COWMutationQ - A Copy-On-Write technique with wait-free progress

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Abstract

Several concurrency primitives are known that can provide concurrent access while allowing the code to be written as if it was single-threaded, with the most common of these being Mutual Exclusion Locks and Reader-Writer Locks, both of which have Blocking progress conditions.

A less used pattern is known as the Copy-On-Write (COW) technique, which requires the protected resource to be *copyable*, and provides Wait-Free Population Oblivious (WFPO) progress for read-only access. Two variants of COW are know, which we name COWLock and COWCAS. In COWLock the mutative operations (writes) are protected by a mutual exclusion lock and then the global reference is updated atomically, thus providing Blocking progress for writes and WFPO progress for reads. In COWCAS the mutative operations change the global reference using a Compare-And-Set (CAS) instruction, thus providing Lock-Free progress for writes and WFPO progress for reads.

We propose a third technique which we named COWMutationQ, where the mutative operations are placed in a Wait-Free queue and are then applied sequentially. This pattern provides Linearizable consistency and Wait-Free progress for writes, while maintaining WFPO progress for reads, making it the first practical generic technique to provide complete wait-free access to an object or data structure, as long as they are cloneable in the case of the object, or shallow-copyable in the case of a data structure.

Categories and Subject Descriptors D.4.1 [Operating Systems]: Mutual Exclusion

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1. Introduction

Although many concurrent data structures are known with lock-free and wait-free progress conditions, many developers still prefer to use generic primitives likes Mutual Exclusion Locks and Reader-Writer Locks to protect concurrently accessible resources. These two primitives provide access with Blocking progress condition, with some algorithms like the Ticket Lock, CLH Lock and Tidex

Lock providing Starvation Freedom (only on x86), while others do not, like the Test-and-set lock.

Recently, two new generic primitives have been discovered that can be applied for resources that are duplicable, namely, Double Instance Locking [6, 7], and the Left-Right pattern [8]. Double Instance Locking (DIL) is composed of two Reader-Writer Locks and one Mutual Exclusion Lock, and can provide Lock-Free access for read-only operations, while still being Blocking for writes. Even when using Locks that are Starvation-Free, this technique does not provide Starvation-Freedom for read operations. The Left-Right pattern allows for Wait-Free Population Oblivious (WFPO) readonly access if a suitable Read-Indicator is used, but it's Blocking for writes. The Left-Right pattern is inherently Starvation-Free and can provide full Starvation-Freedom when a Starvation-Free Lock and Wait-Free Read-Indicators are used. Both the Double Instance Locking and the Left-Right techniques require the usage of two equal resources (duplicates), which is trivial to accomplish for objects in memory, but non-trivial or even impossible for physical resources.

1.1 Copy-On-Write and its variants

Another generic technique is the Copy-On-Write (COW) which is more common in languages with a Garbage Collector (GC) and functional languages. The idea behind COW is that each thread wishing to do a write/mutation, must create a copy of the current resource (logical or physical) and apply the mutations to its own copy, but read-only operations can just follow the current reference to the most recent (stable) copy. Two variants of COW are known, with one serializing mutative operations (COWLock), and the other variant uses a Compare-And-Set (CAS) loop, where each thread attempts to make its own copy visible (COWCAS).

In COWLock, a mutual exclusion lock is used to protect modifications to the main reference, thus guaranteeing that one and only one thread at a time can be copying the current instance and applying the mutations to its own copy, before updating the reference with the newly modified instance. An example implementation in Java of COWLock can be seen in Algorithm 1 as class COWLock, and a practical implementation in Java JDK's java.util.concurrent.CopyOnWriteArrayList [3].

In COWCAS, multiple threads may simultaneously perform a copy of the current instance, and each one apply its own mutation to its copy, followed by a compareAndSet() that eventually succeeds in making its visible to other threads. The threads for which the CAS fails, must read again the *new* reference, make a copy of it, and apply its mutation to the copy. This procedure could go on indefinitely with consecutive failures, thus starving one or multiple threads, but still this approach gives a Lock-Free progress condition for writes. The starvation effect can become particularly unfair if there are multiple mutative methods being called with different

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execution times, in which case, the ones with the longest time to complete will be the most likely to be starved. Every time N threads compete for the CAS, N-1 copies of the current instance will need to be discarded, which which increases memory consumption, and consequent churn on the GC.

