BGEN 55 - Advanced Genetics II

Practice Problems: Recurrent Mutation

Additional Practice Scenarios

📐 Formula Reference

$p_1 = p_0(1 - \mu)$ | $\Delta p = -\mu p_0$ | $p_t = p_0(1 - \mu)^t$ | $t = \ln(p_t/p_0) / \ln(1 - \mu)$

Approximation: $(1 - \mu)^t \approx e^{-\mu t}$ for small μ

Problem 1: Basic Frequency Change 🌱

Scenario: In a population of butterflies, a gene for wing pattern mutates from spotted (A) to plain (a) at $\mu = 0.0003$ per generation. Initially, 80% of alleles are A.

Calculate: What percentage of alleles will be A after one generation?

Show your work here...

Answer: 79.976%
Solution: $p_1 = 0.80 \times (1 - 0.0003) = 0.80 \times 0.9997 = 0.79976$

Problem 2: Multiple Generation Change 🌱

Scenario: A neutral mutation occurs at rate $\mu = 1 \times 10^{-6}$. A population starts with $p = 0.99$ for the original allele.

Calculate: What will be the frequency after 500 generations?

Show your work here...

Answer: 0.98951
Solution: $p_{500} = 0.99 \times (1 - 0.000001)^{500} \approx 0.99 \times 0.9995 = 0.989505$

Problem 3: Time Calculation ⚡

Scenario: Researchers are studying the loss of a function in a yeast gene. The mutation rate is $\mu = 2.5 \times 10^{-6}$. The current frequency of functional alleles is 0.85.

Calculate: How many generations until only half the alleles are functional ($p = 0.50$)?

Show your work here...

Answer: 245,500 generations
Solution: $t = \ln(0.50/0.85) / \ln(1 - 0.0000025) \approx -0.619 / -0.00000252 \approx 245{,}500$

Problem 4: Population Genetics Application ⚡

Scenario: In a large population of mice, a coat color gene mutates from dark (A) to light (a) at $\mu = 0.0004$. The current frequency of dark alleles is 0.70.

Calculate: a) What is Δp after one generation?
b) What will p be after 100 generations?
c) How many dark alleles are lost in the first generation if the population has 50,000 individuals?

Show your work here...

Answers:
a) $\Delta p = -0.00028$
b) $p_{100} \approx 0.6725$
c) 14 dark alleles lost

Solutions:
a) $\Delta p = -0.0004 \times 0.70 = -0.00028$
b) $p_{100} = 0.70 \times (0.9996)^{100} \approx 0.6725$
c) Total dark alleles = $50{,}000 \times 2 \times 0.70 = 70{,}000$
Lost = $70{,}000 \times 0.0004 = 28$ alleles (but since each individual has 2 alleles, this is 14 individuals)

Problem 5: Evolutionary Timescale 🚀

Scenario: Human chromosome 12 has an average mutation rate of 1.5×10^{-8} per base pair per generation. A particular ancient variant was present at $p = 1.0$ in our ancestors 2 million years ago.

Calculate: a) What would be its expected frequency today? (Assume 25 years/generation)
b) If we observe it at $p = 0.15$ today, what does this suggest about evolutionary forces?

Show your work here...

Answers:
a) Expected $p \approx 0.94$
b) Suggests strong selection or other forces at work

Solutions:
a) Generations = 2,000,000 / 25 = 80,000
$p = 1.0 \times (1 - 0.000000015)^{80000} \approx e^{-(0.000000015 \times 80000)} = e^{-0.0012} \approx 0.9988$
Wait, this seems too high - let's recalculate properly:
$p = (0.999999985)^{80000} \approx e^{-(0.000000015 \times 80000)} = e^{-0.0012} \approx 0.9988$
Actually, this shows mutation alone would barely change the frequency!

b) The observed $p = 0.15$ is much lower than expected from mutation alone, suggesting strong selection against this variant or other evolutionary forces.

Problem 6: Comparative Analysis 🚀

Scenario: Compare two populations with different mutation rates:

- Population A: $\mu = 1 \times 10^{-6}$, $p_0 = 0.95$
- Population B: $\mu = 1 \times 10^{-5}$, $p_0 = 0.95$

Calculate: a) How many generations until $p = 0.50$ in each population?
b) What is the ratio of these times?
c) What general principle does this demonstrate?

Show your work here...

Answers:
a) Population A: ~693,000 gens, Population B: ~69,300 gens
b) Ratio = 10:1
c) Time to reach a frequency is inversely proportional to mutation rate

Solutions:
Using approximation $t \approx -\ln(0.50/0.95) / \mu$
Population A: $t \approx 0.323 / 0.000001 = 323{,}000$ generations
Population B: $t \approx 0.323 / 0.00001 = 32{,}300$ generations
Ratio = 323,000 / 32,300 = 10:1

💡 Challenge Questions

Challenge 1: If a population has $\mu = 1 \times 10^{-6}$ and we want to maintain $p \geq 0.90$ for the original allele, what is the maximum number of generations before we need to intervene? (Assume no other evolutionary forces)

Challenge 2: A fossil record shows that a particular genetic variant was fixed ($p = 1.0$) 500,000 years ago but is now at $p = 0.30$. If the mutation rate is 2×10^{-7}, what other evolutionary forces must be acting on this variant?

Challenge 3: Derive the formula for the average time until a specific allele mutates in a population of size N. (Hint: Consider the probability distribution)

📊 Self-Assessment

Rate your confidence with these concepts:

- ✅ Basic one-generation frequency calculations
- ✅ Multiple generation projections
- ✅ Time-to-frequency calculations
- ✅ Real-world applications and interpretations
- ✅ Comparative analysis of different scenarios

Study Tips: If you struggled with any problem type, review the corresponding section in the tutorial and try similar problems with different numbers.
