CHAPTER 10

Adding a Third Variable

Preliminary Exploratory Analyses

10.1 ♦ Three-Variable Research Situations

In previous chapters, we reviewed the bivariate correlation (Pearson r) as an index of the strength of the linear relationship between one independent variable (X) and one dependent variable (Y). This chapter moves beyond the two-variable research situation to ask, "Does our understanding of the nature and strength of the predictive relationship between a predictor variable, X_1, and a dependent variable, Y, *change* when we take a third variable, X_2, into account in our analysis, and if so, how does it change?" In this chapter, X_1 denotes a predictor variable, Y denotes an outcome variable, and X_2 denotes a third variable that may be involved in some manner in the X_1, Y relationship. For example, we will examine whether age (X_1) is predictive of systolic blood pressure (SBP) (Y) when body weight (X_2) is statistically controlled.

We will examine two preliminary exploratory analyses that make it possible to statistically control for scores on the X_2 variable; these analyses make it possible to assess whether controlling for X_2 changes our understanding about whether and how X_1 and Y are related. First, we can split the data file into separate groups based on participant scores on the X_2 control variable and then compute Pearson correlations or bivariate regressions to assess how X_1 and Y are related separately within each group. Although this exploratory procedure is quite simple, it can be very informative. Second, if the assumptions for partial correlation are satisfied, we can compute a partial correlation to describe how X_1 and Y are correlated when scores on X_2 are statistically controlled. The concept of statistical control that is introduced in this chapter continues to be important in later chapters that discuss analyses that include multiple predictor variables.

Partial correlations are sometimes reported as the primary analysis in a journal article. In this textbook, partial correlation analysis is introduced primarily to explain the concept of statistical control. The data analysis methods for the three-variable research situation that are presented in this chapter are suggested as preliminary exploratory analyses that can help a data analyst evaluate what kinds of relationships among variables should be taken into account in later, more complex analyses.

For partial correlation to provide accurate information about the relationship between variables, the following assumptions about the scores on X_1, X_2, and Y must be reasonably well satisfied. Procedures for data screening to identify problems with these assumptions were reviewed in detail in Chapters 7 and 9, and detailed examples of data screening are not repeated here.

1. The scores on X_1, X_2, and Y should be quantitative. It is also acceptable to have predictor and control variables that are dichotomous. Most of the control variables (X_2) that are used as examples in this chapter have a small number of possible score values because this limitation makes it easier to work out in detail the manner in which X_1 and Y are related at each level or score value of X_2. However, the methods described here can be generalized to situations where the X_2 control variable has a large number of score values, as long as X_2 meets the other assumptions for Pearson correlation.
2. Scores on X_1, X_2, and Y should be reasonably normally distributed. For dichotomous predictor or control variables, the closest approximation to a normal distribution occurs when the two groups have an equal number of cases.
3. For each pair of variables (X_1 and X_2, X_1 and Y, and X_2 and Y), the joint distribution of scores should be bivariate normal, and the relation between each pair of variables should be linear. The assumption of linearity is extremely important. If X_1 and Y are nonlinearly related, then Pearson r does not provide a good description of the strength of the association between them.
4. Variances in scores on each variable should be approximately the same across scores on other variables (the homogeneity of variance assumption).
5. The slope that relates X_1 to Y should be equal or homogeneous across levels of the X_2 control variable. (That is, the overall partial correlation does not provide an adequate description of the nature of the X_1, Y relationship when the nature of the X_1, Y relationship differs across levels of X_2 or, in other words, when there is an **interaction** between X_1 and X_2 as predictors of Y.)

When interpreting partial and **semipartial correlations (*sr*)**, factors that can artifactually influence the magnitude of Pearson correlations must be considered. For example, if X_1 and Y both have low measurement reliability, the correlation between X_1 and Y will be attenuated or reduced, and any partial correlation that is calculated using r_{1Y} may also be inaccurate.

We can compute three bivariate (or **zero-order**) Pearson correlations for a set of variables that includes X_1, X_2, and Y. When we say that a correlation is "zero-order," we mean that the answer to the question, "How many other variables were statistically controlled or partialled out when calculating this correlation?" is 0 or none.

r_{Y1} or r_{1Y} denotes the zero-order bivariate Pearson correlation between Y and X_1.

r_{Y2} or r_{2Y} denotes the zero-order correlation between Y and X_2.

r_{12} or r_{21} denotes the zero-order correlation between X_1 and X_2.

Note that the order of the subscripts in the zero-order correlation between X_1 and Y can be changed: The correlation can be either r_{1Y} or r_{Y1}. A zero-order Pearson correlation is symmetrical; that is, the correlation between X_1 and Y is identical to the correlation between Y and X_1. We can compute a partial correlation between X_1 and Y, controlling for one or more variables (such as X_2, X_3, and so forth). In a **first-order partial correlation** between X_1 and Y, controlling for X_2, the term *first order* tells us that only one variable (X_2) was statistically controlled when assessing how X_1 and Y are related. In a **second-order partial correlation**, the association between X_1 and Y is assessed while statistically controlling for two variables; for example, $r_{Y1.23}$ would be read as "the partial correlation between Y and X_1, statistically controlling for X_2 and X_3." In a kth order partial correlation, there are k control variables. This chapter examines first-order partial correlation in detail; the conceptual issues involved in the interpretation of higher-order partial correlations are similar.

The three zero-order correlations listed above (r_{1Y}, r_{2Y}, and r_{12}) provide part of the information that we need to work out the answer to our question, "When we control for, or take into account, a third variable called X_2, how does that change our description of the relation between X_1 and Y?" However, examination of separate scatter plots that show how X_1 and Y are related separately for each level of the X_2 variable provides additional, important information.

In all the following examples, a distinction is made among three variables: an independent or predictor variable (denoted by X_1), a dependent or outcome variable (Y), and a control variable (X_2). The preliminary analyses in this chapter provide ways of exploring whether the nature of the relationship between X_1 and Y changes when you remove, partial out, or statistically control for the X_2 variable.

Associations or correlations of X_2 with X_1 and Y can make it difficult to evaluate the true nature of the relationship between X_1 and Y. This chapter describes numerous ways in which including a single control variable (X_2) in the data analysis may change our understanding of the nature and strength of the association between an X_1 predictor and a Y outcome variable. Terms that describe possible roles for an X_2 or control variable (confound, **mediation,** suppressor, interaction) are explained using hypothetical research examples.

10.2 ♦ First Research Example

Suppose that X_1 = height, Y = vocabulary test score, and X_2 = grade level (Grade 1, 5, or 9). In other words, measures of height and vocabulary are obtained for groups of school children who are in Grades 1, 5, and 9. A scatter plot for hypothetical data, with vocabulary scores plotted in relation to height, appears in Figure 10.1. Case markers are used to identify group membership on the control variable—namely, grade level; that is, scores for first graders appear as "1," scores for fifth graders appear as "5," and scores for ninth graders appear as "9" in this scatter plot. This scatter plot shows an example in which there are three clearly separated groups of scores. Students in Grades 1, 5, and 9 differ so much in both height and vocabulary scores that there is no overlap between the groups on these variables; both height and vocabulary increase across grade levels. Real data can show a similar pattern; this first example shows a much clearer separation between groups than would typically appear in real data.

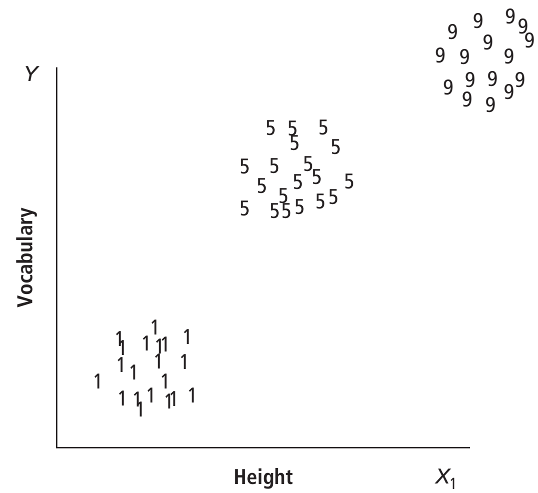


Figure 10.1 ♦ Hypothetical Data That Show a Spurious Correlation Between Y (Vocabulary) and X_1 (Height)

NOTE: Height and vocabulary are correlated only because scores on both variables increase across grade level (Grade Levels are 1, 5, and 9).

When all the data points in Figure 10.1 are examined (ignoring membership in grade-level groups), it appears that the correlation between X_1 (height) and Y (vocabulary) is large and positive ($r_{1Y} \approx +.8$). On the other hand, if you examine the X_1,Y scores within any one grade (e.g., Grade 1 children, whose scores are denoted by "1" in the scatter plot), the r_{1Y} correlation between height and vocabulary scores within each level of X_2 appears to be close to 0. This is an example of one type of situation where the apparent correlation between X_1 and Y is different when you ignore the X_2 variable (grade level) than when you take the X_2 variable into account. When you ignore grade level and compute a Pearson correlation between height and vocabulary for all the scores, there appears to be a strong positive linear association between height and vocabulary. If you take grade level into account by examining the X_1, Y (height, vocabulary) relationship separately within each grade level, there appears to be no correlation between X_1 and Y. Later in this chapter, we will see that one reasonable interpretation for this outcome would be that the correlation between height and vocabulary is "spurious." That is, height and vocabulary are not *directly* associated with each other; however, both height and vocabulary tend to increase across grade levels. The increase in height and vocabulary that occurs from Grades 1 to 5 to 9 creates a pattern in the data such that when all grade levels are treated as one group, height and vocabulary appear to be positively correlated.