An example implementation in Java of COWCAS can be seen in Algorithm 1 as class COWCAS.

Table 1 shows a comparison between the different techniques described in this section and their properties in terms of progress conditions. Regarding the progress of read-only operations, all COW based techniques and the Left-Right pattern can provide WFPO progress. As for mutative operations (writes), only the COWCAS and COWMutationQ are non-blocking, with the COWCAS being easily starvable. Notice that all the COW based techniques are meant to be used with a GC or some kind of memory management technique, and although COWLock and COWCAS can be used with some memory management techniques at the detriment of its progress conditions. For example, Hazard Pointers [4] are lock-free, which means that for read-only operations, we will lose the wait-free progress conditions when using them together with one of the COW techniques.

	Read	Write	Starvation	Needs
	Progress	Progress	Freedom?	GC?
Mutex	Blocking	Blocking	Yes	No
RW-Lock	Blocking	Blocking	Yes	No
DIL	Lock-Free	Blocking	No	No
Left-Right	WFPO	Blocking	Yes	No
COWLock	WFPO	Blocking	Yes	Yes
COWCAS	WFPO	Lock-Free	No	Yes
COWMutationQ	WFPO	Wait-Free	Yes	Yes

Table 1. Comparison table between different generic techniques that provide concurrent access to an object or data structure. The first two columns indicate the progress conditions of each algorithm, the third column shows whether it is possible to do an implementation of the algorithm that provides freedom from starvation, and the fourth column indicates whether the algorithm can be implemented without a Garbage Collector or a memory management system.

Algorithm 1 shows generic code for the two variants COWLock and COWCAS when used to protect access to an object of type C. These were the concrete implementations used in section 3 for the performance evaluations. Notice that both COWLock and COWCAS variants are linearizable [1] for read-only and mutative methods, and both provide Wait-Free Population Oblivious for the read-only methods.

The COW techniques are not particularly scalable, but they are easy to use and to implement, making them a somewhat popular way to provide concurrent access to objects or data structures. It becomes interesting to consider whether a third technique exists that provides wait-free progress conditions for mutative operations without incurring a significant cost in overhead. Such a technique would be interesting not just from a point of view of providing strong guarantees of latency and fairness, while providing ease of use like other COW techniques. In this paper, we present such a technique, which we named COWMutationQ.

The COW technique we have developed makes use of a linearizable wait-free queue to provide linearizable concurrent access to a resource, as long as such a resource is *copyable*, similarly to the COWLock and COWCAS techniques. The idea is to enqueue all mutations in a wait-free queue, like the one devised by Alex Kogan and Erez Petrank [2], and then let each thread make one and only one copy of the current instance, and apply each one of the mutations from the queue, sequentially, until the thread's own mutation

Algorithm 1 Copy-On-Write patterns: COWLock and COWCAS

is applied. When compared with the other two COW techniques, the COWMutationQ has the advantage that a single copy is done of the current instance per thread, like on COWLock, but COWMutationQ allows for each thread's copy to be done in parallel.

One alternate technique that can be mistaken for COWMutationQ, is an asynchronous queue of operations with Futures, which works based on *delegation*. In such a technique, mutations are added to a queue and a Future is kept, to later access the result of the mutation. If that result is needed, then this technique will be blocking. For the case where read-only operations are not placed in the asynchronous queue, then those operations will be wait-free but not be linearizable. Because this technique is blocking when implemented in a linearizable way, we chose not to compare it in our benchmarks.

We present our COWMutationQ variant in detail in Section 2. We show our empirical evaluation in Section 3, and conclude in Section 4.

2. Algorithm

The COWMutationQ technique is composed of the following elements:

- A queue of mutations where each node, MutationNode, stores a function pointer (lambda) to be applied on the object (or data structure) and the arguments with which the function should be called;
- A reference to the tail (last node) of the queue;
- A reference to a Combined instance that contains the current head and object references;

The class definition for the nodes of the queue, MutationNode, and for the Combined, can be seen in Algorithm 2.

