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ME 314 Final Project
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Final Project: Angry Birds Space Game

Project Description

My project was inspired by the physics-based game, Angry Birds Space. The level I created has one bird, two pigs, and two planets. The bird and pigs were modeled as squares with rotational inertia. The planets are fixed and will not be modeled as dynamic bodies (they are assumed to have infinite mass relative to the bird and pigs). Each of the planets will have a gravitational field that acts towards the planet's center. The planets' gravitational fields only act on bodies within a distance R from the planet's center, where R is greater than the radius of the planet's surface[1]. The bird and the pigs can collide with one another and also with the planets' surfaces. Unlike the actual Angry Birds Space game, the pigs do not disappear after being hit by the bird[2]. In addition, I modeled the loss of energy due to drag forces (friction with air) that only act on bodies inside a planet's atmosphere (or gravitational field)[3] and plastic impacts with a coefficient of restitution α. There are nine degrees of freedom in my system. They are the x coordinates, y coordinates, and angles from vertical of the three bodies.

In my original proposal, I said I was going to model a pig that initially resides on a planet's surface and a rock that initially resides in space. However, I made the decision to only model 2 pigs that initially reside in space because I simply ran out of time to calculate the constrained E-L equations for a body resting on a planet.

Frames and Transformations

I defined four frames: the world frame and three translated and then rotated frames, one for each body. I labelled these frames numerically: {0}, {1}, {2}, and {3}. The reason I avoided labelling with letters is because I wanted the frames to correspond with each body's number and configuration variables (for instance, frame {1} corresponds with body 1 with configuration variables x_1, y_1, and θ_1). The transformations between the frames are labelled as g_01, g_02, and g_03 (a transformation from the world frame, {0}, to body frame {#}. Because I modularized my code, the transformation matrices may be difficult to identify in the code without my help pointing them out. Please look for the comment: "### Transformations

g_01=

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x_1(t) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y_1(t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_02=

$$\begin{bmatrix} \cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & x_2(t) \\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & y_2(t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_03=

$$\begin{bmatrix} \cos(\theta_3(t)) & -\sin(\theta_3(t)) & 0 & x_3(t) \\ \sin(\theta_3(t)) & \cos(\theta_3(t)) & 0 & y_3(t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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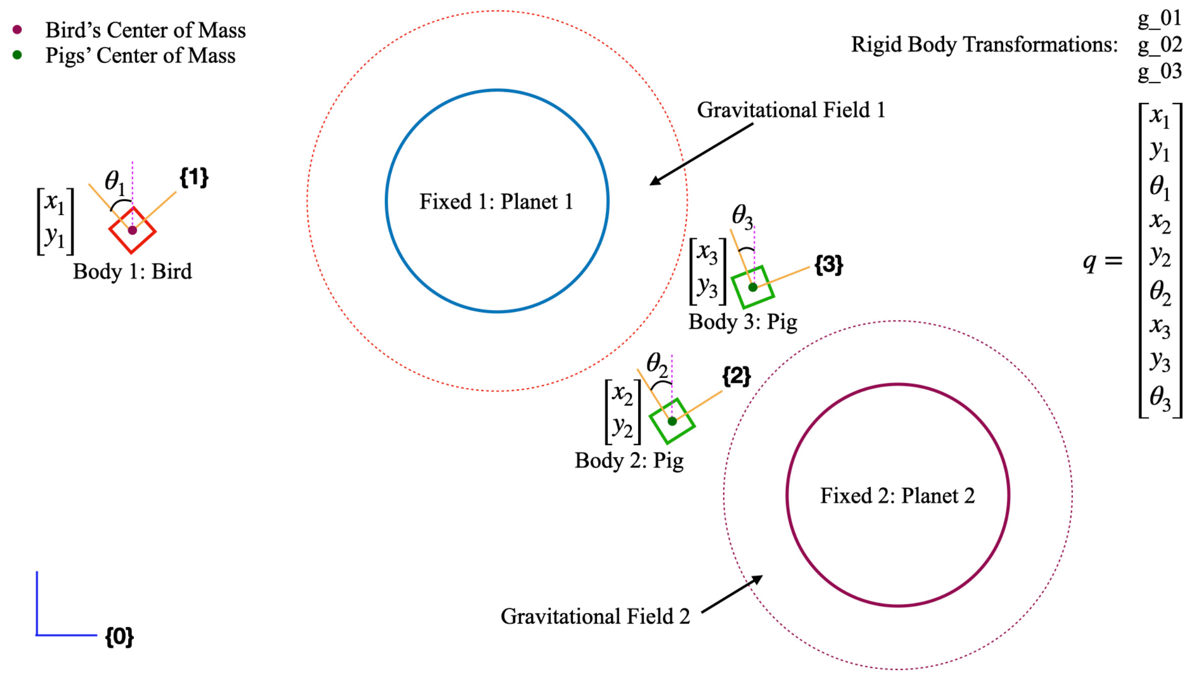
[1] Limiting the effect of each planet's gravitational field leads to more interesting game dynamics. For instance, if the bird is not aimed properly and doesn't enter the gravitational field, the direction of the bird's velocity remains unchanged. Also, the bird, if its velocity is large enough, can enter, curve, and escape a planet's gravitational field.

[2] I want to model the impacts between the bodies. If the pigs can be "defeated," then there are only impacts between the bodies and the planets.

[3] I made the assumption that space is a perfect vacuum and no drag force acts on objects in space. In other words, drag force only acts on bodies within one of the planets' gravitational fields.

Between World and Body Frame ###." An image of the transformation matrices, g_01, g_02, and g_03, is provided.



