy for decades, but recent multisatellite observations seem to be
505 in agreement that they are odd harmonics, probably fundamental[30, 40, 51, 52, 87, 91].
506 They are poloidally polarized, with modenumbers $10 \lesssim m \lesssim 40$ [28, 40, 73, 79, 91].

⁷The term was first used by Alfvén in the 1940s[1].

⁸The Pc4 range is periods of 45 s to 140 s, while Pgs range from 60 s to 200 s[7].

507 Whereas FLRs are waves in space which may produce ground signatures, “giant pulsation” refers to the ground signature specifically⁹. That is, Takahashi’s satellite observation of a sinusoidal, morningside, high- m , fundamental poloidal resonance was not
509
510 classified as a Pg because it did not produce a signal on the ground[88].

511 Due to their remarkably clear waveforms, Pgs are typically found by “visual inspection
512 of magnetometer data”[68]. Over the course of the past century, a number of multi-year
513 (sometimes multi-decade[7]) surveys have totaled nearly one thousand Pg events. On
514 average, a ground magnetometer near 66° magnetic latitude observes \sim 10 Pg events per
515 year[7, 39, 78, 85]. Observations are not distributed uniformly; rather, giant pulsations
516 are most common near the equinox and during times of low solar activity.

517 Unlike Pc4 pulsations in general, Pgs show no particular correlation with storm phase[68].
518 However, they do often occur as the magnetosphere recovers from a substom[68, 79].

519 3.5 Motivations for the Present Work

520 A great deal has been learned — and continues to be learned — through observations of
521 field line resonance. However, the cost-to-benefit ratio is climbing. As summarized in the
522 sections above, FLR behavior depends significantly on harmonic structure, azimuthal
523 modenumber, and polarization — not to mention frequency, spectral width, and so
524 on. With each degree of freedom comes the necessity for an additional simultaneous
525 observation.

526 Similarly, as FLRs have been shown to depend on ever-more-specific magnetospheric
527 conditions, analytical techniques have fallen out of favor. The height-resolved iono-
528 sphere, multidimensional Alfvén speed profile, and inconvenient geometry combine to
529 create a problem beyond the reasonable purview of pencil and paper.

530 That is, the topic of field line resonance is ripe for numerical modeling.

⁹Historically, the wave designations shown in Table 3.1 all referred to ground phenomena. Over time, they have come to describe satellite observations as well, including those without corresponding signatures on the ground.

531 Past models of the magnetosphere have been limited in their consideration of FLRs.
532 Reasons include overly-simplified treatment of the ionospheric boundary, no consider-
533 ation of the plasmapause, limited range in m , and the inability to compute ground
534 signatures. Chapter 5 presents a model which addresses these issues, allowing the com-
535 putation of field line resonance with unparalleled attention to realism.

536 The model allows a bird’s-eye view of the structure and evolution of FLRs. As such,
537 not only can several open questions be addressed, but their answers serve to unify a
538 number of seemingly-disparate properties described in the sections above.

539 The rotation of poloidally-polarized waves to the toroidal mode is investigated. Par-
540 ticular attention is paid to the importance of azimuthal modenumber and ionospheric
541 conductivity. The interplay between said rotation and the transport of energy across
542 field lines — which also depends on azimuthal modenumber — is considered as well.

543 By their nature, drifting particles have the potential to spur wave-particle interactions
544 at all MLT. However, Pc4 observations are strongly peaked on the dayside. In a 2015
545 paper, Dai notes, “It is not clear why noncompressional [high- m] Pc4 poloidal waves,
546 which are presumably driven by instability within the magnetosphere, preferentially
547 occur on the dayside”[16]. Motoba, later that year, echoes, “It is unclear whether other
548 generation mechanisms of fundamental standing waves ... can explain the localization
549 of Pgs in local time”[68]. This, too, is considered numerically: to what degree is field
550 line resonance affected by the (day-night asymmetric) conditions of the ionosphere?

551 An attempt is also made to demystify giant pulsations. It’s been shown that toroidal
552 Pc4s outnumber poloidal ones, and that most poloidal Pc4s are even, so perhaps it
553 should come as no surprise that (poloidal, odd) Pgs are rare. Is it truly the case that
554 Pgs are only “a small subset of fundamental poloidal waves”[88], set apart from the rest
555 by their distinctive properties? Or, said another way, to what degree do the properties
556 associated with Pgs arise in fundamental poloidal waves overall?

⁵⁵⁷ **Chapter 4**

⁵⁵⁸ **Waves in Cold Resistive Plasma**

⁵⁵⁹ Before delving into the implementation of the numerical model, it's instructive to con-
⁵⁶⁰ sider the fundamental equations of waves in a cold, resistive plasma. Specifically, the
⁵⁶¹ present chapter is concerned with waves much slower than the electron cyclotron fre-
⁵⁶² quency. High-frequency waves such as the L and R modes are beyond the scope of the
⁵⁶³ present work — and, in fact, beyond the limits of the model described in Chapter 5.

The evolution of electric and magnetic fields is governed by Ampère's law and Faraday's law. Notably, the present work takes the linearized versions of Maxwell's equations; the vectors \underline{E} and \underline{B} indicate perturbations above the zeroth-order magnetic field.

$$\underline{\epsilon} \cdot \frac{\partial}{\partial t} \underline{E} = \frac{1}{\mu_0} \nabla \times \underline{B} - \underline{J} \quad \frac{\partial}{\partial t} \underline{B} = -\nabla \times \underline{E} \quad (4.1)$$

Current follows Ohm's law. For currents perpendicular to the background magnetic field, Kirchhoff's formulation is sufficient. Conductivity is much higher along the magnetic field lines¹, so it's appropriate to also include the electron inertial term².

$$0 = \underline{\sigma}_{\perp} \cdot \underline{E}_{\perp} - \underline{J}_{\perp} \quad \frac{1}{\nu} \frac{\partial}{\partial t} J_z = \sigma_0 E_z - J_z \quad (4.2)$$

¹The dipole coordinate system is defined rigorously in Section 5.1; at present, it's sufficient to take \hat{z} parallel to the zeroth-order magnetic field, and \hat{x} and \hat{y} perpendicular to \hat{z} (and to each other).

²Electron inertial effects are not included in the model described in Chapter 5, or in the results in Chapter 7; however, their implementation and impact are explored in Chapter 6.

Derivatives in Equations (4.1) and (4.2) can be evaluated by performing a Fourier transform. They can then be combined, eliminating the magnetic field and current terms.

$$\begin{aligned} 0 &= \underline{\underline{E}}_{\perp} + \frac{v_A^2}{\omega^2} (\underline{k} \times \underline{k} \times \underline{E})_{\perp} + \frac{i}{\epsilon_{\perp}\omega} \underline{\sigma}_{\perp} \cdot \underline{\underline{E}}_{\perp} \\ 0 &= E_z + \frac{c^2}{\omega^2} (\underline{k} \times \underline{k} \times \underline{E})_z + \frac{i\omega_P^2}{\omega(\nu - i\omega)} E_z \end{aligned} \quad (4.3)$$

The speed of light, Alfvén speed (including the displacement current correction), plasma frequency, and parallel conductivity are defined in the usual way:

$$c^2 \equiv \frac{1}{\mu_0\epsilon_0} \quad v_A^2 \equiv \frac{1}{\mu_0\epsilon_{\perp}} \quad \omega_P^2 \equiv \frac{ne^2}{m_e\epsilon_0} \quad \sigma_0 \equiv \frac{ne^2}{m_e\nu} \quad (4.4)$$

Where the perpendicular dielectric constant ϵ_{\perp} is analogous to the electric constant ϵ_0 , but for electric fields which are perpendicular to the preferred direction of the dielectric medium. As noted in Equation (3.1), $\epsilon_{\perp} \equiv \frac{\rho}{B^2}$ where ρ is the mass density and B is the magnitude of the (zeroth-order) magnetic field.

Using the vector identity $\underline{k} \times \underline{k} \times \underline{E} = \underline{k} \underline{k} \cdot \underline{E} - k^2 \underline{E}$, Equation (4.3) can be reassembled into a single expression,

$$0 = \left(\underline{\underline{\mathbb{I}}} + \frac{1}{\omega^2} \underline{\underline{V}}^2 \cdot \underline{k} \underline{k} - \frac{k^2}{\omega^2} \underline{\underline{V}}^2 + \frac{i}{\omega} \underline{\underline{\Omega}} \right) \cdot \underline{\underline{E}} \quad (4.5)$$

Where $\underline{\underline{\mathbb{I}}}$ is the identity tensor and in x - y - z coordinates,

$$\underline{\underline{V}}^2 \equiv \begin{bmatrix} v_A^2 & 0 & 0 \\ 0 & v_A^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} \quad \text{and} \quad \underline{\underline{\Omega}} \equiv \begin{bmatrix} \frac{\sigma_P}{\epsilon_{\perp}} & \frac{-\sigma_H}{\epsilon_{\perp}} & 0 \\ \frac{\sigma_H}{\epsilon_{\perp}} & \frac{\sigma_P}{\epsilon_{\perp}} & 0 \\ 0 & 0 & \frac{\omega_P^2}{(\nu - i\omega)} \end{bmatrix} \quad (4.6)$$

The parenthetical expression in Equation (4.5) is the dispersion tensor. Nontrivial solutions exist only when its determinant is zero. This gives rise to a seventh-order polynomial in ω , so rather than a direct solution it's necessary to consider limits of specific interest.

572 In these limits, as explored in the sections below, the wave vector \underline{k} is taken — without
 573 loss of generality — to lie in the x - z plane (that is, k_y is taken to be zero). The distinction
 574 between the two perpendicular components (k_x and k_y) is discussed in Section 4.4.

575 4.1 Guided Propagation

576 The wave vector of a field line resonance aligns closely to the background magnetic
 577 field. By supposing that the two align exactly (that is, taking $k_x = 0$), the parallel and
 578 perpendicular components of the dispersion relation are decoupled.

The parallel-polarized component ($E_x = E_y = 0$) is quadratic, and thus can be solved directly.

$$0 = \omega^2 + i\nu\omega - \omega_P^2 \quad \text{so} \quad \omega = -\frac{i\nu}{2} \pm \sqrt{\omega_P^2 - \frac{\nu^2}{4}} \quad (4.7)$$

It bears noting that the plasma frequency is large — not just compared to Pc4 frequencies, but even compared to the collision frequencies in the ionosphere³. Expanding Equation (4.7) with respect to large ω_P , the dispersion relation becomes

$$\omega^2 = \omega_P^2 \pm i\nu\omega_P + \dots \quad (4.8)$$

579 Equation (4.8) can hardly be called a wave, as it does not depend on the wave vector
 580 \underline{k} . Rather, it is the plasma oscillation⁴: electrons vibrating in response to a charge
 581 separation along the background magnetic field.

582 The plasma oscillation is not specifically relevant to the study of field line resonance.
 583 The two phenomena are separated by six orders of magnitude in frequency. The topic
 584 is revisited in Chapter 6 insomuch as it is inseparable from the electron inertial effects
 585 in Ohm’s law, but it is otherwise not relevant to the work at hand.

³The collision frequency may match or exceed the plasma frequency for a thin slice at the bottom of the ionosphere, where the density of neutral atoms is large; however, it falls off quickly. Above an altitude of 200 km, conservatively[71], the plasma frequency exceeds the collision frequency by an order of magnitude or more.

⁴The plasma oscillation is also called the Langmuir wave.

The perpendicular ($E_z = 0$) components of the dispersion relation give an expression quartic in ω .

$$0 = \omega^4 + 2i\frac{\sigma_P}{\epsilon_\perp}\omega^3 - \left(2k_z^2v_A^2 + \frac{\sigma_P^2 + \sigma_H^2}{\epsilon_\perp^2}\right)\omega^2 - 2ik_z^2v_A^2\frac{\sigma_P}{\epsilon_\perp}\omega + k_z^4v_A^4 \quad (4.9)$$

Like the parallel-polarized component above, Equation (4.9) can be solved directly. Noting that \pm and \oplus are independent,

$$\omega = \left(\frac{\pm\sigma_H - i\sigma_P}{2\epsilon_\perp}\right) \oplus \sqrt{k_z^2v_A^2 + \left(\frac{\pm\sigma_H - i\sigma_P}{2\epsilon_\perp}\right)^2} \quad (4.10)$$

Also like the parallel-polarized component, the exact solution is less useful than its Taylor expansion. The Pedersen and Hall conductivities are large near the ionospheric boundary, but fall off quickly; for the vast majority of the field line, the ratios $\frac{\sigma_P}{\epsilon_\perp}$ and $\frac{\sigma_H}{\epsilon_\perp}$ are small compared to field line resonance frequencies. Expanding with respect to small conductivity gives

$$\omega^2 = k_z^2v_A^2 \oplus k_zv_A\frac{\sigma_H \pm i\sigma_P}{\epsilon_\perp} + \dots \quad (4.11)$$

586 This is the shear Alfvén wave, with a shift in its frequency proportional to the character-
 587 istic conductivity of the ionosphere⁵. The wave travels along the background magnetic
 588 field like a bead on a string, with electric and magnetic field perturbations perpendicular
 589 to the magnetic field line (and to one another).

590 4.2 Compressional Propagation

591 The partner to guided motion is compressional motion; in order for energy to move
 592 across field lines, the wave vector must have a component perpendicular to \hat{z} . If the

⁵Admittedly, it's difficult to say what constitutes a characteristic conductivity. The Pedersen and Hall conductivities vary by several orders of magnitude within the ionosphere, then drop essentially to zero at large radial distance.

593 wave vector is completely perpendicular to the magnetic field line ($k_z = 0$), the parallel
 594 and perpendicular components of the dispersion relation decouple, as in Section 4.1.

The parallel-polarized ($E_x = E_y = 0$) component of the dispersion relation is

$$0 = \omega^3 + i\nu\omega^2 - (k_x^2 c^2 + \omega_P^2) \omega - ik_x^2 c^2 \nu \quad (4.12)$$

Equation (4.12) can be solved directly, though its solution (as a cubic) is far too long to be useful. Expanding with respect to the large plasma frequency, gives

$$\omega^2 = k^2 c^2 + \omega_P^2 - i\nu\omega_P + \dots \quad (4.13)$$

595 This is the O mode, a compressional wave with an electric field perturbation along
 596 the background magnetic field. Like the plasma oscillation discussed in Section 4.1, its
 597 frequency is very large compared to that of a field line resonance, putting it beyond the
 598 concern of the present work.

The perpendicular-polarized ($E_z = 0$) component of the compressional dispersion relation is also cubic.

$$0 = \omega^3 + 2i\frac{\sigma_P}{\epsilon_\perp}\omega^2 - \left(2k_x^2 v_A^2 + \frac{\sigma_P^2 + \sigma_H^2}{\epsilon_\perp^2}\right) \omega - ik_x^2 v_A^2 \frac{\sigma_P}{\epsilon_\perp} \quad (4.14)$$

A wave moving across magnetic field lines near the ionospheric boundary will encounter large $\frac{\sigma_P}{\epsilon_\perp}$ and $\frac{\sigma_H}{\epsilon_\perp}$, while at moderate to high altitudes those ratios will be small. Solving Equation (4.14) in those two limits respectively gives:

$$\omega^2 = k_x^2 v_A^2 \pm ik_x v_A \frac{\sigma_P}{\epsilon_\perp} + \dots \quad \text{and} \quad \omega^2 = k_x^2 v_A^2 + \left(\frac{\sigma_H \pm i\sigma_P}{\epsilon_\perp}\right)^2 + \dots \quad (4.15)$$

599 At first glance, both limits of Equation (4.15) appear to describe compressional Alfvén
 600 waves. The magnetic perturbation is along the background magnetic field — indicating
 601 compression of the frozen-in plasma — while the electric field perturbation is perpen-
 602 dicular to both the magnetic field and the wave vector.

603 However, in the high-conductivity limit, the parenthetical term actually dominates the
 604 Alfvén term, taking values as large as $\sim 10^6$ Hz. Waves at such frequencies are beyond
 605 the scope of the present work. As a matter of interest, however, it bears noting that —
 606 as long as $\nu \ll 10^6$ Hz — $\frac{\sigma_H}{\epsilon_{\perp}}$ reduces to the electron cyclotron frequency, $\frac{eB}{m_e}$.

607 4.3 High Altitude Limit

608 In the limit of large radial distance, it's reasonable to take the collision frequency to
 609 zero; doing so also neglects the Pedersen and Hall conductivities.

Whereas in Sections 4.1 and 4.2 it was the parallel-polarized component of the dispersion relation that factored out from the other two, the high altitude limit decouples the component polarized out of the x - z plane. The y -polarized ($E_x = E_z = 0$) dispersion relation is simply:

$$\omega^2 = k^2 v_A^2 \quad (4.16)$$

610 Equation (4.16) describes an Alfvén wave with an arbitrary angle of propagation. De-
 611 pending on the angle between the wave vector and the background magnetic field, it
 612 could be guided, compressional, or somewhere in between. Regardless of propagation
 613 angle, the electric field perturbation is perpendicular to both the direction of propaga-
 614 tion and the magnetic field perturbation.

The other two components (from $E_y = 0$) of the high altitude dispersion tensor give an expression quadratic in ω^2 :

$$0 = \omega^4 - (k_x^2 c^2 + k_z^2 v_A^2 + \omega_P^2) \omega^2 + k_z^2 v_A^2 \omega_P^2 \quad (4.17)$$

Which can be solved for

$$\omega^2 = \frac{1}{2} (k_x^2 c^2 + k_z^2 v_A^2 + \omega_P^2) \pm \sqrt{\frac{1}{4} (k_x^2 c^2 + k_z^2 v_A^2 + \omega_P^2)^2 - k_z^2 v_A^2 \omega_P^2} \quad (4.18)$$

Noting again that ω_P is very large, the two roots can be expanded to

$$\omega^2 = k_z^2 v_A^2 \left(1 - \frac{k_x^2 c^2 + k_z^2 v_A^2}{\omega_P^2} \right) + \dots \quad \text{and} \quad \omega^2 = k_x^2 c^2 + k_z^2 v_A^2 + \omega_P^2 + \dots \quad (4.19)$$

- 615 The first solution of Equation (4.19) is a shear Alfvén wave, as in Equation (4.11).
- 616 Notably, this form arises only when the parenthetical quantity is close to unity — as it
- 617 is for FLRs. The inertial limit, where frequencies are close to the plasma frequency, is
- 618 beyond the scope of the present work. For that same reason, the second solution (which
- 619 describes an oscillation faster than the plasma frequency) is not further considered.

620 4.4 Implications to the Present Work

- 621 The present section’s findings carry three implications of particular significance to the
- 622 present work.
- 623 First — with the exception of the plasma oscillation and similar modes, which are
- 624 revisited in Chapter 6 — waves propagate at the Alfvén speed. This, in combination
- 625 with the grid configuration, constrains the time step that can be used to model them
- 626 numerically. The time step must be sufficiently small that information traveling at the
- 627 Alfvén speed cannot “skip over” entire grid cells⁶.

Second, the results of the present section allow an estimate of the magnitude of parallel electric field that might be expected from a field line resonance compared to its perpendicular electric field. Between the full dispersion tensor form in Equation (4.5) and the Alfvén wave described by Equation (4.19),

$$\left| \frac{E_z}{E_x} \right| = \frac{k_x k_z c^2}{\omega^2 - k_x^2 c^2 - \omega_P^2} \sim \frac{k^2 c^2}{\omega_P^2} \quad (4.20)$$

- 628 In essence, the relative magnitudes of the parallel and perpendicular electric fields should
- 629 scale quadratically with the ratio of the electron inertial length ($\frac{c}{\omega_P} \sim 1 \text{ km to } 100 \text{ km}$)

⁶As is the convention, the time step is scaled down from the smallest Alfvén zone crossing time by a Courant factor, typically 0.1, meaning that it takes at least ten time steps to cross a grid cell.

and the FLR wavelength ($\frac{1}{k} \sim 10^5$ km). That is, parallel electric fields should be smaller than perpendicular ones by at least six orders of magnitude.

Finally, the dispersion relations shown above indicate how the behavior of a field line resonance should behave as the azimuthal modenumber becomes large.

Whereas the shear Alfvén wave's dispersion relation depends only on the parallel component of the wave vector, the compressional Alfvén wave depends on its magnitude: $\omega^2 = k^2 v_A^2$. If the frequency is smaller than $k v_A$, the wave will become evanescent. The wave vector's magnitude can be no smaller than its smallest component, however, and the azimuthal component scales with the azimuthal modenumber: $k_y \sim \frac{m}{2\pi r}$.

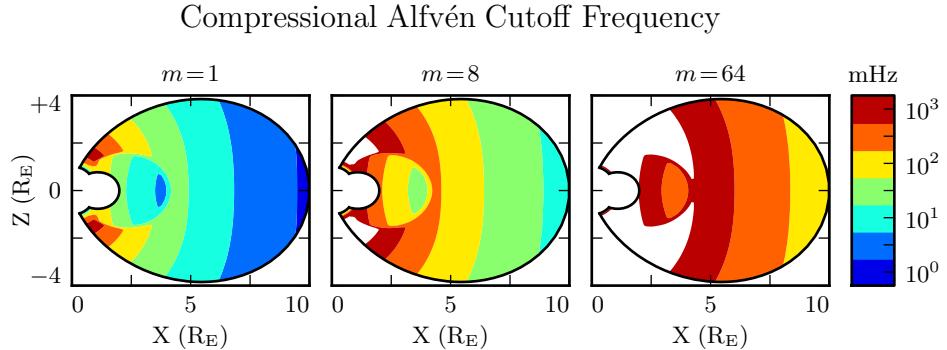


Figure 4.1: Taking $k_y \sim \frac{m}{2\pi r}$ as a lower bound for the magnitude of the wave vector, the above figure demonstrates the lowest frequency at which compressional Alfvén waves may propagate as a function of position and m . Regions shown in white are off the color scale — they have a lower bound on the order of 10^4 mHz or more. The above Alfvén frequency profile is from Kelley[47], for quiet dayside conditions, as discussed in Section 5.2.

This imposes a frequency cutoff on compressional Alfvén waves which scales with the azimuthal modenumber, as shown in Figure 4.1. At small values of m , most of the magnetosphere is conducive to compressional Alfvén waves in the Pc4 frequency range. As m increases, and the wave vector with it, the inner magnetosphere becomes increasingly inaccessible to them.

644 **Chapter 5**

645 **“Tuna Half” Dimensional Model**

646 The present section describes the implementation of Tuna, a new two and a half dimensional
647 Alfvén wave code based largely on work by Lysak[58, 61].

648 The half-integer dimensionality refers to the fact that Tuna’s spatial grid resolves a
649 two-dimensional slice of the magnetosphere, but that electric and magnetic fields —
650 as well as their curls — are three-dimensional vectors. This apparent contradiction is
651 reconciled by the use of a fixed azimuthal modenumber, m . Electric and magnetic fields
652 are taken to be complex-valued, varying azimuthally per $\exp(im\phi)$; derivatives with
653 respect to ϕ are then replaced by a factor of im .

654 Unlike a fully three-dimensional code, Tuna cannot resolve the evolution of wave structures
655 in the azimuthal direction. Furthermore, the model does not allow coupling between
656 the dayside and nightside magnetospheres. What Tuna does offer is efficiency.
657 The model’s economical geometry allows it to include a realistic Earthward boundary:
658 grid spacing on the order of 10 km, a height-resolved ionospheric conductivity tensor,
659 and even the computation of magnetic field signatures at the ground. Such features
660 would pose a prohibitive computational expense in a large global code.

661 Tuna was developed with field line resonance in mind. As discussed in Chapter 3,
662 such waves are azimuthally localized, minimizing the importance of Tuna’s missing half
663 dimension. Moreover, because field line resonances are known to be affected by both

664 the ionosphere and the plasmasphere, a faithful treatment of the inner magnetosphere
665 is a crucial part of studying them numerically.

666 Perhaps the most exciting feature of Tuna is its novel driving mechanism: ring current
667 perturbation. Codes similar to Tuna have traditionally been driven using compressional
668 pulses at the outer boundary[58, 61, 97, 98]. This has precluded the investigation of
669 waves with large azimuthal modenumber — such as giant pulsations — which are guided,
670 and thus must be driven from within the magnetosphere.

671 Tuna’s source code, written in Fortran, is publicly available at <https://github.com/UMM-Space-Physics>. The repository also includes a pair of Python scripts: a test har-
672 ness — which automates programming environment setup, compilation, and interfacing
673 with the supercomputer queue — and a plotter.
674

675 **TODO:** Set up Git repo.

676 5.1 Coordinate System

Field line resonance has traditionally been modeled by straightening the field lines into a rectangular configuration[20, 64], by unrolling the azimuthal coordinate into a cylindrical coordinate system[76], or through the use of dipole coordinates[75]¹:

$$x \equiv -\frac{\sin^2 \theta}{r} \quad y \equiv \phi \quad z \equiv \frac{\cos \theta}{r^2} \quad (5.1)$$

677 Where r , θ , and ϕ take on their usual spherical meanings of radial distance, colatitude,
678 and azimuthal angle respectively.

679 The dipole coordinate x is constant over each L -shell², y is azimuthal angle, and z
680 indexes each field line from south to north. The unit vectors \hat{x} , \hat{y} , and \hat{z} point in
681 the crosswise³ (radially outward at the equator), azimuthal (eastward), and parallel
682 (northward at the equator) directions respectively.

¹The dipole coordinates x , y , and z are sometimes named μ , ϕ , and ν respectively.

²In fact, x is inversely proportional to L .

³In the context of in situ measurements taken near the equatorial plane, \hat{x} is referred to as the radial direction; however, the present work extends the dipole grid to low altitudes, where \hat{x} is often more

Notably, the dipole coordinates in Equation (5.1) are normal to one another. While mathematically convenient, they do not readily accommodate a fixed-altitude boundary at the ionosphere, nor do they allow the dipole magnetic field to intersect the boundary at an oblique angle, as Earth's field does. As a solution, a nonorthogonal set of dipole coordinates was developed numerically by Proehl[74], then formalized analytically by Lysak[58] in terms of their contravariant components:

$$u^1 = -\frac{R_I}{r} \sin^2 \theta \quad u^2 = \phi \quad u^3 = \frac{R_I^2}{r^2} \frac{\cos \theta}{\cos \theta_0} \quad (5.2)$$

683 Above, R_I is the position of the ionosphere relative to Earth's center; it's typically taken
684 to be $1 R_E + 100 \text{ km}$.

685 Like the dipole coordinates x , y , and z , Lysak's coordinates u^1 , u^2 , and u^3 correspond to
686 L -shell, azimuthal angle, and position along a field line respectively. However, compared
687 to z , u^3 has been renormalized using the invariant colatitude⁴ θ_0 . As a result, u^3 takes
688 the value $+1$ at the northern ionospheric boundary and -1 at the southern ionospheric
689 boundary for all field lines.

Because Lysak's coordinate system is not orthogonal — that is, because curves of constant u^1 and curves of constant u^3 can intersect at non-right angles — it's necessary to consider covariant and contravariant basis vectors separately.

$$\hat{e}_i \equiv \frac{\partial}{\partial u^i} \underline{x} \quad \hat{e}^i \equiv \frac{\partial}{\partial \underline{x}} u^i \quad (5.3)$$

Covariant basis vectors \hat{e}_i are normal to the curve defined by constant u^i , while contravariant basis vectors \hat{e}^i are tangent to the coordinate curve (equivalently, \hat{e}^i is normal to the plane defined by constant u^j for all $j \neq i$). These vectors are reciprocal to one

horizontal than vertical. The term “crosswise” is meant to indicate that \hat{x} is defined by the cross product of \hat{y} and \hat{z} .

⁴The invariant colatitude is the colatitude θ at which a field line intersects the ionosphere. It is related to the McIlwain parameter by $\cos \theta_0 \equiv \sqrt{1 - \frac{R_I}{L}}$.

another, and can be combined to give components of the metric tensor $\underline{\underline{g}}$ [19].

$$\hat{e}^i \cdot \hat{e}_j = \delta_j^i \quad g_{ij} \equiv \hat{e}_i \cdot \hat{e}_j \quad g^{ij} \equiv \hat{e}^i \cdot \hat{e}^j \quad (5.4)$$

690 The symbol δ_j^i is the Kronecker delta; the present work also makes use of the Levi-Civita
691 symbol ε^{ijk} and Einstein's convention of implied summation over repeated indeces[22].