Notice that in line 2 of Algorithm 2 we define the return type of BiFunction as ? because each instance of MutationNode can represent a different mutative operation, and therefore have a different return value, i.e. one instance can be for an insertion, another instance for a removal, yet another for an iteration, and all these methods may have different return types.

Algorithm 2 Helper classes of COWMutationQ

```
public static class MutationNode<T,C> {
              final BiFunction<T,C,?> mutation
final T param1;
               final AtomicReference<MutationNode<T,C>>> next;
               \textbf{public} \hspace{0.2cm} \textbf{MutationNode}(T \hspace{0.1cm} \textbf{p1} \hspace{0.1cm}, \hspace{0.1cm} \textbf{BiFunction} \textcolor{red}{<} \textbf{T}, \textbf{C}, ? \textcolor{red}{>} \hspace{0.1cm} \textbf{mutativeFunc}) \hspace{0.1cm} \big\{
                     this . mutation = mutativeFunc;
this . param1 = p1;
                     this.next = new AtomicReference < MutationNode < T, C >> (null);
10
11
12
       }
13
       public static class Combined<T,C> {
               final MutationNode T, C> head
14
15
16
17
18
19
               final C instance;
              public Combined (MutationNode<T.C> head . C instance) {
                     this.head = head;
this.instance = instance;
```

The main steps of this technique when doing a mutation can be summarized from Algorithm 3 as:

- Read the current Combined instance, combinedRef, so we have a reference to the object and to the head of the queue (line 18);
- 2. Create a new MutationNode with the desired mutation and insert it in the queue (lines 17 and 21);
- 3. Clone the object whose reference was obtained from step 1 and start applying to it the mutations in the queue, starting from head until reaching the node this thread inserted in the queue (lines 24 to 30);
- 4. Do CAS on combinedRef from the current value up to the newly created Combined, where head is this thread's MutationNode and the object created on step 3. Retry this step if it fails until it succeeds, or the current instance of combinedRef has an head that is after this thread's MutationNode in the queue (lines 34 to 43);

For read-only operations, it's just a matter of reading the current Combined instance combinedRef and using its reference to the object, as shown in line 13 of Algorithm 3.

Another interesting detail about COWMutationQ is that the combinedRef is first read to obtain the head and only then do we insert our MutationNode in the queue. This guarantees the invariant that the head of the Combined instance will always be before our own MutationNode. This detail is of vital because on the last do/while() loop in applyWrite() we start from that head until we find our MutationNode or until we find the latest tail ltail. If we do not find our node before reaching tail, then it means the current head is after our node, which implies that the current combinedRef contains an instance that has our mutation already applied to it. We can exit applyWrite() with the certainty that our mutation is now visible to other threads. If we find the current tail before our node, then it means that our mutation is not yet visible, and we have to retry the CAS on combinedRef to make our mutation (and previous ones) visible to other threads. A naive approach might consider doing the for() loop in line 40 up to null instead of a local tail (ltail), but this would unfortunately be lock-free and not wait-free. The reason for it being lock-free, is that another thread could be constantly adding a new node to the queue, while the current thread is traversing the queue, which would theoretically go on for ever.

When the maximum number of threads in the application is known (MAX_THREADS), we can do one optimization in the loop in line 40, by traversing at most MAX_THREADS - 1 nodes until we find null or our node. If starting from the head of the current Combined instance we have not reached our node within traversing

