Case Study 4: Computational Methods in Finance

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In [3]:
```
import modulesForCalibration as mfc

import warnings
warnings.filterwarnings("ignore")

import math
import numpy as np
import scipy.integrate as integrate
import pandas as pd

from scipy.optimize import fmin, fmin_bfgs, minimize
from scipy.stats import norm

import cmath
import math

from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
%matplotlib inline

from datetime import datetime
from tqdm import tqdm
from matplotlib import cm
```

Simulate the market data with Heston Stochastic Volatility Model

Let's set the parameters we will consider for the Heston model:

In [4]:
```
S0 = 4100
r = 0.0485
mu = r
params = [0.02, 1.5, 0.05, 0.18, 0.5, 0.04]
```

Here, we chose to considerate the parameter we obtain working on the SP500 data during the last assignment.

In [36]:
```
N = 10*252 # 10 years * 252 trading day per year
T = 10
dt = T/N
```

In [37]:
```
def heston_simulated_prices(params, N, T, S0, r, mu, plot = False):
    kappa  = params[0]
    theta  = params[1]
    sigma  = params[2]
    rho    = params[3]
```

```
    v0     = params[4]

    # Define discretization parameters
    dt = T/N        # time increment
    M = 1           # number of simulations
    print('T:',T,'  N:', N, '  dt:', dt)
    #print(v0)

    # Generate random numbers
    Z1 = norm.rvs(size=(N, M))
    Z2 = rho*Z1 + np.sqrt(1-rho**2)*norm.rvs(size=(N, M))

    # Define arrays to store stock price and volatility paths
    S = np.zeros((N+1, M))
    v = np.zeros((N+1, M))

    # Set initial values
    S[0,:] = S0
    v[0,:] = v0 #theta
    #print(v0)

    # Calculate paths
    for i in range(N):
        v[i+1,:] = np.maximum(0, v[i,:] + kappa*(theta-v[i,:])*dt + sigma*np
        #print(v[i+1,:])
        S[i+1,:] = S[i,:] * np.exp((mu - 0.5*v[i,:])*dt + np.sqrt(v[i,:])*np
        #print(S[i+1,:])

    # Plot results
    if plot == True:
        plt.plot(S)
        plt.title('Simulated Heston Model Stock Price Path')
        plt.xlabel('Time Steps')
        plt.ylabel('Stock Price')
        plt.show()

        plt.plot(v)
        plt.title('Simulated Heston Model volatility Path')
        plt.xlabel('Time Steps')
        plt.ylabel('Volatility')
        plt.show()

    # Reshaping the outputs
    y = np.log(S)
    S = S.T
    S = S[0]
    v = v.T
    v = v[0]
    y = y.T
    y = y[0]

    return S, v, y
```

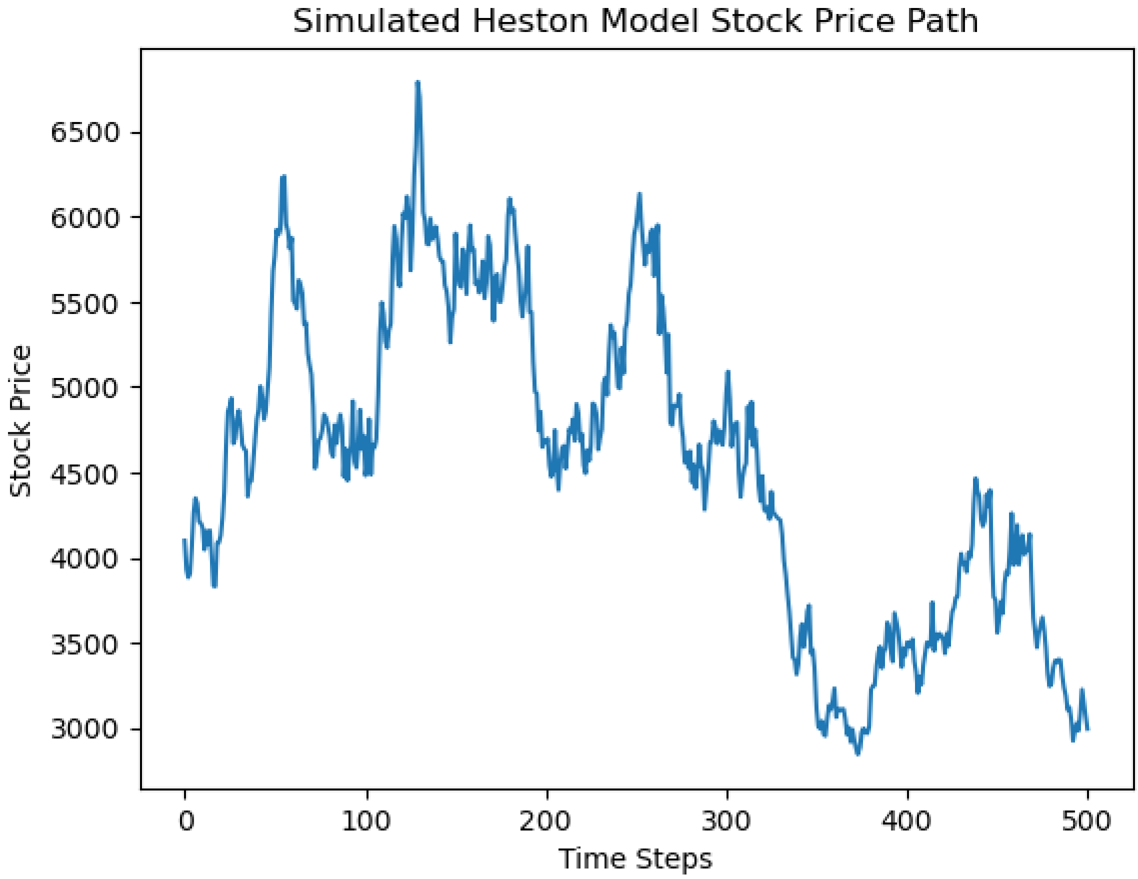
In [38]:

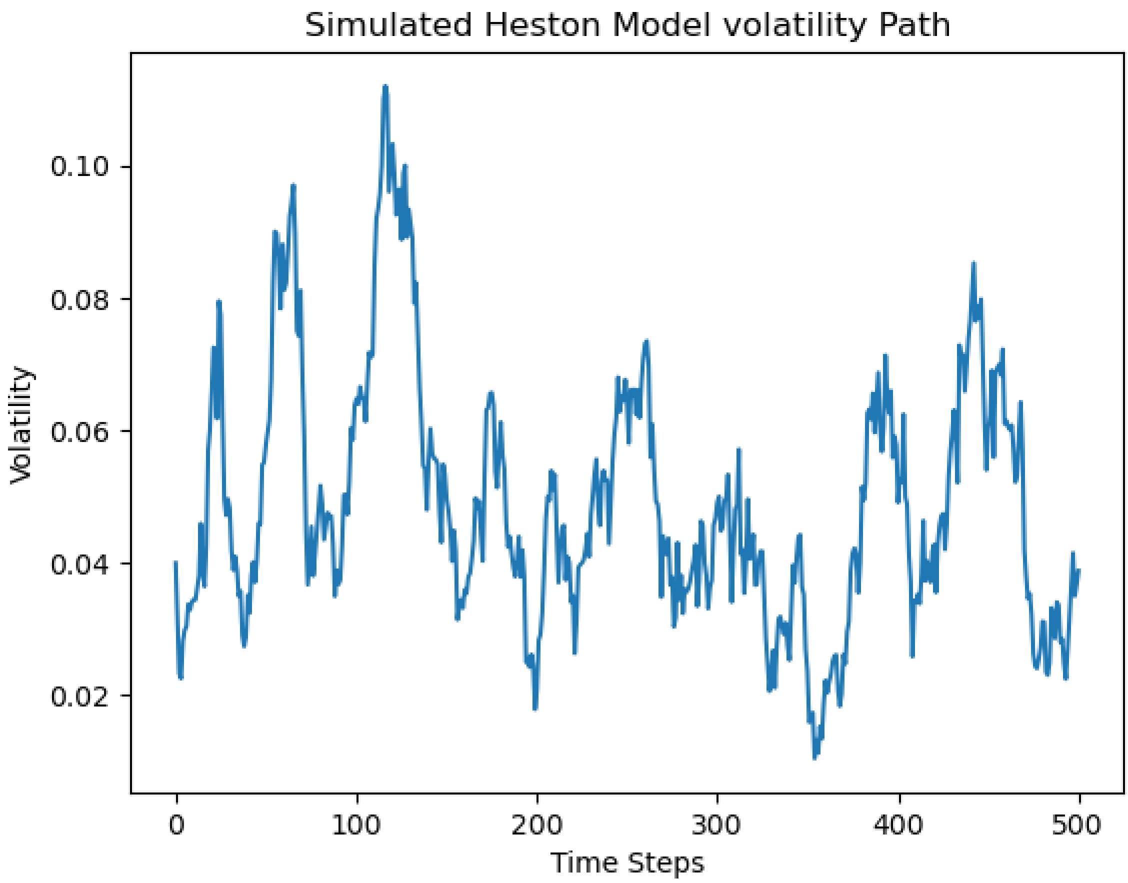
```
#prices, v_, y = heston_simulated_prices(params, N, T, S0, r, mu, plot = Tru
S0 = 4100
r = 0.0485
mu = r
params = [mu, 1.5, 0.05, 0.18, 0.5, 0.04]
#.        mu, kappa, theta, lambda_, rho , v0
N = 500
```

```
T = 10

params_sim = np.zeros(5)
params_sim[0] = params[1]# kappa
params_sim[1] = params[2]#theta
params_sim[2] = params[3]#sigma
params_sim[3] = params[4]#rho
params_sim[4] = params[5]#v0
#print(params_sim)
prices, v_, y = heston_simulated_prices(params_sim, N, T, S0, r, mu, plot =
```

```
T: 10   N: 500   dt: 0.02
```