The metric tensor allows rotation between covariant and contravariant representations of vectors.

$$A_i = g_{ij} A^j \quad \text{and} \quad A^i = g^{ij} A_j \quad \text{where} \quad A_i \equiv \underline{A} \cdot \hat{e}_i \quad \text{and} \quad A^i \equiv \underline{A} \cdot \hat{e}^i \quad (5.5)$$

In addition, the determinant of the metric tensor is used to define cross products and curls⁵.

$$(\underline{A} \times \underline{B})^i = \frac{\varepsilon^{ijk}}{\sqrt{g}} A_j B_k \quad \text{and} \quad (\nabla \times \underline{A})^i = \frac{\varepsilon^{ijk}}{\sqrt{g}} \frac{\partial}{\partial u^j} A_k \quad \text{where} \quad g \equiv \det \underline{\underline{g}} \quad (5.6)$$

Explicit forms of the basis vectors and metric tensor can be found in the appendix of Lysak's 2004 paper[58]. At present, it's sufficient to note the mapping between Lysak's basis vectors and the usual dipole unit vectors.

$$\hat{x} = \frac{1}{\sqrt{g^{11}}} \hat{e}^1 \quad \hat{y} = \frac{1}{\sqrt{g^{22}}} \hat{e}^2 \quad \hat{z} = \frac{1}{\sqrt{g^{33}}} \hat{e}_3 \quad (5.7)$$

The basis vectors can also be mapped to the spherical unit vectors, though Equation (5.8) is valid only at the ionospheric boundary.

$$\hat{\theta} = \frac{1}{\sqrt{g_{11}}} \hat{e}_1 \quad \hat{\phi} = \frac{1}{\sqrt{g_{22}}} \hat{e}_2 \quad \hat{r} = \frac{1}{\sqrt{g_{33}}} \hat{e}^3 \quad (5.8)$$

692 The coordinates converge at the equatorial ionosphere, so an inner boundary is necessary
693 to maintain finite grid spacing. It's typically placed at $L = 2$. The outer boundary is

⁵The square root of the determinant of the metric tensor is called the Jacobian. It's sometimes denoted using the letter J , which is reserved for current in the present work.

694 at $L = 10$. The dipole approximation of Earth’s magnetic field is tenuous at the outer
695 boundary; however, in practice, wave activity is localized inside $L \sim 7$. The grid is
696 spaced uniformly in u^1 , which gives finer resolution close to Earth and coarser resolution
697 at large distances.

698 Spacing in u^3 is set by placing grid points along the outermost field line. The points are
699 closest together at the ionosphere, and grow towards the equator. The spacing increases
700 in a geometric fashion, typically by 3%.

701 Typically, Tuna uses a grid 150 points in u^1 by 350 points in u^3 . The result is a resolution
702 on the order of 10 km at the ionosphere, which increases to the order of 10^3 km at the
703 midpoint of the outermost field line.

704 There are no grid points in u^2 due to the two-and-a-half-dimensional nature of the
705 model. Fields are assumed to vary as $\exp(imu^2)$ — equally, $\exp(im\phi)$ — so derivatives
706 with respect to u^2 are equivalent to a factor of im . In effect, the real component of
707 each field is azimuthally in phase with the (purely real) driving, while imaginary values
708 represent behavior that is azimuthally offset. Azimuthal modenumbers span the range
709 $1 \lesssim m \lesssim 100$, consistent with observations of Pc4 and Pg events[16, 17, 68, 88].

710 The simulation’s time step is set based on the grid spacing. As is the convention, δt is
711 set to match the smallest Alfvén zone crossing time, scaled down by a Courant factor
712 (typically 0.1). It bears noting that the smallest crossing time need not correspond to
713 the smallest zone; the Alfvén speed peaks several thousand kilometers above Earth’s
714 surface, in the ionospheric Alfvén resonator[61]. A typical time step is on the order of
715 10^{-5} s.

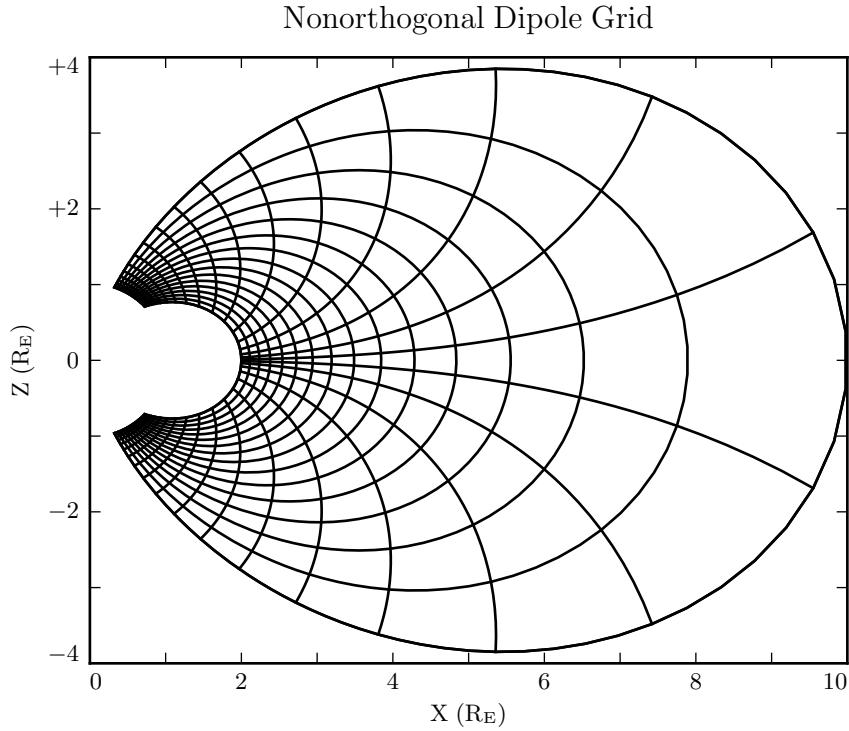


Figure 5.1: The model’s nonorthogonal dipole grid. Every fifth point is shown in each direction. The high concentration of grid points near Earth’s equator is a consequence of the coordinate system, which converges at the equatorial ionosphere. Earth (not shown) is centered at the origin with unit radius.

716 5.2 Physical Parameter Profiles

Tuna models Earth’s magnetic field to be an ideal dipole, so the zeroth-order field strength is written in the usual form:

$$\underline{B}_0 = 3.1 \times 10^4 \text{ nT} \left(\frac{R_E}{r} \right)^3 \left(2 \cos \theta \hat{r} - \sin \theta \hat{\theta} \right) \quad (5.9)$$

Number density is taken to be the sum of an inverse-radial-dependent auroral profile and a plasmaspheric profile dependent on the L -shell[61]:

$$n = n_{AZ} \frac{r_{AZ}}{r} + n_{PS} \exp\left(\frac{-L}{L_{PS}}\right) \left(\frac{1}{2} - \frac{1}{2} \tanh \frac{L - L_{PP}}{\delta L_{PP}} \right) \quad (5.10)$$

⁷¹⁷ Where typical values are shown in Table 5.1.

Table 5.1: Typical Parameters for the Tuna Density Profile

Variable	Value	Description
L_{PS}	1.1	Scale L of the plasmasphere.
L_{PP}	4.0	Location of the plasmapause.
δL_{PP}	0.1	Thickness of the plasmapause.
n_{AZ}	$10 / \text{cm}^3$	Number density at the base of the auroral zone.
n_{PS}	$10^4 / \text{cm}^3$	Number density at the base of the plasmasphere.
r_{AZ}	1 R_E	Scale height of the auroral density distribution.

The perpendicular component of the electric tensor, ϵ_{\perp} , is computed based on Kelley's[47] tabulated low-density values, ϵ_K , which are adjusted to take into account the effects of high density in the plasmapause.

$$\epsilon_{\perp} = \epsilon_K + \frac{Mn}{B_0^2} \quad (5.11)$$

⁷¹⁸ Where M is the mean molecular mass, which is large ($\sim 28 \text{ u}$) at 100 km altitude, then
⁷¹⁹ drops quickly (down to 1 u by $\sim 1000 \text{ km}$)[61].

⁷²⁰ The Alfvén speed profile is computed from the perpendicular electric constant in the
⁷²¹ usual way, $v_A^2 \equiv \frac{1}{\mu_0 \epsilon_{\perp}}$. This form takes into account the effect of the displacement
⁷²² current, which becomes important in regions where the Alfvén speed approaches the
⁷²³ speed of light.

⁷²⁴ While the density profile is held constant for all runs discussed in the present work,
⁷²⁵ the Alfvén speed profile is not. Four different profiles are used for the low-density

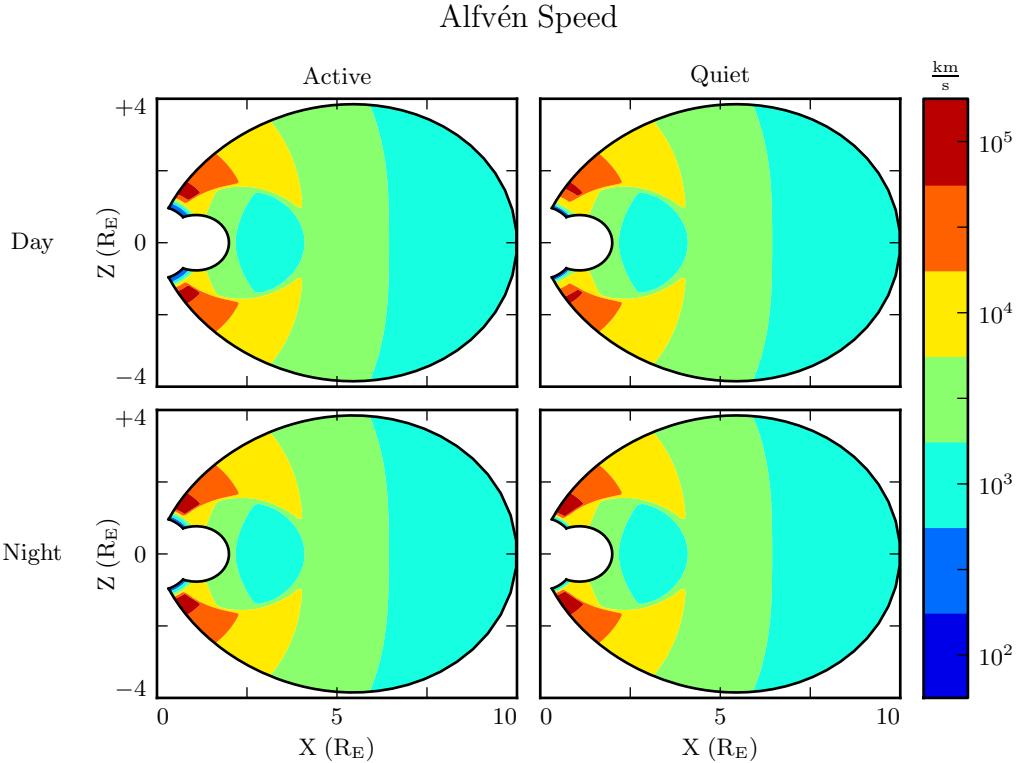


Figure 5.2: Alfvén speed profiles are based on low-density values tabulated by Kelley[47]. They are adjusted to take into account the density of the plasmapause and mass loading near the Earthward boundary.

726 perpendicular electric constant ϵ_K , corresponding to the differing ionospheric conditions
 727 between the dayside and the nightside, and between the high and low points in the
 728 solar cycle. These differences are visible in Figure 5.2, particularly in the size of the
 729 ionospheric Alfvén resonator (the peak in Alfvén speed in the high-latitude ionosphere).

730 Ionospheric conductivity profiles are also based on values tabulated by Kelley, adjusted
 731 by Lysak[61] to take into account the abundance of heavy ions near the Earthward
 732 boundary. Pedersen, Hall, and parallel conductivities are each resolved by altitude, as
 733 shown in Figure 5.3.

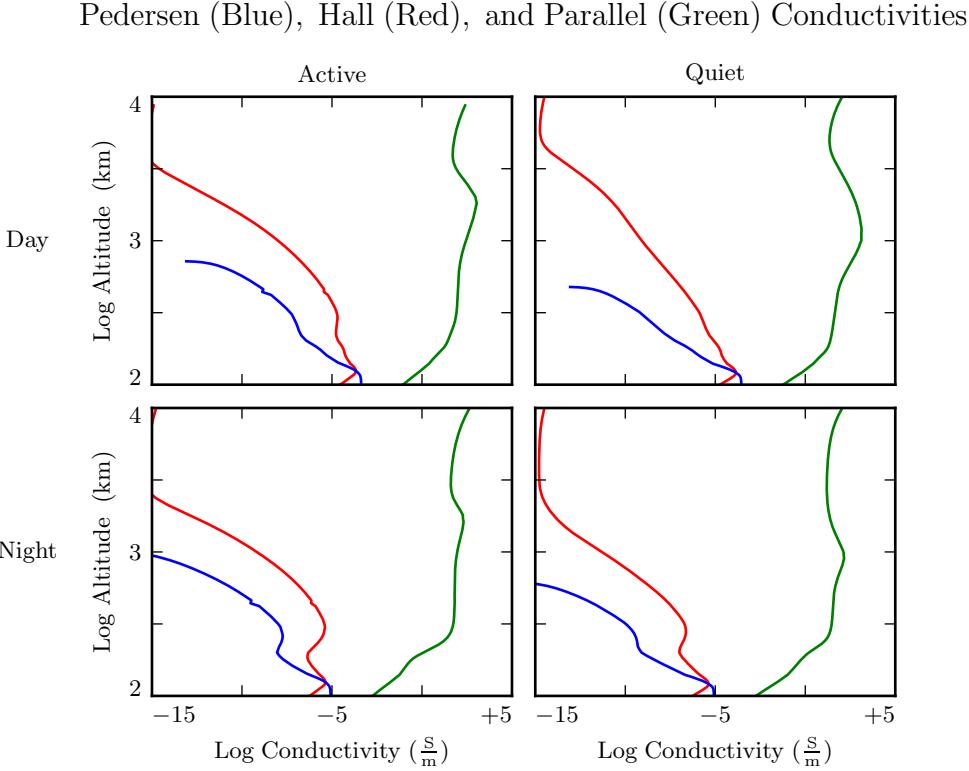


Figure 5.3: Ionospheric conductivity profiles are adapted by Lysak[61] from Kelley's tabulated values[47], in order to take into account the effects of heavy ions at low altitudes. These profiles allow Tuna to simulate magnetosphere-ionosphere coupling under a variety of conditions.

734 Tuna's physical parameter profiles are static over the course of each run. Even so-called
 735 ultra low frequency waves like Pc4 pulsations are fast compared to convective timescales
 736 in the magnetosphere.

737 5.3 Driving

738 Models similar to Tuna have traditionally been driven using compression at the outer
 739 boundary[58, 61, 97, 98]. Such driving acts as a proxy for solar wind compression,
 740 Kelvin-Helmholtz effects at the magnetopause, and so on. However, because of the

741 guided nature of high-modenumber Alfvén waves, simulations driven from the outer
742 boundary are constrained to the consideration of waves with low azimuthal modenumber
743 (equivalently, large azimuthal wavelength).

744 This issue is demonstrated in Figure 5.4. At low modenumber, energy delivered at
745 the outer boundary propagates across field lines in order to stimulate resonances in
746 the inner magnetosphere. However, as modenumber increases, Alfvén waves become
747 increasingly guided, and the inner magnetosphere is unaffected by perturbations at the
748 outer boundary.

749 In order to simulate high-modenumber Alfvén waves in the inner magnetosphere — such
750 as giant pulsations — a new driving mechanism is necessary. Perturbation of the ring
751 current is a natural choice, as Alfvén waves in the Pc4 range are known to interact with
752 ring current particles through drift and drift-bounce resonances. The ring current is a
753 dynamic region, particularly during and after geomagnetic storms and substorms; it's
754 easy to imagine the formation of localized inhomogeneities.

755 In order to estimate an appropriate magnitude for perturbations of the ring current, the
756 Sym-H storm index⁶ is used. The index is measured once per minute, and so cannot
757 directly detect ring current modulations in the Pc4 frequency range. Instead, the index
758 is transformed into the frequency domain, allowing a fit of its pink noise⁷.

759 As shown in Figure 5.5, a Fourier transform of the Sym-H index (taken here from the
760 June 2013 storm) suggests that magnetic field perturbations at Earth's surface due to
761 ring current activity in the Pc4 frequency range could be up to the order of 10^{-2} nT.
762 Supposing that the ring current is centered around $5 R_E$ geocentric, that corresponds to
763 a current on the order of 0.1 MA. Tuna's driving is spread using a Gaussian profile in
764 u^1 (typically centered at $L = 5$) and u^3 (typically centered just off the equator), with a
765 characteristic area of $1 R_E^2$; this gives a current density on the order of $10^{-4} \mu\text{A}/\text{m}^2$.

766 Admittedly, Sym-H is an imperfect tool for estimating the magnitude of localized per-
767 turbations to the ring current, particularly those with high modenumber. As a global

⁶Sym-H is analogous to DST, and the two match each other closely[96]. The crucial difference in this case is that Sym-H is recorded at a higher frequency.

⁷Pink noise, also called $\frac{1}{f}$ noise, refers to the power law decay present in the Fourier transforms of all manner of physical signals.

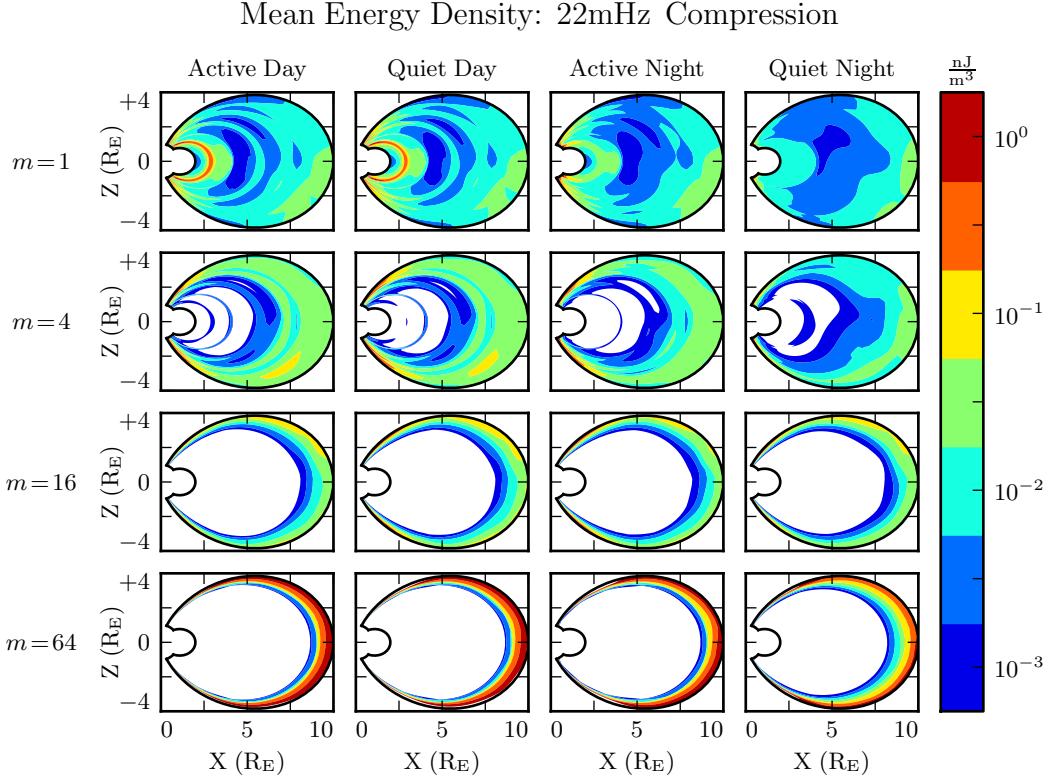


Figure 5.4: Each cell presents the mean energy density over the course of a 300s run, as a function of position, due to compressional driving at the outer boundary. Azimuthal modenumber varies by row, and ionospheric conditions vary by column. For runs with small azimuthal modenumber, it's clear that energy is able to propagate across field lines and stimulate wave activity in the inner magnetosphere. When the modenumber is large, energy is unable to penetrate to the inner magnetosphere. Notably, the large values on the bottom row should be taken with a grain of salt; it's not clear that the model's results are reliable when waves are continuously forced against the boundary.

- 768 index, its values are effectively averaged around Earth. Unlike in situ measurements,
 769 however, this method has the advantage of estimating total driving current (and thus
 770 total energy input).
 771 In the results shown in Chapters 6 and 7, the driving current is sinusoidal, and delivered
 772 purely in the azimuthal direction (representing a perturbation to the magnitude of the

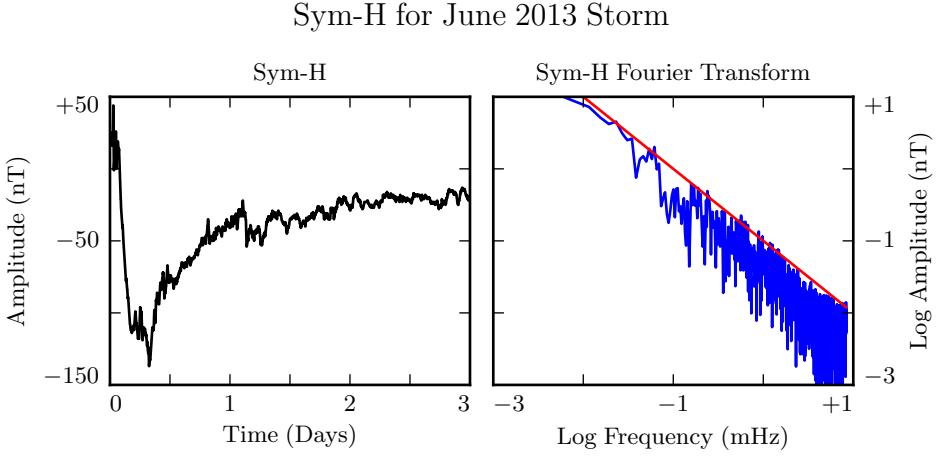


Figure 5.5: The Sym-H storm index[69] measures magnetic perturbations on Earth’s surface due to ring current activity. The amplitude of oscillations in the Pc4 range is estimated by fitting the pink noise in its Fourier transform.

773 ring current, but not to its direction). Tuna also allows several other driving waveforms,
774 and the direction of the current could be modified with trivial modifications to the code.

775 5.4 Maxwell’s Equations

776 Tuna simulates the evolution of electromagnetic waves in accordance with Ampère’s
777 law and Faraday’s law. Computation is carried out on a Yee grid[102]: electric fields
778 and magnetic fields are offset by half a time step, and each field component is defined
779 on either odd or even grid points in each dimension to ensure that curls are computed
780 using centered differences.

The Ohmic current in Ampère’s law is replaced with $\underline{\sigma} \cdot \underline{E}$ per Kirchhoff’s formulation of Ohm’s law. Then, taking \underline{J} to represent the driving current discussed in Section 5.3, Maxwell’s equations are written

$$\frac{\partial}{\partial t} \underline{B} = -\nabla \times \underline{E} \quad \underline{\epsilon} \cdot \frac{\partial}{\partial t} \underline{E} = \frac{1}{\mu_0} \nabla \times \underline{B} - \underline{J} - \underline{\sigma} \cdot \underline{E} \quad (5.12)$$

It is convenient to introduce shorthand for the curl of each field: $\underline{C} \equiv \nabla \times \underline{E}$ and $\underline{F} \equiv \nabla \times \underline{B} - \mu_0 \underline{J}$. Or, recalling Equation (5.6),

$$C^i = \frac{\varepsilon^{ijk}}{\sqrt{g}} \frac{\partial}{\partial u^j} E_k \quad F^i = \frac{\varepsilon^{ijk}}{\sqrt{g}} \frac{\partial}{\partial u^j} B_k - J^i \quad (5.13)$$

In these terms, Faraday's law can simply be written:

$$\frac{\partial}{\partial t} B^i = -C^i \quad (5.14)$$

Writing each component out explicitly, and using the metric tensor (per Equation (5.5)) to eliminate contravariant magnetic field components⁸, Equation (5.14) becomes:

$$\begin{aligned} B_1 &\leftarrow B_1 - g_{11} \delta t C^1 - g_{13} \delta t C^3 \\ B_2 &\leftarrow B_2 - g_{22} \delta t C^2 \\ B_3 &\leftarrow B_3 - g_{31} \delta t C^1 - g_{33} \delta t C^3 \end{aligned} \quad (5.15)$$

⁷⁸¹ Note that the \leftarrow operator is used in Equation (5.15) to indicate assignment, rather than
⁷⁸² equality. Terms on the left are new, while those on the right are old.

Unlike Faraday's law, Ampère's law cannot be trivially solved. Not only does the time derivative of \underline{E} depend on its own future value, but the crosswise and azimuthal components of the equation are coupled by the Hall terms in the conductivity tensor. Fortunately, the electric tensor can be inverted, allowing a solution by integrating factors:

$$\underline{\epsilon} \cdot \frac{\partial}{\partial t} \underline{E} = \frac{1}{\mu_0} \underline{F} - \underline{\sigma} \cdot \underline{E} \quad \text{becomes} \quad \left(\underline{\Omega} + \underline{\epsilon} \frac{\partial}{\partial t} \right) \cdot \underline{E} = \underline{\underline{V}}^2 \cdot \underline{F} \quad (5.16)$$

⁸Fields are stored only in terms of their covariant components, and curls in terms of their contravariant components. This reduces Tuna's memory footprint (since each field need not be stored twice) as well as its computational cost (since time need not be spent rotating between bases). As a result, Tuna's solutions to Maxwell's equations are linear expressions whose coefficients are a mishmash of physical constants and geometric factors. To save time, each coefficient is computed once and stored, rather than being multiplied out from scratch with each time step.

Where $\underline{\underline{\mathbb{I}}}$ is the identity tensor and in x - y - z coordinates⁹,

$$\underline{\underline{V}^2} \equiv \frac{1}{\mu_0} \underline{\underline{\epsilon}}^{-1} = \begin{bmatrix} v_A^2 & 0 & 0 \\ 0 & v_A^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} \quad \text{and} \quad \underline{\underline{\Omega}} \equiv \underline{\underline{\epsilon}}^{-1} \cdot \underline{\underline{\sigma}} = \begin{bmatrix} \frac{\sigma_P}{\epsilon_\perp} & \frac{-\sigma_H}{\epsilon_\perp} & 0 \\ \frac{\sigma_H}{\epsilon_\perp} & \frac{\sigma_P}{\epsilon_\perp} & 0 \\ 0 & 0 & \frac{\sigma_0}{\epsilon_0} \end{bmatrix} \quad (5.17)$$

Multiplying through by $\exp(\underline{\underline{\Omega}} t)$ and applying the product rule, Equation (5.16) becomes¹⁰

$$\frac{\partial}{\partial t} \left(\exp(\underline{\underline{\Omega}} t) \cdot \underline{\underline{E}} \right) = \exp(\underline{\underline{\Omega}} t) \cdot \underline{\underline{V}^2} \cdot \underline{\underline{F}} \quad (5.18)$$

Equation (5.18) can then be integrated over a small time step δt and expressed in terms of the assignment operator introduced in Equation (5.15).

$$\underline{\underline{E}} \leftarrow \exp(-\underline{\underline{\Omega}} \delta t) \cdot \underline{\underline{E}} + \delta t \exp(-\underline{\underline{\Omega}} \frac{\delta t}{2}) \cdot \underline{\underline{V}^2} \cdot \underline{\underline{F}} \quad (5.19)$$

The tensor exponential is evaluated by splitting $\underline{\underline{\Omega}}$ into the sum of its diagonal and Hall components¹¹. The Hall exponential condenses into sines and cosines, giving a rotation around the magnetic field line:

$$\underline{\underline{E}} \leftarrow \exp(-\underline{\underline{\Omega}}' \delta t) \cdot \underline{\underline{R}}_z \left(\frac{-\sigma_H \delta t}{\epsilon_\perp} \right) \cdot \underline{\underline{E}} + \delta t \underline{\underline{V}^2} \cdot \exp(-\underline{\underline{\Omega}}' \frac{\delta t}{2}) \cdot \underline{\underline{R}}_z \left(\frac{-\sigma_H \delta t}{2\epsilon_\perp} \right) \cdot \underline{\underline{F}} \quad (5.20)$$

Where

$$\underline{\underline{R}}_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \underline{\underline{\Omega}}' \equiv \begin{bmatrix} \frac{\sigma_P}{\epsilon_\perp} & 0 & 0 \\ 0 & \frac{\sigma_P}{\epsilon_\perp} & 0 \\ 0 & 0 & \frac{\sigma_0}{\epsilon_0} \end{bmatrix} \quad (5.21)$$

⁹Note the parallel component of the present definition of $\underline{\underline{\Omega}}$ differs slightly from that used in Chapter 4, due to the present chapter's neglect of inertial effects; see Chapter 6.

¹⁰Tensor exponentiation is analogous to scalar exponentiation[37]: $\exp(\underline{\underline{T}}) \equiv \sum_n \frac{1}{n!} \underline{\underline{T}}^n$.