10.3 ♦ Exploratory Statistical Analyses for Three-Variable Research Situations

There are several different possible ways to analyze data when we introduce a third variable X_2 into an analysis of variables X_1 and Y. This chapter describes two preliminary or exploratory analyses that can be helpful in understanding what happens when a control variable X_2 is included in an analysis:

1. We can obtain a bivariate correlation between X_1 and Y separately for subjects who have different scores on X_2. For instance, if X_2 is grade level (coded "1," "5," and "9"), we can compute three separate r_{1Y} correlations between height and vocabulary for the students within Grades 1, 5, and 9.
2. We can obtain a partial correlation between X_1 and Y, controlling for X_2. For example, we can compute the partial correlation between height and vocabulary score, controlling for grade level.

We will examine each of these two approaches to analysis to see what information they can provide.

10.4 ♦ Separate Analysis of X_1, Y Relationship for Each Level of the Control Variable X_2

One simple way to take a third variable X_2 into account when analyzing the relation between X_1 and Y is to divide the dataset into groups based on scores on the X_2 or control variable and then obtain a Pearson correlation or bivariate regression to assess the nature of the relationship between X_1 and Y separately within each of these groups. Suppose that X_2, the variable you want to control for, is grade level, whereas X_1 and Y are continuous, interval/ratio variables (height and vocabulary). Table 10.1 shows a small hypothetical dataset for which the pattern of scores is similar to the pattern in Figure 10.1; that is, both height and vocabulary tend to increase across grade levels.

A first step in the exploratory analysis of these data is to generate an X_1, Y scatter plot with case markers that identify grade level for each child. To do this, make the following SPSS menu selections: <Graph> → <Scatter/Dot>, then choose the "Simple" type of scatter plot. This set of menu selections opens up the SPSS dialog window for the simple scatter plot procedure, which appears in Figure 10.2. As in earlier examples of scatter plots, the names of the variables for the X and Y axes are identified. In addition, the variable grade is moved into the "Set Markers by" window. When the "Set Markers by" command is included, different types of markers are used to identify cases for Grades 1, 5, and 9. (Case markers can be modified using the SPSS Chart Editor to make them more distinctive in size or color.)

The scatter plot generated by these SPSS menu selections appears in Figure 10.3. These data show a pattern similar to the simpler example in Figure 10.1; that is, there are three distinct clusters of scores (all the Grade 1s have low scores on both height and vocabulary, the Grade 5s have intermediate scores on both height and vocabulary, and the Grade 9s have high scores on both height and vocabulary). A visual examination of the scatter plot in Figure 10.3 suggests that height and vocabulary are positively correlated when a correlation is calculated using all the data points, but the correlation between height and vocabulary is approximately 0 within each grade level.

We can obtain bivariate correlations for each pair of variables (height and vocabulary, height and grade, and vocabulary and grade) by making the SPSS menu selections

Table 10.1 ♦ Hypothetical Data for a Research Example Involving a Spurious Correlation Between Height and Vocabulary

Grade	*Height*	*Vocabulary*
1	46	43
1	48	45
1	48	30
1	48	18
1	46	50
1	47	53
1	48	46
1	53	33
1	49	47
1	53	54
1	49	28
1	49	51
1	51	37
1	47	33
1	49	42
1	52	50
5	54	51
5	54	79
5	54	49
5	52	50
5	58	66
5	52	65
5	53	44
5	55	73
5	52	51
5	55	47
5	54	65
5	55	52
5	53	50
5	58	48
5	59	53
5	52	59
9	57	73
9	63	62
9	62	67
9	62	80
9	61	79
9	58	67
9	57	85
9	60	64
9	61	75
9	62	62
9	62	72
9	66	85
9	62	69
9	61	75
9	64	60
9	62	58

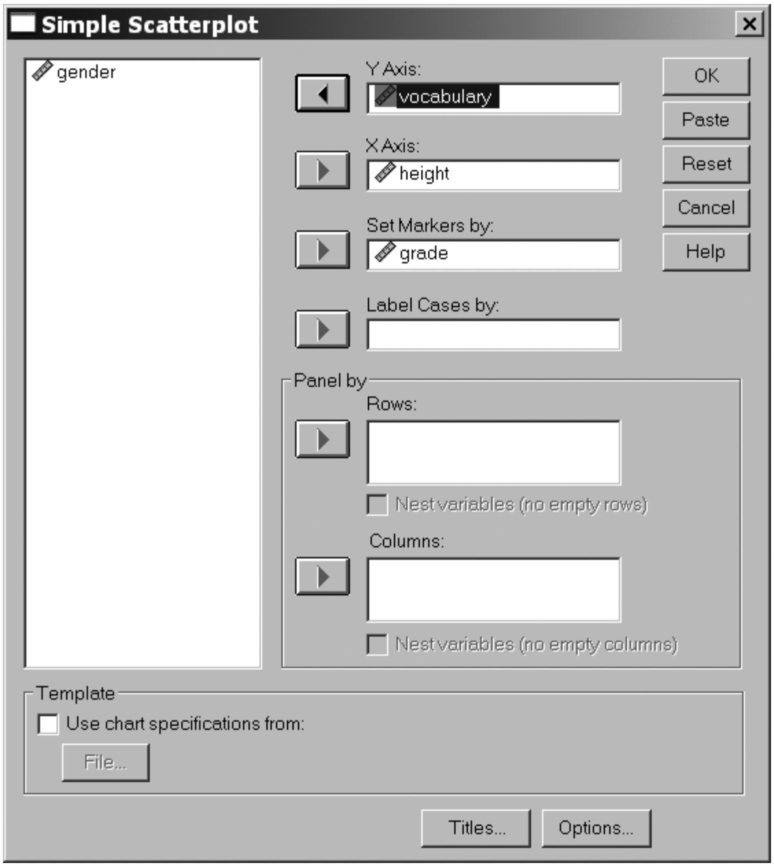


Figure 10.2 ♦ SPSS Dialog Window for Scatter Plot With Case Markers

<Analyze> → <Correlate> → <Bivariate> and entering the three variable names. These three bivariate correlations appear in Figure 10.4.

In this example, all three zero-order bivariate correlations are large and positive. This is consistent with what we see in the scatter plot in Figure 10.3. Height increases across grade levels; vocabulary increases across grade levels; and if we ignore grade and compute a Pearson correlation between height and vocabulary for all students in all three grades, that correlation is also positive.

Does the nature of the relationship between height and vocabulary appear to be different when we statistically control for grade level? One way to answer this question is to obtain the correlation between height and vocabulary separately for each of the three grade levels. We can do this conveniently using the <Split File> command in SPSS.

To obtain the correlations separately for each grade level, the following SPSS menu selections are made (as shown in Figure 10.5): <Data> → <Split File>. The SPSS dialog window for the Split File procedure appears in Figure 10.6. To split the file into separate groups based on scores on the variable grade, the user clicks the radio button for "Organize output by groups" and then enters the name of the grouping variable (in this case, the control variable grade) in the window for "Groups Based on," as shown in Figure 10.6. Once the <Split File>

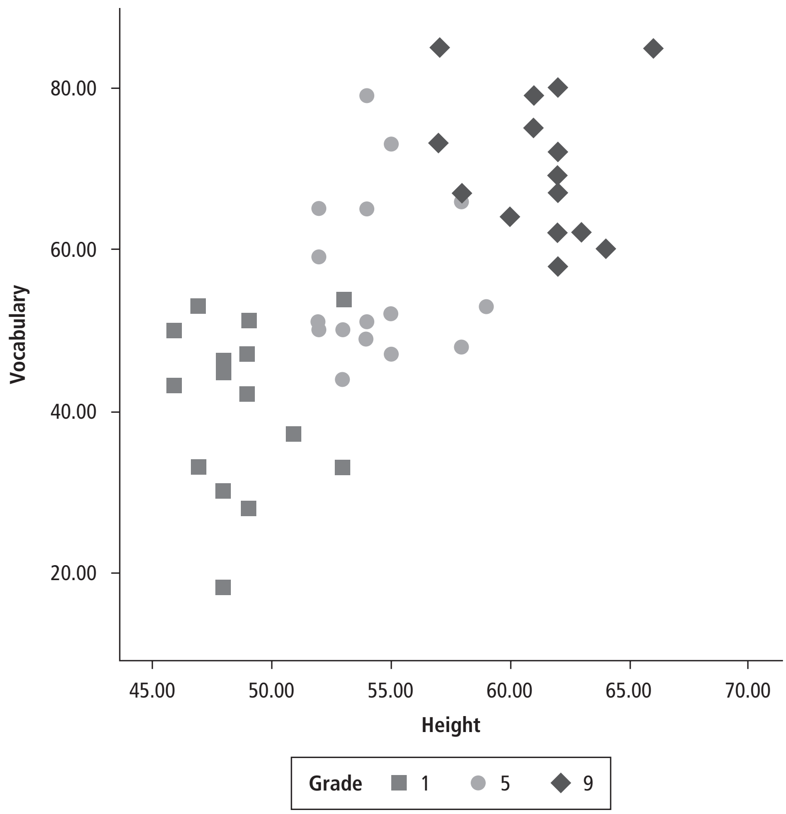


Figure 10.3 ♦ A Bivariate Scatter Plot: Vocabulary (on *Y* Axis) Against Height (on *X* Axis)

NOTE: Case markers identify grade level for each data point (Grades 1, 5, 9).

Correlations

		height	vocabulary	grade
height	Pearson Correlation	1	.716**	.913**
	Sig. (2-tailed)		.000	.000
	N	48	48	48
vocabulary	Pearson Correlation	.716**	1	.787**
	Sig. (2-tailed)	.000		.000
	N	48	48	48
grade	Pearson Correlation	.913**	.787**	1
	Sig. (2-tailed)	.000	.000	
	N	48	48	48

**. Correlation is significant at the 0.01 level (2-tailed).

Figure 10.4 ♦ The Bivariate Zero-Order Pearson Correlations Between All Pairs of Variables: X_1 (Height), Y (Vocabulary), and X_2 (Grade Level) command has been run, any subsequent analyses that are requested are performed and reported separately for each group based on the score value for grade; in this example, each analysis is performed separately for the groups of children in Grades 1, 5, and 9. Note that to carry out subsequent analyses that treat all the scores in the dataset as one group, you need to go back to the Split File dialog window shown in Figure 10.6 and select the radio button for "Analyze all cases, do not create groups."