1. Extended Kalman Filter

In [211…

```
def Extended_Kalman_Filter(params):
    # Declare global variables
    global y_EKF, v_EKF

    # Extract parameters
    mu, kappa, theta, lbda, rho, v0 = params

    dt = T / N  # Calculate time step

    # Initialize matrices
    P = np.matrix([[0.01, 0], [0, 0.01]])
    I = np.identity(2)
    F = np.matrix([[1, -0.5*dt], [0, 1-kappa*dt]])
    U = np.matrix([[np.sqrt(v0*dt), 0], [0, lbda*np.sqrt(v0*dt)]])
    Q = np.matrix([[1, rho], [rho, 1]])
    H = np.matrix([1, 0])

    # Initialize state variables
    x_update = np.matrix([np.log(S0), v0]).T

    # Initialize arrays for storing results
    y_EKF = np.zeros(N)
    v_EKF = np.zeros(N)
    y_EKF[0] = np.log(S0)
    v_EKF[0] = v0

    loglk = 0  # Initialize objective function value
    for i in range(1, N):
        # Prediction step
        x_pred = np.matrix([0, 0], dtype=np.float64).T
```

```
        x_pred[0, 0] = x_update[0, 0] + (mu - 1/2*x_update[1, 0]) * dt
        x_pred[1, 0] = x_update[1, 0] + kappa * (theta - x_update[1, 0]) * c

        P_1 = F * P * F.T + U * Q * U.T

        # Update step
        S = H * P_1 * H.T
        S = S[0, 0]

        e = y[i] - x_pred[0, 0]
        loglk += np.log(abs(S)) + e**2 / S

        K = P_1 * H.T / S
        x_update = x_pred + K * e

        # Apply full truncate scheme to avoid negative volatility values
        x_update[1, 0] = max(1e-5, x_update[1, 0])

        vk = x_update[1, 0]
        U = np.matrix([[np.sqrt(vk*dt), 0], [0, lbda*np.sqrt(vk*dt)]])
        P = (I - K * H) * P_1

        y_EKF[i] = x_update[0, 0]
        v_EKF[i] = x_update[1, 0]

    return loglk #/ N
```

In [199…

```
# Define the objective function for optimization with noise
def objective(params):
    noise = np.random.normal(scale=0.01, size=len(params))  # Add random noi
    params_with_noise = params + noise
    return Extended_Kalman_Filter(params_with_noise)
```