¹¹For tensors, $\exp(\underline{\underline{S}} + \underline{\underline{T}}) = \exp(\underline{\underline{S}}) \exp(\underline{\underline{T}})$ as long as $\underline{\underline{S}} \cdot \underline{\underline{T}} = \underline{\underline{T}} \cdot \underline{\underline{S}}$.

The parallel component of term of Equation (5.20) is simply

$$E_z \leftarrow E_z \exp\left(\frac{-\sigma_0 \delta t}{\epsilon_0}\right) + c^2 \delta t F_z \exp\left(\frac{-\sigma_0 \delta t}{2\epsilon_0}\right) \quad (5.22)$$

Or, using Equation (5.7) to map to the covariant basis,

$$E_3 \leftarrow E_3 \exp\left(\frac{-\sigma_0 \delta t}{\epsilon_0}\right) + c^2 \delta t (g_{31} F^1 + g_{33} F^3) \exp\left(\frac{-\sigma_0 \delta t}{2\epsilon_0}\right) \quad (5.23)$$

783 Tuna's conductivity profile gives a minimum value of $\frac{\sigma_0 \delta t}{\epsilon_0}$ on the order of 10^3 , making
 784 $\exp\left(\frac{-\sigma_0 \delta t}{\epsilon_0}\right)$ far too small to be stored in a double precision variable¹². That is, this
 785 model takes E_3 (and, proportionally, E_z) to be uniformly zero. This issue is revisited
 786 in Chapter 6.

The perpendicular components of Equation (5.20) give the expressions:

$$\begin{aligned} E_1 + \frac{g^{13}}{g^{11}} E_3 &\leftarrow E_1 \cos\left(\frac{-\sigma_H \delta t}{\epsilon_\perp}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_\perp}\right) \\ &+ E_2 \sin\left(\frac{-\sigma_H \delta t}{\epsilon_\perp}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_\perp}\right) \sqrt{\frac{g^{22}}{g^{11}}} \\ &+ E_3 \cos\left(\frac{-\sigma_H \delta t}{\epsilon_\perp}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_\perp}\right) \frac{g^{13}}{g^{11}} \\ &+ F^1 \cos\left(\frac{-\sigma_H \delta t}{2\epsilon_\perp}\right) \exp\left(\frac{-\sigma_P \delta t}{2\epsilon_\perp}\right) \frac{v_A^2 \delta t}{g^{11}} \\ &+ F^2 \sin\left(\frac{-\sigma_H \delta t}{2\epsilon_\perp}\right) \exp\left(\frac{-\sigma_P \delta t}{2\epsilon_\perp}\right) \frac{v_A^2 \delta t}{\sqrt{g^{11} g^{22}}} \end{aligned} \quad (5.24)$$

¹²Not coincidentally, $\frac{\sigma_0}{\epsilon_0}$ can also be written $\frac{\omega_P^2}{\nu}$. At the ionosphere, the collision frequency ν is fast compared to field line resonance timescales, but it's still slow compared to the plasma frequency.

and

$$\begin{aligned}
E_2 \leftarrow & -E_1 \sin\left(\frac{-\sigma_H \delta t}{\epsilon_{\perp}}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_{\perp}}\right) \sqrt{\frac{g^{11}}{g^{22}}} \\
& + E_2 \cos\left(\frac{-\sigma_H \delta t}{\epsilon_{\perp}}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_{\perp}}\right) \\
& - E_3 \sin\left(\frac{-\sigma_H \delta t}{\epsilon_{\perp}}\right) \exp\left(\frac{-\sigma_P \delta t}{\epsilon_{\perp}}\right) \frac{g^{13}}{\sqrt{g^{11} g^{22}}} \\
& - F^1 \sin\left(\frac{-\sigma_H \delta t}{2\epsilon_{\perp}}\right) \exp\left(\frac{-\sigma_P \delta t}{2\epsilon_{\perp}}\right) \frac{v_A^2 \delta t}{\sqrt{g^{11} g^{22}}} \\
& + F^2 \cos\left(\frac{-\sigma_H \delta t}{2\epsilon_{\perp}}\right) \exp\left(\frac{-\sigma_P \delta t}{2\epsilon_{\perp}}\right) \frac{v_A^2 \delta t}{g^{22}}
\end{aligned} \tag{5.25}$$

787 The E_3 terms in Equations (5.24) and (5.25) can be ignored at present. They are
788 revisited in Chapter 6.

789 It bears recalling that the driving current is defined as part of \underline{F} , per Equation (5.13).
790 When the driving current is purely azimuthal ($J^1 = J^3 = 0$), the driving is in principle
791 applied to both the poloidal and the toroidal electric fields through F^2 . However,
792 in practice, the driving applied to the toroidal electric field is vanishingly small. The
793 driving current J^2 is localized around $5 R_E$ geocentric, and $\exp\left(\frac{-\sigma_P \delta t}{2\epsilon_{\perp}}\right)$ drops off quickly
794 with altitude.

795 5.5 Boundary Conditions

796 Dirichlet and Neumann boundary conditions are applied to the electric field components
797 and magnetic field components respectively. That is, electric fields are zero at the inner
798 and outer boundaries, and magnetic fields normal to the inner and outer boundaries are
799 zero.

800 These boundary conditions can in principle cause nonphysical reflections at the bound-
801 ary¹³. In practice, however (with a noted exception in Chapter 7), wave activity is
802 concentrated well within the simulation domain. Simulation results are robust under

¹³See, for example, the bottom row of Figure 5.4.

803 an exchange of Dirichlet and Neumann boundary conditions, though a self-inconsistent
 804 set of boundary conditions (such as applying Neumann conditions to B_1 but Dirichlet
 805 conditions to B_3) quickly causes instability.

806 The Earthward boundary conditions are as follows.

Between the top of the neutral atmosphere and the bottom of the ionosphere, the model includes a thin, horizontal current sheet representing the ionosphere's E layer[58]. By integrating Ampère's law over the layer, it can be shown[27] that the horizontal electric field values at the edge of the grid are determined by the jump in the horizontal magnetic fields:

$$\underline{\underline{\Sigma}} \cdot \underline{E} = \frac{1}{\mu_0} \lim_{\delta r \rightarrow 0} \left[\hat{r} \times \underline{B} \right]_{R_I - \delta r}^{R_I + \delta r} \quad (5.26)$$

The integrated conductivity tensor $\underline{\underline{\Sigma}}$ is written in θ - ϕ coordinates as[58]:

$$\underline{\underline{\Sigma}} \equiv \begin{bmatrix} \frac{\Sigma_0 \Sigma_P}{\Sigma_0 \cos^2 \alpha + \Sigma_P \sin^2 \alpha} & \frac{-\Sigma_0 \Sigma_H}{\Sigma_0 \cos^2 \alpha + \Sigma_P \sin^2 \alpha} \\ \frac{\Sigma_0 \Sigma_H}{\Sigma_0 \cos^2 \alpha + \Sigma_P \sin^2 \alpha} & \Sigma_P + \frac{\Sigma_H^2 \sin^2 \alpha}{\Sigma_0 \cos^2 \alpha + \Sigma_P \sin^2 \alpha} \end{bmatrix} \quad (5.27)$$

807 Where α is the angle between the magnetic field and the vertical direction, given by
 808 $\cos \alpha \equiv \frac{-2 \cos \theta}{\sqrt{1+3 \cos^2 \theta}}$, and Σ_P , Σ_H , and Σ_0 are the height-integrated Pedersen, Hall,
 809 and parallel conductivities respectively. Their values are determined by integrating
 810 Kelley's[47] conductivity profiles from Earth's surface to the ionospheric boundary; val-
 811 ues are shown in Table 5.2.

Table 5.2: Integrated Atmospheric Conductivity (S)

	Σ_0	Σ_P	Σ_H
Active Day	424	0.65	6.03
Quiet Day	284	0.44	4.02
Active Night	9	0.01	0.12
Quiet Night	9	0.01	0.12

812 The atmospheric conductivities of the two nightside profiles are the same, though the
 813 profiles differ significantly at higher altitudes, as shown in Section 5.2.

An expression for the horizontal electric fields at the boundary is obtained by inverting Equation (5.26). After mapping to covariant coordinates per Equation (5.8), and taking $\Sigma \equiv \det \underline{\underline{\Sigma}}$,

$$\begin{aligned} E_1 &\leftarrow \frac{1}{\mu_0 \Sigma} \lim_{\delta r \rightarrow 0} \left[-\Sigma_{\theta\phi} B_1 - \sqrt{\frac{g_{11}}{g_{22}}} \Sigma_{\phi\phi} B_2 \right]_{R_I-\delta r}^{R_I+\delta r} \\ E_2 &\leftarrow \frac{1}{\mu_0 \Sigma} \lim_{\delta r \rightarrow 0} \left[\sqrt{\frac{g_{22}}{g_{11}}} \Sigma_{\theta\theta} B_1 + \Sigma_{\phi\theta} B_2 \right]_{R_I-\delta r}^{R_I+\delta r} \end{aligned} \quad (5.28)$$

814 The atmospheric field is computed in terms of a scalar magnetic potential, Ψ , such
 815 that $\underline{B} = \nabla \Psi$. The neutral atmosphere is considered to be a perfect insulator, giving
 816 $\nabla \times \underline{B} = 0$. Combined with $\nabla \cdot \underline{B} = 0$ (per Maxwell's equations), this ensures that Ψ
 817 satisfies Laplace's equation, $\nabla^2 \Psi = 0$, and thus can be written as a sum of harmonics.

Laplace's equation can be solved analytically; however, a numerical solution is preferable to ensure orthonormality on a discrete and incomplete¹⁴ grid. After separating out the radial and azimuthal dependence in the usual way, the latitudinal component of Laplace's equation is written in terms of $s \equiv -\sin^2 \theta$:

$$(4s^2 + 4s) \frac{d^2}{ds^2} Y_\ell + (4 + 6s) \frac{d}{ds} Y_\ell - \frac{m^2}{s} Y_\ell = \ell (\ell + 1) Y_\ell \quad (5.29)$$

Using centered differences to linearize the derivatives, Equation (5.29) becomes a system of coupled linear equations, one per field line. It is then solved numerically for eigenvalues $\ell (\ell + 1)$ and eigenfunctions Y_ℓ ¹⁵. In terms of the harmonics Y_ℓ , the magnetic potential between the Earth's surface and the top of the atmosphere is written using

¹⁴As discussed in Section 5.1, the grid is constrained to finite L , which excludes the equator as well as the poles.

¹⁵Solving Laplace's equation analytically results in spherical harmonics indexed by both ℓ and m , the separation constants for θ and ϕ respectively. In two and a half dimensions, ϕ is not explicitly resolved, so m is set manually.

coefficients a_ℓ and b_ℓ :

$$\Psi = \sum_\ell \left(a_\ell r^\ell + b_\ell r^{-\ell-1} \right) Y_\ell \quad (5.30)$$

As a boundary condition for Ψ , Earth is assumed to be a perfect conductor. This forces the magnetic field at Earth's surface to be horizontal; that is, $B_r = \frac{\partial}{\partial r} \Psi = 0$. Noting that solutions to Laplace's equation are orthonormal, each element of the sum in Equation (5.30) must be independently zero at R_E . This allows the coefficients a_ℓ and b_ℓ to be expressed in terms of one another.

$$b_\ell = \frac{\ell}{\ell+1} R_E^{2\ell+1} a_\ell \quad (5.31)$$

The current sheet at the top of the atmosphere is assumed to be horizontal, so the radial component of the magnetic field must be the same just above and just below it. Taking the radial derivative of Equation (5.30) at the top of the atmosphere, and eliminating b_ℓ with Equation (5.31), gives

$$B_r = \sum_\ell \ell a_\ell R_I^{\ell-1} \left(1 - \lambda^{2\ell+1} \right) Y_\ell \quad \text{where} \quad \lambda \equiv \frac{R_E}{R_I} \sim 0.975 \quad (5.32)$$

The summation is collapsed by “integrating” over a harmonic¹⁶. Inverse harmonics are obtained by inverting the eigenvector matrix. Then $Y_\ell \cdot Y_{\ell'}^{-1} = \delta_{\ell\ell'}$ by construction.

$$a_\ell = \frac{1}{\ell R_I^{\ell-1}} \frac{B_r \cdot Y_\ell^{-1}}{1 - \lambda^{2\ell+1}} \quad (5.33)$$

Combining Equations (5.30), (5.31), and (5.33) allows the expression of Ψ at the top and bottom of the atmosphere as a linear combination of radial magnetic field components

¹⁶Because the functions are defined on a discrete grid, their inner product is not an integral, but a sum: $B_r \cdot Y_\ell^{-1} \equiv \sum_i B_r[i] Y_\ell^{-1}[i]$.

at the bottom of the ionosphere.

$$\begin{aligned}\Psi_E &= \sum_{\ell} Y_{\ell} \frac{R_I}{\ell (\ell - 1)} \frac{(2\ell - 1) \lambda^{\ell}}{1 - \lambda^{2\ell+1}} B_r \cdot Y_{\ell}^{-1} \\ \Psi_I &= \sum_{\ell} Y_{\ell} \frac{R_I}{\ell (\ell - 1)} \frac{(\ell - 1) + \ell \lambda^{2\ell+1}}{1 - \lambda^{2\ell+1}} B_r \cdot Y_{\ell}^{-1}\end{aligned}\tag{5.34}$$

Horizontal magnetic fields are obtained by taking derivatives of Ψ .

$$B_1 = \frac{\partial}{\partial u^1} \Psi \quad B_2 = \frac{\partial}{\partial u^2} \Psi\tag{5.35}$$

- 818 Horizontal magnetic field values at the top of the atmosphere are used to impose bound-
- 819 ary conditions on the electric fields at the bottom of the ionosphere, per Equation (5.28).
- 820 Those at Earth's surface are valuable because they allow a direct comparison between
- 821 model output and ground magnetometer data, after being mapped to physical coordi-
- 822 nates per Equation (5.8).

823 **Chapter 6**

824 **Electron Inertial Effects**

825 As laid out in Chapter 5, Tuna resolves neither currents nor electric fields parallel to
826 the background magnetic field. This is unfortunate; parallel electric fields generated by
827 kinetic and inertial Alfvén waves (including fundamental field line resonances[77, 93])
828 are a topic of particular interest in the study of auroral particle precipitation.

Parallel currents and electric fields can be added to Tuna through the consideration of electron inertial effects in Ohm's law:

$$0 = \sigma_0 E_z - J_z \quad \text{becomes} \quad \frac{1}{\nu} \frac{\partial}{\partial t} J_z = \sigma_0 E_z - J_z \quad (6.1)$$

With the addition of the electron inertial term, it is necessary to track the parallel current independent of the parallel electric field¹. Solving by integrating factors² gives

$$J_3 \leftarrow J_3 \exp(-\nu \delta t) + \delta t \frac{ne^2}{m_e} E_3 \exp(-\nu \frac{\delta t}{2}) \quad (6.2)$$

¹The parallel current J_z is defined on the same points of the Yee grid as E_z . It is offset in time by half of a time step.

²The integrating factors technique is laid out explicitly for Ampère's law in Section 5.4.

From there, the parallel electric field can be updated directly; Equation (5.23) is replaced by the following:

$$E_3 \leftarrow E_3 + c^2 \delta t (g_{31} F^1 + g_{33} F^3) - \frac{\delta t}{\epsilon_0} J_3 \quad (6.3)$$

829 The present section explores the complications that arise from the addition of the elec-
 830 tron inertial term to Ohm's law, as well as a few results that may be gleaned despite
 831 those complications. Notably — for reasons discussed in Section 6.3 — the results
 832 presented in Chapter 7 do not make use of the effects of electron inertia.

833 Inertial effects have been considered in previous numerical work, such as by Lysak and
 834 Song in 2001[60], but never at the global scale. That work considered waves in the
 835 ionospheric Alfvén resonator, with frequencies of hundreds of mHz, and did not account
 836 for the effects of the dipolar geometry. In fact, in that work, circular polarization
 837 (essentially a superposition of poloidal and toroidal modes) was noted to be a promising
 838 avenue for future work.

839 6.1 The Boris Factor

With the addition of the electron inertial term, a cyclical dependence appears between Ampère's law and Ohm's law:

$$\frac{\partial}{\partial t} E_z \sim -\frac{1}{\epsilon_0} J_z \quad \text{and} \quad \frac{\partial}{\partial t} J_z \sim \frac{ne^2}{m_e} E_z \quad \text{so} \quad \frac{\partial^2}{\partial t^2} E_z \sim -\omega_P^2 E_z \quad (6.4)$$

840 That is, electron inertial effects come hand in hand with the plasma oscillation.
 841 As mentioned in Chapter 4 and shown in Figure 6.1, plasma oscillation is quite fast —
 842 several orders of magnitude smaller than Tuna's time step as determined in Section 5.1
 843 ($\sim 10 \mu\text{s}$). This poses a conundrum. At Tuna's usual time step, the plasma oscillation
 844 becomes unstable within seconds³. On the other hand, reducing the time step by three

³For stability, $\omega_P \delta t < 1$ is necessary.

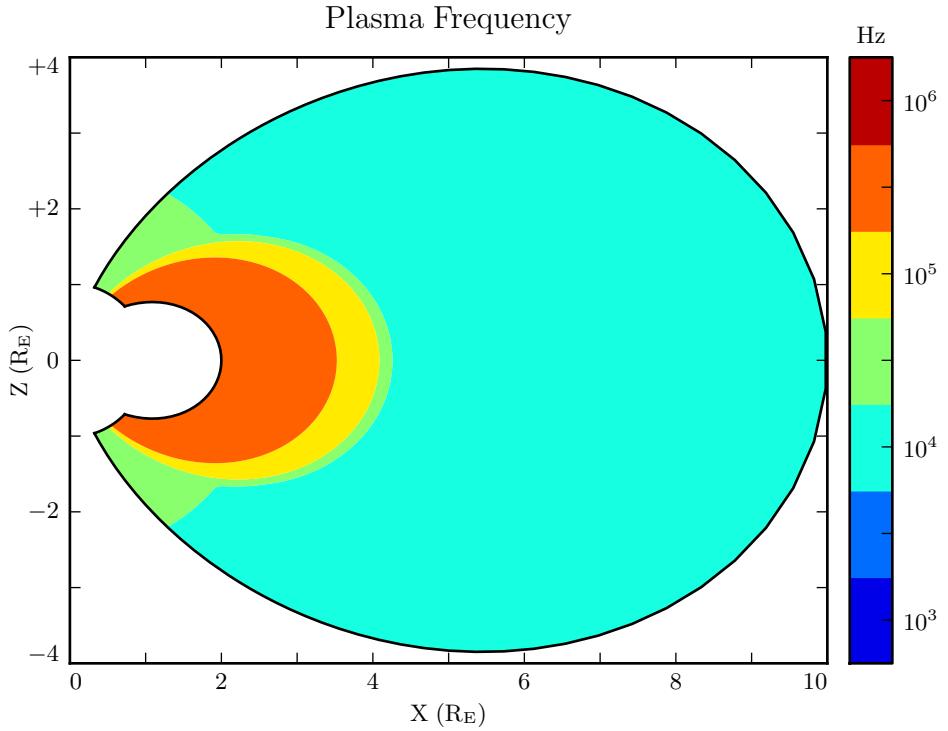


Figure 6.1: The plasma frequency reaches a peak value just under 10^6 Hz near the equator. Outside the plasmasphere, its value is closer to 10^4 Hz, which is still not well-resolved by Tuna's usual time step.

845 orders of magnitude to resolve the plasma oscillation is computationally infeasible; a
 846 run slated for an hour would require six weeks to complete.

847 As it happens, this problem can be solved by artificially increasing the parallel electric
 848 constant above its usual value of ϵ_0 . Doing so lowers both the speed of light and
 849 the plasma frequency within the simulation. This technique — and others like it —
 850 has been widespread in numerical modeling since it was presented by Boris in 1970[6].
 851 The following paraphrases an argument by Lysak and Song[60], outlining its validity
 852 specifically in the case of electron inertial effects.

Supposing that the current and electric field are oscillating at frequency ω , the parallel components of Ampère's law and Ohm's law can be written

$$-i\omega\epsilon_0 E_z = \frac{1}{\mu_0} (\nabla \times \underline{B})_z - J_z \quad -\frac{i\omega}{\nu} J_z = \sigma_0 E_z - J_z \quad (6.5)$$

Then, eliminating the current, the parallel electric field can be related to the curl of the magnetic field by⁴

$$\left(1 - \frac{\omega^2 - i\nu\omega}{\omega_P^2}\right) E_z = \frac{c^2}{\omega_P^2} (\nu - i\omega) (\nabla \times \underline{B})_z \quad (6.6)$$

- 853 In Equation (6.6), $\frac{c}{\omega_P}$ is the electron inertial length. While the speed of light and the
- 854 plasma frequency each depend on ϵ_0 , their ratio does not. This allows an estimation of
- 855 how much the model should be affected by an artificially-large electric constant (and
- 856 thus an artificially-small plasma frequency). So long as $\frac{\omega^2 - i\nu\omega}{\omega_P^2}$ remains small compared
- 857 to unity, the model should behave physically.
- 858 For waves with periods of a minute or so, even perhaps-implausibly large Boris factors
- 859 are allowed; for example, increasing ϵ_0 by a factor of 10^6 gives $\left|\frac{\omega^2 + i\omega\nu}{\omega_P^2}\right| \lesssim 0.01$.

860 6.2 Parallel Currents and Electric Fields

- 861 As discussed in Section 4.4, parallel electric fields in this regime are expected to be at
- 862 least six orders of magnitude smaller than the perpendicular electric fields. Numerical
- 863 results show general agreement: in Figure 6.2, the parallel electric field appears compa-
- 864 rable to its perpendicular counterparts only after its been scaled up by a factor of 10^6 .

865

- 866 As such, the inclusion of electron inertial effects does not appreciably impact the sim-
- 867 ulation's gross behavior. In Faraday's law, $\nabla \times \underline{E}$ is unaffected, to the extent that
- 868 side-by-side magnetic field snapshots with and without electron inertial effects are not
- 869 visibly distinguishable (not shown). In a sense, this is reassuring. It ensures that the

⁴From Equation (4.4), $c^2 \equiv \frac{1}{\mu_0\epsilon_0}$ and $\sigma_0 \equiv \frac{ne^2}{m_e\nu}$ and $\omega_P^2 \equiv \frac{ne^2}{m_e\epsilon_0}$.

Electric Field Snapshots: Quiet Day, 16mHz Current, $m = 16$

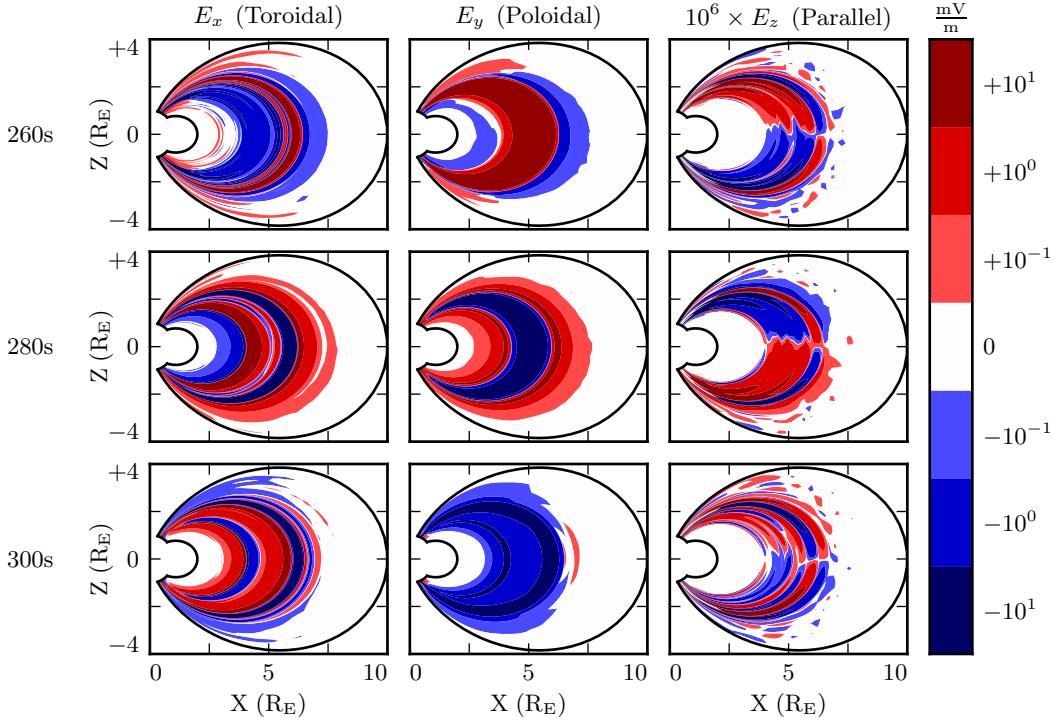


Figure 6.2: Parallel electric fields are smaller than perpendicular electric fields by about six orders of magnitude. As a result, parallel electric fields do not contribute significantly to $\nabla \times \underline{E}$ in Faraday's law.

870 present section does not cast doubt on the results presented in Chapter 7, where electron
 871 inertial effects are neglected.

872 Even if there is no significant feedback through Faraday's law, it's informative to con-
 873 sider the structures that arise in parallel currents and electric fields driven by pertur-
 874 bations in the ring current. For example, in Figure 6.2, the parallel electric field per-
 875 turbation (with maxima near the ionosphere) exhibits the opposite harmonic structure
 876 to the perpendicular electric field components (which peak near the equator).

877 Figure 6.3 shows how parallel currents lines up with the Poynting flux over time. Four
 878 runs are shown, one per row. The horizontal axis is time, and the vertical axis is latitude.
 879 The real and imaginary components of the parallel current are shown in the first and

third columns respectively, while the second and fourth columns show the poloidal and toroidal Poynting flux. Values are taken at an altitude of 1000 km.

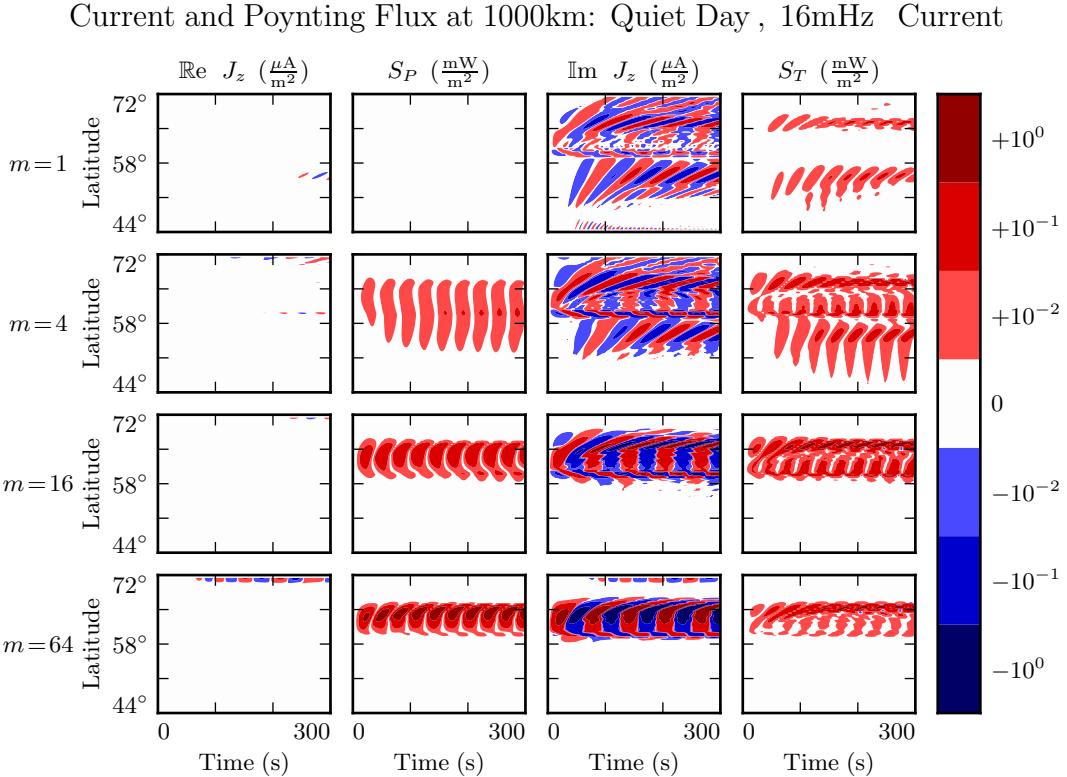


Figure 6.3: Parallel current and Poynting flux is shown for four runs, one per row, measured at an altitude of 1000 km. The parallel current is overwhelmingly imaginary, which implies a connection to the toroidal mode. Appropriately enough, the structure of the parallel current (particularly at low modenumber) seems to resemble the structure of the toroidal mode more than it does that of the poloidal mode. This is likely because the toroidal mode, with its sharp gradients across L -shells, dominates $(\nabla \times \underline{B})_z$.