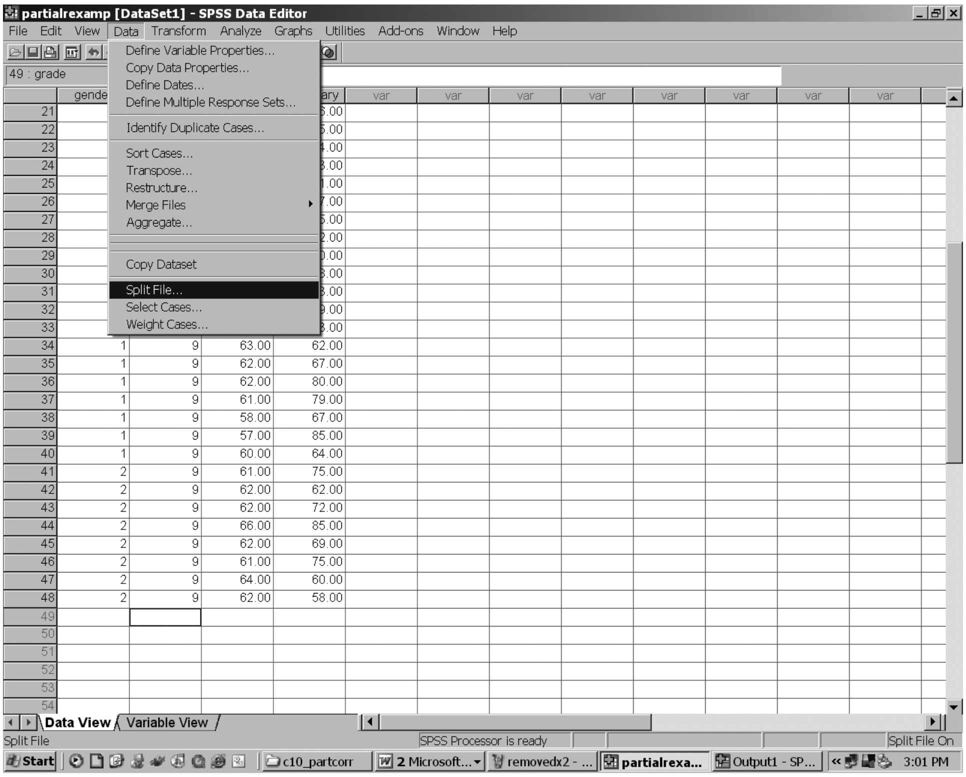


Figure 10.5 ♦ SPSS Menu Selections for the Split File Command

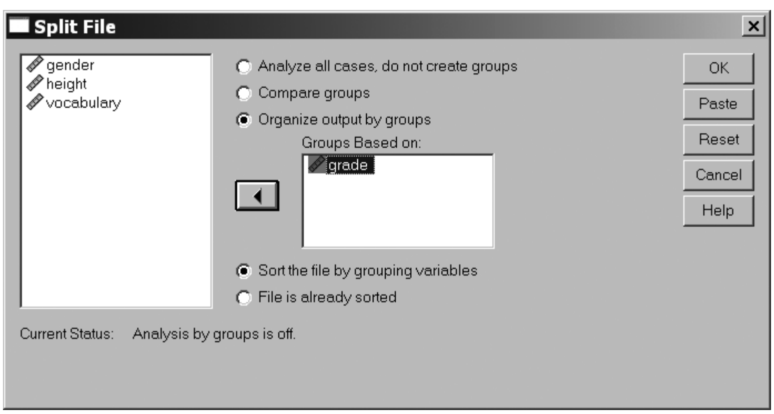


Figure 10.6 ♦ SPSS Dialog Window for Split File: Grade Level X_2 Is the "Control" Variable in This Situation

To obtain the Pearson correlation between height and vocabulary separately within each grade, the user next makes the menu selections <Analyze> → <Correlate> → <Bivariate> and enters the names of the variables (height and vocabulary) in the dialog window for the bivariate correlation procedure. The corresponding output appears in Figure 10.7. Within each grade level, the correlation between height and vocabulary did not differ significantly from 0. To summarize, the overall Pearson correlation between height and vocabulary (combining the scores for students in all three grade levels) was positive and statistically significant (from Figure 10.4, the correlation between height and vocabulary was +.72). After we statistically control for grade level by calculating the height, vocabulary correlation separately for students within each grade level, we find that the correlation between these variables within each grade level was close to 0 (from Figure 10.7, $r = .07$, .03, and −.14, for students in Grade 1, Grade 5, and Grade 9, respectively). Scatter plots are not shown for each grade, but they can provide valuable additional information when sample sizes are larger than in this example.

In this situation, our understanding of the nature of the relationship between height (X_1) and vocabulary (Y) is quite different, depending on whether or not we statistically control for grade (X_2). If we ignore grade, height and vocabulary appear to be positively correlated; if we statistically control for grade, height and vocabulary appear to be uncorrelated.

Another possible outcome when the X_1, Y relationship is examined separately for each value of X_2 is that the slope or correlation between X_1 and Y may differ across levels of X_2. This outcome suggests that there may be an interaction between X_1 and X_2 as predictors of Y. If there is evidence that X_1 and Y have significantly different correlations or slopes across levels of X_2, then it would be misleading to report a single overall partial correlation between X_1 and Y, controlling for X_2. Later chapters in this textbook describe ways to set up analyses that include an interaction between X_1 and X_2 and test the statistical significance of interactions (see Chapters 12 and 13).

10.5 ♦ Partial Correlation Between X_1 and Y, Controlling for X_2

Another way to evaluate the nature of the relationship between X_1 (height) and Y (vocabulary) while statistically controlling for X_2 (grade) is to compute a partial correlation between X_1 and Y, controlling for or partialling out X_2. The following notation is used to denote the partial correlation between Y and X_1, controlling for X_2: $r_{Y1.2}$. The subscript 1 in $r_{Y1.2}$ refers to the predictor variable X_1, and the subscript 2 refers to the control variable X_2. When the subscript is read, pay attention to the position in which each variable is mentioned relative to the "**.**" in the subscript. The period within the subscript divides the subscripted variables into two sets. The variable or variables to the right of the period in the subscript are used as predictors in a regression analysis; these are the variables that are statistically controlled or partialled out. The variable or variables to the left of the period in the subscript are the variables for which the partial correlation is assessed while taking one or more control variables into account. Thus, in $r_{Y1.2}$, the subscript *Y1.2* denotes the partial correlation between X_1 and Y, controlling for X_2.

In the partial correlation, the order in which the variables to the left of the period in the subscript are listed does not signify any difference in the treatment of variables; we could read either $r_{Y1.2}$ or $r_{1Y.2}$ as "the partial correlation between X_1 and Y, controlling for X_2." However, changes in the position of variables (before versus after the period) do reflect

Correlations
grade = 1

Correlations[a]

		height	vocabulary
height	Pearson Correlation	1	.067
	Sig. (2-tailed)		.806
	N	16	16
vocabulary	Pearson Correlation	.067	1
	Sig. (2-tailed)	.806	
	N	16	16

a. grade = 1

grade = 5

Correlations[a]

		height	vocabulary
height	Pearson Correlation	1	.031
	Sig. (2-tailed)		.909
	N	16	16
vocabulary	Pearson Correlation	.031	1
	Sig. (2-tailed)	.909	
	N	16	16

a. grade = 5

grade = 9

Correlations[a]

		height	vocabulary
height	Pearson Correlation	1	-.141
	Sig. (2-tailed)		.603
	N	16	16
vocabulary	Pearson Correlation	-.141	1
	Sig. (2-tailed)	.603	
	N	16	16

a. grade = 9

Figure 10.7 ♦ Height (X_1) and Vocabulary (Y) Correlations for Each Level of Grade (X_2)

a difference in their treatment. For example, we would read $r_{Y2.1}$ as "the partial correlation between X_2 and Y, controlling for X_1."

Another common notation for partial correlation is pr_1. The subscript 1 associated with pr_1 tells us that the partial correlation is for the predictor variable X_1. In this notation, it is implicit that the dependent variable is Y and that other predictor variables, such as X_2, are statistically controlled. Thus, pr_1 is the partial correlation that describes the predictive relation of X_1 to Y when X_2 (and possibly other additional variables) is controlled for.

10.6 ♦ Understanding Partial Correlation as the Use of Bivariate Regression to Remove Variance Predictable by X_2 From Both X_1 and Y

One way to calculate the partial r between X_1 and Y, controlling for X_2, is to carry out the following series of simple and familiar analyses. First, use bivariate regression to obtain the residuals for the prediction of X_1 from X_2; these residuals (X_1^*) represent the parts of the X_1 scores that are not predictable from X_2 or are not correlated with X_2. Second, use bivariate regression to obtain the residuals for the prediction of Y from X_2; these residuals (Y^*) represent the parts of the Y scores that are not predictable from or correlated with X_2. Third, compute a Pearson correlation between the X_1^* and Y^* residuals. This Pearson correlation is equivalent to the partial correlation $r_{Y1.2}$; it tells us how strongly X_1 is correlated with Y when the variance that is predictable from the control variable X_2 has been removed from both X_1 and Y. This method using correlations of regression residuals is not the most convenient computational method, but the use of this approach provides some insight into what it means to statistically control for X_2.

X_1 and Y are the variables of interest. X_2 is the variable we want to statistically control for; we want to remove the variance that is associated with or predictable from X_2 from both the X_1 and the Y variables.

