Math 3900 Spring 2019 Quiz 1 Instructor: J. Haga

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1. Use the Taylor series for $f(x) = \frac{1}{x^2}$ around $x_0 = 1$ to approximate $\frac{1}{1.96}$ to within 10^{-6}. To get full credit, you must clearly show how use of the remainder term informs us of the minimal degree of the Taylor expansion required. What is the interval/radius of convergence for this Taylor series?

Solution: I expanded the function around 1 because of it's proximity to the value we'd like to approximate. I was able to bound the remainder term by 1, because for the function and all it's derivatives, the absolute value $\frac{1}{1.96}$ is less than the absolute value at 1. After calculating 7 remainder terms by hand, I decided to use matlab to determine where the remainder term is less than 10^{-6}.

```
syms x;
f(x) = 1/x^2;
N = 10^(-6);

for i=1:10^6
    f = diff(f);
    r = abs( (f(1)/factorial(i + 1)) * (1 - 1/1.96)^(i + 1) );
    if r ≤ N
        i
        vpa(double(r))
        break
    end
end
```

The result shows that that T_{18} will give us an aproximation of $\frac{1}{1.96}$ within 10^{-6}. I also used matlab to compute this value.

```
f(x) = 1/x^2;
sum = f(1);
for i=1:19
    f = diff(f);
    sum = sum + ( (1 / factorial(i)) * (1/1.96 - 1)^(i) );
    vpa(double(sum))
end
```

$\frac{1}{1.96}$ approximated to 10^{-6} is 0.510204. The radius of convergence for this series is x $\neq$ 0.

2. Determine the Taylor series expansion of $f(x) = x - x^2 + 4x^3 + x^4 - x^5$ around $x_0 = 5$. What is the interval/radius of convergence for this Taylor series? Approximate $f(10)$ to within 10^{-8}. Is the Taylor series of f an infinite power series? Draw a conclusion about the Taylor expansions of polynomials and offer an explanation.

Solution: The Taylor series expansion of $f(x) = x - x^2 + 4x^3 + x^4 - x^5$ around $x_0 = 5$ is

$$-2020 - 2334(x-5) - 1041(x-5)^2 - 226(x-5)^3 - 24(x-5)^4 - (x-5)^5$$

The series converges everywhere. The series is an infinite power series. $f(10)$ approximated to within 10^{-8} is -84066.0, which isn't right. I used the same code as above to get this value. Each successive expansion of a polynimial in a Taylor series is a partial sum of the original function, similar to Riemann sums.