Poloidal and toroidal fields are overwhelmingly real and imaginary respectively, because they are separated from one another by an azimuthal derivative (which carries a factor of i). However, when a wave's polarization is rotated by the Hall conductivity, there is no accompanying rotation in the complex plane — this gives rise to an imaginary component of the poloidal wave and a real component of the toroidal wave.

887 In Figure 6.3, the ionospheric conductivity is small, so the imaginary component of the
888 parallel current dominates. This implies a connection between the parallel current and
889 the toroidal mode, and indeed, the two do exhibit qualitatively similarities. At $m = 4$
890 in particular, the poloidal and toroidal Poynting fluxes are similar in strength much of
891 the time, yet the form of the parallel current strongly resembles that of the toroidal
892 Poynting flux over the poloidal.

893 The toroidal mode's dominant effect on the parallel current at small m is not surprising.
894 As shown in Figure 6.2, toroidal waves vary sharply in L^5 . When the poloidal and
895 toroidal magnetic fields are comparable in magnitude, $\frac{\partial}{\partial x} B_y$ typically exceeds $\frac{\partial}{\partial y} B_x$ (at
896 least for $m \lesssim 32$).

897 Whereas the imaginary component of the parallel current corresponds to that carried
898 into the ionosphere by Alfvén waves, its real component comes from electric fields rotated
899 by the Hall conductivity. Figure 6.4 shows the same four runs as Figure 6.3, but
900 measured at 100 km, the Earthward boundary of the simulation. At that point, the real
901 and imaginary components are similar in magnitude.

902 In Figure 6.4, as in Figure 6.3, the imaginary component of the parallel current prefer-
903entially follows the toroidal Poynting flux. This is particularly apparent at $m = 16$,
904 where the poloidal Poynting flux is clearly stronger, yet the structure of the imaginary
905 current resembles that of the toroidal Poynting flux. The real parallel current, on the
906 other hand, appears to follow the poloidal Poynting flux.

907 Put another way, low- m poloidal waves seem to primarily give rise to field-aligned
908 currents only after being rotated to the toroidal mode by the Hall conductivity. At
909 high modenumber, the two modes contribute comparably to the formation of parallel
910 currents.

It's also possible to consider the effect of parallel currents and electric fields on the
ionosphere's energy budget, per Poynting's theorem:

$$\frac{\partial}{\partial t} u = -\nabla \cdot \underline{E} - \underline{J} \cdot \underline{E} \quad (6.7)$$

⁵The sharp definition in L of the toroidal mode compared to the poloidal mode is also the topic of significant discussion in Chapter 7.

Current and Poynting Flux at 100km: Quiet Day , 16mHz Current

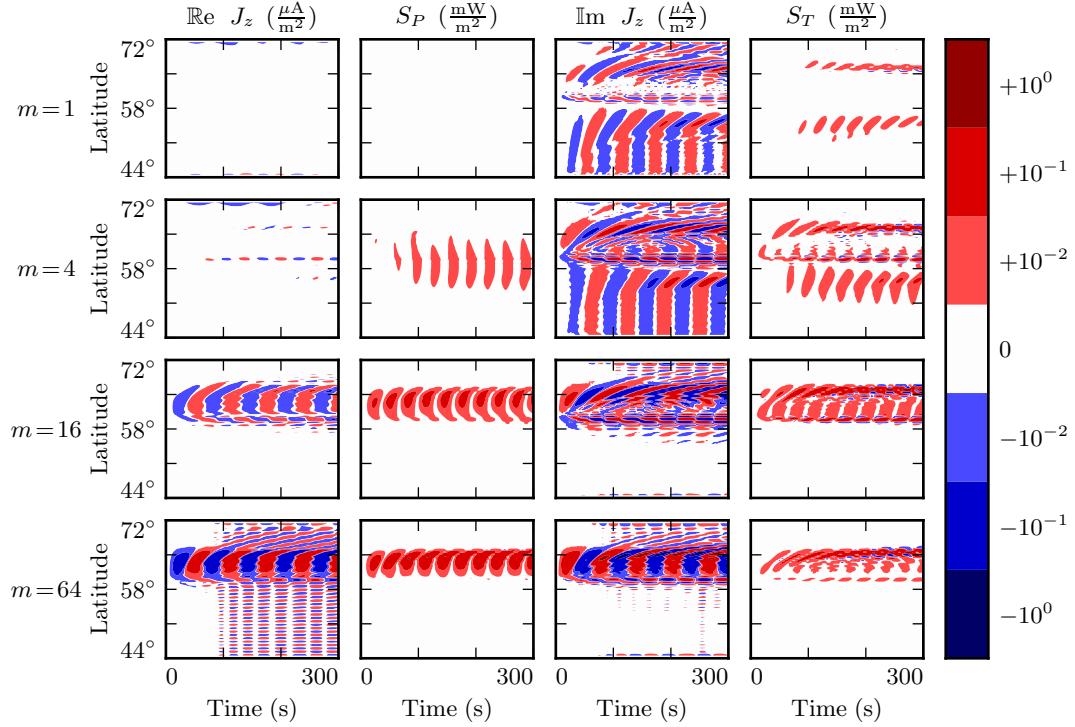


Figure 6.4: The above slices are taken from the same runs shown in Figure 6.3, but at an altitude of 100 km instead of 1000 km. The primary difference between the two altitudes is the strength of the ionospheric Hall conductivity, which directly couples the poloidal and toroidal modes. The Hall-rotated fields give rise to a real component of the parallel current, the structure of which follows the poloidal Poynting flux (as it rotates to the toroidal mode).

911 The magnitude of the parallel current tops out over $1 \mu\text{A}/\text{m}^2$, just shy of the up-to-tens
 912 of $\mu\text{A}/\text{m}^2$ inferred from ground observations and seen in situ[8, 45, 80]. However, this
 913 current is not a significant contributor to ionospheric Joule dissipation. As shown in
 914 Figure 6.5, the energy deposited in the ionosphere by the Poynting flux matches closely
 915 with the energy lost to Joule dissipation — as it should, to conserve energy. But,
 916 according to the model, Pedersen and Hall are dominant. The parallel component of
 917 $\underline{J} \cdot \underline{E}$ is smaller by several orders of magnitude.

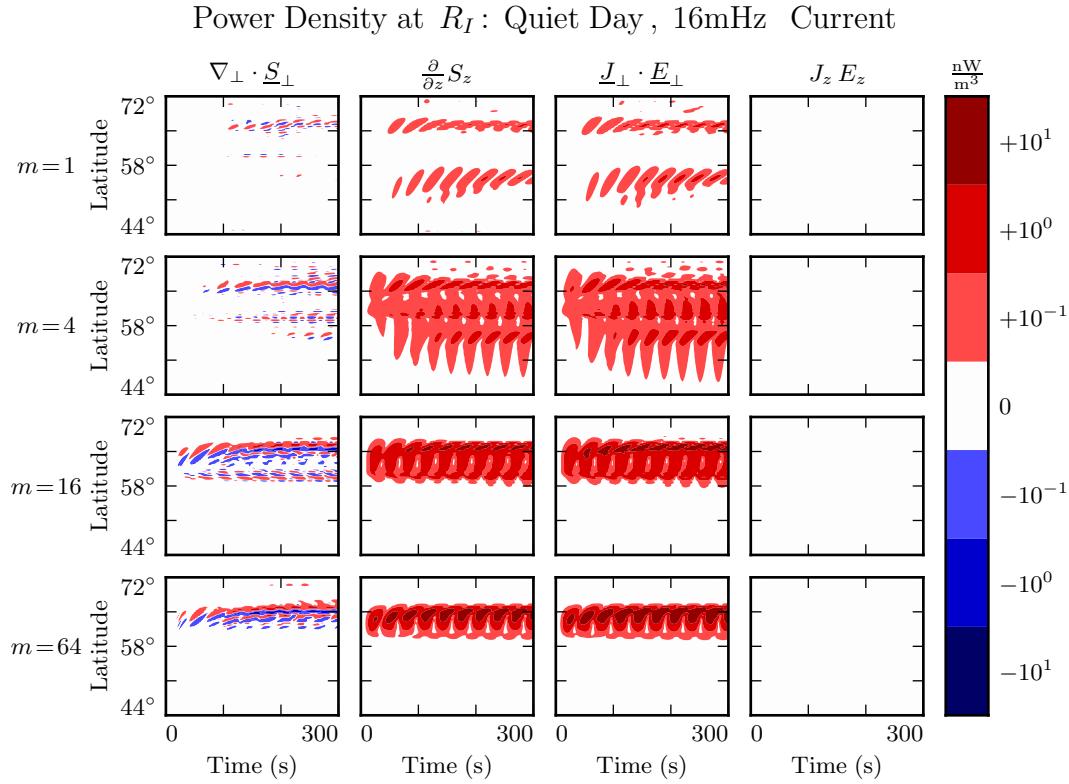


Figure 6.5: While field-aligned currents can be of significant size, they are not particularly good at depositing energy in the ionosphere. Energy deposited by the Poynting flux matches closely with Joule dissipation from the perpendicular currents, while $J_z E_z$ is smaller by several orders of magnitude.

918 6.3 Inertial Length Scales

- 919 It's not quite fair to compare the parallel and perpendicular contributions to $\nabla \times \underline{E}$ as
920 is done in Section 6.2. Perpendicular electric fields are on the order of 1 mV/m, with
921 wavelengths on the order of 10^5 km; they cause magnetic fields to change at a rate of
922 around 0.1 nT/s. Parallel electric fields, closer to 10^{-6} mV/m, would need to vary over
923 length scales of 0.1 km to match with that.
- 924 Such scales are believable. The characteristic length scale of the plasma oscillation is the
925 electron inertial length, $\frac{c}{\omega_P}$, which is on the order of 1 km in the auroral ionosphere and

926 0.1 km in the low-altitude plasmasphere. However, Tuna’s grid out bottoms out closer
 927 to 10 km. That is, with the inclusion of electron inertial effects, the grid is too coarse
 928 to resolve all of the waves expected to be present. The model is prone to instability as
 929 a result — for example, “wiggles” are visible in the bottom-left subplot of Figure 6.4.

930 Figure 6.6 shows a run with perpendicular resolution smaller than the electron inertial
 931 length, side by side with an analogous run on the typical grid described in Chapter 5. In
 932 order to carry out the inertial-scale run, several concessions were made to computational
 933 cost. The run simulates only a duration of 100 s (other figures in the present chapter,
 934 and those in Chapter 7, show 300 s), and the grid covers only the auroral latitudes from
 935 $L = 5$ to $L = 7$.

Parallel Electric Fields: Quiet Day, 100s of 16mHz Current, $m = 16$

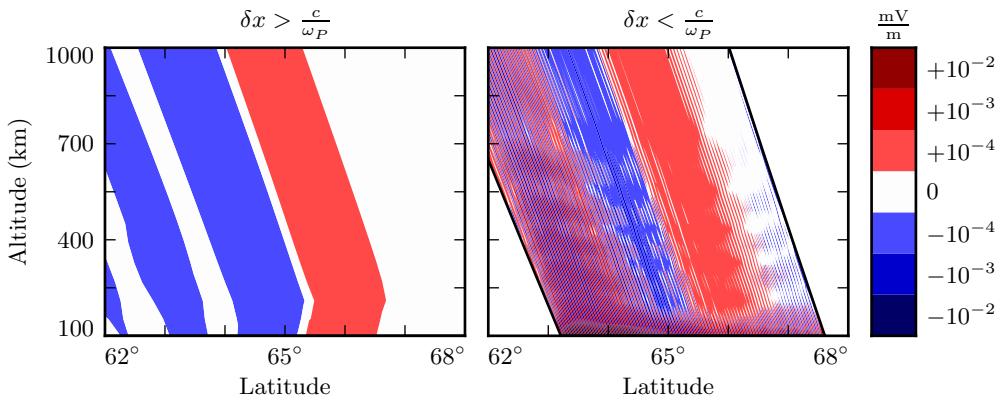


Figure 6.6: The parallel electric field develops significant structure when the perpendicular grid resolution is smaller than the electron inertial length. Unfortunately, such runs are prohibitively expensive. The subplot on the right — which still fails to resolve wave structure properly — represents a 100-fold increase in computational time compared to that on the left.

936 Even so, the run presents a significant computational expense. Spread over 16 cores,
 937 a 100 s run on Tuna’s usual grid takes well under an hour. The inertial-scale run
 938 barely finished in 96 hours, the cutoff at the Minnesota Supercomputing Institute. This
 939 is because runtime goes as the inverse square of grid resolution; not only does finer

resolution require more grid cells, but it also gives rise to proportionally smaller crossing times, imposing a smaller time step.

The snapshot shown in Figure 6.6 uses a perpendicular grid resolution of 0.7 km at the Earthward edge, which just satisfies the Nyquist rate for the minimum inertial length of 1.7 km. It’s still too coarse. There is clearly some small-scale structure developing in the ionosphere, but it’s not well resolved. The large number of “wiggles” portends an imminent crash.

6.4 Discussion

The present chapter is a proof of concept: the addition of electron inertial effects to Tuna presents a promising first-principles-based approach to the investigation of parallel currents and electric fields associated with field line resonances. Electric fields arise which are consistent in magnitude with those predicted by the dispersion relation, and parallel currents fall within an order of magnitude or so of observed values, even when inertial length scales are not properly resolved.

Results in Section 6.2 suggest a disparity between low- m poloidal and toroidal FLRs in terms of the parallel current response. At low altitude, where the two modes are directly coupled by the Hall conductivity, both seem to be accompanied by parallel currents. However, in regions of low Hall conductivity, parallel currents appear to preferentially accompany toroidal waves. This is likely a result of the toroidal mode’s sharp gradients across L -shells.

Future work could consider the relationship between the dynamic height-integrated potentials and the accompanying parallel currents, specifically with respect to the Knight relation[50]. Inertial effects could also be accompanied by test particles, in order to gauge the precipitation that would be expected to accompany global Alfvénic potential structures.

Unfortunately, simulations are prone to instability when inertial length scales are not properly resolved. And, at least at present, resolving those scales poses a prohibitive computational expense. For this reason, the consideration of inertial effects is limited to

968 the present chapter; results in Chapter 7 make use of the core version of Tuna presented
969 in Chapter 5, which does not include the effects of electron inertia.

970 Notably, the addition or omission of parallel currents and electric fields does not appear
971 to significantly alter the behavior of perpendicular electric fields or magnetic fields.
972 Because the parallel electric fields are relatively small, $\nabla \times \underline{E}$ is essentially unaffected
973 by their inclusion. Joule dissipation from parallel currents also does not seem to be a
974 significant in comparison to that from Pedersen and Hall currents.

₉₇₅ **Chapter 7**

₉₇₆ **Numerical Results**

₉₇₇ A primary motivation for the development of Tuna is the fact that FLRs vary in interesting ways as a function of azimuthal modenumber, and that existing numerical models
₉₇₈ are limited in their ability to examine such behaviors. The present chapter discusses
₉₇₉ the core results that the model has so far produced.
₉₈₀

₉₈₁ **7.1 Modenumber and Compression**

₉₈₂ It's well known that the poloidal FLR mode is compressional at low modenumber, but
₉₈₃ guided at high modenumber. However, the relationship is not well quantified. Theoretical
₉₈₄ work has historically been concerned with the limits $m \rightarrow 0$ and $m \rightarrow \infty$ [14, 76], and
₉₈₅ only a handful of satellite observations have explicitly considered an event's azimuthal
₉₈₆ modenumber[17, 68, 88]. Using results from Tuna, the present section examines the
₉₈₇ strength of the poloidal wave's compressional component at an ensemble of finite mod-
₉₈₈ enumbers.

₉₈₉ Figures 7.1 and 7.2 show magnetic field snapshots taken from a pair of runs; the first
₉₉₀ uses a small azimuthal modenumber, and the second uses a large one. The runs are
₉₉₁ otherwise identical: both simulations use the quiet dayside ionospheric profile, and both
₉₉₂ are driven at 22 mHz.

993 The differences between the two runs are striking. At low modenumber, wave activity
994 is visible throughout the simulation domain. Structure in the poloidal magnetic field is
995 only vaguely governed by the dipole geometry, and the compressional magnetic field is
996 comparably strong to the two perpendicular components.

997 In contrast, at high modenumber, the poloidal magnetic field is localized to $L \sim 5$, where
998 the driving is delivered. The compressional field is weaker than the poloidal field by
999 at least an order of magnitude. A third-harmonic poloidal mode is visible at the outer
1000 boundary — its magnitude is just barely large enough to be visible on the logarithmic
1001 scale. The gap between $L \sim 5$ (where 22 mHz matches a first-harmonic FLR) and
1002 $L \sim 10$ (where 22 mHz matches a third-harmonic FLR) speaks to the evanescence of
1003 non-guided waves above the compressional Alfvén cutoff frequency¹.

1004 In both the low- m and high- m runs, toroidal activity is more or less coincident with
1005 poloidal activity — as is to be expected, since the driving is purely poloidal, and so
1006 the poloidal mode must be the source of the toroidal mode. It is further notable that
1007 the toroidal mode is sharply guided. Particularly in Figure 7.2, strong, narrow, toroidal
1008 FLRs of opposite phase can be seen oscillating very close to one another. Strong poloidal
1009 waves, in contrast, are smeared in L .

1010 Snapshots are not shown for runs carried out using the other ionospheric profiles (active
1011 day, quiet night, and active night). The morphology of their waves is qualitatively
1012 similar. The differences between the profiles is considered in Sections 7.2 to 7.4.

1013 Figure 7.3 quantifies the compressional component of the poloidal mode as a function of
1014 modenumber. Each subplot corresponds to a different run of Tuna; the runs shown in
1015 Figures 7.1 and 7.2 are in the rightmost column, second from the top and second from the
1016 bottom respectively. The red line indicates the ratio between the RMS compressional
1017 magnetic field and the RMS poloidal magnetic field; both averages are taken over the
1018 entire simulation “volume” each time step. Mean values are shown in black.

1019 At $m = 1$, the compressional and poloidal magnetic fields are comparable in magnitude.
1020 As m increases, however, the compressional component quickly falls off. The compressional
1021 component is half the strength of the poloidal component at $m \sim 5$, and a quarter

¹See Section 4.4.

1022 by $m \sim 10$. Similar behavior is seen using the active dayside and active nightside pro-
1023 files (not shown). On the quiet nightside (not shown), the compressional component of
1024 the poloidal mode does not fall off quite as sharply; $\left| \frac{B_z}{B_x} \right|$ falls to 50 % at $m \sim 8$ and to
1025 25 % at $m \sim 16$.

1026 A slight frequency dependence is apparent across each row in Figure 7.3. Compressional
1027 coupling falls off slower for waves at higher frequency. This is because higher-frequency
1028 waves are that much closer to the cutoff frequency, and so their propagation across
1029 L -shells is that much less evanescent.

1030 Notably, the waves considered in the present work are fundamental harmonics. The
1031 compressional behavior of the poloidal mode may vary for the (more-common) second
1032 harmonic: Radoski suggests that the asymptotic value of $\left| \frac{B_z}{B_x} \right|$ is inversely proportional
1033 to the harmonic number[76].

Magnetic Field Snapshots: Quiet Day , 22mHz Current, $m = 2$

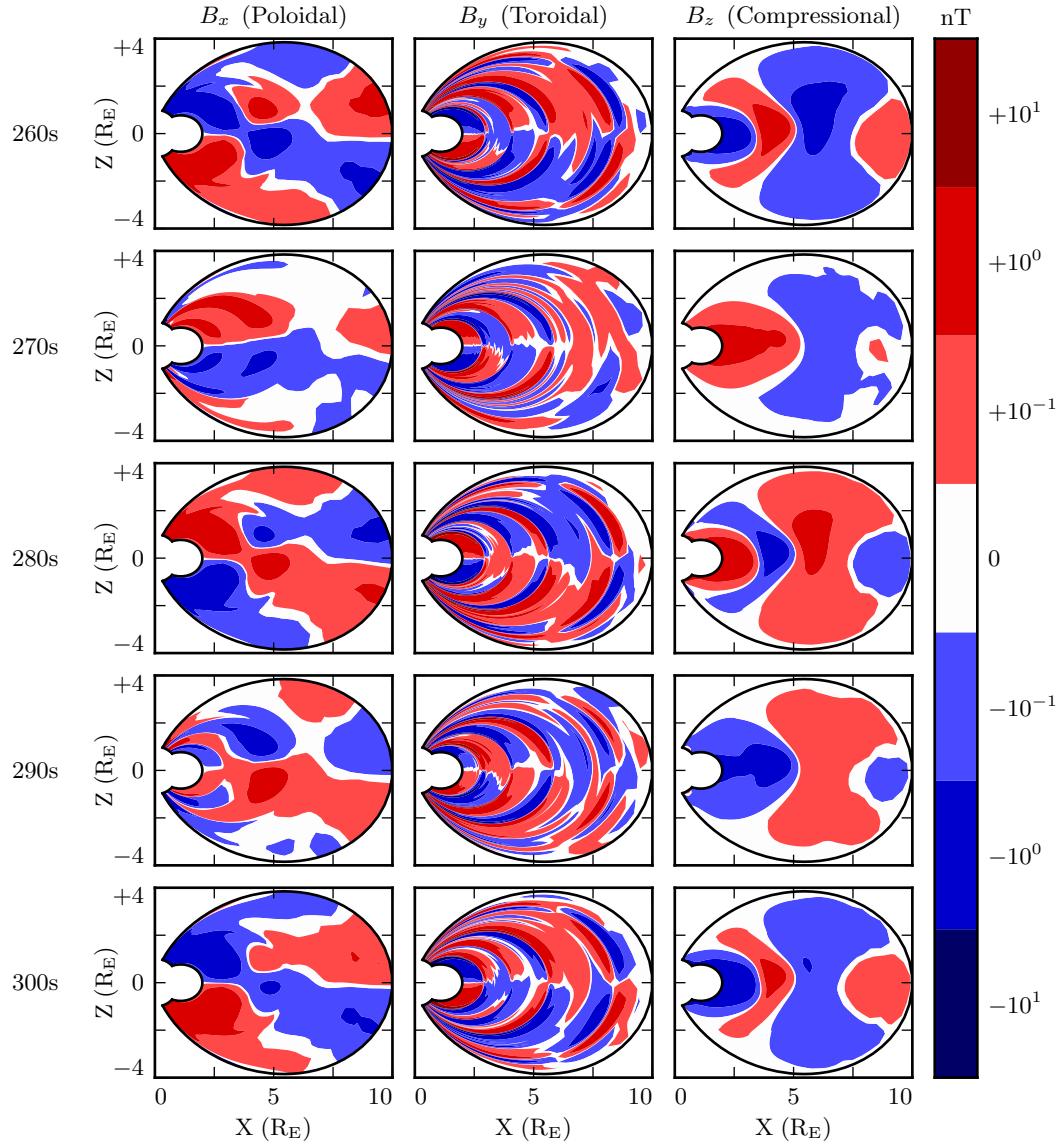


Figure 7.1: Each row in the above figure is a snapshot in time. The three columns show the simulated poloidal, toroidal, and compressional magnetic fields. Due to the run's low azimuthal modenumber, the poloidal mode has a significant compressional component. This is visible both in the fact that B_z is comparable in size to B_x , and in that structure in B_x is only vaguely guided by the geometry of the magnetic field. Toroidal waves, in contrast, are sharply guided.

Magnetic Field Snapshots: Quiet Day , 22mHz Current, $m = 32$

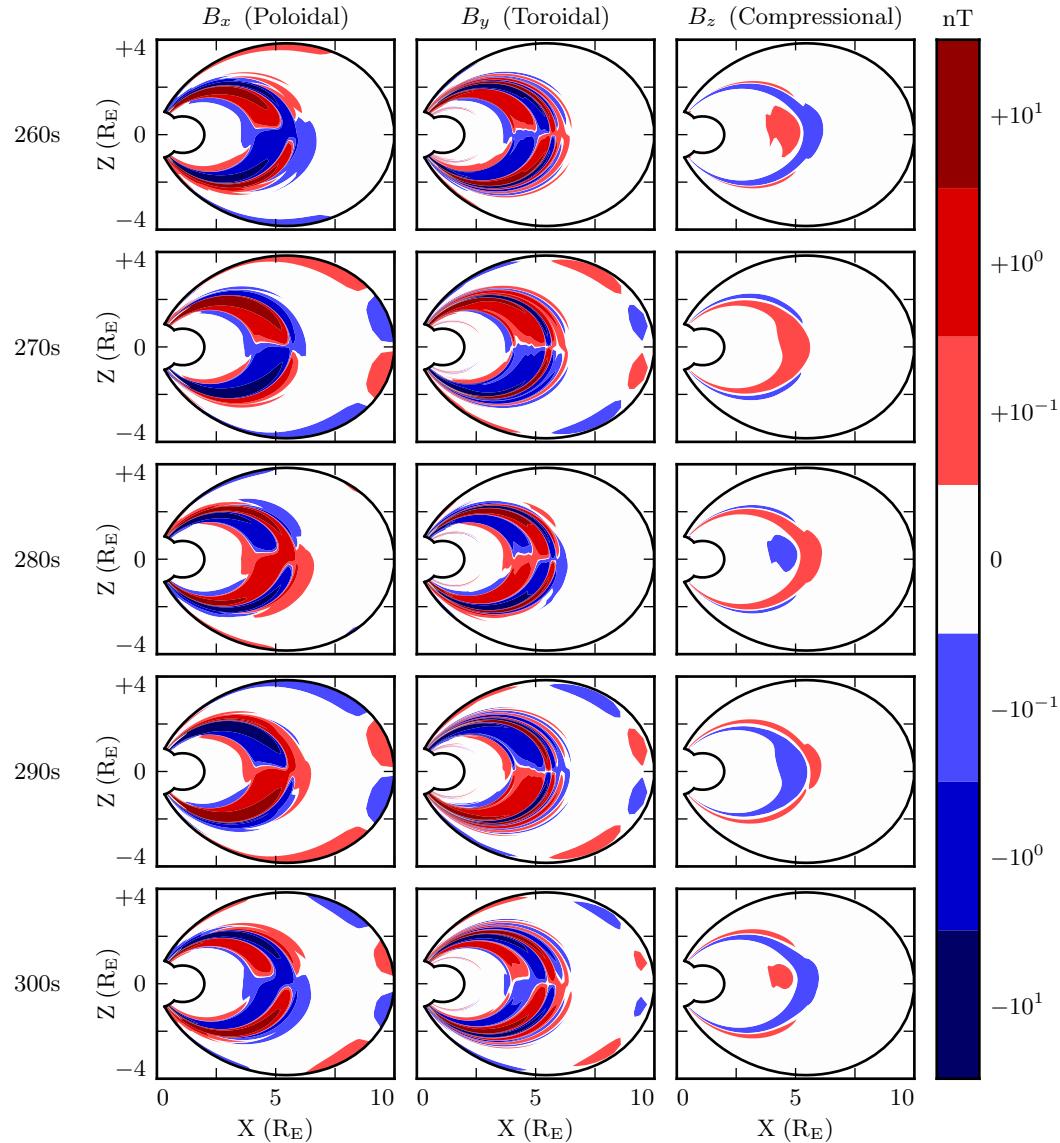


Figure 7.2: The above figure is analogous to Figure 7.1, but the run uses a large azimuthal modenumber. The change has a dramatic effect. The poloidal wave is concentrated much more sharply in L , and its compressional component is weakener by an order of magnitude. Regardless of modenumber, toroidal waves exist at a range of L shells similar to poloidal waves, and show sharp definition across L -shells.

