1. (50 points) Use equation 3.10 of the text to determine the interpolatory polynomial for the data

$$f(1.4) = 2.37 \qquad f(1.8) = 3.351 \qquad f(1.9) = 0.233 \qquad f(2.5) = 4.572.$$

Determine the value of this polynomial at $x = 2$. Show all of your steps.

Solution: We started by computing all of the divided differences for our data, starting with the data:

$f[x_0] = f(x_0),\ f[x_1] = f(x_1),\ f[x_2] = f(x_2),\ f[x_3] = f(x_3)$

and then using the equations:

$f[x_0, x_1] = \frac{f[x_1]-f[x_0]}{x_1-x_0}$	-	-
$f[x_1, x_2] = \frac{f[x_2]-f[x_1]}{x_2-x_1}$	$f[x_0, x_1, x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$	-
$f[x_2, x_3] = \frac{f[x_3]-f[x_2]}{x_3-x_2}$	$f[x_1, x_2, x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$	$f[x_0, x_1, x_2, x_3] = \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$

After computing, we got the divided differences to be:

$x_0 = 1.4$	$f[x_0] = 2.37$	-	-	-
$x_1 = 1.8$	$f[x_1] = 3.351$	$f[x_0, x_1] = 2.4525$	-	-
$x_2 = 1.9$	$f[x_2] = 0.233$	$f[x_1, x_2] = -31.18$	$f[x_0, x_1, x_2] = -67.265$	-
$x_3 = 2.5$	$f[x_3] = 4.572$	$f[x_2, x_3] = 7.23167$	$f[x_1, x_2, x_3] = 54.8738$	$f[x_0, x_1, x_2, x_3] = 111.035$

Given equation 3.10, our interpolatory polynomial is

$$P_3(x) = 2.37 + 2.4525(x-1.4) - 67.265(x-1.4)(x-1.8) + 111.035(x-1.4)(x-1.8)(x-1.9)$$

Evaluating $P_3(2)$ gives us -2.89788.

2. (50 points) Follow the pseudo-code expressed in Algorithm 3.2 of the text, to write a MATLAB script which will perform Newton's Divided Difference method to determine the interpolatory polynomial for the data in the previous problem. Show that your script verifies the value of $P(2)$.

Solution: This is our MATLAB code:

```
xValues = [1.4 1.8 1.9 2.5];
yValues = [2.37 3.351 0.233 4.572];

syms P(x);
P(x) = NIDD(xValues, yValues);
double(P(2))

function P = NIDD(xValues, yValues)
    n = length(xValues);
    F = zeros(n,n);
    F(1:n,1) = yValues;
    for i = 2:n
        for j = 2:i
            F(i,j) = ...
                (F(i,j-1)-F(i-1,j-1))/(xValues(i)-xValues((i-j)+1));
        end
    end

    syms P(x);
    P = F(1,1);
    for i=2:n
        T = F(i,i);
        for j=1:i-1
            T = T * (x - xValues(j));
        end
        P = P + T;
    end
end
```

We get

$$P(2) = -2.8979,$$

which is the same as our answer from part 1.