Calculations

Euler-Lagrange Equations

To calculate the E-L equations, I first found the body velocities by unhatting the product of the inverse of the transformation matrices and the time derivative of the transformation matrices. Then, I calculated the inertia matrices by taking a triple integral over the volume of a cube. Using the body velocities and inertia matrices, I calculated the kinetic energy using the formula we derived in class ($KE = 0.5V^{b^T} I_{6\times6} V^b$). The gravitational potential energy for an object in a planet's gravitational field is equal to $V = -\dfrac{km}{r}$ where k is a parameter relating to the strength of the planet's gravitational pull, m is the mass of the body, and r is the distance between the center of mass of the body and the center of the planet. The Lagrangian is equal to the kinetic energy minus the potential energy.

External Forces

Within each planet's gravitational field, the bodies experience a drag force. Friction can be defined as the negative velocity gradient of the general dissipation function[4] $D = \dfrac{1}{n+1}\sum_j c_j v_j^{n+1}$. Drag force is proportional to the square of a body's velocity. Hence, the dissipation function for drag is $D = \dfrac{1}{6}\rho C_D A v^3$ where ρ is the density of the air, C_D is the drag

[4] Source: https://profoundphysics.com/friction-in-lagrangian-mechanics/.

coefficient (~0.8 for cubes), A is the reference area (s^2 for cubes), and v is the velocity of the body. The forced E-L equations can now be written as $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = -\frac{\partial D}{\partial \dot{q}}$.

Impact Update Laws

To reduce the amount of work I had to do, I made the assumption that impacts between two bodies can only occur when they both occupy the same planet's gravitational field or space. In nearly all cases, this assumption holds. With this assumption, I only need to write down three sets of impact equations: (1) impacts in space, (2) impacts in planet 1's gravitational field, and (3) impacts in planet 2's gravitational field. I can choose which Lagrangian (with the correct potential energy for the impacting bodies) to use since I know where the impact is occurring.

There are three bodies (all of which are squares) and two planets. Therefore, there are 72 impact conditions or $\phi(q)$ equations. I wrote an algorithm to calculate these constraints for me to minimize the risk of human error. For the impact conditions between a corner of body 1 and a side of body 2, $\phi(q)$ looks like $\phi(q) = (p_1 - p_2)^2 + (p_2 - p_3)^2 - (p_1 - p_3)^2$ where p_1 and p_3 are the coordinates of body 2's corners that, when connected, form a side and p_2 is the coordinate of body 1's corner. For the impact conditions between the corners of a body and a planet's surface, $\phi(q)$ looks like $\phi(q) = (x - x_{cen})^2 + (y - y_{cen})^2 - R^2$ where x and y are coordinates of the body's corner, x_{cen} and y_{cen} are coordinates of the planet's center, and R is the radius of the planet. The impact equations are $\left\{ -\frac{\partial L}{\partial \dot{q}}\Big|_{\tau^-}^{\tau^+} = \lambda \nabla \phi(q)\,, H\Big|_{\tau^-}^{\tau^+} = 0\right\}$. Due the complexity of the impact equations, the solutions for $\dot{q}(\tau^+)$ were solved for numerically at the time of the impact. To model plastic impacts between the bodies and the planet, I used the heuristic we discussed in class: I solved for the elastic case, replaced λ with $\alpha\lambda$, and then solved again for $\dot{q}(\tau^+)$.

Simulation Outcome

I ran multiple simulations each with different initial conditions to check that the impacts between all the bodies were working properly. I will only describe and rationalize the simulations titled "1 Pig Impact," "Test W/ Alpha=1.0," and "Test W/ Alpha=0.85" in the video. The other simulations appear reasonable for the same reasons I will give for the simulations I choose to describe below.

In the "1 Pig Impact" simulation, the pigs are initially at rest in space. Both are outside both planet's gravitational field. At first, they remain stationary since no gravitational forces or drag forces exist outside the planets' gravitational field. The bird is also initially in space but is traveling with a positive horizontal velocity. As expected, the bird continues to travel in the same direction, at the same speed, since, in space, there are no gravitational forces or drag forces acting on the bird. When the bird enters the gravitational field of planet 1, it begins to curve as excepted due to the gravitational force acting inwards towards the center of the planet. I also observe that the bird slows down slightly due to the drag force now acting on the bird. The initial horizontal velocity of the bird is great enough that the bird is able to escape planet 1's gravitational field after curving a bit. Once it enters space again, the bird travels at the same speed and direction it had when it left planet 1's gravitational field. Shortly after, the bird collides with one of the pigs, which sends the pig flying towards planet 2. When the pig enters planet 2's gravitational field it begins accelerating towards the surface and impacts the surface with considerable speed. The pig bounces off the surface, escapes planet 2's gravitational field, and flies off into space. This makes sense because most of the kinetic energy from the pig's impact with the bird was transferred to the pig, providing it enough energy to impact with planet 2's surface and escape the planet's gravitational pull. Meanwhile, the bird, after colliding with the pig, is redirected towards planet 1. Having lost most of its kinetic energy to the pig, the bird bounces along the surface of planet 1, pulled towards the center of planet 1 by gravity and unable to escape planet 1's gravitational field. The second pig does not experience impacts with the other pig or the bird, so it remains stationary.

To check that the plastic impacts were working, I compared two simulations ("Test W/ Alpha=1.0" and "Test W/ Alpha=0.85") with the same initial conditions but different values of α, the coefficient of restitution. In the simulation with $\alpha = 0.85$, the bird loses energy upon contact with the planet's surface and bounces to a lower height compared to the simulation with $\alpha = 1.0$. Overall, all the simulations I ran behaved in the way I expected and support that my code works.