Algorithm 3 COWMutationQ

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```
class COWMutationQ<T,R,C> {
           final AtomicReference<Combined<T,C>>> combinedRef;
final AtomicReference<MutationNode> tail;
           public COWMutationQ(C instance) {
                         sentinel = new MutationNode (null null):
                      \textbf{public} <\!\!R\!\!> R \ applyRead(T \ paraml \,, \ BiFunction <\!\!T, C, R\!\!> readOnlyFunc) \ \big\{
                       return readOnylFunc.apply(paraml, combinedRef.get().instance):
           public <R> R applyWrite(T paraml, BiFunction<T,C,R> mutativeFunc) {
                      MutationNode\(T, \infty\) myode = new MutationNode\(T, \infty\) (Combined\(\times\) (C
                       addToTail(myNode);
                       // Clone the current instance and apply all mutations up until our node is reached
                       final C mutatedInstance = (C)curComb.instance.copyOf();
for (MutationNode<T,C> mn = curComb.head.next.get(); mn! = myNode; mn = mn.next.get()) {
    mn.mutation.apply(mn.paraml, mutatedInstance);
                       // Save the return value of the last mutation (ours).
                       // We don't care about the other return values.
final R retValue = mutativeFunc.apply(paraml, mutatedInstance);
                      // Create a new Combined with all the mutations up to ours (inclusive)
// and try to CAS the ref to it until it has our mutation.
final Combined<T,C>newComb = new Combined<T,C>(myNode, mutatedInstance);
                       while (!casRef(curComb, newComb));
                       // Our mutation is now visible to other threads (through combinedRef)
                       return retValue;
```

MAX_THREADS - 1 nodes, it means that the current thread's node must be *before* the current head and therefore, its mutation is already visible to other threads. This guarantee comes from the fact that, at most, MAX_THREADS - 1 threads can insert a new mutation node between the head and the current thread's node without the current Combined being updated, because for any given thread to insert a new node in the queue, it must have at least updated the current Combined combinedRef's head to point to its previous mutation.

One caveat of COWMutationQ when compared with COWLock and COWCAS, is that the parameters passed to the mutative operation must be (logically) immutable. Suppose for an instant that a thread that is applying the mutation with a given param p1 has successfully CAS'ed its Combined instance and made its mutation visible to other threads, and so now it can return to the calling method, and after doing so, it changes the contents of p1. It could then happen that there is still some thread that is applying the mutations in sequence and is still going to apply the mutation of the previous thread, with the p1 parameter, and in doing so, not only apply the wrong parameter, but also break mutual exclusivity. In practice, re-using the parameters passed to such methods is rarely a necessity and even then, a copy can be done of the parameters themselves if needed.

2.1 Progress Conditions

A method is Wait-Free [1] if it guarantees that every call finishes its execution in a finite number of steps.

We shall now show that apart from the call to addToTail() in line 21 of Algorithm 3 whose progress depends on the queue/list chosen, the method applyWrite() is Wait-Free. To do this, we will show that the method applyWrite() will always complete in a finite number of steps.

The for() loop in lines 25 and 26 of Algorithm 3 will complete in a finite number of steps because there can only be a finite number of MutationNode instances between the current Combined instance head and our node. However, the number of MutationNodes traversed may end up being larger than the number of active threads because one or several threads may introduce multiple nodes between the time it takes the thread to read combinedRef and to insert its MutationNode in the queue.

The for() loop in lines 40 and 41 of Algorithm 3 will traverse the queue up until it finds this thread's myNode or the end of the queue, tail, thus completing in a finite number of steps.

The do/while() loop starting in line 35 of Algorithm 3 contains a loop that completes in a finite number of steps. It will complete when ltail or myNode is found, or when the compareAndSet() is successful in line 43, which will fail at most the MAX_THREADS-1, thus completing in a finite number of steps.