Compressional Coupling to the Poloidal Mode: Quiet Day

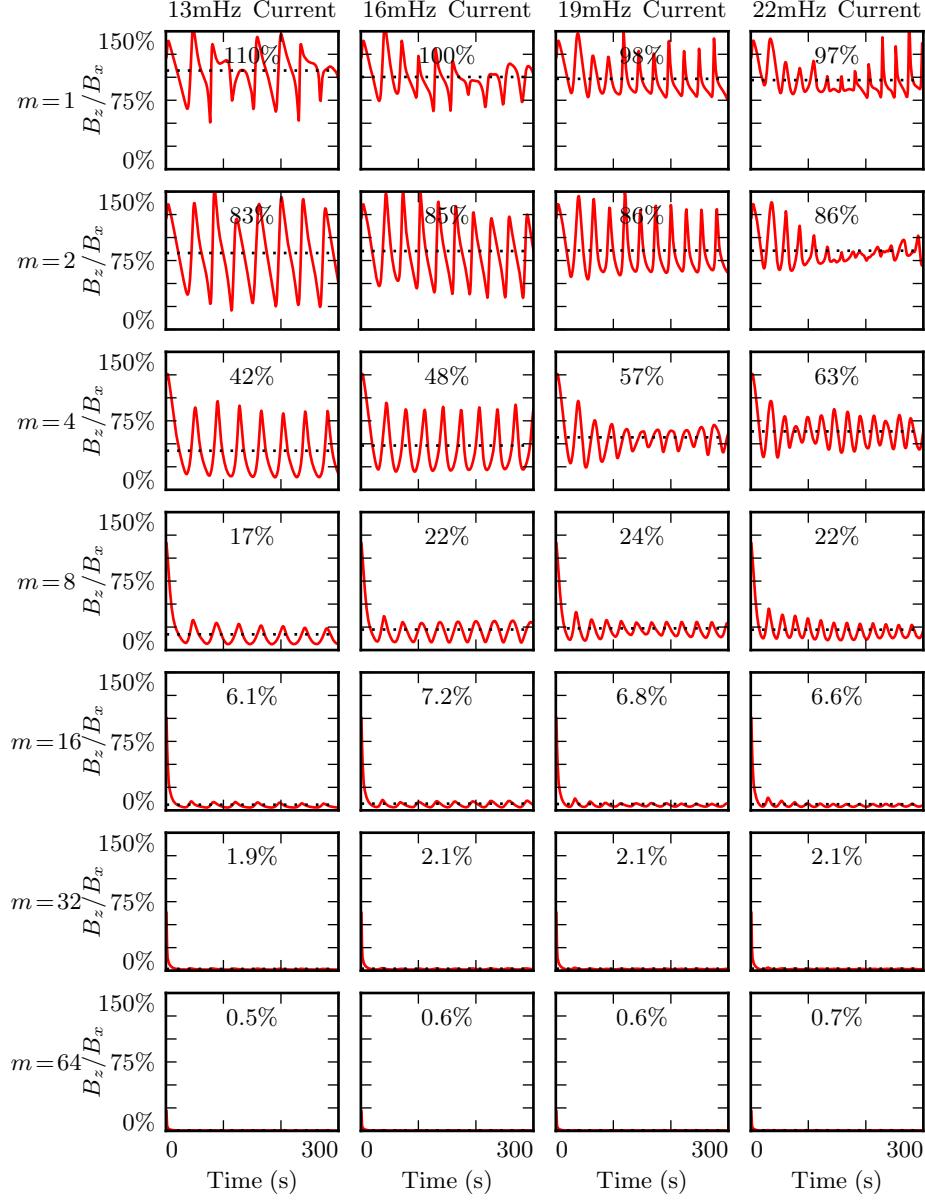


Figure 7.3: Each subplot above corresponds to a different run; the runs shown in Figures 7.1 and 7.2 are in the rightmost column, second from the top and second from the bottom respectively. Red lines indicate the ratio between the RMS compressional and poloidal magnetic fields. Mean values are shown in black. The compressional field is comparable to the poloidal field at $m = 1$, but falls quickly.

1034 **7.2 Resonance and Rotation on the Dayside**

1035 In his 1974 paper, Radoski argues that a poloidally-polarized wave should asymptotically
1036 rotate to the toroidal polarization[76] as a result of the curved derivative in the
1037 meridional plane. The question of finite poloidal lifetimes is considered further in a
1038 1995 paper by Mann and Wright[64], using a straightened field line and an Alfvén speed
1039 gradient in the “radial” direction. They also found a rotation over time from poloidal to
1040 toroidal polarization, with the characteristic time proportional to the azimuthal mode-
1041 number.

1042 The present section builds on the aforementioned results by relaxing several of their non-
1043 physical assumptions. Tuna’s geometry is more realistic than Radoski’s half-cylinder or
1044 the box model used by Mann and Wright. Previous work has considered the evolution
1045 of an initial condition, while the simulations shown below include driving delivered
1046 over time. In addition, Tuna features a finite, height-resolved ionospheric conductivity
1047 profile, rather than the perfectly-reflecting boundaries used in the past.

Each subplot in Figure 7.4 is analogous to Figure 3 in Mann and Wright’s paper[64]. Blue lines show the total energy in the poloidal mode as a function of time. Red lines show toroidal energy. Runs are organized analogous to those in Figure 7.3: drive frequency is constant down each column, and azimuthal modenum is constant across each row. Axis bounds are held constant across all subplots. The poloidal and toroidal energy are computed by integrating over the electromagnetic energy density, per Poynting’s theorem:

$$U_P = \int \frac{dV}{2\mu_0} \left(B_x^2 + \frac{1}{v_A^2} E_y^2 \right) \quad U_T = \int \frac{dV}{2\mu_0} \left(B_y^2 + \frac{1}{v_A^2} E_x^2 \right) \quad (7.1)$$

1048 Where the differential volume dV is computed using the Jacobian² to account for Tuna’s
1049 unusual geometry. The integral is evaluated in u^1 and u^3 but not u^2 (Tuna’s missing
1050 half-dimension), which gives energy in units of gigajoule per radian. More than anything
1051 else, this serves as a reminder that Pc4 pulsations are localized in MLT.

²See Section 5.1.

1052 The 28 runs shown in Figure 7.4 use an ionospheric profile corresponding to the dayside
1053 during times of low solar activity, where the conductivity is relatively high. The active
1054 and quiet dayside profiles are briefly contrasted in Section 7.4; for the most part, the
1055 focus of the present work is on the difference between the dayside and the nightside
1056 (Section 7.3). Differences between the two dayside profiles are small in comparison.

1057 The fact that red (toroidal) lines appear at all in Figure 7.4 speaks to a net rotation
1058 of energy from the poloidal mode to the toroidal. As discussed in Section 5.3, Tuna's
1059 driving is delivered purely into the poloidal electric field, reflecting a perturbation in
1060 the magnitude of the ring current.

1061 As expected, the rotation from poloidal to toroidal is slowest at large azimuthal mode-
1062 numbers. The toroidal energy overtakes the poloidal energy within a single drive period
1063 at small m ; at $m = 64$, the most of the energy is in the poloidal mode for ~ 10 peri-
1064 ods. However, the relationship between azimuthal modenumber and rotation timescale
1065 is not linear, as was suggested by Mann and Wright. Instead, in a practical setting, the
1066 rotation is fastest at $m \sim 4$.

1067 This is explained by the compressional character of the poloidal mode. At very low
1068 modenumber, energy in the poloidal mode moves readily across L -shells. A significant
1069 fraction of that energy is lost to the outer boundary before rotating to the toroidal
1070 mode. At high modenumber, compressional propagation is evanescent, so all energy
1071 in the poloidal mode must ultimately rotate to the toroidal mode or be lost to Joule
1072 dissipation.

1073 Joule dissipation is a major player in the system's energy economy. However, due to the
1074 highly conductive dayside ionosphere, dissipation timescales are in the tens of Pc4 wave
1075 periods. Energy loss through Joule dissipation asymptotically balances energy input
1076 from driving, but most of that energy is not lost until after it has rotated from the
1077 poloidal mode to the toroidal. As such, in most runs shown in Figure 7.4, the energy
1078 content of the toroidal mode asymptotically exceeds that of the poloidal mode.

1079 The asymptotic energy content of the system also depends on how well the drive fre-
1080 quency matches the local eigenfrequency. If the two do not match, energy is lost to
1081 destructive interference between the standing wave and the driving.

1082 In principle, energy moves between the poloidal and toroidal modes due to their direct
1083 coupling through the ionospheric Hall conductivity. In practice, this effect does not
1084 move large amounts of energy. When the runs shown in Figure 7.4 are repeated with
1085 the Hall conductivity set to zero, the resulting energy curves are not visibly different
1086 (not shown).

1087 The low- m runs at 19 mHz merit additional discussion. These runs accumulate energy
1088 over a large number of wave periods, while the low- m waves at 13 mHz, 16 mHz, and
1089 22 mHz do not. This effect is likely nonphysical. At 19 mHz, a third-harmonic resonance
1090 forms very close to the outer boundary, and is likely enhanced by nonphysical reflections
1091 against the simulation boundary.

1092 The presence of individual harmonics can be seen in the contours shown in Figures 7.5
1093 and 7.6. These figures show the same runs as Figure 7.4, arranged in the same way on
1094 the page. However, instead of showing the total energy integrated over the simulation
1095 domain, the energy densities are averaged over the volume of each flux tube individually.
1096 Figure 7.5 shows contours of poloidal energy density and Figure 7.6 shows toroidal
1097 energy density.

1098 The top few rows of Figure 7.5 confirm that the poloidal mode's compressional nature is
1099 to blame for its failure to accumulate energy at low modenumber. Waves move so readily
1100 across field lines that no visible amount of energy builds up at $L \sim 5$, the location of the
1101 driving. Some energy moves inward, and is trapped by the peak in Alfvén speed just
1102 inside the plasmapause, while the rest moves to the outer boundary. The time spent
1103 moving across field lines counts against the poloidal mode's finite lifetime, inhibiting
1104 the buildup of poloidal energy density even at L -shells where the wave matches the local
1105 eigenfrequency.

1106 As m increases, the energy distribution becomes more concentrated in L , though indi-
1107 vidual features remain fairly broad. At $m = 8$, runs at 13 mHz and 16 mHz are inclined
1108 to build up energy just inside the plasmapause, while those at 19 mHz and 22 mHz res-
1109 onate just outside the plasmapause; in all four cases, the energy is spread over a range
1110 of at least 1 in L .

1111 The peak energy density in the bottom-right run (22 mHz driving, $m = 64$) is by far the
1112 largest of any run in Figure 7.5. The azimuthal modenumber is large, so the poloidal
1113 mode is purely guided; energy is not smeared across multiple L -shells. And, crucially, the
1114 frequency of the driving matches closely with the Alfvén frequency at $L \sim 5$. Other runs
1115 on the bottom row are also guided, but they reach lower asymptotic energy densities
1116 because of a mismatch between the drive frequency and the local eigenfrequency —
1117 resulting in destructive interference between the standing wave and its driver.

1118 Giant pulsations are typically seen at ~ 10 mHz, well below the 22 mHz poloidal peak
1119 shown in Figure 7.5. Part of the discrepancy is likely due to the position of the driving.
1120 Pgs are most common at latitudes of $\sim 66^\circ$, which maps out to $L \sim 6$, whereas these
1121 runs are driven at $L \sim 5$. The size of the plasmapause also has a significant effect.
1122 When the runs in Figure 7.5 are repeated with the plasmapause at $L = 5$ instead of
1123 $L = 4$, the strongest resonance (driven at $L \sim 5$) drops from 22 mHz to 16 mHz (not
1124 shown).

1125 Whereas the poloidal contours are smeared over a swath of L -shells (though the high- m
1126 runs less so), the toroidal contours in Figure 7.6 appear only where the wave frequency
1127 matches the local eigenfrequency. A horizontal line drawn through the Alfvén speed
1128 frequency profiles (recall Figure 3.1) intersects the profile up to three times: once as
1129 the Alfvén frequency drops through the Pc4 range from its low-latitude peak, again as
1130 the Alfvén frequency rises sharply at the plasmapause, and a third time as the Alfvén
1131 frequency drops asymptotically. Toroidal waves can be seen resonating at all three of
1132 these locations in the $m = 4$, 22 mHz run in Figure 7.6, along with a third harmonic at
1133 large L . This is consistent with observations: toroidal resonances are noted for having
1134 frequencies which depend strongly on L , in contrast to the poloidal mode's less-strict
1135 relationship between frequency and location.

1136 In only one of the runs shown in Figure 7.5 does the poloidal mode attain an energy
1137 density on the order of 10^{-1} nJ/m³. On the other hand, the toroidal mode reaches
1138 $\sim 10^{-1}$ nJ/m³ in six of the runs in Figure 7.6. That is, the poloidal mode only exhibits
1139 a high energy density on the dayside only when conditions are ideal; the toroidal mode
1140 isn't nearly so particular.

Poloidal (Blue) and Toroidal (Red) Energy: Quiet Day

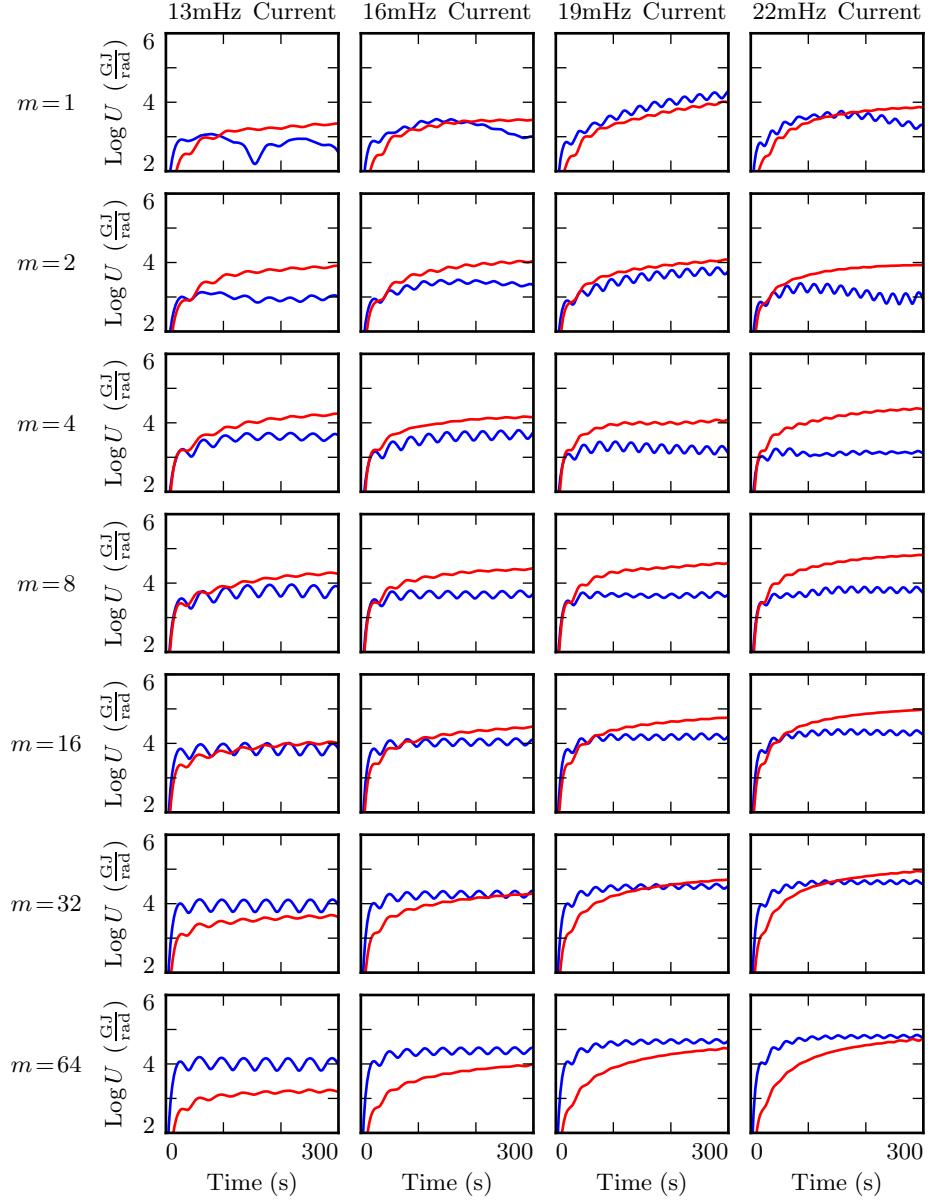


Figure 7.4: Each subplot above shows the poloidal (blue) and toroidal (red) energy for a simulation as a function of time. Each row contains four simulations, each with the same azimuthal modenumber; the seven rows in each column share a drive frequency. Driving is purely poloidal, but energy rotates asymptotically to the toroidal mode, and rotation is slowest at high modenumber.

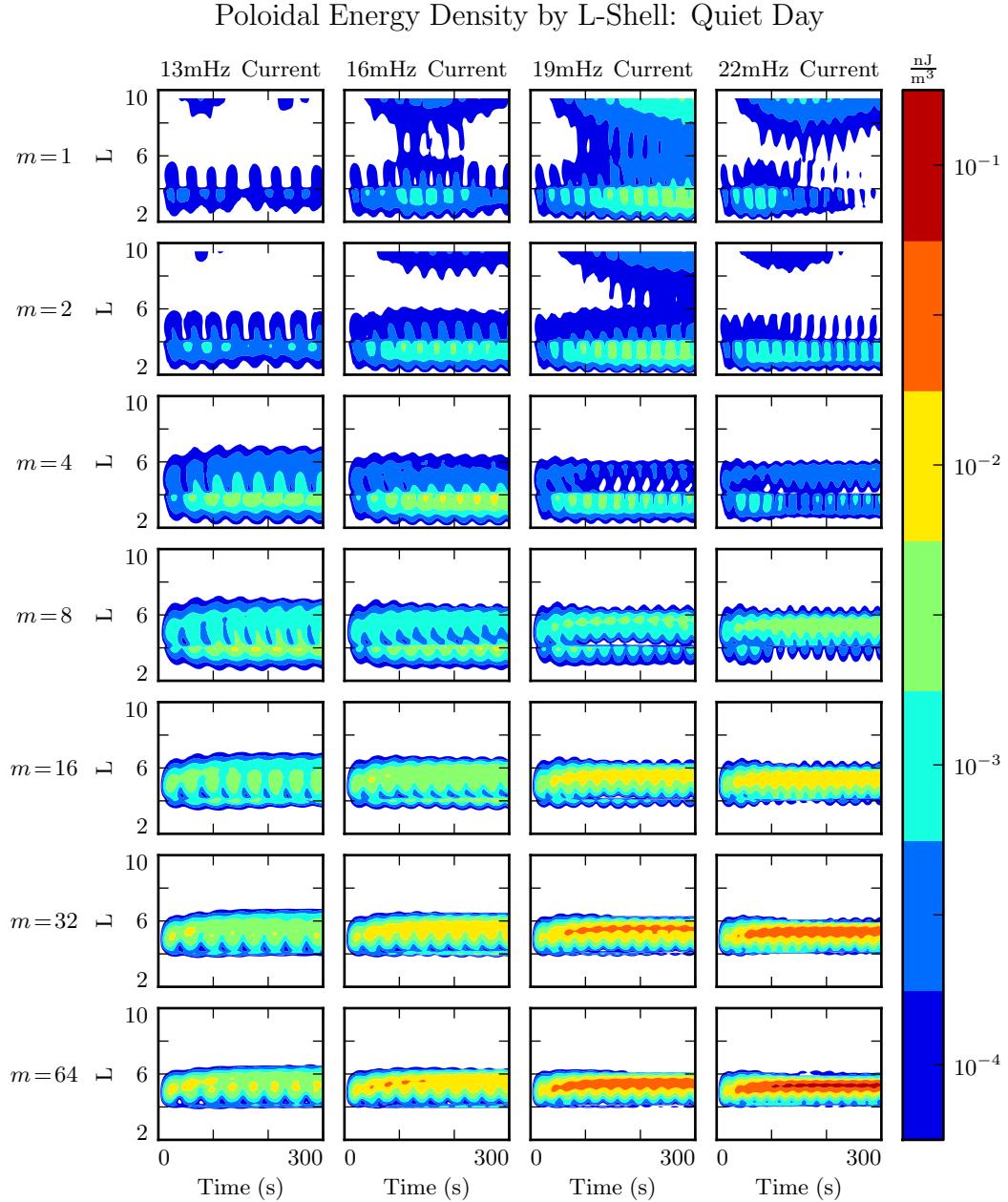


Figure 7.5: At low modenumber (top rows), the compressional nature of the poloidal mode allows energy to escape the simulation. At high modenumber (lower rows), the poloidal mode is guided; energy is trapped at the L -shell where it's injected, and rotation to the toroidal mode is slow — ideal conditions for resonance. But energy buildup is lackluster except where the drive frequency matches the local eigenfrequency (best in the rightmost row).

Toroidal Energy Density by L-Shell: Quiet Day

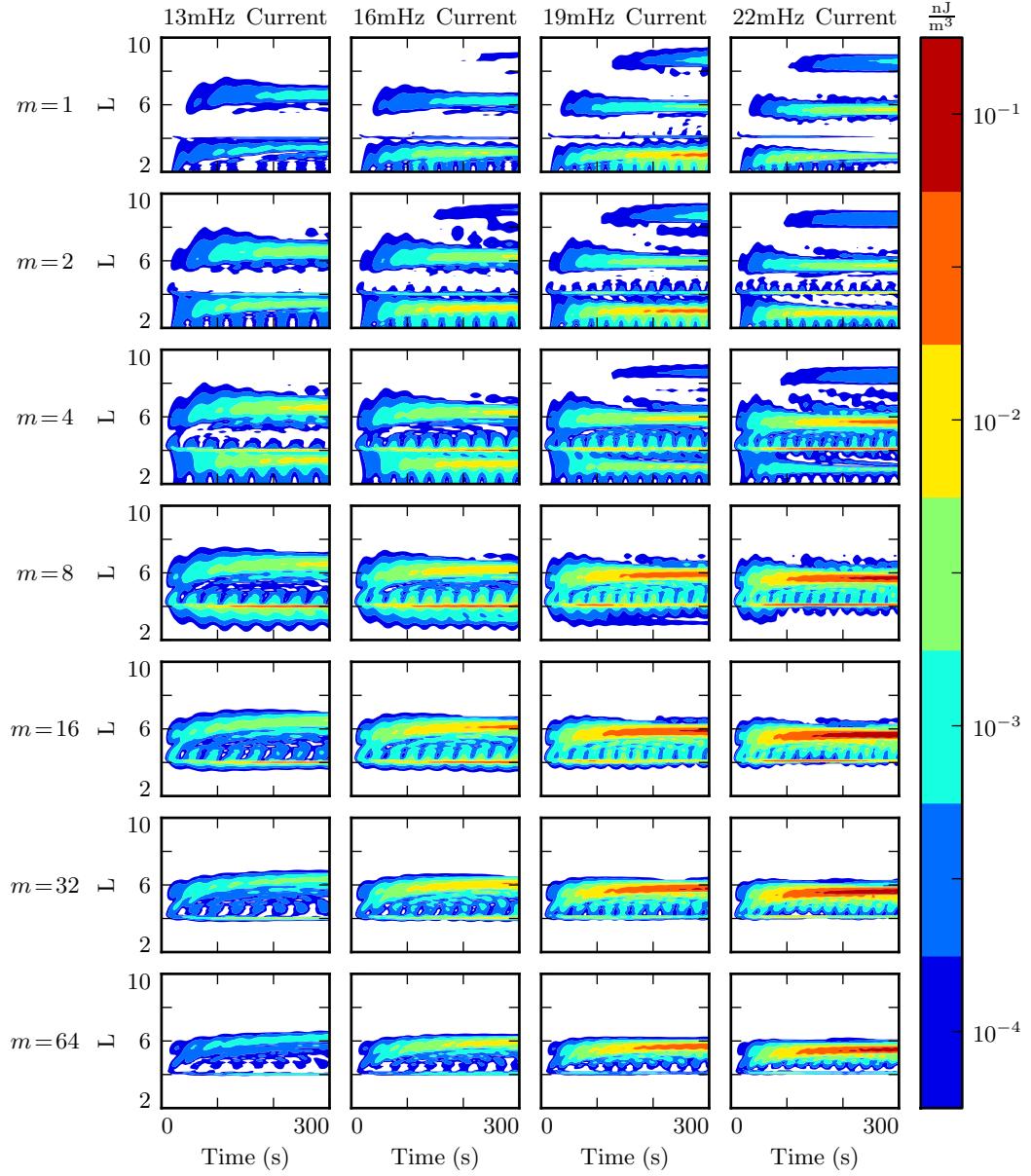


Figure 7.6: Whereas the poloidal mode is smeared in L due to its compressional nature, the toroidal mode is focused at L shells where it's resonant. In general, when the conductivity is high, the toroidal mode also exhibits a higher asymptotic energy density than the poloidal mode (Figure 7.5).

1141 7.3 Resonance and Rotation on the Nightside

- 1142 Compared to the dayside ionosphere employed in Section 7.2, conductivity on the night-
1143 side is much lower. Runs in the present section use Tuna’s ionospheric profile corre-
1144 sponding to the nightside during quiet solar conditions. The two nightside profiles are
1145 briefly compared in Section 7.4, but for the most part the present work is concerned
1146 with the behavior of the nightside compared to that on the dayside.
- 1147 Other than the change in ionospheric profile, Figures 7.7 to 7.9 are analogous to Fig-
1148 ures 7.4 to 7.6. Each subplot corresponds to a different 300s run of Tuna. Drive
1149 frequency is constant down each column, and azimuthal modenumber is constant across
1150 each row.
- 1151 The low conductivity on the nightside gives rise to strong Joule dissipation. Waves are
1152 damped out in just a few bounces, so asymptotic energy values are reached quickly.
1153 No combination of frequency and modenumber gives rise to the accumulation of energy
1154 over multiple drive periods.
- 1155 As on the dayside, rotation of energy from the poloidal to toroidal mode is fastest at
1156 $m \sim 4$. Unlike the dayside, however, dissipation on the nightside is fast compared to
1157 the rotation of energy to the toroidal mode. Toroidal energy does not asymptotically
1158 exceed the poloidal energy by a significant margin in any run shown in Figure 7.7. At
1159 $m = 64$, where the rotation timescale is slowest, no more than 10% of the energy in the
1160 poloidal mode rotates to the toroidal mode before being lost.
- 1161 Also similar to the dayside, low- m runs driven at 19 mHz and 22 mHz resonate very
1162 close to the outer boundary. As before, the buildup of energy is likely nonphysical.
- 1163 Poloidal contours on the nightside (Figure 7.8) are weaker than those on the dayside,
1164 and build up energy over less time, but otherwise similar. At low modenumber, poloidal
1165 energy propagates across L -shells, preventing the significant accumulation of energy
1166 anywhere. As the modenumber increases, energy is contained near the driving at $L \sim 5$.
1167 The strongest response is seen at 13 mHz on the bottom row, where the modenumber is
1168 at its largest and the frequency matches closest with the local eigenfrequency. Even in

1169 that case, dissipation timescales are comparable to the oscillation period, so the wave
1170 only persists in the presence of continuous driving.

1171 Toroidal energy contours on the nightside exhibit significantly different behavior from
1172 those on the dayside.

1173 At low modenumber, the nightside toroidal mode (Figure 7.9) contains less energy
1174 than on the dayside, but it still shows some preference for sharp resonances where the
1175 drive frequency matches the local Alfvén frequency. At moderate modenumbers, as on
1176 the dayside, the toroidal mode is more or less comparable in strength to its poloidal
1177 counterpart. It's only at high modenumber that the difference between the dayside
1178 and nightside toroidal contours become truly dramatic. Whereas on the dayside, most
1179 energy is asymptotically deposited in the toroidal mode, on the nightside most poloidal
1180 energy is dissipated faster than the poloidal-to-toroidal rotation timescale. At $m = 64$,
1181 where the poloidal mode is at its strongest, the toroidal mode is at its weakest; it barely
1182 registers, even on Figure 7.9's log scale.