First, we perform a simple bivariate regression to predict X_1 from X_2: X_1' is the predicted score for X_1 based on X_2; that is, X_1' is the part of X_1 that is related to or predictable from X_2:

$$X_1' = b_0 + bX_2. \tag{10.1}$$

The residuals from this regression, denoted by X_1^*, are calculated by finding the difference between the actual value of X_1 and the predicted value of X_1' for each case: $X_1^* = (X_1 - X_1')$. The X_1^* residual is the part of the X_1 score that is *not* predictable from or related to X_2.

Next, we perform a similar regression to predict Y from X_2:

$$Y' = b_0 + bX_2. \tag{10.2}$$

Then, we take the residuals Y^*, where $Y^* = (Y - Y')$. Y^* gives us the part of Y that is *not* related to or predictable from X_2. (Note that the b_0 and b coefficients will have different numerical values for the regressions in Equations 10.1 and 10.2.)

This method can be used to compute the partial correlation between height and vocabulary, controlling for grade level, for the data in the previous research example, where X_1 = height, Y = vocabulary, and X_2 is the control variable grade.

As described in Chapter 9, we can run the SPSS regression procedure and use the <Save> command to save computational results, such as the unstandardized residuals for each case; these appear as new variables in the SPSS worksheet. Figure 10.8 shows the SPSS Regression dialog window to run the regression that is specified in Equation 10.1 (to predict X_1 from X_2—in this example, height from grade). Figure 10.9 shows the SPSS Data View worksheet after performing the regressions in Equations 10.1 (predicting height from grade) and 10.2 (predicting vocabulary score from grade). The residuals from these two separate regressions were saved as new variables and renamed. RES_1, renamed Resid_Height, refers to the part of the scores on the X_1 variable, height, that were not predictable from the control or X_2 variable, grade; RES_2, renamed Resid_Voc, refers to the part of the scores on the Y variable, vocabulary, that were not predictable from the control variable, grade. Resid_Height corresponds to X_1^*, and Resid_Voc corresponds to Y^* in the previous general description of this analysis (Figure 10.10). Finally, we can obtain the bivariate Pearson correlation between these two new variables, Resid_Height and Resid_Voc (X_1^* and Y^*). The correlation between these residuals, $r = -.012$ in Figure 10.11, corresponds to the value of the partial correlation between X_1 and Y, controlling for or partialling out X_2. Note that X_2 is partialled out or removed from both variables. This partial $r = -.012$ tells us that X_1 (height) is not significantly correlated with Y (vocabulary) when variance that is predictable from grade level (X_2) has been removed from or partialled out of *both* the X_1 and the Y variables.

10.7 ♦ Computation of Partial *r* From Bivariate Pearson Correlations

There is a simpler direct method for the computation of the partial r between X_1 and Y, controlling for X_2, based on the values of the three bivariate correlations

r_{Y1}, the correlation between Y and X_1,

r_{Y2}, the correlation between Y and X_2, and

r_{12}, the correlation between X_1 and X_2.

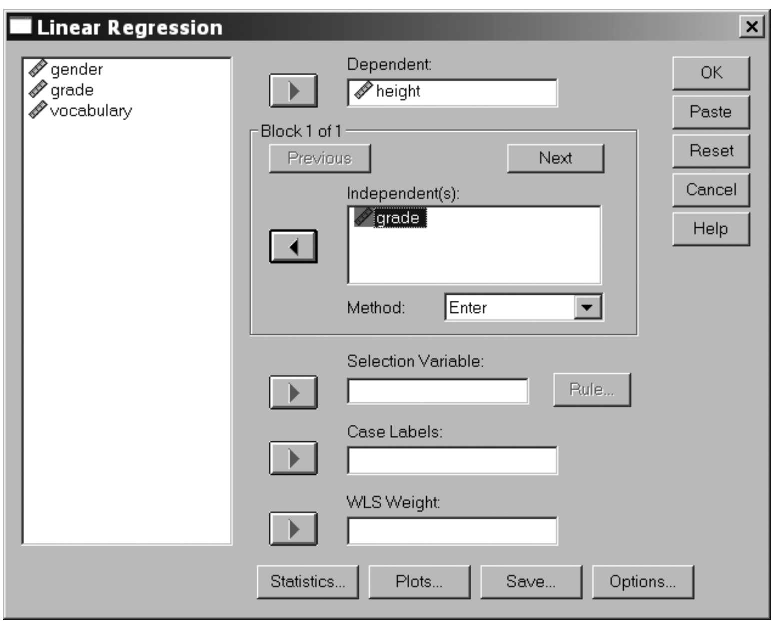
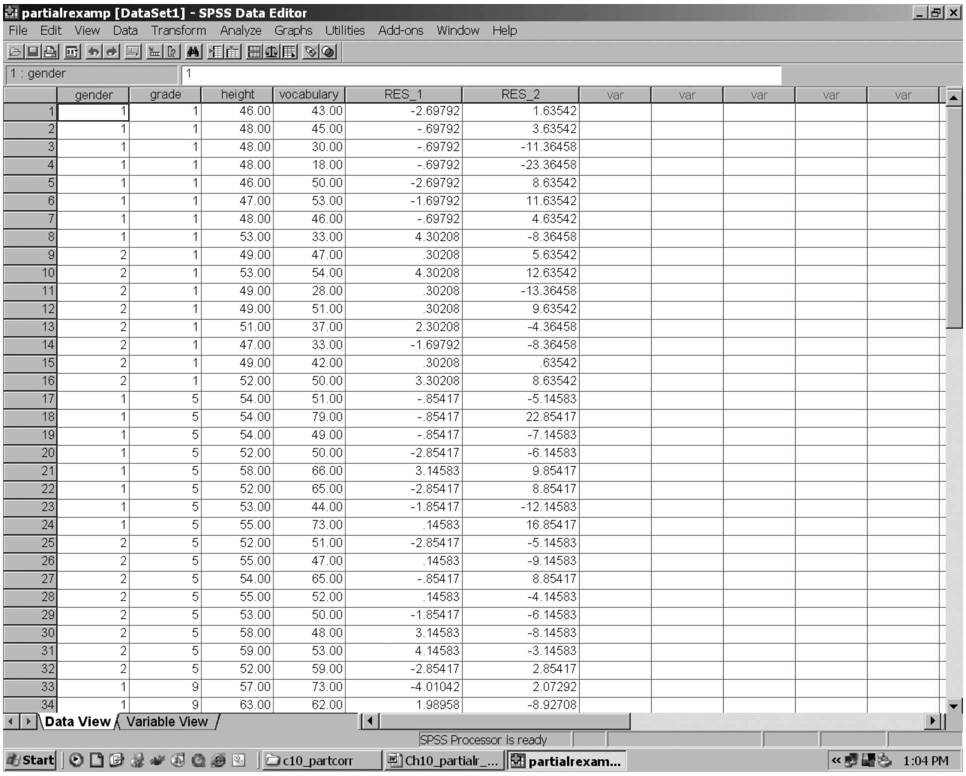


Figure 10.8 ♦ Bivariate Regression to Predict Height (X_1) From Grade in School (X_2)

NOTE: The unstandardized residuals from this regression were saved as RES_1 and renamed Resid_Height. A bivariate regression was also performed to predict vocabulary from grade; the residuals from this regression were saved as RES_2 and then renamed Resid_Voc.



	gender	grade	height	vocabulary	RES_1	RES_2	var	var	var	var	var
1	1	1	46.00	43.00	-2.69792	1.63542					
2	1	1	48.00	45.00	-.69792	3.63542					
3	1	1	48.00	30.00	-.69792	-11.36458					
4	1	1	48.00	18.00	-.69792	-23.36458					
5	1	1	46.00	50.00	-2.69792	8.63542					
6	1	1	47.00	53.00	-1.69792	11.63542					
7	1	1	48.00	46.00	-.69792	4.63542					
8	1	1	53.00	33.00	4.30208	-8.36458					
9	2	1	49.00	47.00	.30208	5.63542					
10	2	1	53.00	54.00	4.30208	12.63542					
11	2	1	49.00	28.00	.30208	-13.36458					
12	2	1	49.00	51.00	.30208	9.63542					
13	2	1	51.00	37.00	2.30208	-4.36458					
14	2	1	47.00	33.00	-1.69792	-8.36458					
15	2	1	49.00	42.00	.30208	.63542					
16	2	1	52.00	50.00	3.30208	8.63542					
17	1	5	54.00	51.00	-.85417	-5.14583					
18	1	5	54.00	79.00	-.85417	22.85417					
19	1	5	54.00	49.00	-.85417	-7.14583					
20	1	5	52.00	50.00	-2.85417	-6.14583					
21	1	5	58.00	66.00	3.14583	9.85417					
22	1	5	52.00	65.00	-2.85417	8.85417					
23	1	5	53.00	44.00	-1.85417	-12.14583					
24	1	5	55.00	73.00	.14583	16.85417					
25	2	5	52.00	51.00	-2.85417	-5.14583					
26	2	5	55.00	47.00	.14583	-9.14583					
27	2	5	54.00	65.00	-.85417	8.85417					
28	2	5	55.00	52.00	.14583	-4.14583					
29	2	5	53.00	50.00	-1.85417	-6.14583					
30	2	5	58.00	48.00	3.14583	-8.14583					
31	2	5	59.00	53.00	4.14583	-3.14583					
32	2	5	52.00	59.00	-2.85417	2.85417					
33	1	9	57.00	73.00	-4.01042	2.07292					
34	1	9	63.00	62.00	1.98958	-8.92708					

Figure 10.9 ♦ SPSS Data View Worksheet

NOTE: RES_1 and RES_2 are the saved unstandardized residuals for the prediction of height from grade and vocabulary from grade; these were renamed Resid_Height and Resid_Voc.

