

## CMPUT 274 - Tangible Computing

### Morning Problem: Increasing Matrix

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#### Description

A sequence of integers is **strictly increasing** if each number is strictly greater than the previous number. A matrix (a 2-dimensional array) is **strictly increasing** if *all rows* and *all columns* of the matrix are strictly increasing.

You are given two  $n \times m$  matrices containing integers. Let the number in the  $i$ -th row and  $j$ -th column of the first matrix be denoted  $a_{i,j}$ . Similarly, let the number in the  $i$ -th row and  $j$ -th column of the second matrix be denoted  $b_{i,j}$ .

In one operation, you can swap  $a_{i,j}$  with  $b_{i,j}$  for any  $i, j$ . In other words, you can swap two numbers in different matrices **if they are located in the same positions**.

Your goal is to make both matrices strictly increasing by performing some number of operations (possibly none). Determine if it is possible to do this. If it is, print “Possible”, otherwise, print “Impossible”.

#### Input

The first line of input contains two integers  $n$  and  $m$  ( $1 \leq n, m \leq 100$ ) - the dimensions of the matrices.

Each of the next  $n$  lines contains  $m$  integers  $a_{i,1}, a_{i,2}, \dots, a_{i,m}$  ( $0 \leq a_{i,j} \leq 10^9$ ) - the number located in position  $(i, j)$  in the first matrix.

Each of the next  $n$  lines contains  $m$  integers  $b_{i,1}, b_{i,2}, \dots, b_{i,m}$  ( $0 \leq b_{i,j} \leq 10^9$ ) - the number located in position  $(i, j)$  in the second matrix.

#### Output

Output “Impossible” if you cannot form two strictly increasing matrices. Output “Possible” otherwise.

#### Sample Input 1

```
2 2
2 10
11 5
9 4
3 12
```

#### Sample Output 1

```
Possible
```

**Explanation** The two matrices are

$$\begin{bmatrix} 2 & 10 \\ 11 & 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 9 & 4 \\ 3 & 12 \end{bmatrix}.$$

If we swap their upper-right entries and then their lower-left entries, we get the following matrices:

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix}.$$

Notice both matrices are strictly increasing. That is, all rows are strictly increasing and all columns are strictly increasing, as required.

### Sample Input 2

```
2 3
2 4 5
4 5 6
3 6 7
8 10 11
```

### Sample Output 2

```
Possible
```

### Explanation

The two matrices are already strictly increasing in all rows and all columns. So we don't even have to perform any swapping.

### Sample Input 3

```
3 2
1 3
2 4
5 10
3 1
3 6
4 8
```

### Sample Output 2

```
Impossible
```

### Explanation

No matter how we swap entries, the matrix that has the number 3 in the upper-left corner will not have its top row being a strictly increasing sequence.