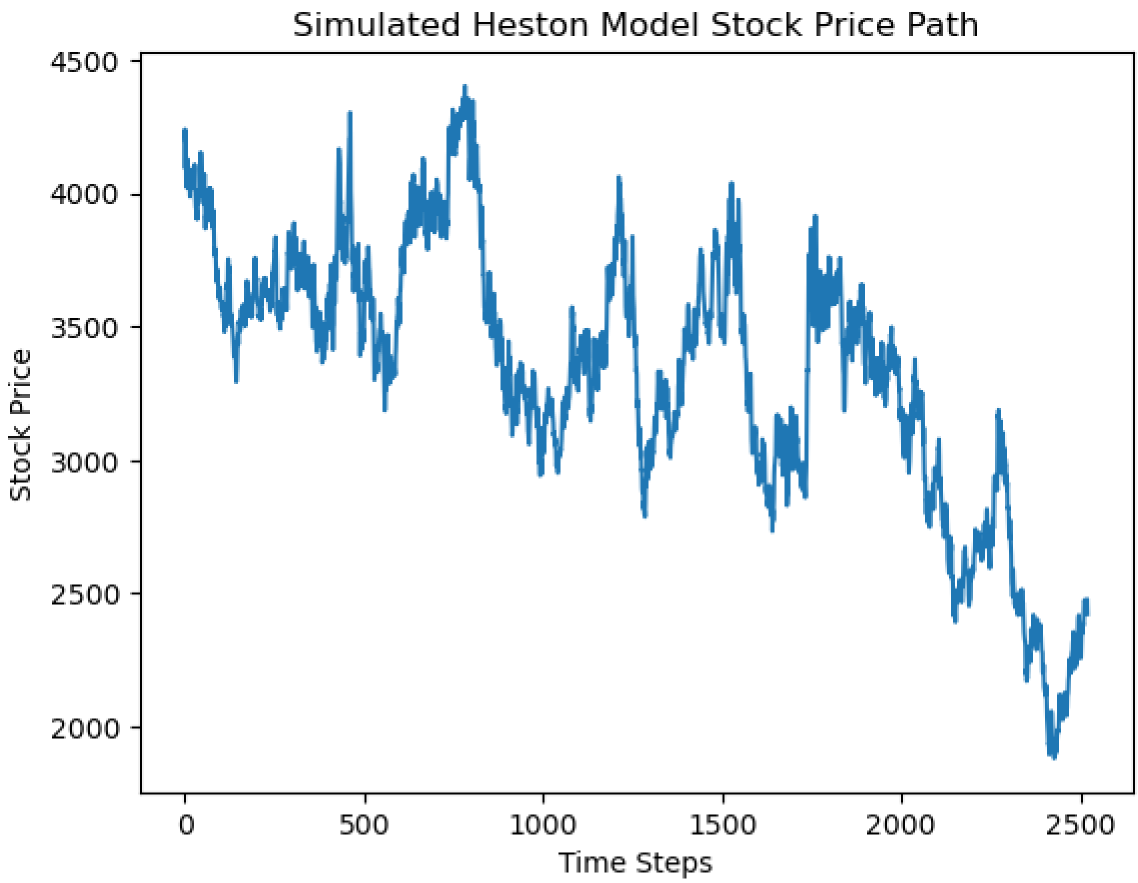
In [212…

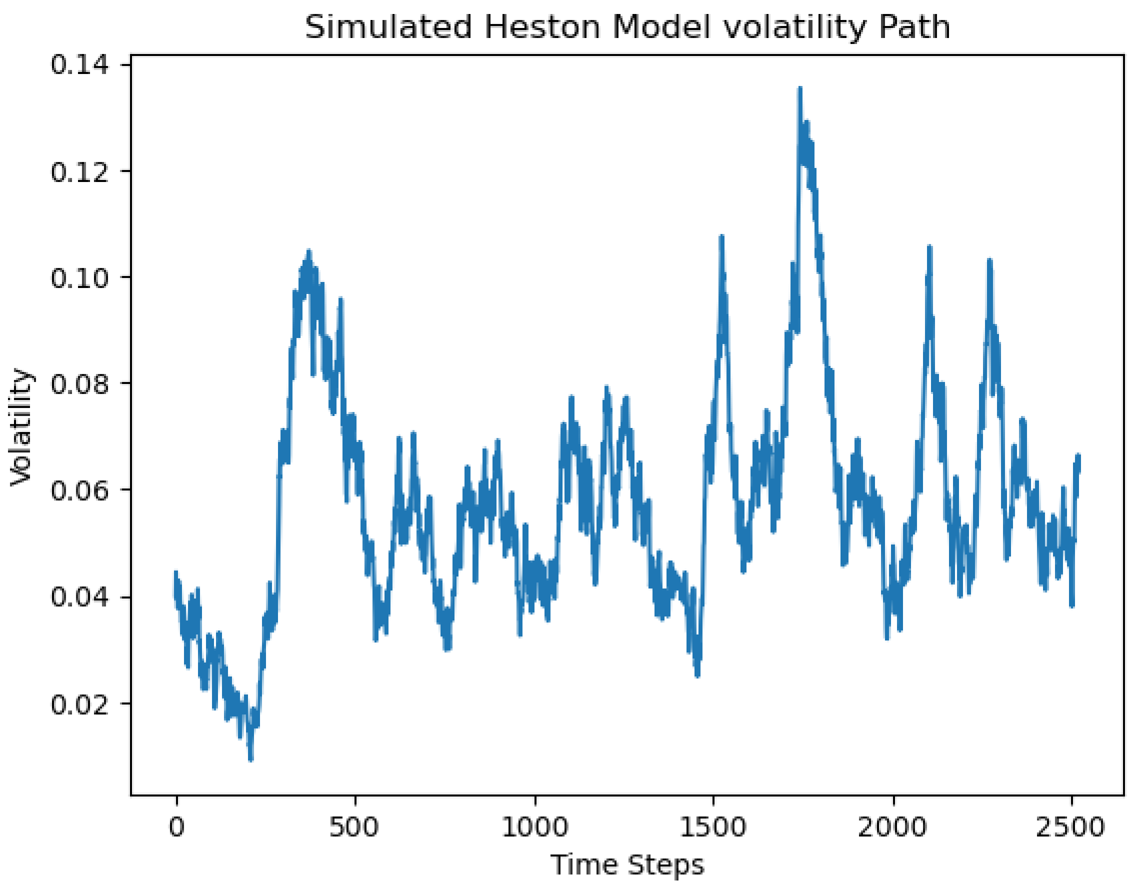
```
S0 = 4100
r = 0.0485
mu = r
params = [mu, 1.5, 0.05, 0.18, 0.5, 0.04]
#.        mu, kappa, theta, lambda_, rho , v0
N = 10*252 # 10 years * 252 trading day per year
T = 10

params_sim = np.zeros(5)
params_sim[0] = params[1]# kappa
params_sim[1] = params[2]#theta
params_sim[2] = params[3]#sigma
params_sim[3] = params[4]#rho
params_sim[4] = params[5]#v0
#print(params_sim)
prices, v_, y = heston_simulated_prices(params_sim, N, T, S0, r, mu, plot =

dt = T/N
#print(dt)
```

```
T: 10   N: 2520   dt: 0.003968253968253968
```





Set the initial guess for the optimization :

In [219…

```
# true : [0.0485, 1.5, 0.05, 0.18, 0.5, 0.04]
params_0 = [0.1, 0.1, 0.1, 0.1, 0.1, 0.05]
params_0 = [0.02, 1.5, 0.05, 0.18, 0.5, 0.1]
params_0 = [0.06, 1.3, 0.07, 0.3, 0.6, 0.04]
```

Use the Nelder-Mead algorithm to minimize the function

In [220…
```
def opt_param_research():

    def callback(x):
        print("Current parameter vector:", x)
        print("Function value: ", Extended_Kalman_Filter(x))

    #constraint1 = {'type': 'ineq', 'fun': lambda x: x}#[0] - 1}
    #constraint2 = {'type': 'ineq', 'fun': lambda x: 2 - x[0]}
    #constraints = [constraint1]#, constraint2]
    bounds = [(0.000001, 30)]*6
    #bounds[1] = (0,2)
    bounds[4] = (-1,1)
    xopt = minimize(Extended_Kalman_Filter, params_0, callback=callback, met
    #xopt_noise =minimize(objective, params_0, callback=callback, method='Ne

    #result_2 = minimize(obj_function_ext_KF_m, params_0, bounds=bounds, cal
    #xopt, fopt, _, _, _ = fmin(ext_Kalman_filter, params_0, maxiter=100, ca

    #result = fmin(ext_Kalman_filter, params_0, callback=callback)
    print(80*'=')
    print('Optimal parameter set:')
    print(xopt)
    print(80*'=')

    return xopt
```