Notice also that this algorithm is Linearizable [1]. A mutative operation becomes visible to other threads — regardless of them doing read-only operations or mutations — when a successful CAS is done in combinedRef (line 43) by the thread wishing to apply the mutation, or by a thread for which its MutationNode was queued *after* the node with the mutative operation in question. The linearization points are in lines 13, 18 and 43 of Algorithm 3.

2.2 Lock-Free variant

As shown on the previous section, the COWMutationQ technique can be used with different queues. One possibility is to use a lock-free queue like the one developed by Maged Michael and Michael Scott [5], which is a simple and efficient algorithm. This approach means that mutations will be lock-free, which may seem counterproductive in comparison to using a COWCAS which has the same progress conditions, but notice that COWCAS can be extremely unfair if concurrent threads attempt different mutative operations (with different expected completion times), while on the COWMutationQ with a lock-free queue, the only part of the algorithm that may incur in starvation is the insertion in the queue, which takes the same time irrespective of the mutative operation that is placed in the node, and it is just as (un)likely to starve as inserting items into a ConcurrentLinkedQueue from j.u.c, which is based on the same algorithm. The simplicity of implementation makes this approach attractive, and as we will see on section 3, the throughput is very similar to the wait-free variant.

2.3 Heuristics

We will now compare the three COW techniques, by looking at the expected time that the mutation can take to be visible (assuming fairness)

If we define T_C as the time is takes to make a copy of the current instance, T_M the time it takes to apply the mutation, T_Q as the time it takes to create a mutation node and insert it into the queue, and T_L the time needed to lock and unlock the mutual exclusion lock used by COWLock, then we can define the following times for each of the techniques, where N is the number of threads:

- COWLock: $T_{COWLock} = (T_C + T_M + T_L)N/2;$
- COWCAS: $T_{COWCAS} = (T_C + T_M)N/2$ with the time it takes for a compareAndSet() being small enough to be neglected by comparison;
- COWMutationQ: $T_{COWMutationQ} = T_C + T_Q + T_M N$;

When the mutation itself is the most time consuming part $(T_M >> T_C, T_Q, T_L)$, then all three techniques will have similar results in throughput with a small advantage to COWCAS because it doesn't spend time in the mutex or adding nodes to a queue. If the copy-/clone is the most time consuming part $(T_C >> T_M, T_Q, T_L)$,

then COWMutationQ will take a total time of about T_C , while COWLock and COWCAS will take about $T_CN/2$, because both COWLock and COWCAS require on average a copy of the instance to be made for every mutation, while on COWMutationQ we need to do a single copy of the instance for about every N competing threads. This means that, assuming T_L and T_Q are small in comparison to T_C and T_M , when $T_M >> T_C$, it is preferable to use COWLock or COWCAS, but if $T_C > T_M$ then COWMutationQ should provide better results, particularly as the number of threads N increases. This analysis does not take into account the time taken to reclaim memory of objects that are no longer used.

3. Experimental Results

In order to compare the different COW techniques, we created a micro benchmark where each of these techniques is used to protect access to a mutable Red-Black TreeSet taken from java.util. Each loop iteration of the microbenchmark consists of randomly selecting either a read-only operation or a read-write operation, depending on the percentage of writes for that particular run. A read-only operation consists in randomly selecting one of the items in the TreeSet and calling contains(item). A read-write operation consists of randomly selecting one of the items in the TreeSet and then removing it from the set and adding it back. In terms of number of operations, the read-write operations are accounted as 2 due to having two separate method calls.

We ran the microbenchmark on a dual AMD Opteron 6272 with a total of 32 cores, running Windows 7 with JDK 8 (u45). Each data point shown in figure 1 is the median of 5 runs, with each run measured over a period of 20 seconds. The COW based techniques have a need for a Garbage Collector (GC), so we decided to use Java as the language to do the comparison on.

We ran our micro benchmarks for the following implementations:

- COWLock: TreeSet instances protected with the COWLock technique using a ReentrantLock from j.u.c, as described in Algorithm 1;
- COWCAS: TreeSet instances protected with the COWCAS technique, as described in Algorithm 1;