Poloidal (Blue) and Toroidal (Red) Energy: Quiet Night

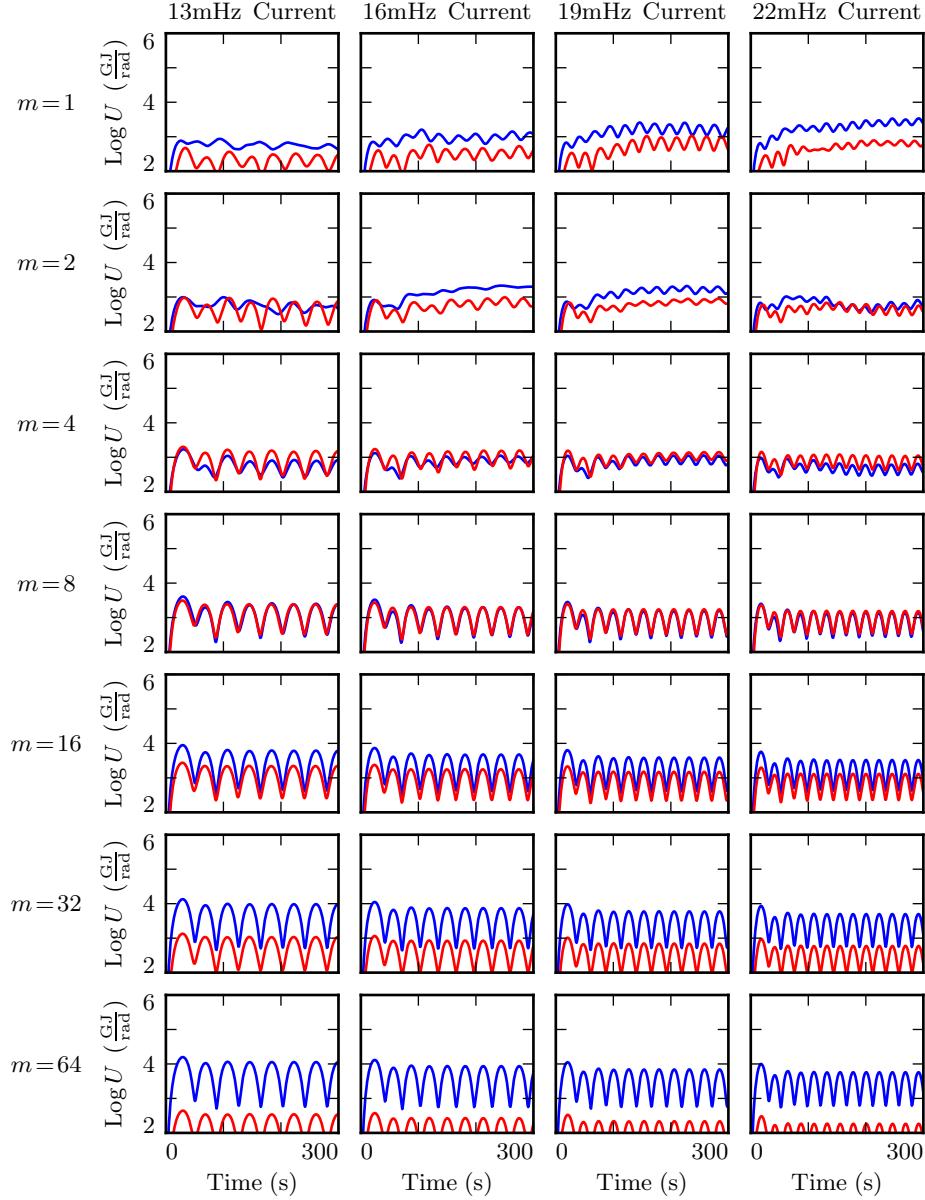


Figure 7.7: The energy content of each FLR on the nightside resembles that of a damped, driven oscillator. Energy is periodically added to the system, but most is lost too fast to rotate to the toroidal mode, particularly at high modenumber. There is no significant buildup of energy over multiple periods. Runs at $m = 1$ (top row) are an apparent exception, likely due to a nonphysical interaction with the boundary.

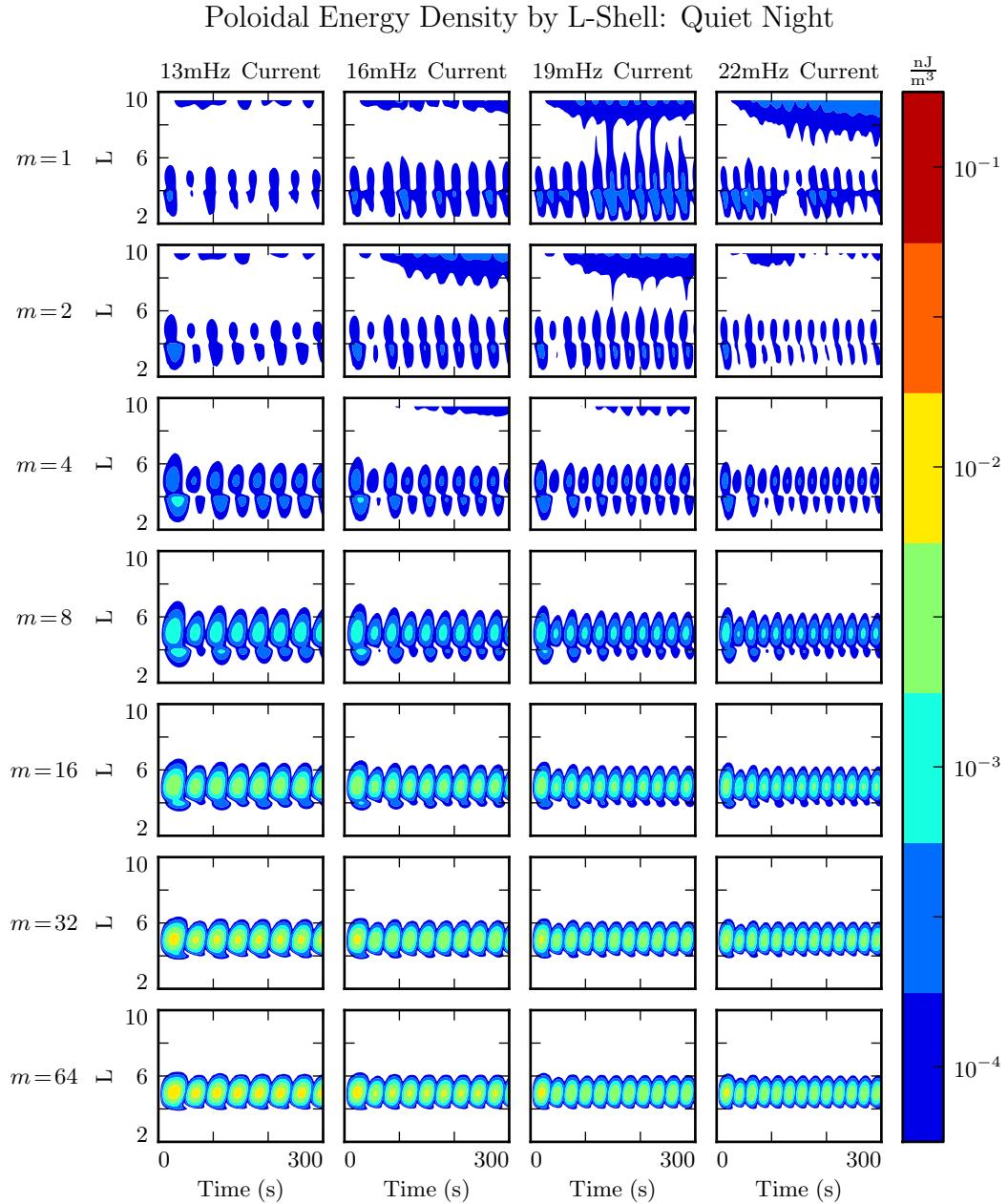


Figure 7.8: As on the dayside (Figure 7.5), low-modenumber poloidal waves (top rows) readily propagate across L -shells and escape the simulation domain. Energy density builds up most effectively at high modenumbers, where the poloidal mode is guided, and poloidal-to-toroidal rotation is slow. Even in this case, however, dissipation is fast enough to prevent energy from accumulating over multiple drive periods.

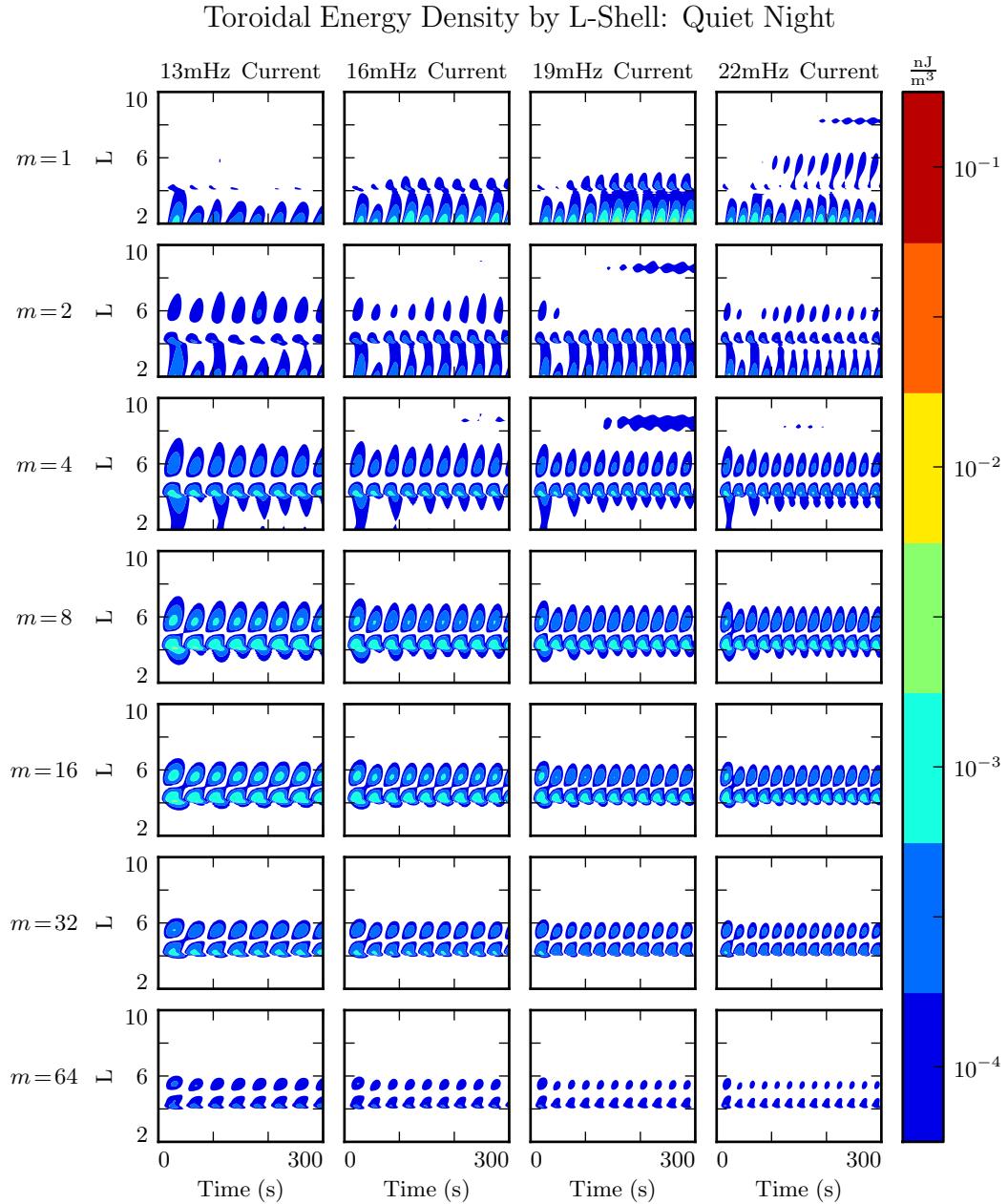


Figure 7.9: On the dayside (Figure 7.6), most energy rotates asymptotically to the toroidal mode. On the night side, the same is not true, since the poloidal mode quickly loses energy to Joule dissipation. At high modenumber, where the poloidal-to-toroidal rotation timescale is in the tens of wave periods, almost all of the energy is dissipated from the poloidal mode rather than rotating to the toroidal mode.

1183 7.4 Ground Signatures and Giant Pulsations

1184 While the majority of the action is in space, the majority of FLR observations have
1185 historically been ground-based. The present section explores simulations (including
1186 those discussed in Sections 7.2 and 7.3) in terms of their ground signatures rather than
1187 their integrated energy distributions.

1188 Figures 7.10 and 7.11 show fourteen runs each, two per row. Contours give magnetic
1189 fields at the ground, plotted against time on the horizontal axis and latitude on the
1190 vertical axis. Modenumber is held constant across each row, as in the above sections;
1191 columns show north-south and east-west ground signatures using an ionospheric profile
1192 for active (first and second columns respectively) and quiet (third and fourth columns).

1193 As noted in Chapter 3, the magnetic polarization of a low frequency Alfven wave is
1194 rotated by $\sim 90^\circ$ as it passes through the ionosphere. The east-west field on the ground
1195 (B_ϕ) corresponds to the poloidal polarization in space, and the north-south field on the
1196 ground (B_θ) corresponds to the toroidal mode.

1197 The most striking feature of Figures 7.10 and 7.11 is the modenumber dependence.
1198 As modenumber increases, the magnetic field signatures become sharply localized in
1199 latitude. At high m , ground signatures are concentrated between 60° and 70° , peaking
1200 near 64° , roughly coincident with the foot point of the $L = 5$ field line; ionospheric
1201 ducting is not significant in the Pc4 regime.

1202 At low modenumber, magnetic signatures are weak on the ground because the waves
1203 in space are also weak. At high modenumber, waves in space are strong, but so is the
1204 attenuation of magnetic signatures by the atmosphere³. The “sweet spot” at which
1205 magnetic ground signatures are maximized falls at $m = 16$ to $m = 32$.

1206 Tuna shows stronger ground signatures on the dayside than on the nightside, more or
1207 less in proportion with the difference in magnitude in space. Energy on the dayside
1208 (which depends on field magnitude squared) peaks an order of magnitude larger than
1209 that on the nightside. Peak ground signatures on the dayside are larger by a factor of
1210 four: 45 nT compared to 11 nT. On both the dayside and the nightside, peak ground

³See Equation (3.3).

1211 signatures are in B_ϕ , the east-west magnetic field component; both peaks are also at
1212 $m = 16$, and both are seen in runs using the ionospheric profile for quiet solar activity.

1213 It's further notable that the ground signatures — particularly those on the nightside
1214 — exhibit a change in chirality based on latitude. At low latitude, B_θ leads B_ϕ , which
1215 creates a counterclockwise signature on the ground (in the northern hemisphere). At
1216 high latitude, the phase is reversed, resulting in a clockwise ground signature.

1217 These results match well with the properties associated with Pgs: east-west polarization,
1218 latitude-dependent chirality, peak latitude of $\sim 66^\circ$, and azimuthal modenumber
1219 of 16 to 35. Just about the only properties missing are the azimuthal drift (which is
1220 beyond the scope of the present model) and the distribution in MLT. Pgs are most
1221 commonly observed pre-dawn, but morning and evening ionospheric profiles are not
1222 presently implemented for Tuna.

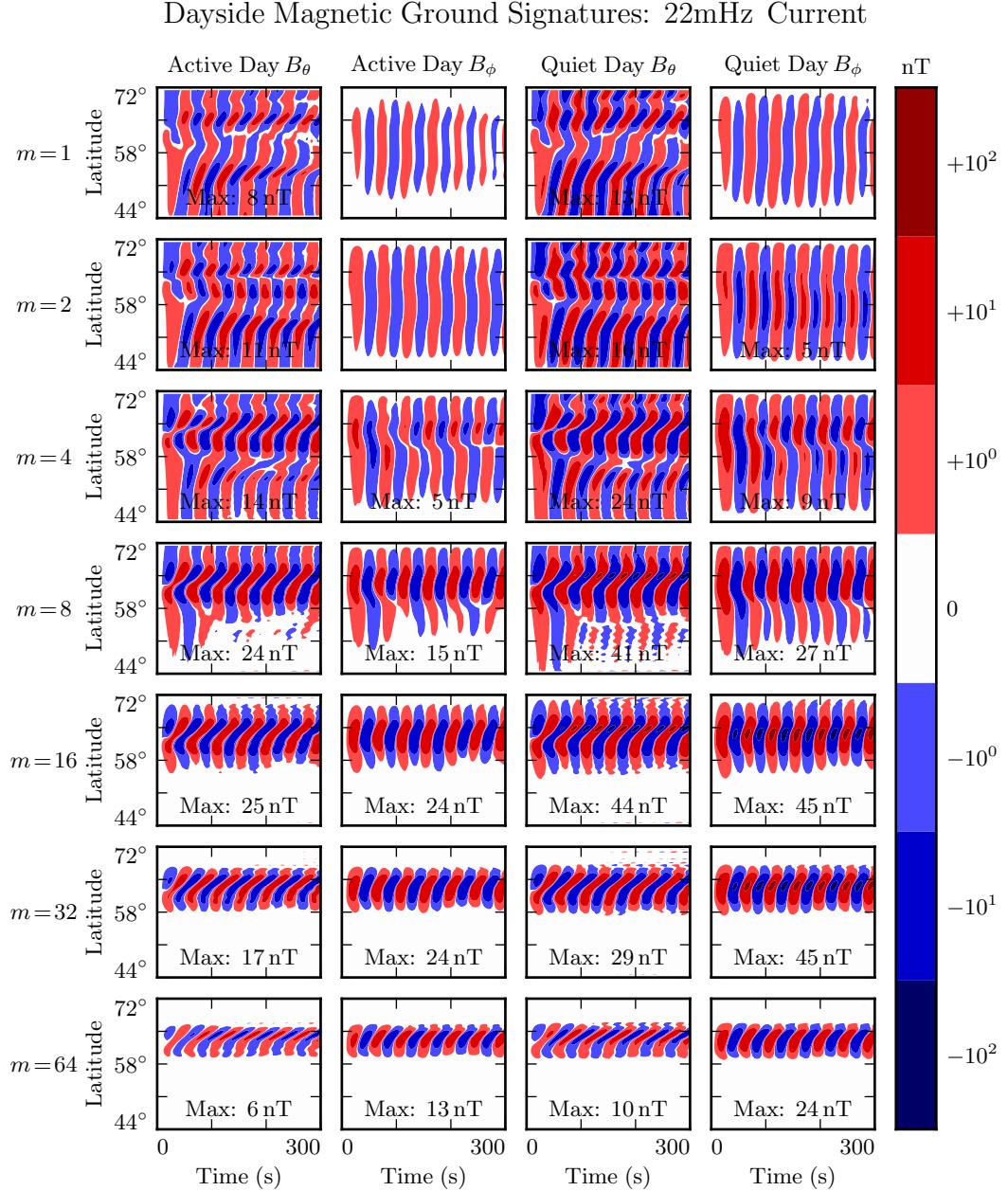


Figure 7.10: The magnetic ground signatures are shown for fourteen runs, two per row. Azimuthal modenumber is constant across each row. Polarization and ionospheric profile vary by column, per the headers. Ground signatures at low modenumber are weak because the waves in space are weak, while those at high modenumber are attenuated by the atmosphere. Considering both effects, ground signatures seem to be maximized at $m = 16$ to $m = 32$. Peak amplitudes above 3 nT are marked.

Nightside Magnetic Ground Signatures: 13mHz Current

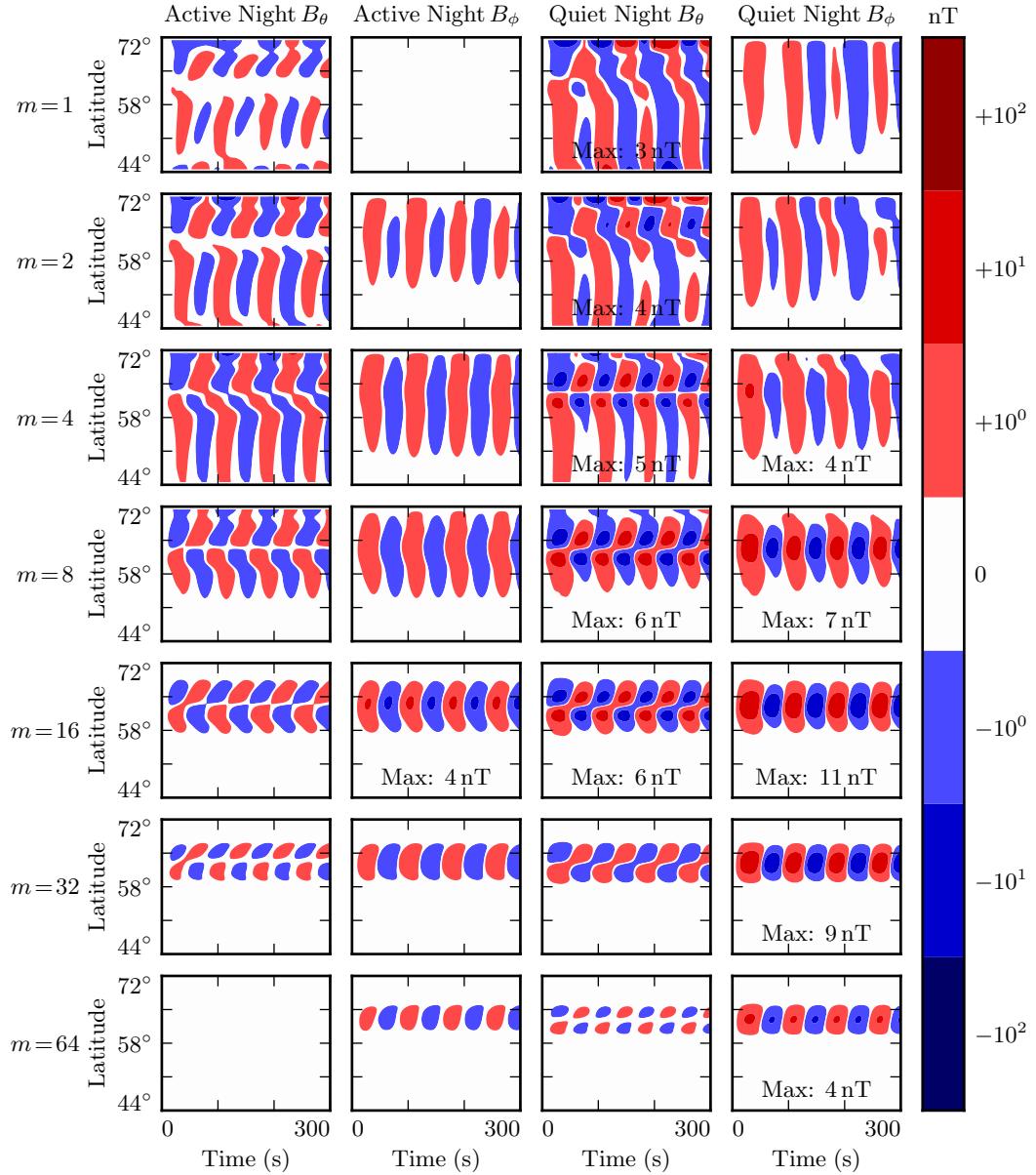


Figure 7.11: Nightside ground signatures are less strongly peaked than those on the dayside, but qualitative features are the same: the strongest signals are in B_ϕ , peaked over just a few degrees in latitude, at a modenumber of 16 or 32, under quiet ionospheric conditions.

1223 **7.5 Discussion**

1224 The above results show agreement with a number of past FLR studies. In addition,
1225 several novel connections are suggested between known properties of FLRs.

1226 The compressibility of poloidal FLRs at low modenumber, but not high modenumber,
1227 is reproduced and quantified. At $m \sim 1$, the poloidal and compressional components of
1228 an FLR in the Pc4 range are comparable in magnitude. At $m \gtrsim 6$, $\left| \frac{B_z}{B_x} \right| \lesssim \frac{1}{2}$, and at
1229 $m \gtrsim 12$, $\left| \frac{B_z}{B_x} \right| \lesssim \frac{1}{4}$.

1230 The present results also suggest that compressional character of poloidal Pc4s is to blame
1231 for the weak relationship between L and frequency, compared to that seen in toroidal
1232 events. Toroidal resonances are defined sharply in L regardless of modenumber, while
1233 poloidal resonances are smeared in L — particularly at low m , but to some degree at
1234 high m as well.

1235 The asymptotic rotation of energy from the poloidal mode to the toroidal mode is
1236 reproduced; at small m , the rotation timescale is comparable to a wave period, while at
1237 large modenumber it's on the order of 10 periods. On the dayside, little energy is lost
1238 to Joule dissipation on rotation timescales. In contrast, on the nightside, dissipation
1239 timescales are comparable to wave periods. In high- m nightside runs, even in the case
1240 of continuous driving near the local eigenfrequency, the movement of energy from the
1241 poloidal mode to the toroidal mode is vanishingly small.

1242 Ground signatures at low modenumber are shown to be weak because waves in space are
1243 weak; this is particularly true for poloidal waves — a non-guided wave can't very well
1244 resonate along a field line — but also true of toroidal waves insomuch as poloidal waves
1245 are their source. FLRs resonate most strongly at high m , but high- m signatures are
1246 also attenuated by the atmosphere. The balance between the two effects falls around m
1247 of 16 to 32. It's further suggested that a high- m driver will cause a weak resonance in
1248 place rather than tunneling across field lines to a matching eigenfrequency, and that the
1249 same driving should give rise to stronger ground signatures during times of low solar
1250 activity, on both the dayside and the nightside.

1251 The findings together suggest, awkwardly, that the morphology of giant pulsations re-
1252 veals relatively little about their origins.

1253 One can consider a hypothetical magnetosphere subject to constant driving: broadband
1254 in frequency, broadband in modenumber, just outside the plasmapause. Low- m poloidal
1255 waves will quickly rotate to the toroidal mode (and/or propagate away). High- m waves
1256 will resonate in place, accumulating energy over time, and giving rise to “multiharmonic
1257 toroidal waves”[87]; Fourier components that do not match the local eigenfrequency
1258 will quickly asymptote. Waves with very high modenumbers will be attenuated by
1259 the ionosphere. The response on the ground will be counterclockwise at low latitude,
1260 clockwise at high latitude, peaked at $16 \lesssim m \lesssim 32$, mostly east-west polarized, and
1261 notably stronger during quiet solar conditions. In other words, the measurements will
1262 look very much like a giant pulsation.

1263 The present results offer no explanation as to the tendency of giant pulsations to drift
1264 azimuthally, or to appear pre-dawn in MLT — though the latter is addressed by the
1265 observational results in Chapter 8.

1266 **Chapter 8**

1267 **Van Allen Probe Observations**

1268 The results presented in Chapter 7 are interesting on their own, but become particularly
1269 valuable when combined with observational data. Unfortunately, only a small number
1270 of studies to date have explored how Pc4 observation rate is affected by the harmonic
1271 and polarization structure of those waves. While Pc4 pulsations have previously been
1272 studied in terms of both harmonic[4, 14, 26, 43, 81, 90] and polarization[2, 16, 17, 52, 56],
1273 no past survey has characterized each event in terms of both properties.

1274 This has largely been due to observational constraints. The classification of a wave's
1275 harmonic is best carried out by computing the phase offset of the magnetic and electric
1276 field waveforms, simultaneous in situ measurements of which have only recently become
1277 available since the launch of THEMIS[3] in 2007 and the Van Allen Probes[84] in 2012.
1278 The Van Allen Probes are particularly well-suited to the study of Pc4 pulsations as
1279 their apogee of $L \sim 6$ coincides closely with eigenfrequencies in the Pc4 range.

1280 The present chapter uses data from the Van Allen Probes mission to survey the oc-
1281 currence rate of FLRs in the Pc4 range as a function of parity and polarization, as
1282 well as magnitude, frequency, and phase. The tools used to perform the present anal-
1283 ysis — SPEDAS and the SPICE kernel — are publicly available. They, along with
1284 the Python routines used to download, filter, and plot the data, can be found at
1285 <https://github.com/UMN-Space-Physics>.

1286 TODO: Set up Git repo.

1287 8.1 Sampling Bias and Event Selection

1288 The present analysis makes use of Van Allen Probe data from October 2012 to August
1289 2015 — the entire range available at the time of writing. Between the two probes, that's
1290 just over 2000 days of observation.

1291 For the purposes of Pc4 pulsations, it's reasonable to consider the two probes to be
1292 independent observers. Nearly all Pc4 events occur near apogee ($L \gtrsim 5$), at which
1293 point the two probes are several hours apart in MLT. Pc4 events are typically not large
1294 enough to be seen by both probes simultaneously, and not long enough in duration to
1295 be seen by two probes passing through the same region of space several hours apart.

1296 Electric and magnetic field waveforms are collected using the probes' EFW[100] and
1297 EMFISIS instruments respectively. Values are cleaned up by averaging over the ten-
1298 second spin period. Three-dimensional electric field data is then obtained using the
1299 $\underline{E} \cdot \underline{B} = 0$ assumption. Notably, this assumption is taken only when the probe's spin
1300 plane is offset from the magnetic field by at least 15° . The rest of the data — about
1301 half — is discarded, which introduces a sampling bias against the flanks.

1302 A further bias is introduced by the probes' non-integer number of precessions around
1303 Earth. As of July 2014, apogee had precessed once around Earth[16]. The present work
1304 considers roughly one and a half precessions; the nightside has been sampled at apogee
1305 twice as often as the dayside.

1306 The spatial distribution of usable data — that is, data for which three-dimensional
1307 electric and magnetic fields are available — is shown in Figure 8.1. Bins are unitary in
1308 L and in MLT. The distribution of the data in magnetic latitude is not shown; the Van
1309 Allen Probes are localized to within $\sim 10^\circ$ of the equatorial plane.

1310 Field measurements are transformed from GSE coordinates into the same dipole coordi-
1311 nates used in Chapters 5 and 7. The z axis (parallel to the background magnetic field)
1312 is estimated using a ten-minute running average of the magnetic field measurements.