In [221…
```
result_EKF = opt_param_research()
```

```
Current parameter vector: [0.0615  1.3325  0.07175 0.3075  0.54    0.041  ]
Function value:  -18574.696118761647
Current parameter vector: [0.0615  1.3325  0.07175 0.3075  0.54    0.041  ]
Function value:  -18574.696118761647
Current parameter vector: [0.0615  1.3325  0.07175 0.3075  0.54    0.041  ]
Function value:  -18574.696118761647
Current parameter vector: [0.0605     1.38666667 0.07466667 0.29666667 0.54
666667 0.03766667]
Function value:  -18583.5072747409
Current parameter vector: [0.0605     1.38666667 0.07466667 0.29666667 0.54
666667 0.03766667]
Function value:  -18583.5072747409
Current parameter vector: [0.0605     1.38666667 0.07466667 0.29666667 0.54
666667 0.03766667]
Function value:  -18583.5072747409
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06133333 1.45527778 0.07369444 0.29111111 0.45
777778 0.04088889]
Function value:  -18588.43820846982
Current parameter vector: [0.06429321 1.54555556 0.07155556 0.29924897 0.42
853909 0.03717764]
Function value:  -18596.043725327778
Current parameter vector: [0.06429321 1.54555556 0.07155556 0.29924897 0.42
853909 0.03717764]
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853909 0.03717764]
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Current parameter vector: [0.06429321 1.54555556 0.07155556 0.29924897 0.42
853909 0.03717764]
Function value:  -18596.043725327778
Current parameter vector: [0.06429321 1.54555556 0.07155556 0.29924897 0.42
853909 0.03717764]
Function value:  -18596.043725327778
Current parameter vector: [0.06263072 1.61933813 0.06854287 0.28614074 0.43
100385 0.03671291]
Function value:  -18600.15329127393
Current parameter vector: [0.0696436  1.72784979 0.06583745 0.29696831 0.31
173211 0.03460983]
Function value:  -18603.637949274245
Current parameter vector: [0.0696436  1.72784979 0.06583745 0.29696831 0.31
173211 0.03460983]
Function value:  -18603.637949274245
Current parameter vector: [0.0696436  1.72784979 0.06583745 0.29696831 0.31
```

```
173211 0.03460983]
Function value:  -18603.637949274245
Current parameter vector: [0.0666243  1.80824531 0.06456596 0.29542065 0.33
90924  0.03398798]
Function value:  -18604.56171581265
Current parameter vector: [0.0666243  1.80824531 0.06456596 0.29542065 0.33
90924  0.03398798]
Function value:  -18604.56171581265
Current parameter vector: [0.0666243  1.80824531 0.06456596 0.29542065 0.33
90924  0.03398798]
Function value:  -18604.56171581265
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90924  0.03398798]
Function value:  -18604.56171581265
Current parameter vector: [0.0666243  1.80824531 0.06456596 0.29542065 0.33
90924  0.03398798]
Function value:  -18604.56171581265
Current parameter vector: [0.0666243  1.80824531 0.06456596 0.29542065 0.33
90924  0.03398798]
Function value:  -18604.56171581265
Current parameter vector: [0.06806589 1.78531062 0.06378107 0.29667082 0.35
099712 0.03196741]
Function value:  -18604.607333898377
Current parameter vector: [0.06569758 1.78422826 0.06503129 0.29009689 0.34
155404 0.03308928]
Function value:  -18605.110117266748
Current parameter vector: [0.06941321 1.84537899 0.06412378 0.3017672  0.28
33234  0.03118948]
Function value:  -18605.142688659787
Current parameter vector: [0.06941321 1.84537899 0.06412378 0.3017672  0.28
33234  0.03118948]
Function value:  -18605.142688659787
Current parameter vector: [0.06941321 1.84537899 0.06412378 0.3017672  0.28
33234  0.03118948]
Function value:  -18605.142688659787
Current parameter vector: [0.06941321 1.84537899 0.06412378 0.3017672  0.28
33234  0.03118948]
Function value:  -18605.142688659787
Current parameter vector: [0.07151678 1.79553558 0.06411776 0.29662244 0.29
757002 0.02843466]
Function value:  -18606.641566214697
Current parameter vector: [0.07151678 1.79553558 0.06411776 0.29662244 0.29
757002 0.02843466]
Function value:  -18606.641566214697
Current parameter vector: [0.07151678 1.79553558 0.06411776 0.29662244 0.29
757002 0.02843466]
Function value:  -18606.641566214697
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757002 0.02843466]
Function value:  -18606.641566214697
Current parameter vector: [0.07151678 1.79553558 0.06411776 0.29662244 0.29
757002 0.02843466]
Function value:  -18606.641566214697
Current parameter vector: [0.07536361 1.83626605 0.06394267 0.3079646  0.27
862058 0.02557274]
Function value:  -18606.964633574695
Current parameter vector: [0.07536361 1.83626605 0.06394267 0.3079646  0.27
862058 0.02557274]
Function value:  -18606.964633574695
Current parameter vector: [0.07202834 1.79100564 0.06549153 0.2966501  0.31
344252 0.02632022]
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Function value:  -18607.23513428986
Current parameter vector: [0.07202834 1.79100564 0.06549153 0.2966501  0.31
344252 0.02632022]
Function value:  -18607.23513428986
Current parameter vector: [0.07589744 1.83131994 0.06344312 0.30219409 0.28
406633 0.0227217 ]
Function value:  -18607.300190455928
Current parameter vector: [0.07811693 1.78591659 0.06424787 0.30449672 0.28
067728 0.02282795]
Function value:  -18607.464861650213
Current parameter vector: [0.07727365 1.82186428 0.06431645 0.30477857 0.30
539147 0.02188089]
Function value:  -18607.82460234764
Current parameter vector: [0.07727365 1.82186428 0.06431645 0.30477857 0.30
539147 0.02188089]
Function value:  -18607.82460234764
Current parameter vector: [0.07727365 1.82186428 0.06431645 0.30477857 0.30
539147 0.02188089]
Function value:  -18607.82460234764
Current parameter vector: [0.07512987 1.75148442 0.06545359 0.29476628 0.32
796491 0.0222509 ]
Function value:  -18608.243910302157
Current parameter vector: [0.07512987 1.75148442 0.06545359 0.29476628 0.32
796491 0.0222509 ]
Function value:  -18608.243910302157
Current parameter vector: [0.07512987 1.75148442 0.06545359 0.29476628 0.32
796491 0.0222509 ]
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796491 0.0222509 ]
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796491 0.0222509 ]
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796491 0.0222509 ]
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796491 0.0222509 ]
Function value:  -18608.243910302157
Current parameter vector: [0.07512987 1.75148442 0.06545359 0.29476628 0.32
796491 0.0222509 ]
Function value:  -18608.243910302157
Current parameter vector: [0.07512987 1.75148442 0.06545359 0.29476628 0.32
796491 0.0222509 ]
Function value:  -18608.243910302157
Current parameter vector: [0.07542546 1.7455393  0.06555336 0.29483184 0.33
634751 0.02065308]
Function value:  -18608.267910590548
```

```
Current parameter vector: [0.07703999 1.72609785 0.06572555 0.29616824 0.32
716452 0.02052323]
Function value:  -18608.270548974055
Current parameter vector: [0.07480402 1.72165924 0.06568394 0.29197681 0.34
109707 0.02211956]
Function value:  -18608.32979332549
Current parameter vector: [0.07480402 1.72165924 0.06568394 0.29197681 0.34
109707 0.02211956]
Function value:  -18608.32979332549
Current parameter vector: [0.07480402 1.72165924 0.06568394 0.29197681 0.34
109707 0.02211956]
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109707 0.02211956]
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109707 0.02211956]
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109707 0.02211956]
Function value:  -18608.32979332549
Current parameter vector: [0.07480402 1.72165924 0.06568394 0.29197681 0.34
109707 0.02211956]
Function value:  -18608.32979332549
Current parameter vector: [0.07480402 1.72165924 0.06568394 0.29197681 0.34
109707 0.02211956]
Function value:  -18608.32979332549
Current parameter vector: [0.07390152 1.72039658 0.06554802 0.28534762 0.33
50847  0.02150864]
Function value:  -18608.37580534408
Current parameter vector: [0.07390152 1.72039658 0.06554802 0.28534762 0.33
50847  0.02150864]
Function value:  -18608.37580534408
Current parameter vector: [0.07482587 1.68186128 0.06606293 0.28513297 0.34
760359 0.02110112]
Function value:  -18608.415801042436
Current parameter vector: [0.07482587 1.68186128 0.06606293 0.28513297 0.34
760359 0.02110112]
Function value:  -18608.415801042436
Current parameter vector: [0.07482587 1.68186128 0.06606293 0.28513297 0.34
760359 0.02110112]
Function value:  -18608.415801042436
Current parameter vector: [0.07192639 1.69499858 0.06573318 0.27839671 0.35
208214 0.02128199]
Function value:  -18608.49617309586
Current parameter vector: [0.07192639 1.69499858 0.06573318 0.27839671 0.35
208214 0.02128199]
Function value:  -18608.49617309586
Current parameter vector: [0.07192639 1.69499858 0.06573318 0.27839671 0.35
208214 0.02128199]
Function value:  -18608.49617309586
Current parameter vector: [0.07021446 1.69202272 0.06526045 0.26816781 0.35
308311 0.0208137 ]
Function value:  -18608.572212699808
Current parameter vector: [0.07021446 1.69202272 0.06526045 0.26816781 0.35
308311 0.0208137 ]
Function value:  -18608.572212699808
Current parameter vector: [0.06989052 1.65770223 0.06543858 0.26243645 0.35
839839 0.01948642]
Function value:  -18608.599638505904
Current parameter vector: [0.06472443 1.6537023  0.06508875 0.24694094 0.36
```

```
81922  0.02106688]
Function value:  -18608.83067363608
Current parameter vector: [0.06472443 1.6537023  0.06508875 0.24694094 0.36
81922  0.02106688]
Function value:  -18608.83067363608
Current parameter vector: [0.06472443 1.6537023  0.06508875 0.24694094 0.36
81922  0.02106688]
Function value:  -18608.83067363608
Current parameter vector: [0.06472443 1.6537023  0.06508875 0.24694094 0.36
81922  0.02106688]
Function value:  -18608.83067363608
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81922  0.02106688]
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Current parameter vector: [0.06157667 1.62050758 0.06482133 0.22937226 0.38
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Current parameter vector: [0.06157667 1.62050758 0.06482133 0.22937226 0.38
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Current parameter vector: [0.05655496 1.60936155 0.06477468 0.2154934  0.40
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441994 0.02083958]
Function value:  -18609.164230657734
Current parameter vector: [0.05406724 1.60132648 0.06448772 0.20666383 0.40
35325  0.02091033]
Function value:  -18609.16616366312
Current parameter vector: [0.05406724 1.60132648 0.06448772 0.20666383 0.40
35325  0.02091033]
Function value:  -18609.16616366312
Current parameter vector: [0.04977622 1.57244363 0.06430123 0.18940962 0.42
144585 0.02197114]
Function value:  -18609.16911898547
Current parameter vector: [0.05626329 1.56533358 0.06532619 0.21345232 0.41
54124  0.02174456]
Function value:  -18609.203612469883
Current parameter vector: [0.04492611 1.6024584  0.06350905 0.1795651  0.43
94318  0.02160711]
Function value:  -18609.34277877319
Current parameter vector: [0.04492611 1.6024584  0.06350905 0.1795651  0.43
94318  0.02160711]
Function value:  -18609.34277877319
============================================================================
=====
Optimal parameter set:
 final_simplex: (array([[0.04492611, 1.6024584 , 0.06350905, 0.1795651 , 0.
4394318 ,
        0.02160711],
       [0.04798195, 1.57002185, 0.06428683, 0.18426726, 0.43152123,
        0.02130408],
       [0.05626329, 1.56533358, 0.06532619, 0.21345232, 0.4154124 ,
        0.02174456],
       [0.04977622, 1.57244363, 0.06430123, 0.18940962, 0.42144585,
```

```
 0.02197114],
[0.05406724, 1.60132648, 0.06448772, 0.20666383, 0.4035325 ,
 0.02091033],
[0.05655496, 1.60936155, 0.06477468, 0.2154934 , 0.40441994,
 0.02083958],
[0.05036551, 1.55956357, 0.06423068, 0.18622377, 0.4232059 ,
 0.01932867]]), array([-18609.34277877, -18609.26307107, -18609.2036
1247, -18609.16911899,
       -18609.16616366, -18609.16423066, -18609.16193958]))
           fun: -18609.34277877319
       message: 'Maximum number of iterations has been exceeded.'
          nfev: 159
           nit: 100
        status: 2
       success: False
             x: array([0.04492611, 1.6024584 , 0.06350905, 0.1795651 , 0.43
94318 ,
       0.02160711])
============================================================================
=====
```

