VE281

Data Structures and Algorithms

Graph Search

Learning Objectives:

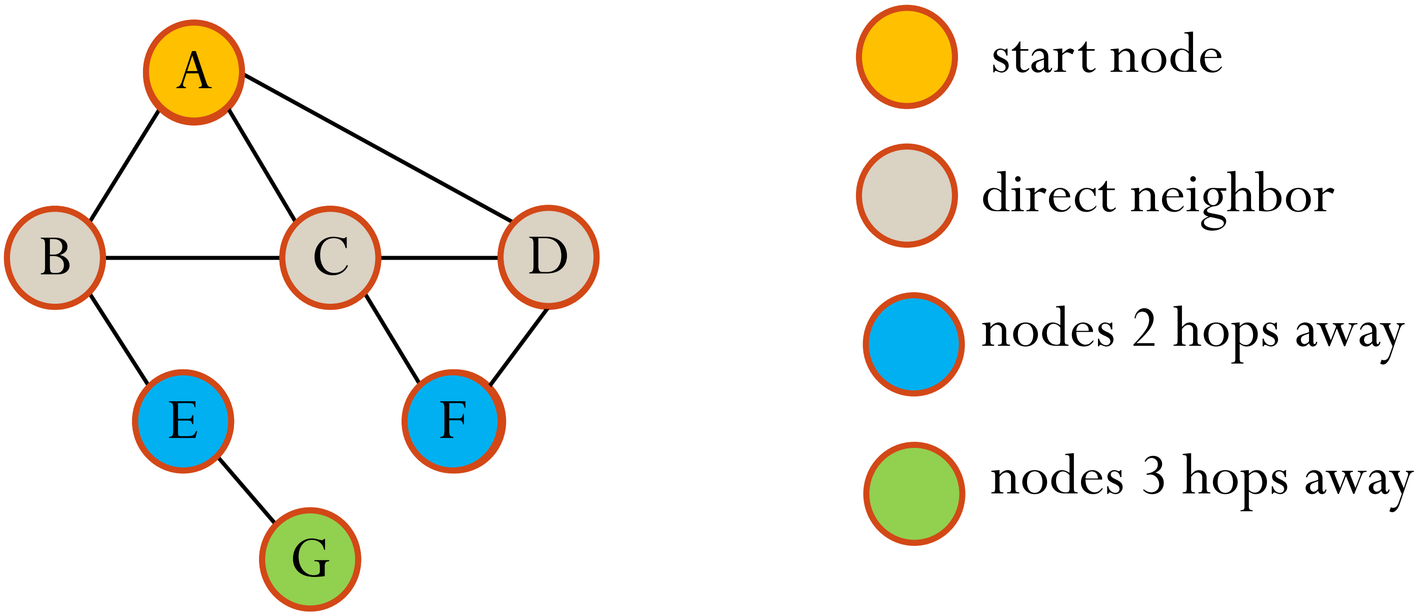
- Know two widely-used graph search algorithms, breadth-first search and depth-first search
- Know their runtime complexity

Graph Search

- A node u is **reachable** from a node v if and only if there is a path from v to u.
- A graph search method starts at a given node v and visits **every** node that is **reachable** from v.
- Many graph problems are solved using a search method.
  - Find a path from one node to another.
  - Find if the graph is connected.
- Commonly used search methods:
  - Breadth-first search.
  - Depth-first search.

Breadth-First Search (BFS)

- Given a start node, visit all directly connected neighbors first, then nodes 2 hops away, 3 hops away, and so on.



A→B→C→D→E→F→G

Breadth-First Search (BFS)