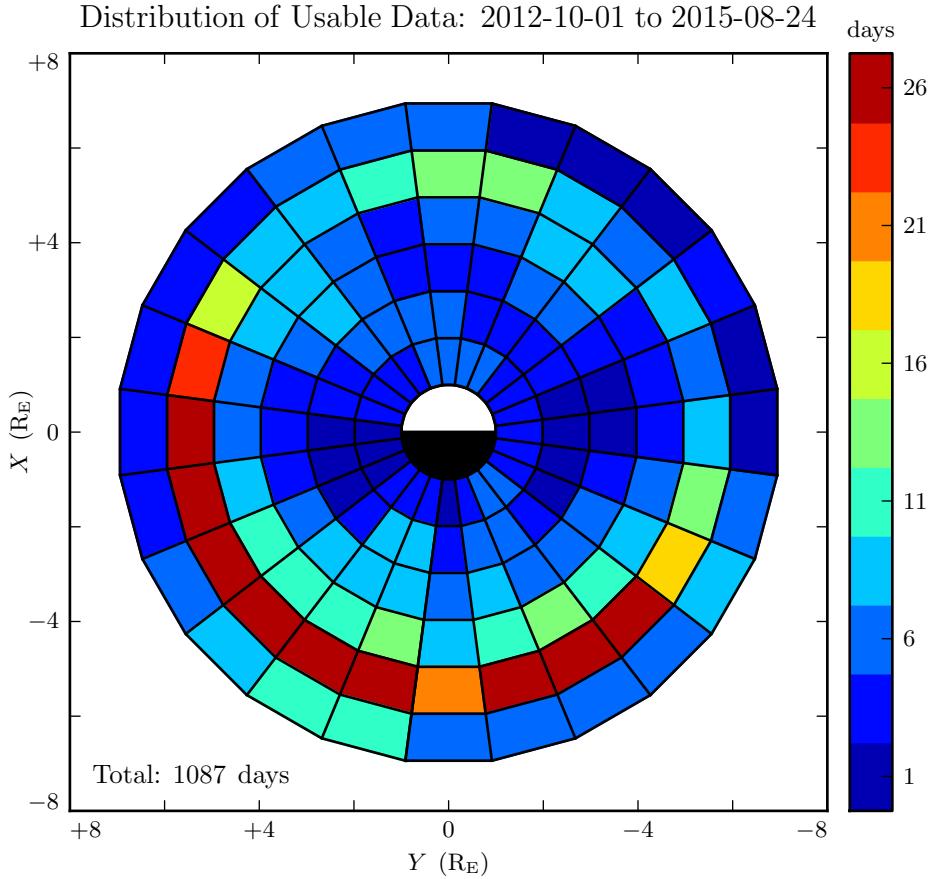


Figure 8.1: Three-dimensional electric field values are computed by assuming $\underline{E} \cdot \underline{B} = 0$. Data is discarded whenever the magnetic field falls within 15° of the spin plane, which introduces a bias against the flanks. Furthermore, the probes have completed one and a half precessions around Earth; the dayside has been sampled once at apogee, and the nightside twice.

- ¹³¹³ The y axis is set parallel to $\hat{z} \times \underline{r}$, where \underline{r} is the probe's geocentric position vector.
- ¹³¹⁴ The x axis is then defined per $\hat{x} \equiv \hat{y} \times \hat{z}$. This scheme guarantees that the axes are
- ¹³¹⁵ right-handed and pairwise orthogonal[56].
- ¹³¹⁶ The ~ 1000 days of usable data are considered half an hour at a time, which gives a fre-
- ¹³¹⁷ quency resolution of ~ 0.5 mHz in the discrete Fourier transform. Spectra are computed
- ¹³¹⁸ for all six field components: \tilde{B}_x , \tilde{B}_y , \tilde{B}_z , \tilde{E}_x , \tilde{E}_y , and \tilde{E}_z . The background magnetic

1319 field is subtracted before transforming the magnetic field components, leaving only the
 1320 perturbation along each axis¹. Each waveform is also shifted vertically so that its mean
 1321 over the thirty minute event is zero.

Frequency-domain Poynting flux is computed from the electric and magnetic field transforms. A factor of L^3 compensates the compression of the flux tube, so that the resulting values are effective at the ionosphere. Poloidal and toroidal Poynting flux, respectively, are given by:

$$\tilde{S}_P \equiv -\frac{L^3}{\mu_0} \tilde{E}_y \tilde{B}_x^* \quad \tilde{S}_T \equiv \frac{L^3}{\mu_0} \tilde{E}_x \tilde{B}_y^* \quad (8.1)$$

1322 The poloidal and toroidal channels are independently checked for Pc4 waves. For each
 1323 channel, a Gaussian profile is fit to the magnitude of the Poynting flux, $|\tilde{S}(\omega)|$. If the
 1324 fit fails to converge, or if the peak of the Gaussian does not fall within 5 mHz of the
 1325 peak value of \tilde{S} , the event is discarded. Events are also discarded if their frequencies
 1326 fall outside the Pc4 frequency range (7 mHz to 25 mHz) or if their amplitudes fall below
 1327 10^{-2} mW/m² (out of consideration for instrument sensitivity).

1328 Events are discarded if their parity is ambiguous. The electric field and the magnetic
 1329 field must be coherent at a level of 0.9 or better (judged at the discrete Fourier transform
 1330 point closest to the peak of the Gaussian fit). Any event within 3° of the magnetic
 1331 equator is also not used; as discussed in Chapter 3, in order to distinguish an odd mode
 1332 from an even mode, it's necessary to know whether the observation is made north or
 1333 south of the equator.

1334 A visual inspection of events shows that those with broad “peaks” in their spectra
 1335 are typically not peaked at all — they are noisy spectra with several spectral features
 1336 grouped just closely enough to trick the fitting routine. A threshold is set at a FWHM
 1337 of 3 mHz (equally, a standard deviation of 1.27 mHz). Any event with a Gaussian fit
 1338 broader than that is discarded.

1339 Notably, events are not filtered on their phase — that is, on the division of their energy
 1340 between standing and traveling waves. This is the topic of Section 8.5.

¹As in Chapters 4 to 7, B_x , refers not to the full magnetic field in the x direction, but to the perturbation in the x direction from the zeroth-order magnetic field. The same is true for B_y and B_z .

1341 8.2 Events by Mode

1342 The filters described in Section 8.1 yield 762 Pc4 events, the spatial distribution of
1343 which is shown in Figure 8.2. In each bin, the event count is normalized to the amount
1344 of usable data (Figure 8.1). Bins shown in white contain zero events. The rate in the
1345 bottom corner is an overall mean; it's an estimate of how often Pc4 events would be
1346 observed if the sampling were distributed uniformly in space.

1347 Consistent with previous work, Pc4 events peak on the dayside and are rarely observed
1348 at $L < 4$. Nearly 30 % of the usable data shown in Figure 8.1 is taken at $L < 4$, yet
1349 only 16 of the 762 events (2 %) appear there.

1350 On the other hand, the present work runs contrary to Dai's 2015 result in terms of Pc4
1351 event rates with respect to the plasmapause (not shown). His analysis found (poloidal)
1352 Pc4 pulsations to be comparably common inside and outside the plasmapause[16]. In
1353 the present work, only 40 of the 762 events (5 %) fall inside the plasmasphere, despite
1354 the fact that 40 % of the available data falls within the plasmasphere. The disparity is
1355 not likely due to a difference in sampling — Dai's work, like the present work, uses data
1356 from the Van Allen Probes mission. Rather, the difference is likely due to a difference
1357 in how the plasmapause is defined. Dai identifies the plasmapause by the maximum
1358 gradient in electron number density, while the present work takes an electron density of
1359 100 /cm^3 to mark the plasmapause².

1360 The same events in Figure 8.2 are shown again in Figure 8.3, partitioned by polarization
1361 and parity.

1362 The distribution of even poloidal events in Figure 8.3 is consistent with that reported
1363 by Dai[16]: the observation rate is peaked at noon, and smeared across the dusk side.
1364 Notably, Dai's work focused on even poloidal waves. While he did not explicitly remove
1365 odd events from his sample, he did introduce a threshold in the magnetic field. This
1366 threshold is preferentially satisfied by even waves (which have a magnetic field antinode
1367 near the equator) compared to odd waves (which have a magnetic field node). Dai

²Per ongoing work by Thaller.

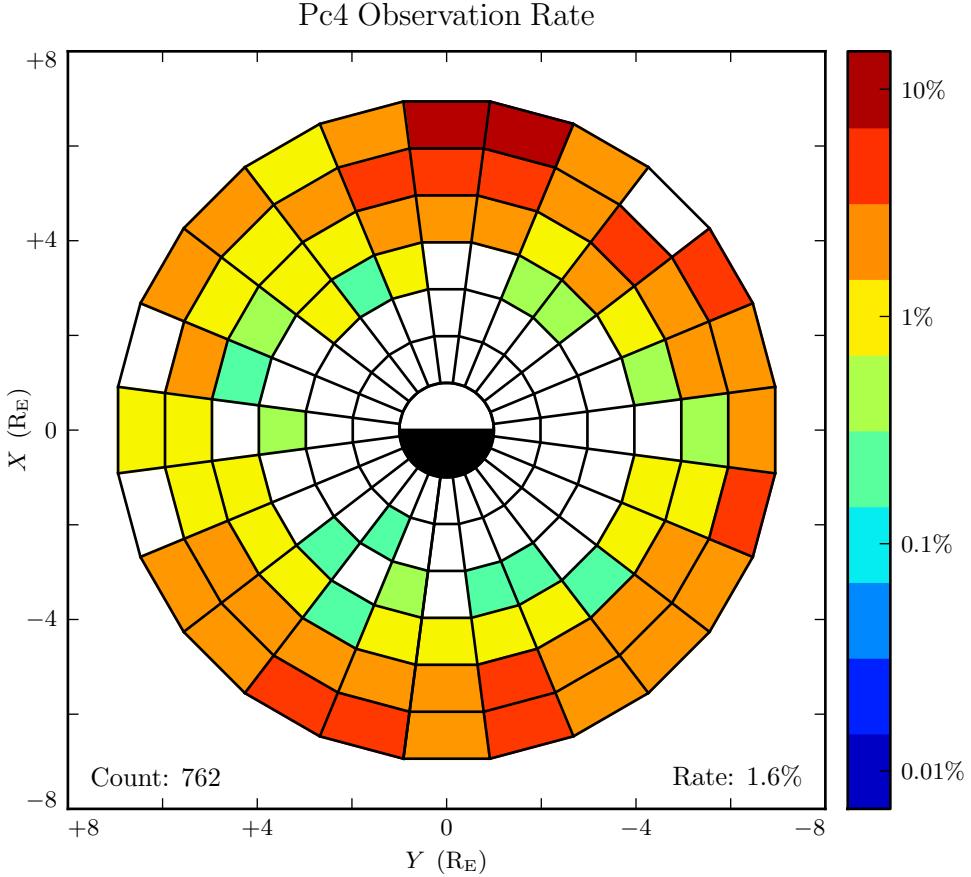


Figure 8.2: The above figure shows the spatial distribution of all 762 observed Pc4 events. Counts are normalized by the amount of usable data in each bin. The value in the bottom-right corner is the mean of the rate in each bin, with the rate in each bin weighed by the area of that bin. Events where the poloidal and toroidal channel both trigger ($\sim 10\%$ of events) are counted as only a single event. Bins shown in white contain zero events.

1368 characterized the parity of only a quarter of his events; among those, he found even
 1369 harmonics to outnumber odd harmonics ten-to-one.

1370 In fact — to the degree that they can be straightforwardly compared — the distributions
 1371 in Figure 8.3 also show agreement with work by Anderson[2] (using AMPTE/CCE),
 1372 Kokubun[52] (using ATS6), Liu[56] (using THEMIS), and Motoba[68] (using GOES).
 1373 Toroidal events dominate overall, and are primarily seen on the morning side. Poloidal

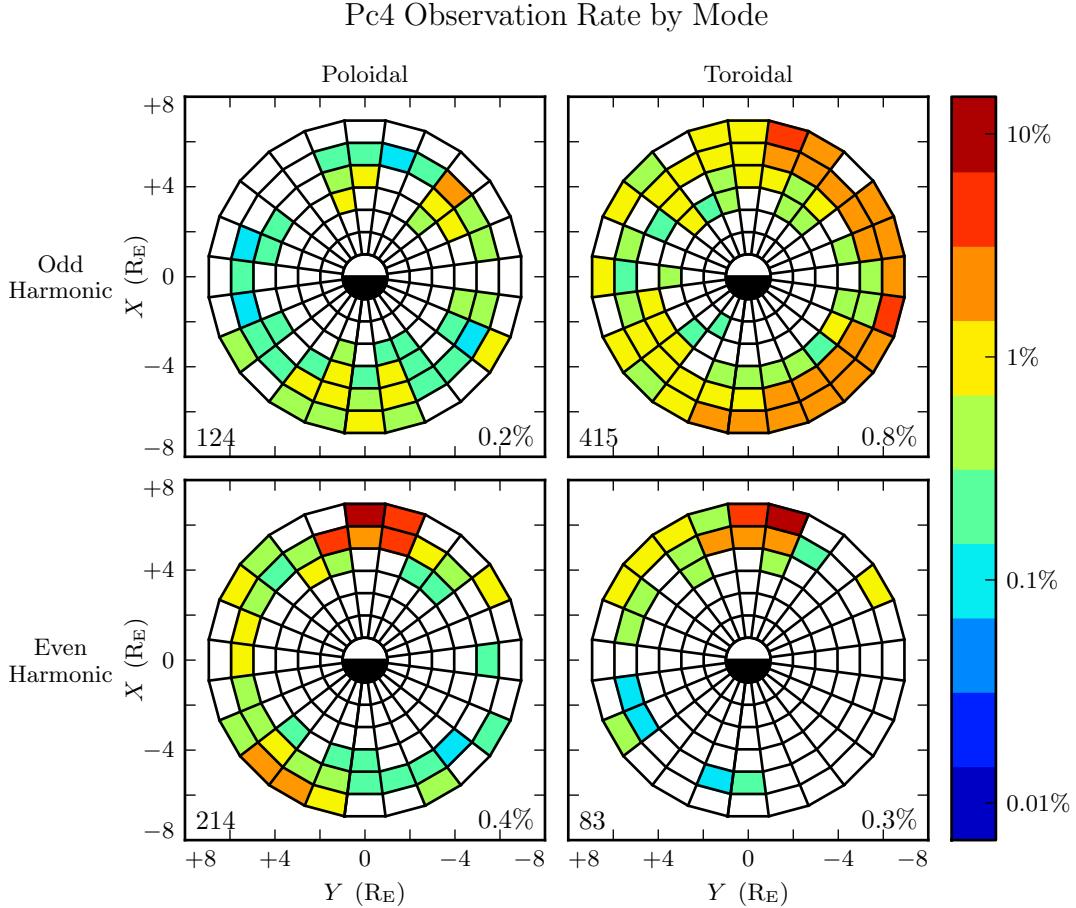


Figure 8.3: The above figure shows the spatial distribution for the same 762 events shown in Figure 8.2, partitioned by polarization and parity. The selection criteria described in Section 8.1 ensure that both properties are known for all events. Event counts are normalized by the time spent by the amount of usable data in each bin. Counts shown in the bottom-left corners do not sum to 762 because some events trigger on both the poloidal channel and the toroidal channel.

- ¹³⁷⁴ events are spread broadly in MLT, with a peak near noon and odd harmonics in the
- ¹³⁷⁵ early morning.
- ¹³⁷⁶ Crucially, the present work can offer insight into how previous results fit together. Unlike
- ¹³⁷⁷ events considered in previous works, those shown in Figure 8.3 have all been categorized

1378 in terms of both polarization and parity. And, crucially, the selection process has not
1379 introduced a bias with respect to polarization or parity (at least not an obvious one).

1380 The even events shown in Figure 8.3 show good agreement with the numerical results in
1381 Chapter 7. The even poloidal and even toroidal distributions are qualitatively similar,
1382 as might be expected if even poloidal waves served as a source for even toroidal waves.
1383 Even poloidal waves are more prevalent, suggesting a typical event duration comparable
1384 to the poloidal-to-toroidal rotation timescale. And even toroidal events are skewed
1385 dayward compared to even poloidal events, suggesting that poloidal-to-toroidal rotation
1386 is inhibited by increased Joule dissipation on the nightside.

1387 The same can be said to some extent for the odd events in Figure 8.3, though the trends
1388 are less strong. Odd poloidal and odd toroidal events are both scarce on the dusk flank.
1389 On the dawn flank, poloidal events skew nightward, while toroidal events are spread
1390 broadly — that is, they are skewed dayward compared to the poloidal events. However,
1391 it's unclear why odd toroidal events outnumber odd poloidal events to such a degree.

1392 When the 762 events are broken down by mode in Figure 8.3, the result is 124 odd
1393 poloidal events, 214 even poloidal events, 415 odd toroidal events, and 83 even toroidal
1394 events — a total of 836 events. The total is greater than 762 because in \sim 10 % of
1395 events, the poloidal and toroidal channels trigger independently. Such cases are marked
1396 as a single event in Figure 8.2, but the toroidal and poloidal events are both shown in
1397 Figure 8.3.

1398 Double-triggering can be taken as a vague proxy for event quality. When the channels
1399 both trigger independently, the two events almost always (71 of 74 events) exhibit the
1400 same parity. This suggests a poloidal wave with sufficient power, and a sufficient narrow
1401 spectral peak, that it can still be seen after much of its energy has rotated to the toroidal
1402 mode.

1403 The spatial distribution of double events is shown in Figure 8.4. The left column shows
1404 events observed with $DST \geq -30$ nT, normalized by the amount of usable data at
1405 $DST \geq -30$ nT. The right column shows events at $DST < -30$ nT, normalized by the
1406 amount of data with $DST < -30$ nT.

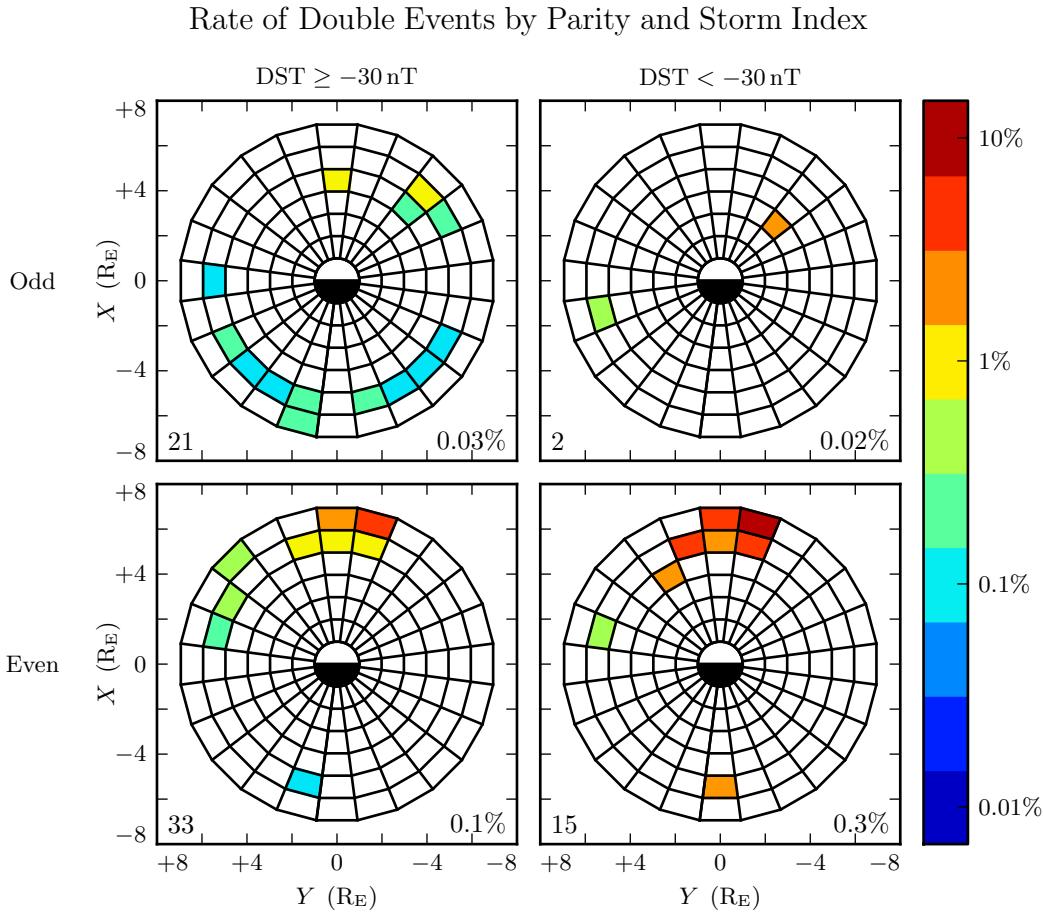


Figure 8.4: A double event is a simultaneous triggering of the poloidal and toroidal channels on the same probe. In such cases, the two channels almost always exhibit the same parity. Double events serve as a vague proxy for event quality — a poloidal event with sufficient strength and clarity to be seen even after much of its energy has rotated to the toroidal mode. Odd double events are spread broadly; even events are concentrated near noon, and become more common during geomagnetically active times.

¹⁴⁰⁷ Odd double-triggering events are spread broadly in MLT. They rarely occur twice on
¹⁴⁰⁸ the same day; the 23 events shown take place over 20 different dates. Odd double events
¹⁴⁰⁹ occur at similar rates regardless of DST.

¹⁴¹⁰ Even-harmonic double-triggering events, on the other hand, are mostly seen near noon,
¹⁴¹¹ and are significantly more common during geomagnetically active times. Even events

1412 are also more concentrated than odd ones. The 48 even-harmonic double-events shown
1413 in the bottom row of Figure 8.4 are spread over 20 days, and 35 of them are spread over
1414 just 7 days. This clustering — where the poloidal and toroidal channel both trigger for
1415 five to ten half-hour events in the same day — is prevalent regardless of DST.

1416 8.3 Events by Amplitude

1417 One might reasonable be concerned that the spatial distributions presented in Figure 8.3
1418 are dominated by these small events, while Pc4 events large enough to be noteworthy
1419 follow a different distribution entirely.

1420 The distribution of event magnitudes is presented in Figure 8.5, graded based on the
1421 peak of the Gaussian fit of each event’s Poynting flux, $|\tilde{S}(\omega)|$. Mean and median
1422 values are listed for each mode. Most events are small, with Poynting flux well below
1423 0.1 mW/m^2 when mapped to the ionosphere. Only a handful of events — 3 out of 762
1424 — exceed 1 mW/m^2 , typically taken to be the threshold at which visible auroral arcs
1425 form.

1426 Perhaps the most notable feature of Figure 8.5 is the relative uniformity of the distri-
1427 bution of even poloidal events. If a higher magnitude threshold is imposed, as shown in
1428 Figure 8.6, the proportion of even poloidal events rises.

1429 The spatial bins in Figure 8.6 are larger than those in Section 8.2; this change reflects
1430 an effort to keep the number of events large compared to the number of bins, even when
1431 considering relatively small subsets of the data. The larger bins — two hours wide in
1432 MLT and divided at $L = 5$ radially — are also used in Sections 8.4 and 8.5. All of the
1433 large-binned bullseye plots also share a common logarithmic color bar.

1434 All else being equal, one might expect the amplitude distribution of even toroidal events
1435 to mimic that of even poloidal events, since poloidal waves asymptotically rotate to
1436 toroidal waves. However, this does not seem to be the case. The mean and median
1437 magnitudes are more or less consistent for even toroidal events, odd toroidal events,
1438 and odd poloidal events, while even poloidal events are twice as large by those metrics.
1439 This would seem to imply that large even poloidal modes have disproportionately high

Amplitude Distribution of Pc4 Events by Mode

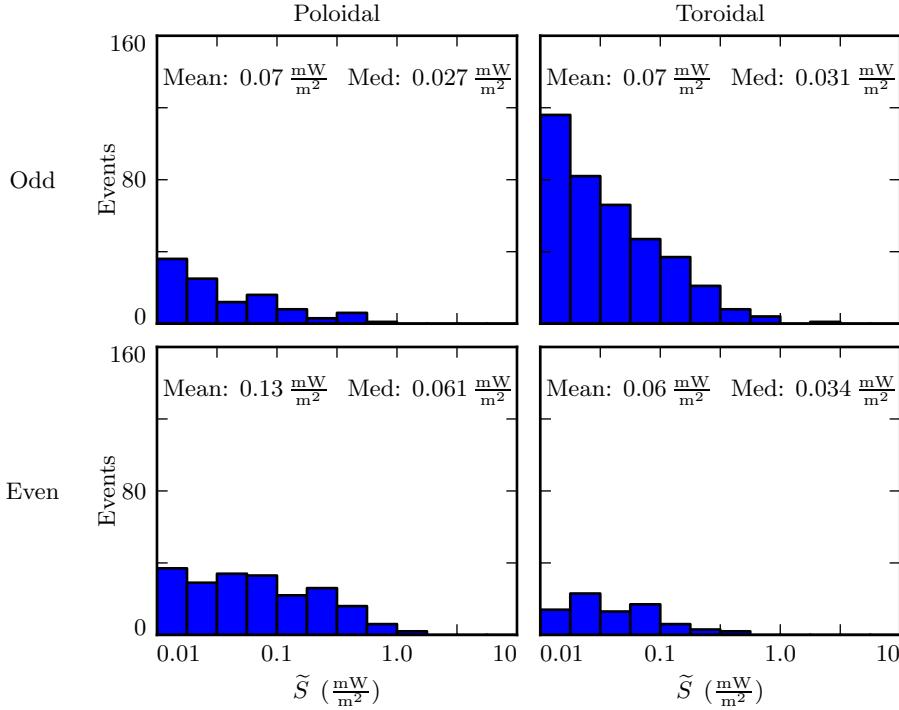


Figure 8.5: Amplitude distribution is shown for Pc4 events by parity and polarization, based on the peak of the spectrum's Gaussian fit. Odd poloidal events, odd toroidal events, and even toroidal events fall off sharply with increasing amplitude, while the even poloidal events are distributed more broadly — the mean and median of the even poloidal distribution are twice as large as those of the others.

modenumbers, and thus deliver energy to the toroidal mode less efficiently. This explanation is unsatisfying, however; Figure 8.6 shows that even poloidal and toroidal modes both become more concentrated near noon at high amplitude, suggesting a common origin.

Distribution of Pc4 Events by Mode and Amplitude

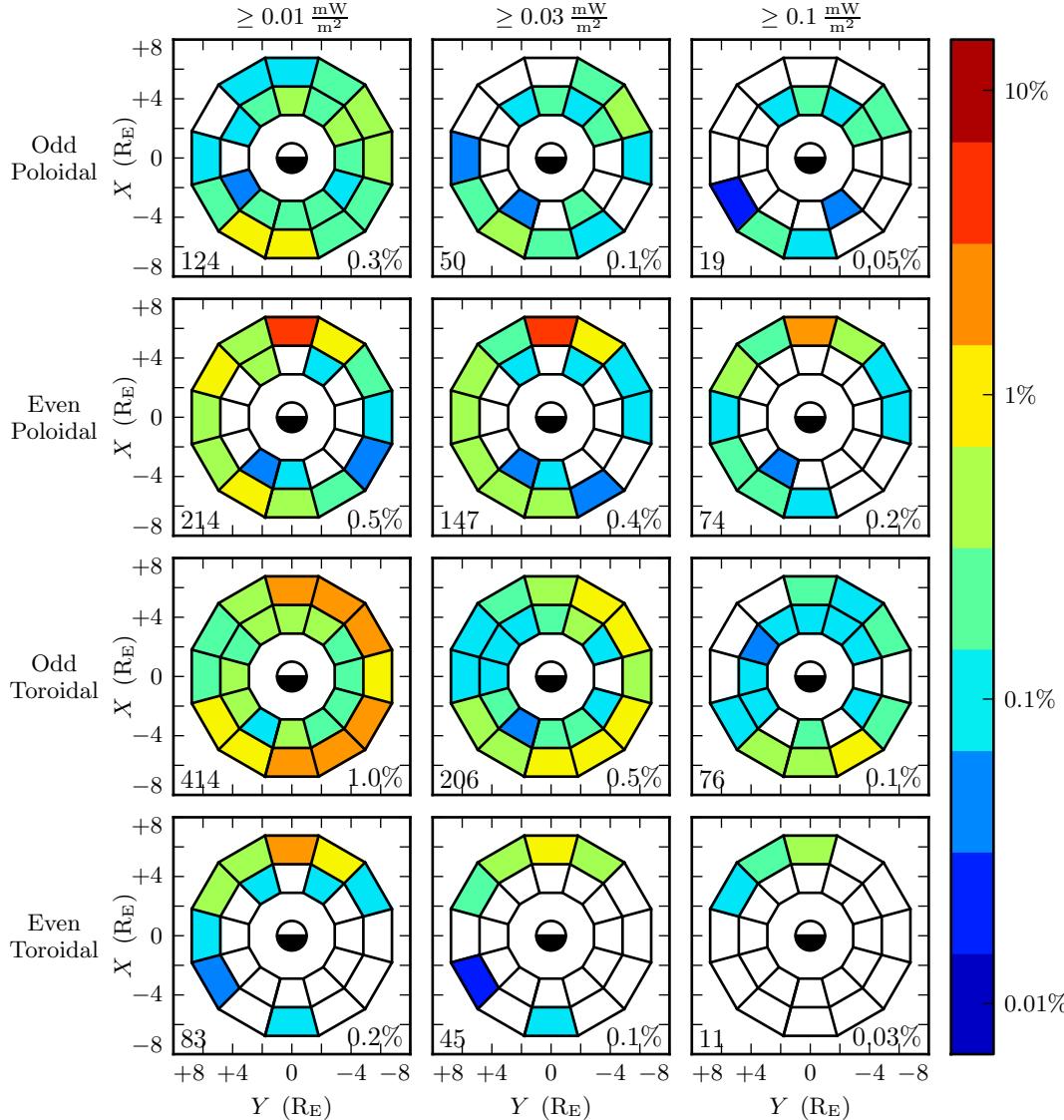


Figure 8.6: The above figure shows the distribution of Pc4 event observations by mode. Event magnitude cutoff is constant down each column, and increases from left to right. Stronger even events appear to become more concentrated on the dayside as the amplitude increases. Even poloidal events also become significantly more numerous relative to the other three modes, from 26 % at a cutoff of 0.01 mW/m^2 to 41 % above 0.1 mW/m^2 .