In [222…
```
print(result_EKF.x)
```

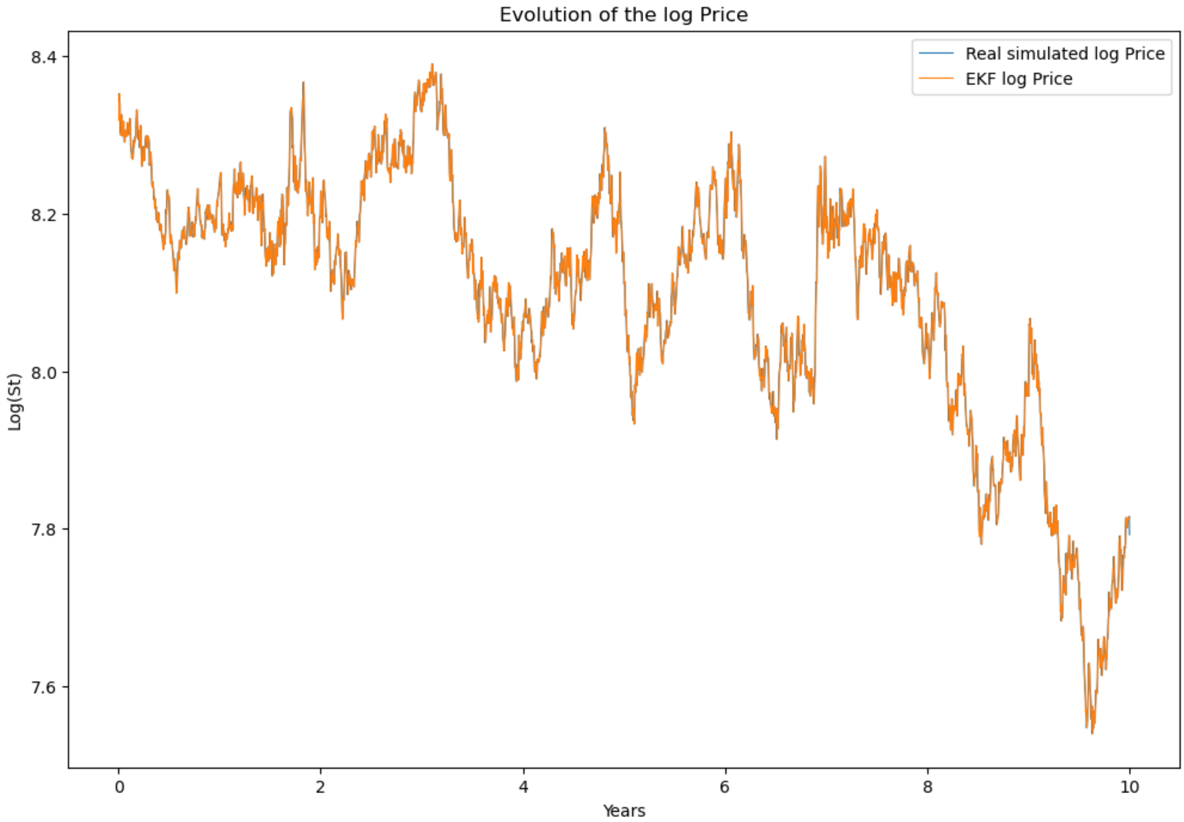
```
[0.04492611 1.6024584  0.06350905 0.1795651  0.4394318  0.02160711]
```

In [180…
```
print(params)
```

```
[0.0485, 1.5, 0.05, 0.18, 0.5, 0.04]
```

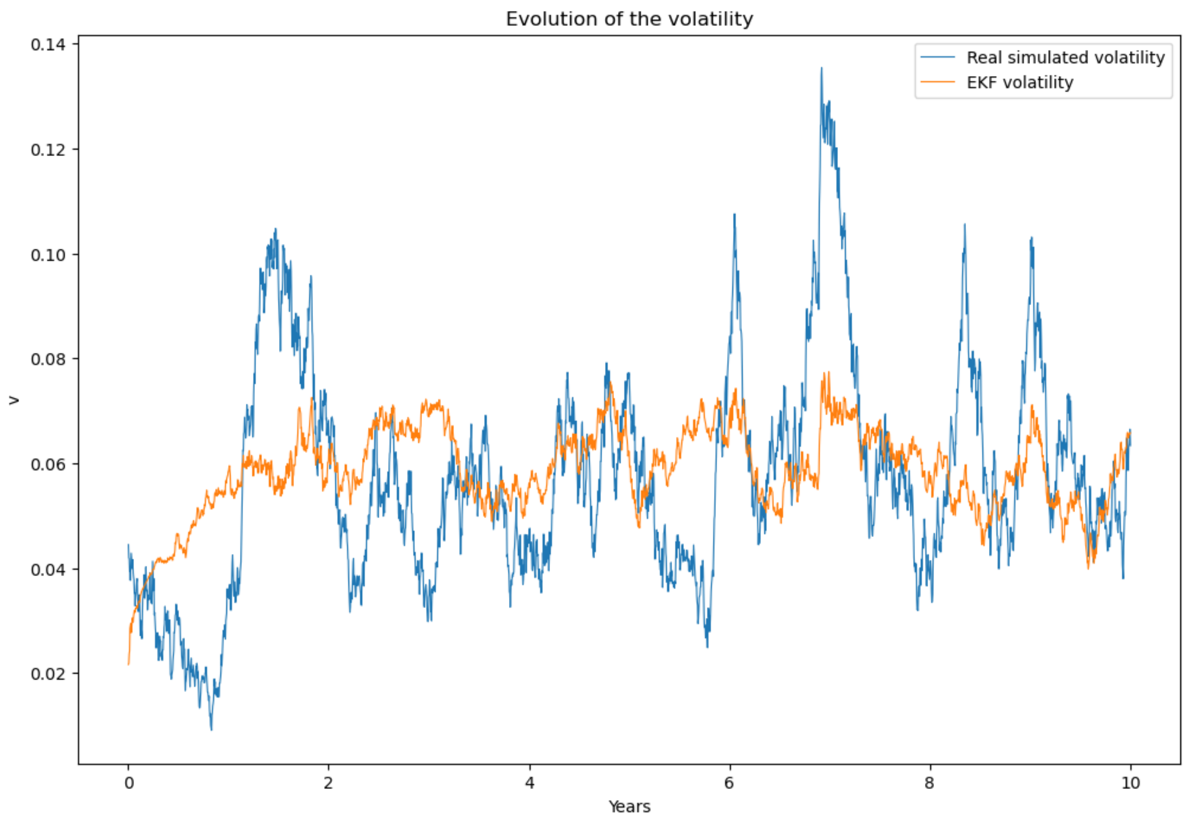
Plotting the results:

In [223…
```
plt.figure(figsize=(12,8))
years = np.arange(y.shape[-1]) * dt
plt.plot(years[1:], y[1:], label = 'Real simulated log Price', linewidth=0.8
plt.plot(years[1:], y_EKF, label = 'EKF log Price', linewidth=0.8)
plt.plot()
plt.title('Evolution of the log Price')
plt.ylabel('Log(St)')
plt.xlabel('Years')
plt.legend()
plt.show()
```