Implementation

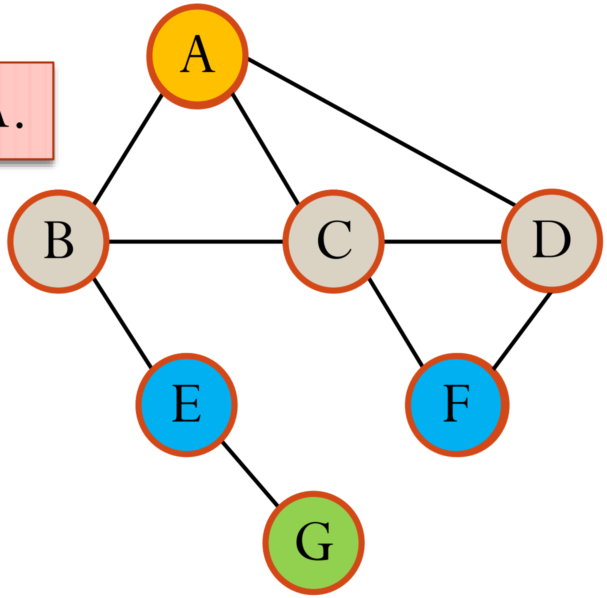
- BFS can be implemented using a queue.

```
BFS(s) {
  queue q; // An empty queue
  visit s and mark s as visited;
  q.enqueue(s);
  while(!q.isEmpty()) {
    v = q.dequeue();
    for(each node u adjacent to v) {
      if(u is not visited) {
        visit u and mark u as visited;
        q.enqueue(u);
      }
    }
  }
}
```

Breadth-First Search (BFS)

Example

Start node is node A.



```
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  queue q; // An empty queue
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  q.enqueue(s);
  while(!q.isEmpty()) {
    v = q.dequeue();
    for(each node u adjacent to v) {
      if(u is not visited) {
        visit u and mark u as visited;
        q.enqueue(u);
      }
    }
  }
}
```

Queue: A B C D E F G

Visit Order: A B C D E F G

Breadth-First Search (BFS)

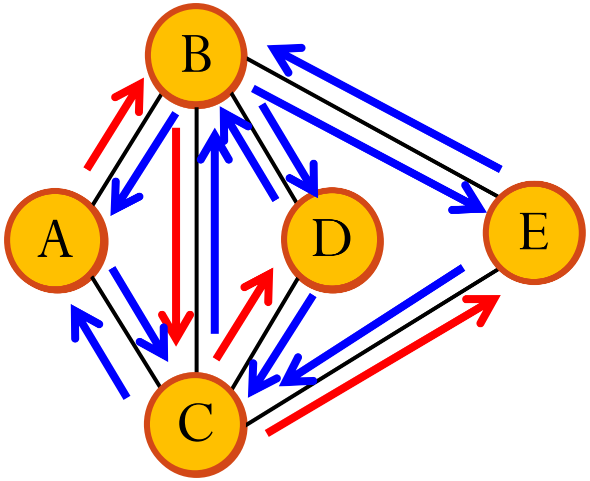
Time Complexity

- If graph is implemented as **adjacency matrix**:
  - Visit each node exactly once: $O(V)$.
  - The row of each node in the adjacency matrix is scanned once: $O(|V|)$ for each node.
  - Total running time: $O(|V|^2)$.
- If graph is implemented as **adjacency list**:
  - Visit each node exactly once: $O(|V|)$.
  - Adjacency list of each node is scanned once.
  - Size of entire adjacency list is $2|E|$ for undirected graph and $|E|$ for directed graph.
  - Total running time: $O(|V| + |E|)$.

Depth-First Search (DFS)

```
DFS(v) {
  visit v;
  mark v as visited;
  for(each node u adjacent to v)
    if(u is not visited) DFS(u);
}
```

- How to mark a node “visited”?
  - Keep a “visited” field in the node

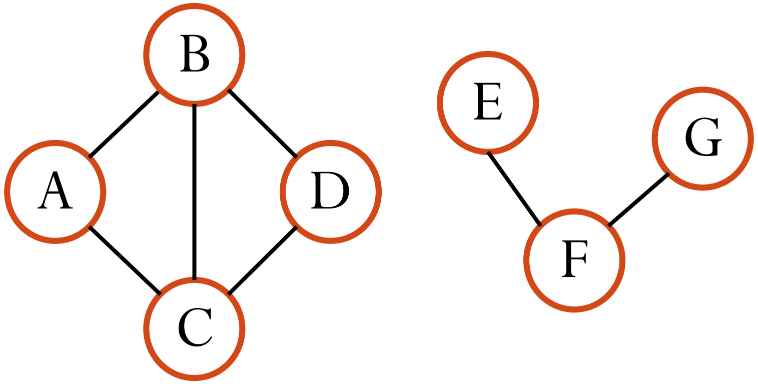


Time complexity?

Same as BFS

Traverse All the Nodes in a Graph

- The graph may not be connected. How can we traverse all the nodes in the graph?



```
for(each node v in the graph)
   if(v is not visited)
      DFS(v);
```