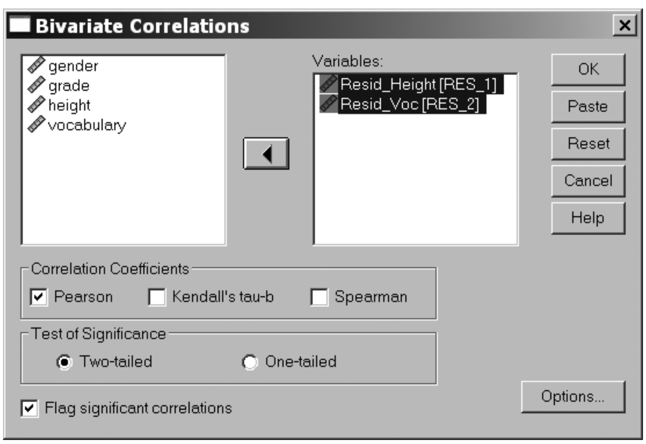


Figure 10.10 ♦ Correlation Between Residuals for Prediction of Height From Grade and Residuals for Prediction of Vocabulary From Grade

Correlations

		Resid_Height	Resid_Voc
Resid_Height	Pearson Correlation	1	-.012
	Sig. (2-tailed)		.937
	N	48	48
Resid_Voc	Pearson Correlation	-.012	1
	Sig. (2-tailed)	.937	
	N	48	48

Figure 10.11 ♦ Correlations Between Residuals of Regressions in Which the Control Variable Grade (X_2) Was Used to Predict Scores on the Other Variables (X_1, Height and Y, Vocabulary)

NOTE: The variable Resid_Height contains the residuals from the bivariate regression to predict height (X_1) from grade (X_2). The variable Resid_Voc contains the residuals from the bivariate regression to predict vocabulary (Y) from grade (X_2). These residuals correspond to the parts of the X_1 and Y scores that are not related to or not predictable from grade (X_2).

The formula to calculate the partial r between X_1 and Y, controlling for X_2, directly from the Pearson correlations is as follows:

$$pr_1 = r_{Y1.2} = \frac{r_{1Y} - (r_{12} \times r_{2Y})}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{2Y}^2}}. \tag{10.3}$$

In the preceding example, where X_1 = height, Y = vocabulary, and X_2 = grade, the corresponding bivariate correlations were $r_{1Y} = +.716$, $r_{2Y} = +.787$, and $r_{12} = +.913$. If these values are substituted into Equation 10.3, the partial correlation $r_{Y1.2}$ is as follows:

$$\begin{aligned}\frac{+.716 - (.913 \times .787)}{\sqrt{1 - .913^2}\sqrt{1 - .787^2}} &= \frac{.716 - .71853}{\sqrt{.166431}\sqrt{.380631}} \\ &= \frac{-.00253}{(.4049595) \times (.6169529)} \\ &= \frac{-.00253}{.2498409} \approx -.010.\end{aligned}$$

Within rounding error, this value of −.010 agrees with the value that was obtained from the correlation of residuals from the two bivariate regressions reported in Figure 10.11. In practice, it is rarely necessary to calculate a partial correlation by hand. However, it is sometimes useful to use Equation 10.3 to calculate partial correlations as a secondary analysis based on tables of bivariate correlations reported in journal articles.

The most convenient method of obtaining a partial correlation when you have access to the original data is using the Partial Correlations procedure in SPSS.

The SPSS menu selections, <Analyze> → <Correlate> → <Partial>, shown in Figure 10.12 open up the Partial Correlations dialog window, which appears in Figure 10.13. The names of the predictor and outcome variables (height and vocabulary) are entered in the window that is headed Variables. The name of the control variable, grade, is entered in the window under the heading "Controlling for." (Note that more than one variable can be placed in this window; that is, we can include more than one control variable.) The output for this procedure appears in Figure 10.14, where the value of the partial correlation between height and vocabulary, controlling for grade, is given as $r_{1Y.2} = -.012$; this partial correlation is not significantly different from 0 (and is identical to the correlation between Resid_Height and Resid_Voc reported in Figure 10.11).

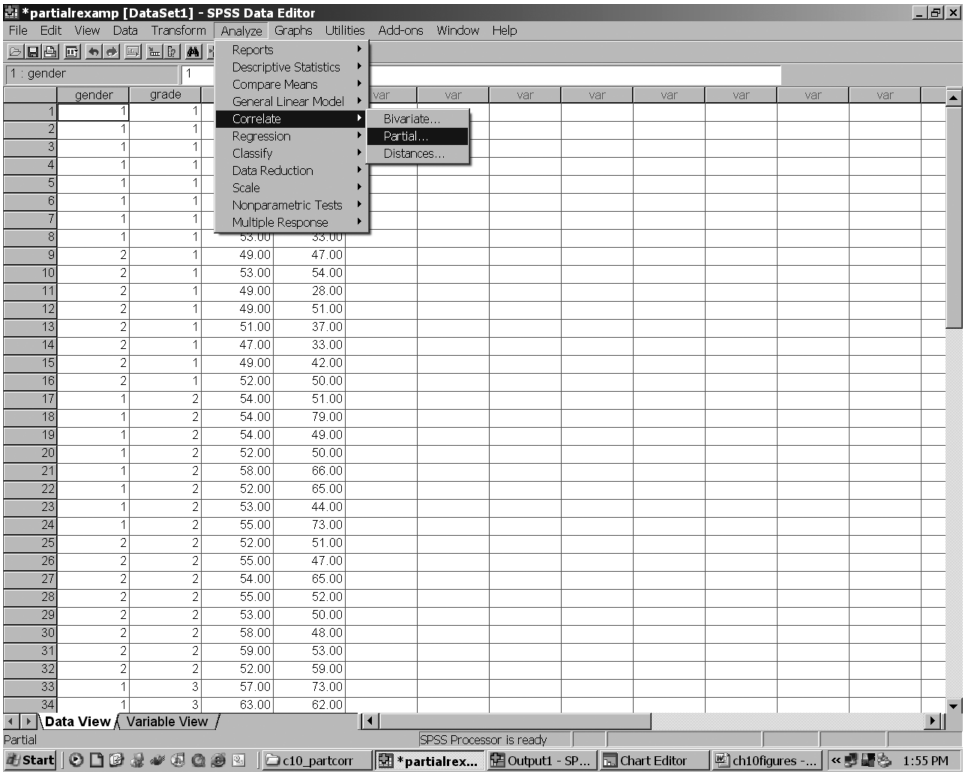


Figure 10.12 ♦ SPSS Data View Worksheet With Menu Selections for Partial Correlation

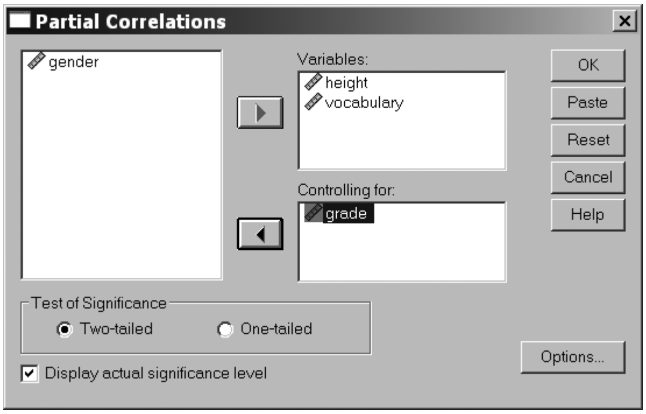


Figure 10.13 ♦ SPSS Dialog Window for the Partial Correlations Procedure

Correlations

Control Variables			height	vocabulary
grade	height	Correlation	1.000	-.012
		Significance (2-tailed)	.	.938
		df	0	45
	vocabulary	Correlation	-.012	1.000
		Significance (2-tailed)	.938	.
		df	45	0

Figure 10.14 ♦ Output From the SPSS Partial Correlations Procedure: First-Order Partial Correlation Between Height (X_1) and Vocabulary (Y), Controlling for Grade Level (X_2)

NOTE: The partial correlation between height and vocabulary, controlling for grade ($r = -.012$), is identical to the correlation between Resid_Height and Resid_Voc ($r = -.012$), which appeared in Figure 10.11.

10.8 ♦ Intuitive Approach to Understanding Partial *r*

In Chapter 7, we saw how the value of a bivariate correlation r_{1Y} is related to the pattern of points in the X_1, Y scatter plot. In some three-variable research situations, we can understand the outcome for partial correlation by looking at the pattern of points in the X_1, Y scatter plot. The partial correlation between X_1 and Y, controlling for X_2, is *approximately* the average of the r_{1Y} values obtained by correlating X_1 and Y separately within each group defined by scores on the X_2 variable. (This is not an exact computational method for partial correlation; however, thinking about partial correlation in this way sometimes helps us understand the way the three variables are interrelated.) In Figure 10.7, the correlations between height and vocabulary for Grades 1, 5, and 9 were $r_1 = .067$, $r_5 = .031$, and $r_9 = -.141$, respectively. If you compute the average of these three within-group correlations, the mean of the within-group correlations *approximately* equals the partial correlation reported earlier for these data:

$$\frac{r_1 + r_5 + r_9}{3} = \frac{.067 + .031 - .141}{3} = -.014,$$

which is approximately equal to the partial r of $-.012$ reported by SPSS from the Partial Correlations procedure. The correspondence between the mean of the within-group correlations and the overall partial r will not always be as close as it was in this example. One possible interpretation of the partial r is as the "average" correlation between X_1 and Y across different groups or levels that are based on scores on X_2. It is easier to visualize this in situations where the X_2 control variable has a small number of levels or groups, and so, in this example, the control variable, grade, was limited to three levels. However, all three variables (X_1, Y, and X_2) can be quantitative or interval/ratio level of measurement, with many possible score values for each variable.