In [224…

```
plt.figure(figsize=(12,8))
plt.plot(years[1:], v_[1:], label = 'Real simulated volatility', linewidth=0
plt.plot(years[1:], v_EKF, label = 'EKF volatility', linewidth=0.8)
plt.plot()
plt.title('Evolution of the volatility')
plt.ylabel('v')
plt.xlabel('Years')
plt.legend()
plt.show()
```



2. Particle Filtering

2.1 Preparing the different functions

In [57]:
```
def proposal_distribution(N, v_prev, dy, params):
    mu, kappa, theta, lbda, rho = params
    # Calculate mean and standard deviation of the proposal distribution
    m = v_prev + kappa * (theta - v_prev) * dt + lbda * rho * (dy - (mu - 1/
    s = lbda * np.sqrt(v_prev * (1 - rho**2) * dt)
    # Sample N particles from the proposal distribution
    return norm.rvs(m, s, N)

def likelihood(y, x, v_prev, y_prev, params):
    mu, kappa, theta, lbda, rho = params
    # Calculate the mean and standard deviation of the measurement distribut
    m = y_prev + (mu - 1/2 * x) * dt
    s = np.sqrt(v_prev * dt)
    # Calculate the likelihood
    return norm.pdf(y, m, s)

def transition_proba(x, v_prev, params):
    mu, kappa, theta, lbda, rho = params
    # Calculate the mean and standard deviation of the transition distributi
    m = 1 / (1 + 1/2 * lbda * rho * dt) * (v_prev + kappa * (theta - v_prev)
    s = 1 / (1 + 1/2 * lbda * rho * dt) * lbda * np.sqrt(v_prev * dt)
    # Calculate the transition probability
    return norm.pdf(x, m, s)

def propo(x, v_prev, dy, params):
    mu, kappa, theta, lbda, rho = params
    # Calculate the mean and standard deviation of the proposal distribution
    m = v_prev + kappa*(theta-v_prev)*dt + lbda*rho*(dy - (mu-1/2*v_prev)*dt
    s = lbda*np.sqrt(v_prev*(1-rho**2)*dt)
    return norm.pdf(x, m, s)

def parameter_states_init(N, bounds):
    current_params = np.zeros((len(bounds), N))
    b0, b1, b2, b3, b4 = bounds
    # Initialize each parameter state using uniform random values within the
    current_params[0] = np.random.rand(N) * (b0[1] - b0[0]) + b0[0]
    current_params[1] = np.random.rand(N) * (b1[1] - b1[0]) + b1[0]
    current_params[2] = np.random.rand(N) * (b2[1] - b2[0]) + b2[0]
    current_params[3] = np.random.rand(N) * (b3[1] - b3[0]) + b3[0]
    current_params[4] = np.random.rand(N) * (b4[1] - b4[0]) + b4[0]

    return current_params

def resample_state(particles, w):
    N = len(particles)
    c_sum = np.cumsum(w)
    c_sum[-1] = 1.
    # Select new particles by randomly sampling from the cumulative sum
    idx = np.searchsorted(c_sum, np.random.rand(N))
    particles[:] = particles[idx]
    # Assign equal w to the new particles
    new_w = np.ones(len(w)) / len(w)

    return particles, new_w
```

```
def resample(v_pred, w, current_params):
        current_params[0], _ = resample_state(current_params[0], w)
        current_params[1], _ = resample_state(current_params[1], w)
        current_params[2], _ = resample_state(current_params[2], w)
        current_params[3], _ = resample_state(current_params[3], w)
        current_params[4], _ = resample_state(current_params[4], w)
        v_pred, w = resample_state(v_pred, w)
        return v_pred, w, current_params

def prediction_density(y, y_prev, x, mu):
    m = y_prev + (mu-1/2*x)*dt
    s = np.sqrt(x*dt)
    return norm.pdf(y, m, s)

def prediction_density_v(v, v_prev, dy, lbda,rho, theta, kappa):
    # Transition
    m = 1/(1+1/2*lbda*rho*dt) * (v_prev + kappa*(theta-v_prev)*dt + 1/2*lbda
    #print('m',m)
    s = (1/(1+1/2*lbda*rho*dt) * lbda * np.sqrt(v_prev*dt))
    return norm.pdf(v, m, s)

def inv_froeb(w):
        return 1. / np.sum(np.square(w))
```

In [58]:

```
def predict(v_pred, particles, y_prev, mu):
    # Generate predicted observations based on the predicted states
    y_hat = y_prev + (mu - 1 / 2 * v_pred) * dt + np.sqrt(particles * dt) *
    # Calculate the prediction density for each predicted observation
    prediction_densities = [prediction_density(y_hat[k], y_prev, v_pred, mu)
    # Calculate the average prediction density for each predicted observatic
    pdf_y_hat = np.array([np.mean(density) for density in prediction_densiti
    # Normalize the prediction densities
    pdf_y_hat = pdf_y_hat / np.sum(pdf_y_hat)
    # Calculate the weighted sum of the predicted observations
    return np.sum(pdf_y_hat * y_hat)

def predict_v(v_pred, particles, v_prev, mu, lbda,rho, theta, kappa, w, para
    v_hat = v_prev + (theta-1/2*particles)*dt + lbda*rho*(((mu-1/2*particles
    pdf_v_hat = np.array([np.mean(prediction_density_v(v_hat[k], particles[k
    pdf_v_hat = pdf_v_hat/sum(pdf_v_hat)
    return np.sum(pdf_v_hat * v_hat)
```

In [166…

```
def particle_filtering(params, N):
    global y_PF, v_PF, v_PF_bis

    # Unpack the model parameters
    mu, kappa, theta, lbda, rho, v0 = params

    print(params[:-1])

    # Initialize the current parameter states
    current_params = parameter_states_init(N, params[:-1])

    # Initialize the arrays to store the estimated states
    y_PF = np.zeros(N)
    v_PF = np.zeros(N)
    v_PF_bis = np.zeros(N)

    y_PF[0] = y[0]
    v_PF[0] = v0
```

```
    v_PF_bis[0] = v0

    # Initialize the weights
    w = np.array([1 / N] * N)

    # Initialize the particles
    particles = norm.rvs(v0, 0.02, N)
    particles = np.maximum(1e-4, particles)

    # Initialize the array to store the parameter steps
    params_steps = np.zeros((len(params) - 1, len(y)))
    params_steps.transpose()[0] = np.mean(current_params, axis=1)
    print(N)

    for i in range(1, N):
        dy = y[i] - y[i - 1]

        # Particle prediction step
        v_pred = proposal_distribution(N, particles, dy, current_params)
        v_pred = np.maximum(1e-3, v_pred)

        # Likelihood calculation
        Li = likelihood(y[i], v_pred, particles, y[i - 1], current_params)
        I = propo(v_pred, particles, dy, current_params)
        T = transition_proba(v_pred, particles, current_params)

        # Update the weights
        w = w * (Li * T / I)
        w = w / np.sum(w)

        # Resampling step
        if inv_froeb(w) < 0.75 * N:
            print('Resampling')
            v_pred, w, current_params = resample(v_pred, w, current_params)

        # State estimation step
        y_hat = predict(v_pred, particles, y[i - 1], np.mean(current_params[
        y_PF[i] = y_hat
        v_PF_bis[i] = predict_v(v_pred, particles, v_PF[i - 1], np.mean(curr
                                np.mean(current_params[3]), np.mean(current_
                                np.mean(current_params[2]), np.mean(current_
        v_PF[i] = np.sum(v_pred * w)
        particles = v_pred
        params_steps.transpose()[i] = np.sum(np.multiply(current_params, w[r

        print("Iteration {} completed".format(i))
    last_param = np.sum(np.multiply(current_params, w[np.newaxis, :]), axis=
    return (v_PF, v_PF_bis, params_steps, y_PF, last_param)
```