- COWMutationQ WF: TreeSet instances protected with the COWMutationQ as described in Algorithm 3, where the queue/list is based on the wait-free queue by Kogan and Petrank [2];
- COWMutationQ LF: TreeSet instances protected with the COWMutationQ as described in Algorithm 3, where the queue/list is based on the lock-free queue by Michael and Scott [5];

Overall, the COWMutationQ matches or exceeds the other two COW variants in throughput, with the only exception being for a low number of threads competing to perform mutations on a very small data structure, like the scenario shown in the top leftmost plot in figure 1. If we look at the scenario with 1000 items in the middle column or the one for 10000 items in the tree in the rightmost column of figure 1, we see that the two COWMutationQ variants have better throughput in all scenarios except the read-only workload (0% Writes) where all COW techniques have the same performance, i.e. linear scalability with the number of threads up until the number of cores is reached.

In our microbenchmark we tested with a Red-Black tree which means that the number of steps to perform a mutation is $O(\log_2(n))$ with n being the number of items in the tree, while the number of steps to copy the tree should be O(n). As shown in section 2.3 the relative time it takes to complete the copy of the instance and the mutation will affect which of the COW methods will give a better overall throughput, which means that using them to encapsulate other data structures with different relative ratios of mutation-to-

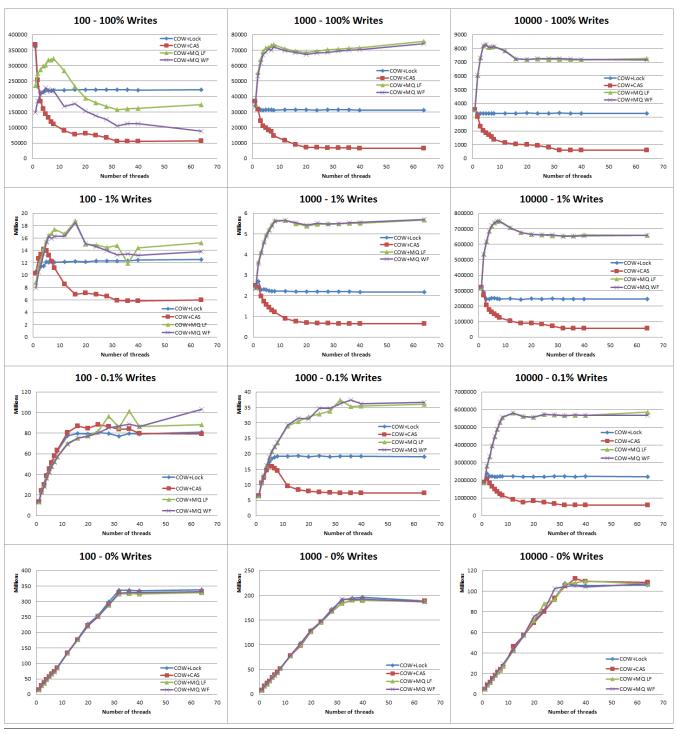


Figure 1. Plots comparing the four different COW techniques, when used to protect a TreeSet with 100, 1000, and 10000 items in the tree respectively. The vertical axis on each plot shows the total number of operations per second (contains()+add()+remove()).

copy, may provide different behaviors than the ones shown in these plots.

3.1 Latency measurements

Using the same machine as on the previous section, we ran a latency measuring benchmark that consisted of creating 16 threads with 8 of them doing only contains() and the remaining 8 threads

doing add() and remove() operations. The TreeSet contains 1000 items. The time taken to complete each individual call to one of these three operations is measured using System.nanotime() and saved as an histogram. Table 2 shows the results as a function of the percentage of events that take the indicated amount of time, or less, to complete.

	COW	COW	COW	COW
	CAS	Lock	MQ (LF)	MQ (WF)
contains() 90%	1	0<	3	3
contains() 99%	6	1	9	8
contains() 99.9%	11	2	14	13
contains() 99.99%	17	9	21	19
add() 90%	341	55	185	182
add() 99%	> 1000	74	235	230
add() 99.9%	>1000	>1000	298	285
remove() 90%	347	55	185	182
remove() 99%	> 1000	73	235	230
remove() 99.9%	>1000	>1000	298	285

Table 2. Latency values in microseconds for the different COW algorithms. Lower values are better. The two rightmost columns are for the lock-free and wait-free variants of COWMutationQ.

Using the COWCAS as an example, the table can be read as follows: 99% of the calls to the contains() method take 6 microseconds or less to complete.

On the first four lines of table 2 we show the results for contains(), a read-only operation. In this case, COWLock is the best of all four algorithms, but we should take into consideration that the COWMutationQ are not too far behind, and that this difference is due to COWLock doing less mutative operations, and as such, there will be less cache misses when doing a read-only operation.

On the lines for the latencies of add() and remove(), we can see that the COWMutationQ variants have at least one order of magnitude below COWCAS and COWLock for the 99.9%, as is to be expected due to their progress conditions. When it comes to the 99.9%, the COWMutationQ with a wait-free queue is, unsurprisingly, the best.

4. Conclusion

This paper presents a new Copy-On-Write based technique which, unlike previous COW techniques, provides a wait-free progress condition for mutations (read-write operations). Similarly to the other COW techniques, this is a generic pattern that can be applied to any resource that is *copyable*, and thus provide wait-free concurrent access to any object or data structure.

Although our implementation uses lambdas, COWMutationQ can be used in languages with just function pointers, or even by encoding the method as an integer in the MutationNode and then using a case/switch to decide which method to use for each node.

Due to its higher throughput and lower latency for all operations when compared to the other COW techniques, we believe that COWMutationQ is a useful and interesting technique.

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