In this example, examination of the partial correlation, controlling for grade level, helps us see that the correlation between height and vocabulary was completely due to the fact that both height and vocabulary increased from Grade 1 to 5 to 9. Height and vocabulary increase with age, and the observed zero-order correlation between height and vocabulary arose only because each of these variables is related to the third variable X_2 (grade level or age). When we statistically control for grade level, the X_1, Y relationship disappears: That is, for children who are within the same grade level, there is no relation between height and vocabulary. In this situation, we could say that the X_1, Y correlation was "completely accounted for" or "completely explained away" by the X_2 variable. The apparent association between height and vocabulary is completely explained away by the association of these two variables with grade level. We could also say that the X_1, Y or height, vocabulary correlation was "completely spurious." A correlation between two variables is said to be spurious if there is no direct association between X_1 and Y and if X_1 and Y are correlated only because both these variables are caused by or are correlated with some other variable (X_2).

In the example above, partial correlation provides an understanding of the way X_1 and Y are related that is quite different from that obtained using the simple zero-order correlation. If we control for X_2, and the partial correlation between X_1 and Y, controlling for X_2, $r_{Y1.2}$, is equal to or close to 0, we may conclude that X_1 and Y are not directly related.

10.9 ♦ Significance Tests, Confidence Intervals, and Statistical Power for Partial Correlations

10.9.1 ♦ Statistical Significance of Partial *r*

The null hypothesis that a partial correlation equals 0 can be tested by setting up a *t* ratio that is similar to the test for the statistical significance of an individual zero-order Pearson correlation. The SPSS Partial Correlations procedure provides this statistical significance test; SPSS reports an exact *p* value for the statistical significance of partial *r*. The degrees of freedom (*df*) for a partial correlation are $N - k$, where k is the total number of variables that are involved in the partial correlation and N is the number of cases or participants.

10.9.2 ♦ Confidence Intervals for Partial *r*

Most textbooks do not present detailed formulas for standard errors or confidence intervals for partial correlations. Olkin and Finn (1995) provided formulas for computation of the standard error for partial correlations; however, the formulas are complicated and not easy to work with by hand. SPSS does not provide standard errors or confidence interval estimates for partial correlations. SPSS add-on programs, such as ZumaStat (www.zumastat.com), can be used to obtain confidence intervals for partial correlations.

10.9.3 ♦ Effect Size, Statistical Power, and Sample Size Guidelines for Partial *r*

Like Pearson r (and r^2), the partial correlation $r_{Y1.2}$ and squared partial correlation $r^2_{Y1.2}$ can be interpreted directly as information about effect size or strength of association between variables. The effect-size labels for the values of Pearson r and r^2 that appeared in Table 5.2 can reasonably be used to describe the effect sizes that correspond to partial correlations.

In terms of statistical power and sample size, the guidelines about sample size needed to detect zero-order Pearson correlations provided in Chapter 7 can be used to set lower limits for the sample size requirements for partial correlation. However, when a researcher wants to explore the association between X_1 and Y separately for groups that have different scores on an X_2 control variable, the minimum N that might be required to do a good job may be much higher. Table 7.4 provides approximate sample sizes needed to achieve various levels of statistical power for different population effect sizes; the population effect size in this table is given in terms of ρ^2. For example, based on Table 7.5, a researcher who believes that the population effect size (squared correlation) for a pair of variables X_1 and Y is of the order of $\rho^2 = .25$ would need a sample size of $N = 28$ to have statistical power of about .80 for a zero-order Pearson correlation between X_1 and Y.

Assessment of the relationship between X_1 and Y that takes a control variable X_2 into account should probably use a larger N than the minimum value suggested in Table 7.5. If the X_2 control variable is SES and the X_2 variable has three different values, the population effect size for the X_1, Y relationship is of the order of $\rho^2_{Y1.2} = .25$, and the researcher wants to obtain good quality information about the nature of the relationship between X_1 and Y separately within each level of SES, it would be helpful to have a minimum sample size of $N = 28$ within *each* of the three levels of SES: in other words, a total N of $3 \times 28 = 84$. Using a larger value of N (which takes the number of levels of the X_2 variable into account) may help the researcher obtain a reasonably good assessment of the nature and significance of the relationship between X_1 and Y within each group, based on scores on the X_2 variable. However, when the X_2 control variable has dozens or hundreds of possible values, it may not be possible to have a sufficiently large N for each possible value of X_2 to provide an accurate description of the nature of the relationship between X_1 and Y at each level of X_2. This issue should be kept in mind when interpreting results from studies where the control variable has a very large number of possible score values.

10.10 ♦ Interpretation of Various Outcomes for $r_{Y1.2}$ and r_{Y1}

When we compare the size and sign of the zero-order correlation between X_1 and Y with the size and sign of the partial correlation between X_1 and Y, controlling for X_2, several different outcomes are possible. The following sections identify some possible interpretations for each of these outcomes. The value of r_{1Y}, the zero-order correlation between X_1 and Y, can range from −1 to +1. The value of $r_{1Y.2}$, the partial correlation between X_1 and Y, controlling for X_2, can also range from −1 to +1, and in principle, any combination of values of r_{1Y} and $r_{1Y.2}$ can occur (although some outcomes are much more common than others). The diagram in Figure 10.15 shows the square that corresponds to all possible combinations of values for r_{1Y} and $r_{1Y.2}$; this is divided into regions labeled *a*, *b*, *c*, *d*, and *e*. The combinations of r_{1Y} and $r_{1Y.2}$ values that correspond to each of these regions in Figure 10.15 have different possible interpretations, as discussed in the next few sections.

For each region that appears in Figure 10.15, the corresponding values of the zero-order correlation and partial correlation and the nature of possible interpretations are as follows:

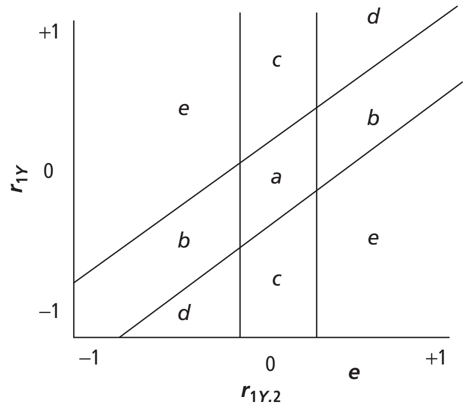


Figure 10.15 ♦ Possible Values for the Partial Correlation $r_{1Y.2}$ (on the Horizontal Axis) and the Zero-Order Pearson Correlation r_{1Y} (on the Vertical Axis)

SOURCE: Adapted from Davis (1971).

Area a. This corresponds to outcomes where the zero-order r_{Y1} is approximately equal to 0 and the partial $r_{Y1.2}$ is also approximately equal to 0. Based on this outcome, we might conclude that X_1 is not (linearly) related to Y whether X_2 is controlled or not. (Note that if the X_1, Y scatter plot shows an indication of a curvilinear relation, a different statistic may be needed to describe the corresponding relationship.)

Area b. This corresponds to outcomes where the value of the partial $r_{Y1.2}$ is approximately equal to the value of the zero-order r_{1Y}, and both the partial $r_{Y1.2}$ and the zero-order correlation are not equal to 0. Based on this outcome, we might conclude that controlling for X_2 does not change the apparent strength and nature of the relationship of X_1 with Y; we might say that X_2 is "irrelevant" to understanding the X_1, Y relation.

Area c. This corresponds to the outcome where the zero-order correlation r_{Y1} is significantly different from 0 but the partial correlation $r_{Y1.2}$ is not significantly different from 0. In other words, X_1 and Y appear to be related if you ignore X_2, but their relationship disappears when X_2 is statistically controlled. There are at least two different interpretations for this outcome. One possible interpretation is that the correlation between X_1 and Y is spurious; both X_1 and Y might have been caused by X_2 or might be correlated with X_2. In this situation, we might say that the X_2 variable completely "accounts for" or "explains away" the apparent association between X_1 and Y. Alternatively, it is possible that X_2 mediates a causal relationship between X_1 and Y. That is, there could be a causal chain such that first X_1 causes X_2 and then X_2 causes Y. If X_1 affects Y *only* through its intermediate effect on X_2, we might say that its effect on Y is completely mediated by X_2. Unfortunately, correlation analysis alone cannot determine which of these two interpretations for Area *c* outcomes is more appropriate. Unless there is additional evidence to support the plausibility of a mediated causal sequence, it is more conservative to interpret this outcome as evidence of a spurious correlation between X_1 and Y. (Chapter 11 will present methods that provide a more rigorous test for the mediated **causal model**, but even these methods will not lead to conclusive proof that the variables are causally associated.)

Area d. This outcome occurs when the partial correlation, $r_{Y1.2}$, is smaller than the zero-order correlation, r_{1Y} and has the same sign as r_{1Y}, but the partial correlation is significantly different from 0. This is a very common outcome. (Note that the likelihood of outcomes *a, b, c, d*, and *e* does not correspond to the proportion of areas that these outcomes occupy in the graph in Figure 10.15.) For outcomes that correspond to Area *d*, we might say that X_2 "partly explains" the relationship of X_1 to Y; when X_2 is controlled, X_1 and Y are still related, but the relationship is weaker than when X_2 is ignored. These values for partial and zero-order correlation suggest that there could be both a direct path from X_1 to Y and an indirect path from X_1 to Y via X_2. Alternatively, under certain conditions, we might also interpret this outcome as being consistent with the theory that X_2 "partly mediates" a causal connection between X_1 and Y.

Area e. This outcome occurs when the partial correlation $r_{Y1.2}$ is opposite to the zero-order r_{1Y} in sign and/or when $r_{Y1.2}$ is larger than r_{1Y} in absolute value. In other words, controlling for X_2 either makes the X_1, Y relationship stronger or changes the direction of the relationship. When this happens, we often say that X_2 is a **suppressor variable**; that is, the effect of X_2 (if it is ignored) is to suppress or alter the apparent relationship between X_1 and Y; only when X_2 is controlled do you see the "true" relationship. Although Area *e* corresponds to a large part of the diagram in Figure 10.15, this is not a very common outcome, and the interpretation of this type of outcome can be difficult.