2.2 Running the algorithm

In [232…

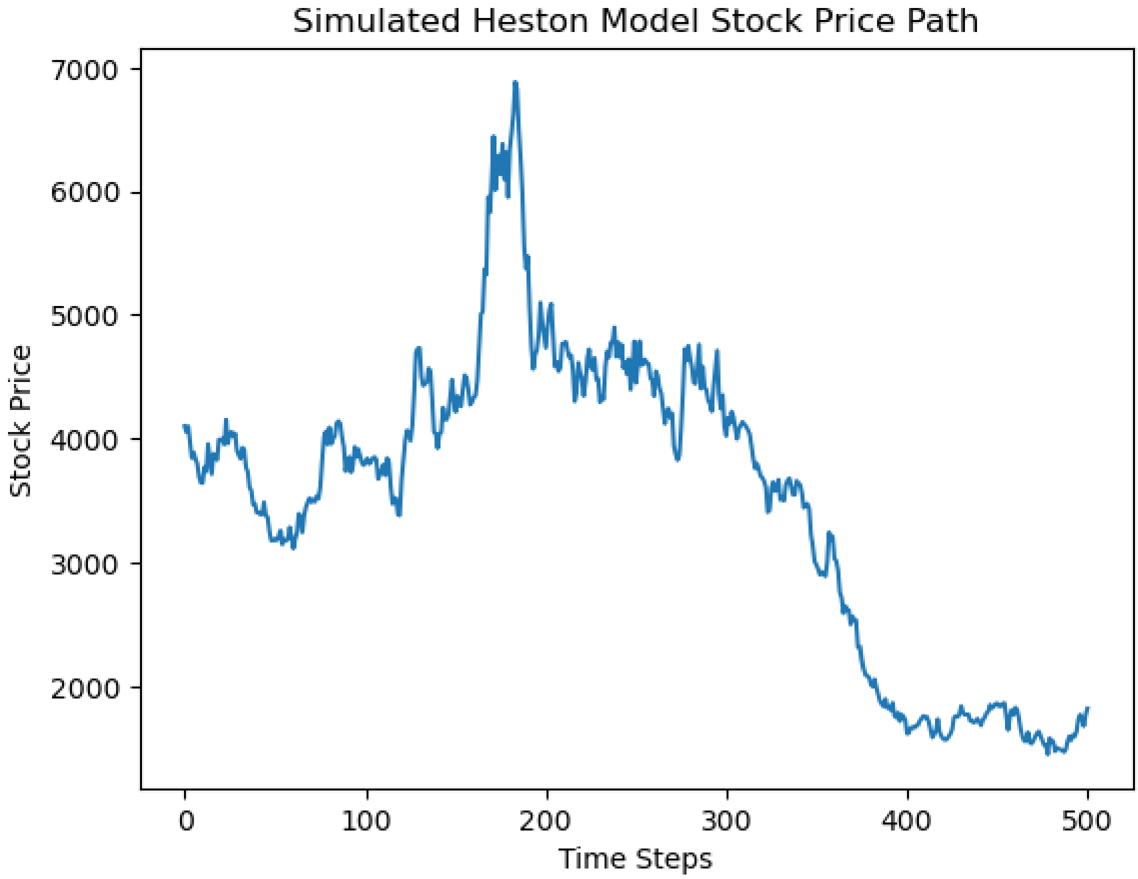
```
S0 = 4100
r = 0.0485
mu = r
params = [mu, 1.5, 0.05, 0.18, 0.5, 0.04]
#.          mu, kappa, theta, lbda, rho , v0
N = 500
T = 10

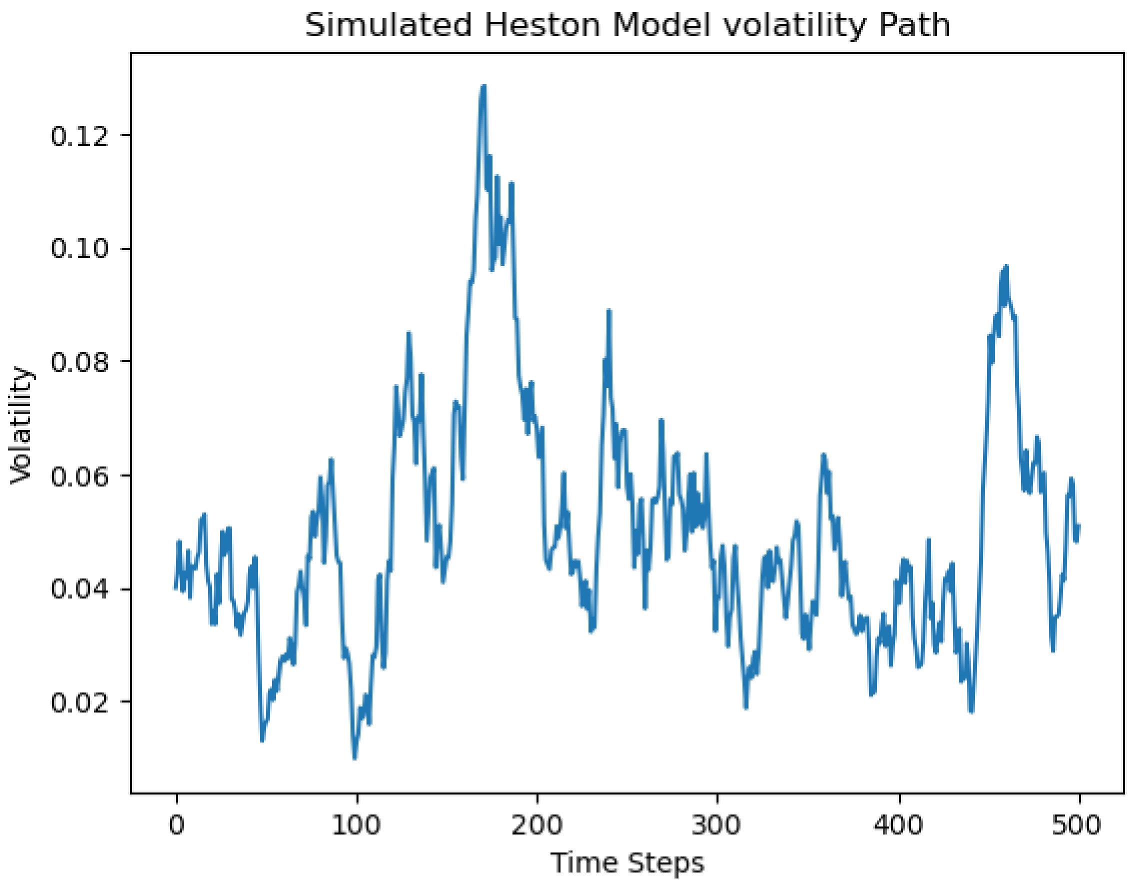
params_sim = np.zeros(5)
```

```
params_sim[0] = params[1]# kappa
params_sim[1] = params[2]#theta
params_sim[2] = params[3]#sigma
params_sim[3] = params[4]#rho
params_sim[4] = params[5]#v0
#print(params_sim)
prices, v_, y = heston_simulated_prices(params_sim, N, T, S0, r, mu, plot =

dt = T/N
#print(dt)
```

```
T: 10   N: 500   dt: 0.02
```





In [233…
```
print(params)
```

```
[0.0485, 1.5, 0.05, 0.18, 0.5, 0.04]
```

In [234…
```
mu = (0.01, 0.05)
kappa = (0.5, 3)
theta = (0.02, 0.2)
lambda_ = (0.01, 0.91)
rho = (-0.5, 1)
v_0 = params[-1]

params_0 = np.array([[mu, kappa, theta, lambda_, rho, v_0]])
params_0 =[mu, kappa, theta, lambda_, rho, v_0]
```

In [242…
```
v_PF_opt, v_bis, param_steps, y_PF_opt, last_param = particle_filtering(para
```

```
[(0.01, 0.05), (0.5, 3), (0.02, 0.2), (0.01, 0.91), (-0.5, 1)]
500
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In [243…

