BGEN 55 - Advanced Genetics II

Modeling One-Way Mutation

Interactive Tutorial: Recurrent Mutation Model

🎯 Tutorial Overview

This interactive tutorial will guide you through the mathematical modeling of recurrent one-way mutation. You'll learn to:

- Set up the one-way mutation model
- Derive the fundamental equations
- Calculate allele frequency changes over time
- Understand the concept of equilibrium
- Apply the model to real genetic scenarios

Learning Goal: By the end of this tutorial, you will be able to model and predict how allele frequencies change under constant mutation pressure.

🧮 Step 1: Model Setup

1 **Define the Variables**

p = frequency of allele A

q = frequency of allele a

μ = mutation rate from A → a per generation

$\mathbf{p_0}$ = initial frequency of A

2 **Model Assumptions**

- Infinite population size (no genetic drift)
- No selection (all genotypes have equal fitness)
- No migration
- Random mating
- One-way mutation only (A → a)
- Constant mutation rate μ

📊 **Visual Representation:**

Allele A → Allele a at rate μ

📐 Step 2: Deriving the Equations

3 Frequency After One Generation

In each generation, a fraction μ of A alleles mutates to become a alleles.

$$p_1 = p_0 - (p_0 \times \mu)$$
$$p_1 = p_0(1 - \mu)$$

Example: If $p_0 = 0.8$ and $\mu = 0.001$
$p_1 = 0.8 \times (1 - 0.001) = 0.8 \times 0.999 = 0.7992$

4 Change in Frequency (Δp)

The change in allele frequency per generation:

$$\Delta p = p_1 - p_0$$
$$\Delta p = p_0(1 - \mu) - p_0$$
$$\Delta p = -\mu p_0$$

Key Insight: The change in frequency is always negative and proportional to both the mutation rate and the current frequency.

⏰ Step 3: Multiple Generations

5 Frequency After t Generations

To find the frequency after multiple generations, we apply the formula repeatedly:

$$p_2 = p_1(1 - \mu) = p_0(1 - \mu)^2$$
$$p_3 = p_2(1 - \mu) = p_0(1 - \mu)^3$$
$$...$$
$$p_t = p_0(1 - \mu)^t$$

6 Understanding the Pattern

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Generation 0: p₀
Generation 1: p₀(1 - μ)
Generation 2: p₀(1 - μ)²
Generation 3: p₀(1 - μ)³
...
Generation t: p₀(1 - μ)ᵗ
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Numerical Example:
$p_0 = 1.0$, $\mu = 0.0001$, $t = 1000$ generations
$p_{1000} = 1.0 \times (1 - 0.0001)^{1000}$
$p_{1000} = 1.0 \times (0.9999)^{1000} \approx 0.9048$

⚖️ Step 4: Equilibrium Analysis

7 Finding Equilibrium

Equilibrium occurs when allele frequencies no longer change ($\Delta p = 0$).

$$\Delta p = -\mu\hat{p} = 0$$
$$\text{Therefore: } \hat{p} = 0$$

Important: In the one-way mutation model, the equilibrium frequency of A is zero. All A alleles eventually mutate to a alleles.

8 Time to Equilibrium

While equilibrium is $p = 0$, we often want to know how long it takes to reach a certain frequency.

$$p_t = p_0(1 - \mu)^t$$
$$\text{Solve for t: } t = \log(p_t/p_0) / \log(1 - \mu)$$

Example: How many generations to go from $p = 1.0$ to $p = 0.5$ with $\mu = 0.0001$?
$t = \log(0.5/1.0) / \log(1 - 0.0001)$
$t = \log(0.5) / \log(0.9999) \approx 6931$ generations

🔢 Step 5: Practical Applications

Practice Problem 1

A population starts with only A alleles ($p = 1.0$). The mutation rate from A → a is $\mu = 2 \times 10^{-5}$ per generation.

Calculate: The frequency of A after 500 generations.

Solution:
$p_{500} = 1.0 \times (1 - 0.00002)^{500}$
$p_{500} = (0.99998)^{500} \approx 0.9900$

Practice Problem 2

In a different population, $p_0 = 0.95$ and $\mu = 1 \times 10^{-4}$.

Calculate: How many generations until $p = 0.50$?

Solution:
$t = \log(0.50/0.95) / \log(1 - 0.0001)$
$t = \log(0.5263) / \log(0.9999) \approx 6395$ generations

Real-World Context: These calculations show why mutation alone is a very slow evolutionary force. Even with relatively high mutation rates, significant changes take thousands of generations.

🎯 Step 6: Key Insights

9 Summary of Important Points

- Mutation pressure alone changes allele frequencies very slowly
- The rate of change depends on both μ and current frequency
- Equilibrium is only reached after infinite time ($p = 0$)
- Most real populations never reach mutation-only equilibrium due to other forces
- Mutation provides the variation that other evolutionary forces act upon

10 Limitations of the Model

- Assumes no reverse mutation (a → A)
- Ignores selection, drift, and migration
- Assumes constant mutation rate
- Infinite population assumption

Next Steps: In the next module, we'll add selection to this model and explore mutation-selection balance.

📝 Quick Reference

Fundamental Equations:

$$p_1 = p_0(1 - \mu)$$
$$\Delta p = -\mu p_0$$
$$p_t = p_0(1 - \mu)^t$$
$$\hat{p} = 0 \text{ (equilibrium)}$$

💡 **Pro Tip:**

For small values of μ, you can approximate $(1 - \mu)^t \approx e^{-\mu t}$. This is often easier to calculate and gives similar results.