The preceding discussion mentions some possible interpretations for outcomes for partial *r* that involve noncausal associations between variables and other interpretations for values of partial *r* that involve hypothesized causal associations between variables. Of course, the existence of a significant zero-order or partial correlation between a pair of variables is not conclusive evidence that the variables are causally connected. However, researchers sometimes do have causal hypotheses in mind when they select the variables to measure in nonexperimental studies, and the outcomes from correlational analyses can be interpreted as (limited) information about the plausibility of some possible causal hypotheses. Readers who do not expect to work with causal models in any form may want to skip Sections 10.11 through 10.13. However, those who plan to study more advanced methods, such as structural equation modeling, will find the introduction to causal models presented in the next few sections useful background for these more advanced analytic methods.

The term *model* can have many different meanings in different contexts. In this textbook, the term *model* generally refers to either (a) a theory about possible causal and noncausal associations among variables, often presented in the form of path diagrams, or (b) an equation to predict Y from scores on one or more X predictor variables; for example, a regression to predict Y from scores on X_1 can be called a regression model. The choice of variables to include as predictors in regression equations is sometimes (although not always) guided by an implicit causal model. The following sections briefly explain the nature of the "causal" models that can be hypothesized to describe the relationships between two variables or among three variables.

10.11 ♦ Two-Variable Causal Models

One possible model for the association between an X_1 predictor and a Y outcome variable is that the variables are not associated, either causally or noncausally. The first row in Table 10.2 illustrates this situation. If r_{1Y} is equal to or nearly equal to 0, we do not need to include a **causal** or **noncausal path** between X_1 and Y in a path model that shows how X_1 and Y are related. If X_1 and Y are not systematically related; for example, if X_1 and Y have a correlation of 0, they probably are not related either causally or noncausally, and the causal model does not need to include a direct path between X_1 and Y.[1]

What statistical result would be evidence consistent with a causal model that assumes no relation between X_1 and Y? Let's assume that X_1 and Y are both quantitative variables and that all the assumptions for Pearson r are satisfied. In this situation, we can use Pearson r to evaluate whether scores on X_1 and Y are systematically related. If we obtain a correlation between X_1 and Y of $r_{1Y} = 0$ or $r_{1Y} \approx 0$, we would tentatively conclude that there is no association (either causal or noncausal) between X_1 and Y.

There are three possible ways in which X_1 and Y could be associated. First, X_1 and Y could be noncausally associated; they could be correlated or confounded but with neither variable being a cause of the other. The second row in Table 10.2 illustrates the path model for this situation; when X_1 and Y are noncausally associated, this noncausal association is represented by a bidirectional arrow. In some textbooks, the arrow that represents a noncausal association is straight rather than curved (as shown here). It does not matter whether the arrow is straight or curved; the key thing to note is whether the arrow is bidirectional or unidirectional. Second, we might hypothesize that X_1 causes Y; this causal hypothesis is represented by a unidirectional arrow or path that points away from the cause (X_1) toward the outcome (Y), as shown in the third row of Table 10.2. Third, we might hypothesize that Y causes X_1; this causal hypothesis is represented by a unidirectional arrow or path that leads from Y to X_1, as shown in the last row of Table 10.2.

Any hypothesized theoretical association between X_1 and Y (whether it is noncausal, whether X_1 causes Y, or whether Y causes X_1) should lead to the existence of a systematic statistical association (such as a Pearson r significantly different from 0) between X_1 and Y. In this example, where Pearson r is assumed to be an appropriate index of association, variables that are either causally or noncausally associated with each other should have correlations that are significantly different from 0. However, when we find a nonzero correlation between X_1 and Y, it is difficult to decide which one of these three models (X_1 and Y are noncausally related, X_1 causes Y, or Y causes X_1) provides the correct explanation. To further complicate matters, the relationship between X_1 and Y may involve additional variables; X_1 and Y may be correlated with each other because they are both causally influenced by (or noncausally correlated with) some third variable, X_2, for example.

Table 10.2 ♦ Four Possible Hypothesized Paths Between Two Variables (X_1 and Y)

Verbal Description of the Relationship Between X_1 and Y	*Path Model for X_1 and Y*	*Comment on Path Model*	*Corresponding Value of the r_{1Y} Correlation*
X_1 and Y are not associated in any way (either causally or noncausally)	$X_1 \quad Y$	No arrow or path between X_1 and Y.	$r_{1Y} = 0$ or $r_{1Y} \approx 0$
X_1 and Y are associated but not in a causal way. X_1 and Y co-occur, or are confounded, but neither variable is the cause of the other	$X_1 \leftrightarrow Y$	Bidirectional arrow or path between X_1 and Y. We use a bidirectional path when our theory says that X_1 is predictive of Y, or is correlated with Y, but X_1 is not a cause of Y.	$r_{1Y} \neq 0$
X_1 is a cause of Y	$X_1 \rightarrow Y$	Unidirectional arrow that points from the cause (X_1) toward the outcome or effect (Y) or path from X_1 to Y. We use this unidirectional "causal" path when our theory involves the hypothesis that X_1 causes Y.	$r_{1Y} \neq 0$
Y is a cause of X_1	$Y \rightarrow X_1$	Unidirectional arrow that points from the cause (Y) toward the outcome or effect (X_1). We use this unidirectional causal path when our theory involves the hypothesis that Y causes X_1.	$r_{1Y} \neq 0$

Although we can propose theoretical models that involve hypothesized causal connections, the data that we collect in typical nonexperimental studies only yield information about correlations. Correlations can be judged consistent or inconsistent with hypothetical causal models; however, finding correlations that are consistent with a specific causal model does not constitute proof of that particular model. Usually, there are several causal models that are equally consistent with an observed set of correlations.

If we find that the correlation r_{1Y} is not significantly different from 0 and/or is too small to be of any practical or theoretical importance, we would interpret that as evidence that that particular situation is more consistent with a model that has no path between X_1 and Y (as in the first row of Table 10.2) than with any of the other three models that do have paths between X_1 and Y. However, obtaining a value of r_{1Y} that is too small to be statistically significant does not conclusively rule out models that involve causal connections between X_1 and Y. We can obtain a value of r_{1Y} that is too small to be statistically significant in situations where X_1 really does influence Y, but this small correlation could be either due to sampling error or due to artifacts such as attenuation of correlation due to unreliability of the measurement or due to a nonlinear association between X_1 and Y.

On the other hand, if we find that the r_{1Y} correlation is statistically significant and large enough to be of some practical or theoretical importance, we might tentatively decide that this outcome is more consistent with one of the three models that include a path between X_1 and Y (as shown in Rows 2–4 of Table 10.2) than with the model that has no path between X_1 and Y (as in Row 1 of Table 10.2). A significant r_{1Y} correlation suggests that there may be a direct (or an indirect) path between X_1 and Y; however, we would need additional information to decide which hypothesis is the most plausible: that X_1 causes Y, that Y causes X_1, that X_1 and Y are noncausally associated, or that X_1 and Y are connected through their relationships with other variables.

In this chapter, we shall also see that when we take a third variable (X_2) into account, the correlation between X_1 and Y can change in many different ways. For a correlation to provide accurate information about the nature and strength of the association between X_1 and Y, we must have a correctly specified model. A model is correctly specified if it includes all the variables that need to be taken into account (because they are involved in causal or noncausal associations between X_1 and Y) and, also, if it does not include any variables that should not be taken into account. A good theory can be helpful in identifying the variables that should be included in a model; however, we can never be certain that the hypothetical causal model that we are using to select variables is correctly specified; it is always possible that we have omitted a variable that should have been taken into account.

For all these reasons, we cannot interpret correlations or partial correlations as either conclusive proof or conclusive disproof of a specific causal model. We can only evaluate whether correlation and partial correlation values are consistent with the results we would expect to obtain given different causal models. Experimental research designs provide more rigorous means to test causal hypotheses than tests that involve correlational analysis of nonexperimental data.

10.12 ♦ Three-Variable Models: Some Possible Patterns of Association Among X_1, Y, and X_2

When there were just two variables (X_1 and Y) in the model, there were four different possible hypotheses about the nature of the relationship between X_1 and Y—namely, X_1 and Y are not directly related either causally or noncausally, X_1 and Y are noncausally associated or confounded, X_1 causes Y, or Y causes X_1 (as shown in Table 10.2). When we expand a model to include a third variable, the number of possible models that can be considered becomes much larger. There are three pairs of variables

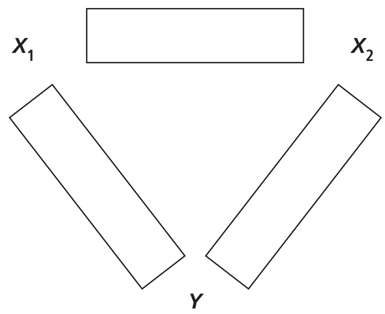


Figure 10.16 ♦ The Set of All Logically Possible Hypothesized "Causal" Models for a Set of Three Variables (X_1, X_2, and Y).

NOTE: The set can be obtained by filling in each of the rectangles with one of the four types of paths described in Table 10.2

(X_1 and X_2, X_1 and Y, and X_2 and Y), and each pair of variables can be related in any of the four ways that were described in Table 10.2. Each rectangle in Figure 10.16 can be filled in with any of the four possible types of path (no relation, noncausal association, or two different directions of cause). The next few sections of this chapter describe some of the different types of causal models that might be proposed as hypotheses for the relationships among three variables.