```
last_param
```

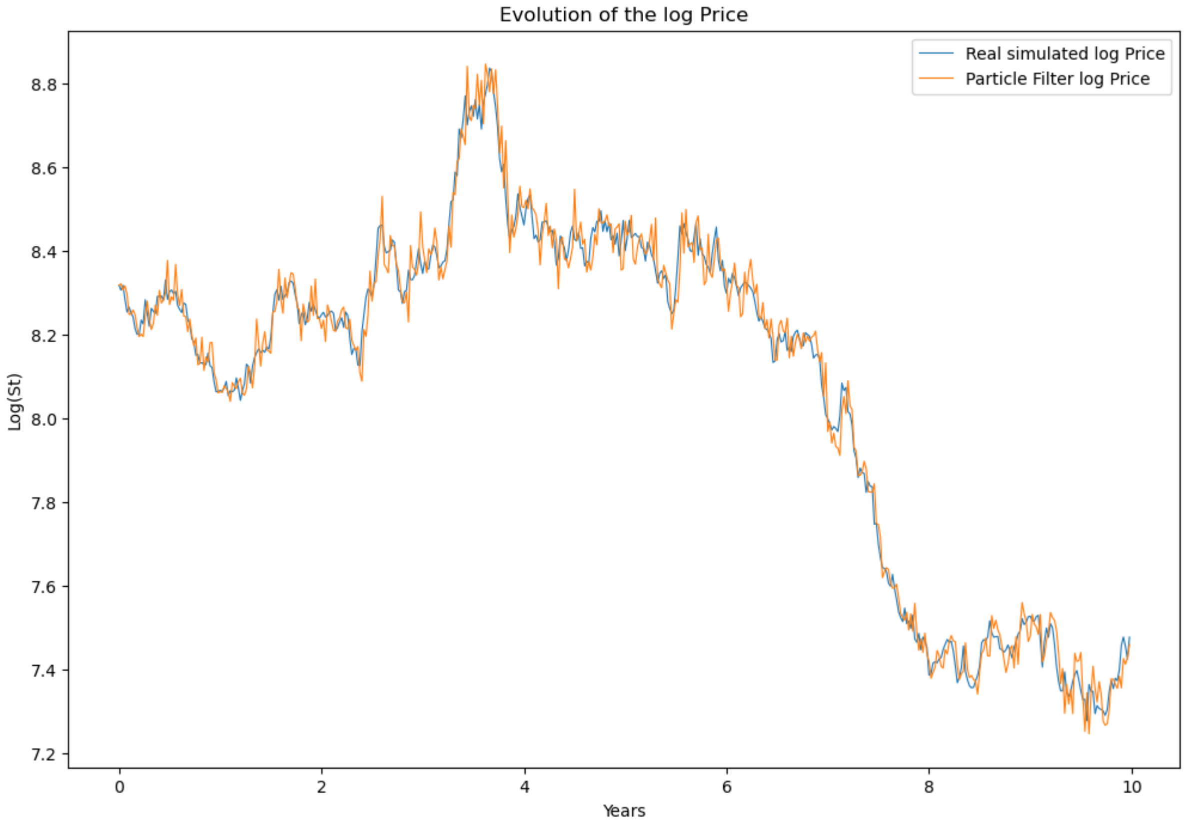
Out[243]:

```
array([0.02730858, 1.62033214, 0.06120653, 0.25795131, 0.93447779])
```

2.3 Plotting the results:

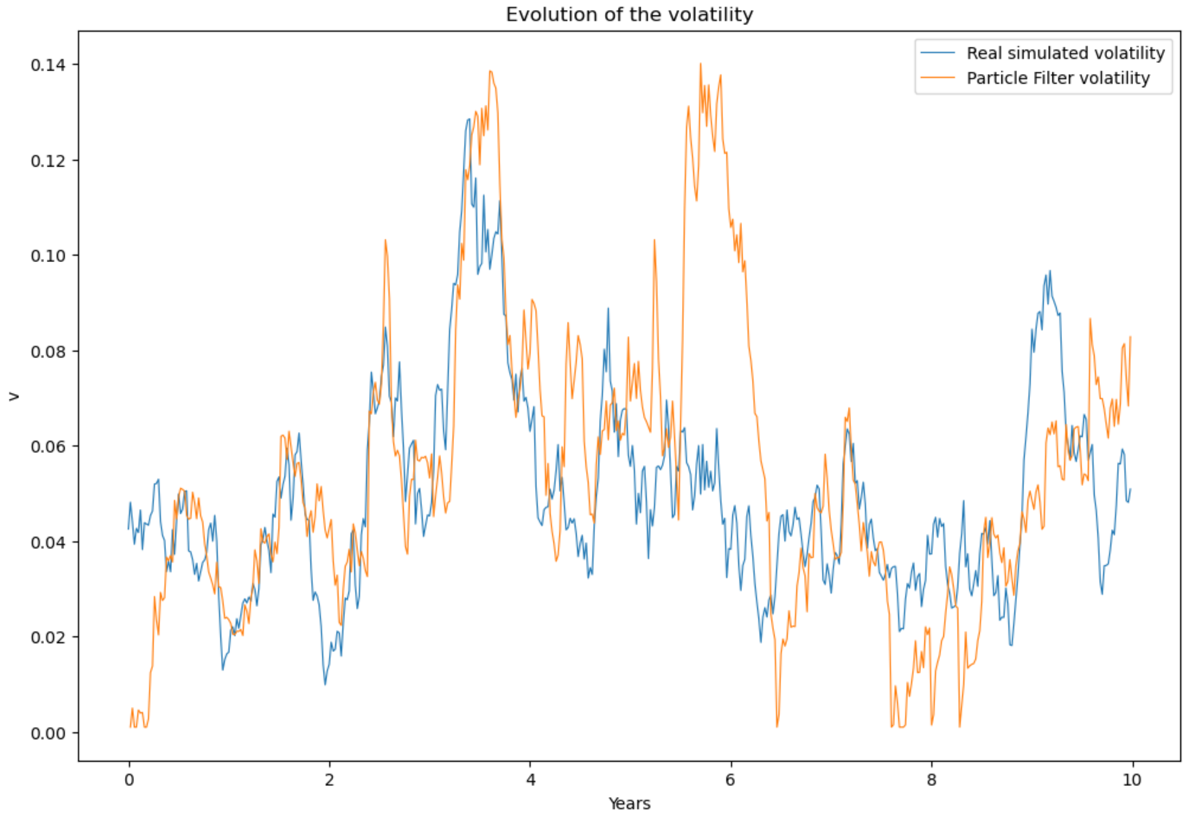
In [244…

```
plt.figure(figsize=(12,8))
years = np.arange(obs.shape[-1]) * (T/N)
plt.plot(years, y[:-1], label = 'Real simulated log Price', linewidth=0.8)
plt.plot(years, y_PF_opt, label = 'Particle Filter log Price', linewidth=0.8
plt.plot()
plt.title('Evolution of the log Price')
plt.ylabel('Log(St)')
plt.xlabel('Years')
plt.legend()
plt.show()
```



In [245…

```
plt.figure(figsize=(12,8))
years = np.arange(v.shape[-1]) * (T/N)
plt.plot(years, v_[1:], label = 'Real simulated volatility', linewidth=0.8)
plt.plot(years[1:], v_PF_opt[1:], label = 'Particle Filter volatility', line
#plt.plot(years[1:], v_PF_bis[1:], label = 'Particle Filter volatility', lin
plt.plot()
plt.title('Evolution of the volatility')
plt.ylabel('v')
plt.xlabel('Years')
plt.legend()
plt.show()
```



Result

In [246…

```
p_PF = np.append(last_param,0.04)
p_EKF = result_EKF.x
```

In [247…

```
params_result = pd.DataFrame({})
params_result['From'] = ['Simulation (true values)','Extended Kalman Filter'
params_name = ['mu', 'kappa', 'theta', 'lambda_', 'rho', 'v_0']

for i in range(len(params)):
    params_result[params_name[i]] = [params[i],p_EKF[i], p_PF[i]]
params_result
```

Out[247]:

	From	mu	kappa	theta	lambda_	rho	v_0
0	Simulation (true values)	0.048500	1.500000	0.050000	0.180000	0.500000	0.040000
1	Extended Kalman Filter	0.044926	1.602458	0.063509	0.179565	0.439432	0.021607
2	Particle filter	0.027309	1.620332	0.061207	0.257951	0.934478	0.040000

We can see that both model get close to real parameters but struggle a bit. For instance the mu estimate of the particle filtering is quite far from the real one. It is worth noting that the two models worked on different data as the two algorithms running on same data would have lead to long computation times. We did not get the parameters back very precisely.

Looking at the graphs, we observe that the two models are able to stick to the price path but are quite noisy around the true volatility paths simulated.

Therefore, in the context of parameter estimation in the Heston stochastic volatility model, both EKF and PF can be used. However, the choice could depend on the specific characteristics of the model, the available data, and the desired accuracy of the estimation. Looking back at the way the models should perform, if the model is relatively linear and Gaussian, and the noise is well-behaved, the EKF may provide accurate estimates. On the other hand, if the model is nonlinear or the noise is non-Gaussian, the PF may be more appropriate to capture the complexity of the distribution and provide robust estimates.