1444 8.4 Events by Frequency

1445 The difference in magnetospheric conditions between the dayside and the nightside
1446 suggest that different eigenfrequencies should arise between dayside and nightside res-
1447 onances at the same L -shell. In fact, this phenomenon has been observed directly;
1448 the frequencies of azimuthally-drifting FLRs have been shown to change over time[68].
1449 The effect is attributed to the difference in mass loading (and thus Alfvén speed) as a
1450 function of MLT.

1451 This effect was furthermore apparent in the numerical results shown in Chapter 7, where
1452 Alfvén speeds on the dayside (based on empirical profiles) gave rise to significantly higher
1453 eigenfrequencies than those on the nightside.

1454 In Figure 8.7, events at 11 mHz to 17 mHz (center column) do seem to be shifted night-
1455 ward compared to those at 7 mHz to 11 mHz (left column), but the effect is far less
1456 pronounced than what is suggested by Sections 7.2 and 7.3.

1457 As might be expected, even events are more prevalent than (mostly fundamental) odd
1458 events higher in the Pc4 range. Events at 7 mHz to 11 mHz (left column) outnumber
1459 those at 17 mHz to 25 mHz (right column) ten-to-one or more for odd events. Among
1460 even events, the comparison is more like three-to-one.

1461 The spatial distribution of odd toroidal events above 17 mHz warrants specific consid-
1462 eration. Whereas odd toroidal events overall show an overwhelming preference for the
1463 morning side, those at the top of the Pc4 frequency band instead appear from noon
1464 to dusk. It's possible that this distribution is a consequence of the small number of
1465 events (25). More likely, however, is that these are third-harmonic events, and that
1466 their source more closely resembles the source for second-harmonic waves than it does
1467 first harmonics.

1468 The frequency distribution for each mode is shown in Figure 8.8. The most distinctive
1469 feature, certainly, is the frequency peak in the odd toroidal mode near 9 mHz. This is
1470 in line with the idea that toroidal waves exhibit frequencies that depend sharply on L ,
1471 as discussed in Chapter 7. While the Van Allen Probes' orbits do cover a large range of

¹⁴⁷² L -shells, their observations (and thus the selected events) are concentrated near apogee
¹⁴⁷³ at $L \sim 6$.

Distribution of Pc4 Events by Mode and Frequency

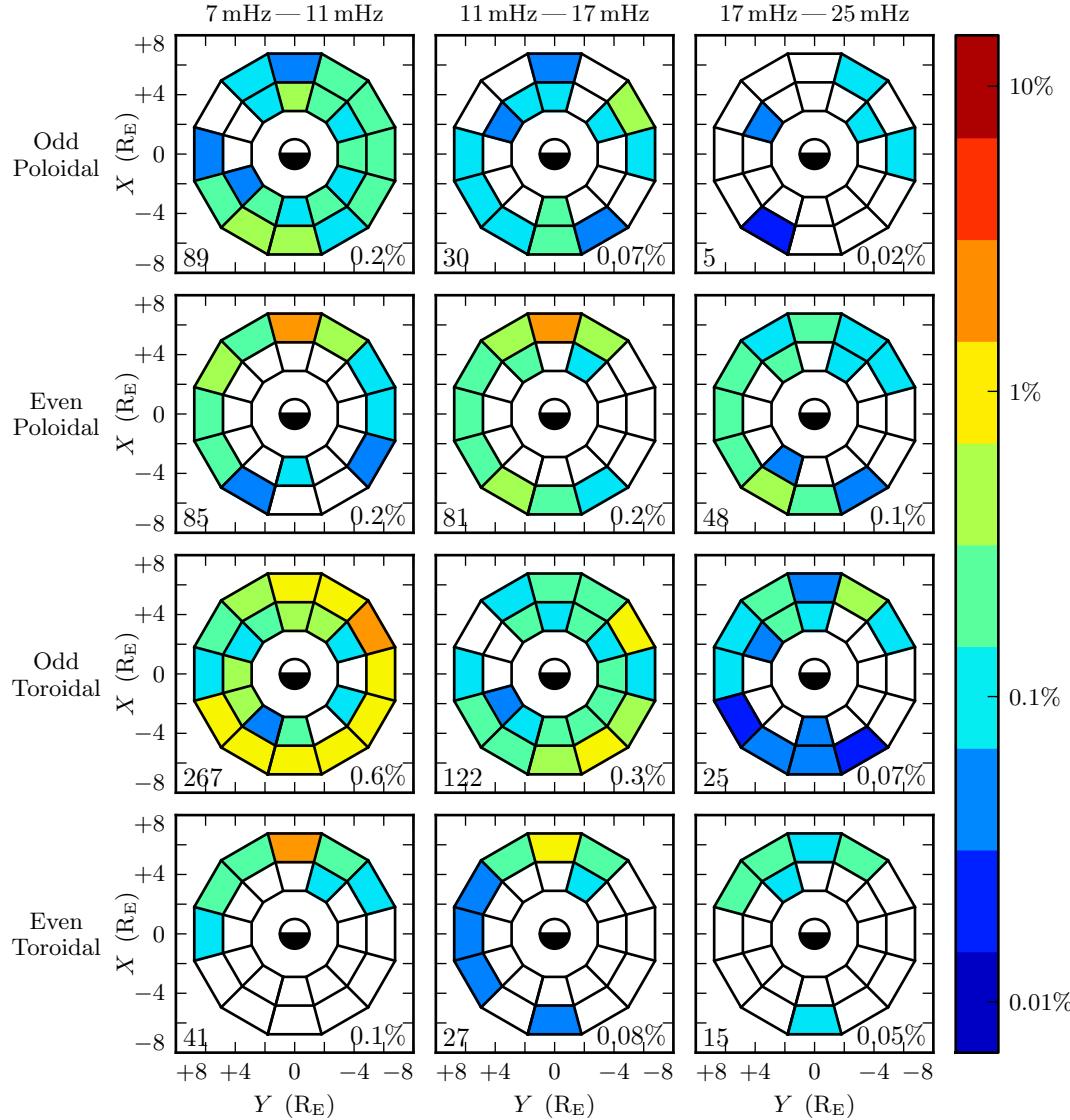


Figure 8.7: Event distributions above are shown in terms of mode (row) as well as event frequency (column). Mid-frequency Pc4 events are shifted somewhat nightward compared to low-frequency Pc4 events, as might be expected from the dayside's faster Alfvén speed. At the top of the Pc4 band, the distribution of odd toroidal events takes on a decidedly different character; this is likely because these events are third harmonics rather than first harmonics.

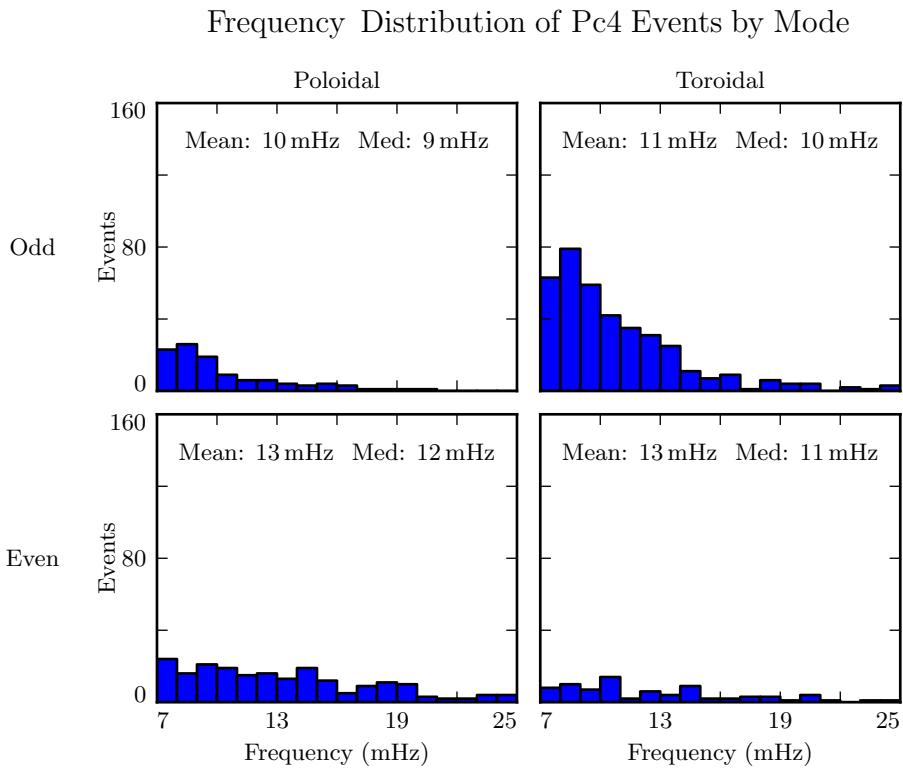


Figure 8.8: Frequency distributions are shown for all events, divided by harmonic and polarization. Odd toroidal events exhibit a particularly sharp peak in frequency, which is consistent with the toroidal mode's strong correlation with the local eigenfrequency. Poloidal modes appear more spread out in frequency, which is also consistent with past observations and with the numerical results in Chapter 7.

¹⁴⁷⁴ 8.5 Events by Phase

¹⁴⁷⁵ The phase of a wave — that is, the phase offset between a wave's electric and magnetic
¹⁴⁷⁶ fields — indicates how its energy is partitioned between the standing and traveling
¹⁴⁷⁷ wave modes. An ideal standing wave has a phase of $\pm 90^\circ$, and thus its Poynting flux is
¹⁴⁷⁸ completely imaginary. A traveling wave, on the other hand, has electric and magnetic
¹⁴⁷⁹ fields in phase (or in antiphase), and is associated with a net movement of energy,
¹⁴⁸⁰ usually toward the ionosphere.

¹⁴⁸¹ Wave phase is a topic of significant interest, since it allows an estimate to be made of
¹⁴⁸² the wave's lifetime. And, because phase can only be determined using simultaneous
¹⁴⁸³ electric and magnetic field measurements, it has only recently become observable.

The energy per unit volume, and the rate at which energy is carried out of that volume
by Poynting flux, are respectively given by:

$$U = \frac{R^3}{2\mu_0} B^2 \quad \frac{\partial}{\partial t} U = \frac{R^2}{\mu_0} EB \cos \varphi \quad (8.2)$$

¹⁴⁸⁴ Where B , E , and R are the characteristic magnetic field magnitude, electric field mag-
¹⁴⁸⁵ nitude, and length scale. The phase, $\varphi \equiv \arctan \frac{\text{Im} \tilde{S}}{\text{Re} \tilde{S}}$, enters because only real Poynting
¹⁴⁸⁶ flux carries energy.

The ratio of the two quantities in Equation (8.2) gives a characteristic timescale over
which energy leaves the system

$$\tau \equiv \frac{BR}{2E \cos \varphi} \quad (8.3)$$

¹⁴⁸⁷ In the present case, magnetic fields are on the order of 1 nT and electric fields are on
¹⁴⁸⁸ the order of 1 mV/m. A reasonable scale length might be 10^4 km, the distance traversed
¹⁴⁸⁹ by the probe over the course of a half-hour event near apogee (notably, back-to-back
¹⁴⁹⁰ events are unusual).

¹⁴⁹¹ At a phase of 80° , this timescale is comparable to a Pc4 wave period. At 135° , where
¹⁴⁹² energy is divided evenly between the standing and traveling wave, the timescale is just

1493 7 seconds. A wave with a phase so far from 90° would quickly vanish unless it were
1494 constantly being replenished.

1495 An example of just such an event is shown in Figure 8.9. The left column shows
1496 electric and magnetic field waveforms in blue and red respectively. The right shows
1497 the corresponding spectra: imaginary Poynting flux in magenta (corresponding to the
1498 strength of the standing wave) and real Poynting flux in green (for the traveling wave).
1499 The black line is a Gaussian fit to the magnitude of the Poynting flux.

1500 The poloidal channel shows a mostly-standing wave, with a phase of 79° . The coherent
1501 activity in the compressional magnetic field implies a low azimuthal modenumber, and
1502 thus a fast rotation of energy from the poloidal mode to the toroidal mode. It's likely
1503 the rotation of energy from the poloidal mode contributes significantly to the toroidal
1504 mode's lifetime; the toroidal wave's phase is 130° , so its energy should be carried away
1505 quickly by Poynting flux.

1506 The selection process described in Section 8.1 does not explicitly consider phase. How-
1507 ever, the discrete Fourier transform is performed over a half-hour time span. An event
1508 with a comparatively short lifetime would be unlikely to register. It's unsurprising to
1509 see the events in Figure 8.10 are tightly clustered near 90° .

1510 It's further notable in Figure 8.10 that the odd events are more spread out in phase
1511 than the even events. Near the equator, odd modes have an electric field antinode and
1512 a magnetic field node; per Equation (8.3), an odd mode's lifetime should be longer than
1513 that of an even mode with the same phase.

1514 Unlike amplitude (Section 8.3) and frequency (Section 8.4), events of different phase do
1515 not seem to exhibit different spatial distributions, as shown in Figure 8.11. Comparisons
1516 are limited by the small event counts in several of the subplots; however, coarsely
1517 speaking, events with phases of 75° and worse (left column) show spatial distributions
1518 more or less in proportion with events phased 85° or better (right column). Figure 8.10
1519 uses the absolute value of each event's phase, as does Figure 8.11.

Waveforms and Spectra: Odd Poloidal Wave and Odd Toroidal Wave

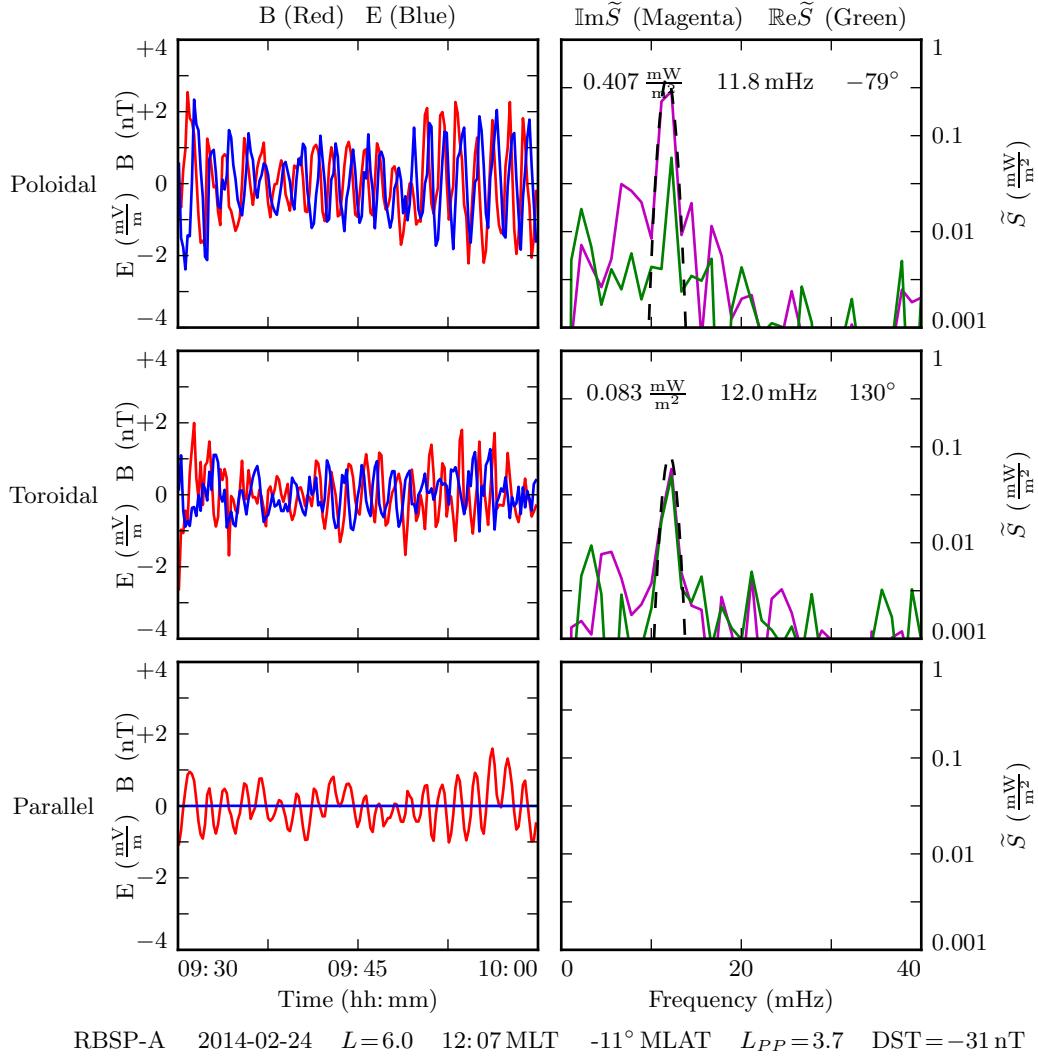


Figure 8.9: The above is a double event, where the poloidal and toroidal channels have been independently selected as events. The poloidal channel shows a wave with most of its energy in the standing wave (phase of 79°). The toroidal mode has a significant traveling component (phase of 130°). The compressional activity implies a low modenumber, which would cause energy to rotate quickly from the poloidal mode to the toroidal mode — evidently at a sufficient rate to replenish the losses due to the traveling mode's real Poynting flux.

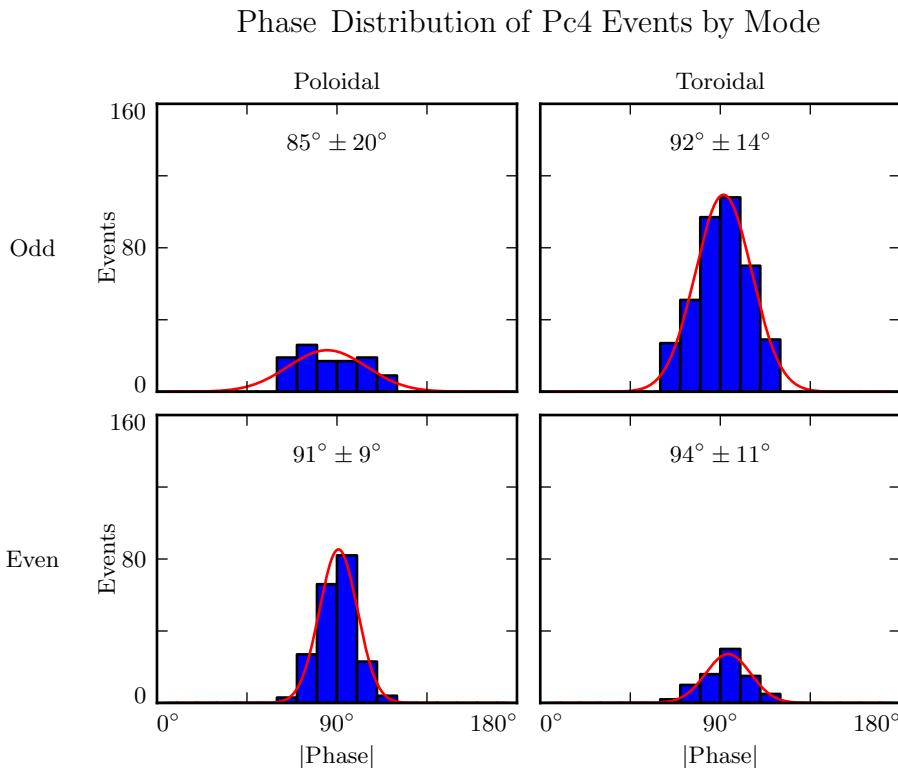


Figure 8.10: The (absolute) phase of the selected Pc4 events is shown above. All modes show phase distributions peaked around 90° . This reflects the fact that a significant traveling wave component quickly carries energy away from an FLR. Odd events are spread more broadly in phase than even events. This is consistent with the odd modes' electric field antinode near the equator, where events are observed; the characteristic loss timescale depends on $\frac{B}{E}$ per Equation (8.3).

Distribution of Pc4 Events by Mode and Phase

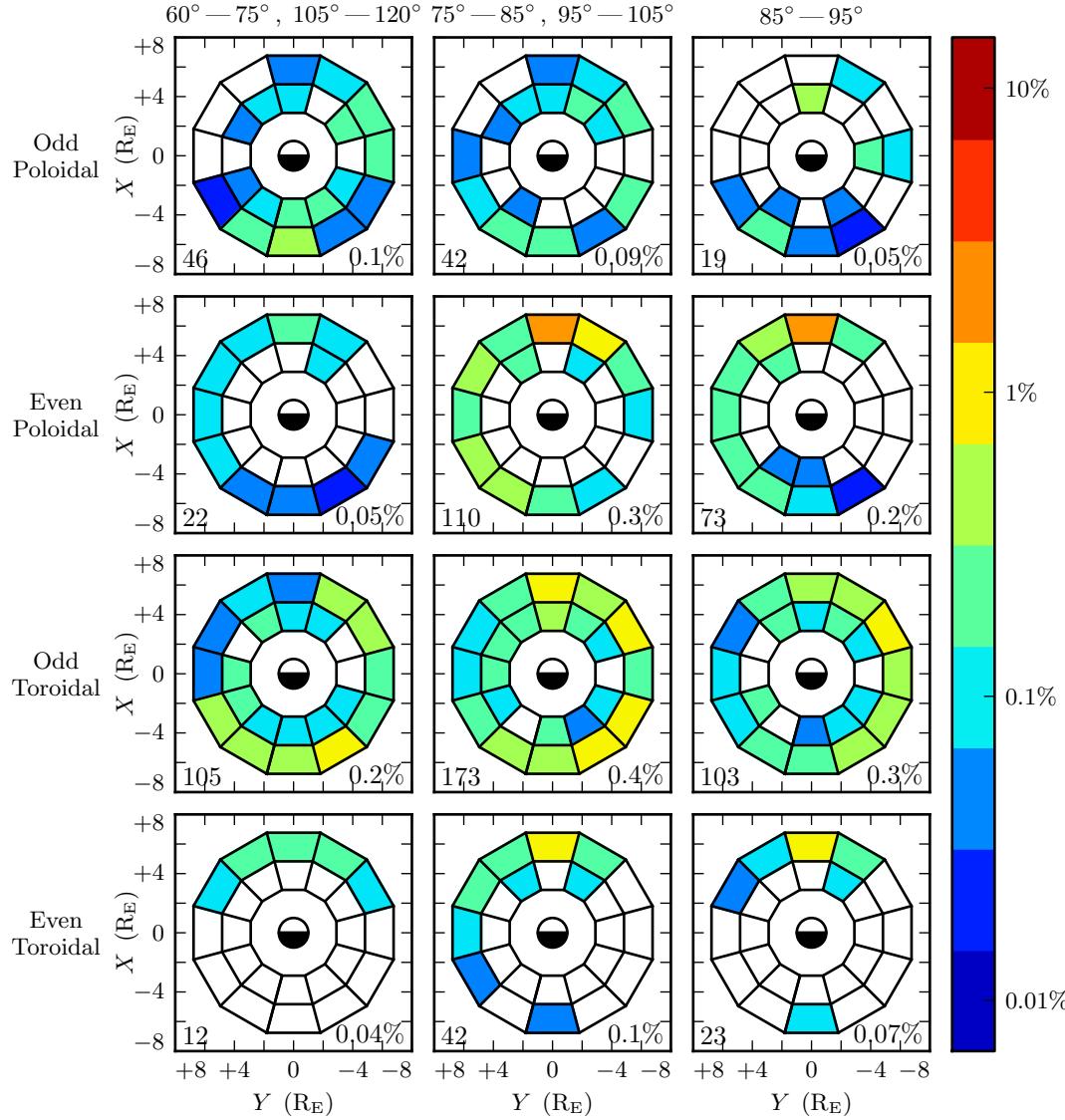


Figure 8.11: The observation rate of events is shown above, divided by (absolute) phase as well as mode. The closer a phase to 90° , the more of an event's energy is in the standing wave, rather than the traveling wave. The spatial distribution of events is more or less consistent between waves with phases very close to 90° and those with a significant traveling wave component.

1520 **8.6 Discussion**

1521 The present chapter gives a survey of ~ 800 thirty-minute Pc4 events, each characterized
1522 in terms of both parity and polarization, and selected in a way that does not introduce
1523 an apparent bias in either property. No past study has so thoroughly disentangled the
1524 parity and polarization of these waves.

1525 Coarsely speaking, event distributions are found to be consistent with past surveys.
1526 Toroidal events dominate overall, and are primarily seen on the morning side. Poloidal
1527 events are spread broadly in MLT, with a peak near noon and distinctive odd harmonics
1528 in the early morning. From there, the simultaneous consideration of harmonic and
1529 polarization, combined with the numerical results from Chapter 7, offers significant
1530 insight.

1531 The near-noon peak of poloidal Pc4 events is shown to be due to even events (a majority
1532 subset). Odd poloidal events occur preferentially near midnight and across the morning
1533 side. Similarly, toroidal events are mostly odd, and it is specifically the odd toroidal
1534 events which occur on the morningside, while even toroidal events peak near noon.

1535 The spatial distribution of even poloidal events looks much like the spatial distribution of
1536 even toroidal events, except that the toroidal distribution is skewed dayward compared
1537 to the poloidal. The same can be said of the odd events. This is consistent (per
1538 Chapter 7) with poloidal events as an effective source for (same-parity) toroidal events
1539 on the dayside, and a less-effective source on the nightside.

1540 Curiously, only a small minority (17 %) of toroidal events are found to be odd, while a
1541 majority (63 %) of poloidal events are even. This disparity may offer clues to the source
1542 of these waves.

1543 All events are found to follow a similar amplitude distribution, except for even poloidal
1544 events, which are notably larger. The cause is unclear.

1545 The $\sim 6\%$ of odd toroidal Pc4s at the top of the frequency range (17 mHz to 25 mHz)
1546 are found to exhibit a qualitatively different spatial distribution from the rest. It's likely
1547 that these waves are third harmonics, and that their excitation mechanism more closely
1548 resembles that of second harmonics than it does first harmonics.

1549 Event phase is also considered. Most events of both polarizations are shown to have
1550 absolute phase in the range 80° to 100° , indicating that the traveling component of
1551 $\text{Pc}4$ pulsations is small compared to the standing component. Odd events are found to
1552 be spread more broadly in phase; this is likely a consequence of being measured near
1553 the equator, where (due to the electric field antinode) the lifetime of an odd event is
1554 significantly larger than that of an even event with the same phase.

1555 **Chapter 9**

1556 **Conclusion**

1557 **9.1 Code Development**

1558 TODO: Code development... Chapters 5 and 6

1559 TODO: Make the Git repository public, and link to it.

1560 Arbitrary deformation of grid. Get $\hat{e}_i = \frac{\partial}{\partial u^i} \underline{x}$, then $g_{ij} = \hat{e}_i \cdot \hat{e}_j$, then invert the metric
1561 tensor for contravariant components.

1562 MPI. Time to compute vs time to broadcast. This might make sense for inertial length
1563 scales.

1564 Better ionospheric profiles. Distinction between the dawn and dusk flanks. Maybe even
1565 update the conductivity based on energy deposition — precipitation causes ionization!

1566 IRI ionosphere model. Solar illumination effects.

1567 **9.2 Numerical Work**

1568 TODO: Numerical results... Chapter 7

1569 TODO: Re-summarize the Discussion sections, I guess.

1570 More complicated driving. Higher harmonics, non-sinusoidal waveforms. Maybe even
1571 drive based on events?

1572 Look at the phase of waves in Tuna. How much is standing/traveling?

1573 **9.3 Van Allen Probe Pc4 Survey**

1574 **TODO:** Observational results... Chapter 8

1575 **TODO:** Link to the Git repository.

1576 Basically just do everything over again, twice as well, once the probes have finished
1577 sampling the dayside again.

1578

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