Using data from a nonexperimental study, we cannot prove or disprove any of these models. However, we can interpret some outcomes for correlation and partial correlation as being either consistent with or not consistent with some of the logically possible models. This may make it possible, in some research situations, to reduce the set of models that could be considered as plausible explanations for the relationships among variables.

The causal models that might be considered reasonable candidates for various possible combinations of values of r_{Y1} and $r_{Y1.2}$ are discussed in greater detail in the following sections.

10.12.1 ♦ *X_1* and *Y* Are Not Related Whether You Control for *X_2* or Not

One possible hypothetical model is that none of the three variables (X_1, X_2, and Y) is either causally or noncausally related to the others (see Figure 10.17). If we obtain Pearson r values for r_{12}, r_{1Y}, and r_{2Y} that are not significantly different from 0 (and all the three correlations are too small to be of any practical or theoretical importance), those correlations would be consistent with a model that has no paths among any of the three pairs of variables, as shown in Figure 10.17. The partial correlation between X_1 and Y, controlling for X_2, would also be 0 or very close to 0 in this situation. A researcher who

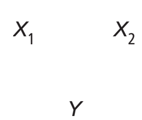


Figure 10.17 ♦ No Direct Paths (Either Causal or Noncausal) Between Any Pairs of Variables

obtains values close to 0 for all the bivariate (and partial) correlations would probably conclude that none of the variables is related to the others either causally or noncausally. (Of course, this conclusion could be incorrect if the variables are related nonlinearly, because Pearson r is not appropriate to assess the strength of nonlinear relations.)

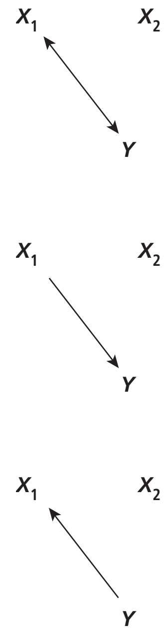


Figure 10.18 ♦ A Theoretical Causal Model in Which X_1 Is Either Causally or Noncausally Related to Y and X_2 Is Not Related to Either X_1 or Y

NOTE: Possible interpretation: X_2 is "irrelevant" to the X_1, Y relationship. It would be reasonable to hypothesize that there is some direct path between X_1 and Y, but we would need information beyond the existence of a significant correlation to evaluate whether the correlation occurred because X_1 and Y are noncausally associated, because X_1 causes Y, or because Y causes X_1. For the pattern of associations shown in these three-path models, we would expect that $r_{1Y} \neq 0$, $r_{12} \approx 0$, $r_{2Y} \approx 0$, and $r_{1Y.2} \approx r_{1Y}$.

10.12.2 ♦ X_2 Is Irrelevant to the X_1, Y Relationship

A second possible theoretical model is that X_1 is either causally or noncausally related to Y and that the X_2 variable is "irrelevant" to the X_1, Y relationship. If this model is correct, then we should obtain a statistically significant correlation between X_1 and Y (large enough to be of some practical or theoretical importance). The correlations of X_1 and Y with X_2 should be 0 or close to 0. The partial correlation between X_1 and Y, controlling for X_2, should be approximately equal to the zero-order correlation between X_1 and Y; that is, $r_{1Y.2} \approx r_{1Y}$. Figure 10.18 shows three different hypothetical causal models that would be logically consistent with this set of correlation values.

10.12.3 ♦ When You Control for X_2, the X_1, Y Correlation Drops to 0 or Close to 0

As mentioned earlier, there are two quite different possible explanations for this outcome. The causal models that are consistent with this outcome do not need to include a direct path between the X_1 and Y variables. However, there are many different possible models that include causal and/or noncausal paths between X_1 and X_2 and X_2 and Y, and these could point to two quite different interpretations. One possible explanation is that the X_1, Y correlation may be completely accounted for by X_2 (or completely spurious). Another possible explanation is that there may be a causal association between X_1 and Y that is "completely mediated" by X_2. However, several additional conditions should be met before we consider the mediated causal model a likely explanation, as described by Baron and Kenny (1986) and discussed in Chapter 11 of this book.

10.12.3.1 ♦ *Completely Spurious Correlation*

To illustrate spurious correlations, consider the previous research example where height (X_1) was positively correlated with vocabulary (Y). It does not make sense to think that there is some direct connection (causal or otherwise) between height and vocabulary. However, both these variables are related to grade (X_2). We could argue that there is a noncausal correlation or confound between height and grade level and between vocabulary and grade level. Alternatively, we could propose that a maturation process that occurs from Grade 1 to 5 to 9 "causes" increases in both height and vocabulary. The theoretical causal models that appear in Figure 10.19 illustrate these two hypotheses. In the top diagram, there is no direct path between height and vocabulary (but these variables are correlated with each other in the observed data because both these variables are positively correlated with grade). In the bottom diagram, the unidirectional arrows represent the hypothesis that grade level or the associated physical and cognitive maturation causes increases in both height and vocabulary. In either case, the model suggests that the correlation between height and vocabulary is entirely "accounted for" by their relationship with the X_2 variable. If all the variance associated with the X_2 variable is removed (through a partial correlation $r_{Y1.2}$, for example), the association between height and vocabulary disappears. It is, therefore, reasonable in this case to say that the correlation between height and vocabulary was completely accounted for or explained by grade level or that the correlation between height and vocabulary was completely spurious (the variables were correlated with each other only because they were both associated with grade level).

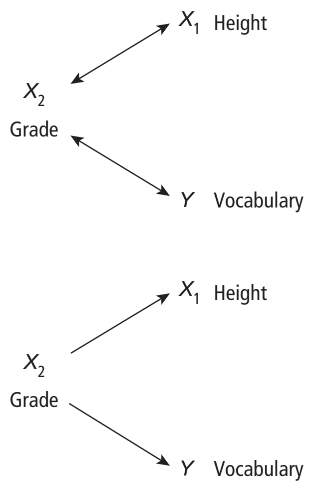


Figure 10.19 ♦ Two Possible Theoretical Models for the Relationships Among X_1 (Height), Y (Vocabulary Score), and X_2 (Grade Level) That Are Consistent With the Observed Correlations and Partial Correlations Obtained for These Variables

NOTE: In both these models, there is no direct path between X_1 and Y. Any observed correlation between X_1 and Y occurs because both X_1 and Y are noncausally associated with X_2 (as shown in the top diagram) or because both X_1 and Y are caused by X_2 (as shown in the bottom diagram). In this example, any correlation between X_1 and Y is spurious; X_1 and Y are correlated only because they are both correlated with X_2.

Some examples of spurious correlation intentionally involve foolish or improbable variables. For example, ice cream sales may increase as temperatures rise; homicide rates may also increase as temperatures rise. If we control for temperature, the correlation between ice cream sales and homicide rates drops to 0, so we would conclude that there is no direct relationship between ice cream sales and homicide but that the association of each of these variables with outdoor temperature (X_2) creates the spurious or misleading appearance of a connection between ice cream consumption and homicide.

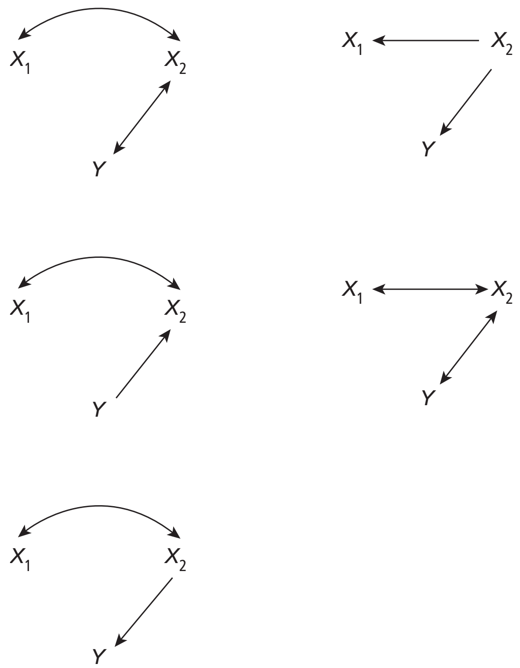


Figure 10.20 ♦ A Theoretical Causal Model in Which X_1 Has No Direct Association With Y, but Because X_1 Is Correlated With X_2 (and X_2 Is Either Causally or Noncausally Associated With Y), X_1 Is Also Correlated With Y

NOTE: In this situation we might say that the observed correlation between X_1 and Y is spurious; it is entirely due to the association of X_1 with X_2 and the association of X_2 with Y. Another possible interpretation for the path model in the lower left-hand corner of the figureis that X_1 is completely redundant with X_2 as a predictor of Y; X_1 provides no predictive information about Y that is not already available in the X_2 variable. A pattern of correlations that would be consistent with these path models is as follows: $r_{1Y} \neq 0$, but $r_{1Y.2} \approx 0$. (However, the mediated causal models in Figure 10.20 would also be consistent with these correlation values.)

There are actually many different theoretical or causal models that would be equally consistent with the outcome values $r_{Y1} \neq 0$ and $r_{Y1.2} = 0$. Some of these models appear in Figure 10.20. The characteristic that all these models for the $r_{Y1} \neq 0$ and $r_{Y1.2} = 0$ situation have in common is that they do *not* include a direct path between X_1 and Y (either causal or noncausal). In every path model in Figure 10.20, the observed correlation between X_1 and Y is due to an association (either causal or noncausal) between X_1 and X_2 and between X_2 and Y. Figure 10.20 omits the special case of models that involve mediated causal sequences, such as $X_1 \rightarrow X_2 \rightarrow Y$; the next section provides a brief preliminary discussion of mediated causal models.

10.12.3.2 ♦ *Completely Mediated Association Between X_1 and Y*

There are some research situations in which it makes sense to hypothesize that there is a causal sequence such that first X_1 causes X_2 and then X_2 causes Y. A possible example is the following: Increases in age (X_1) might cause increases in body weight

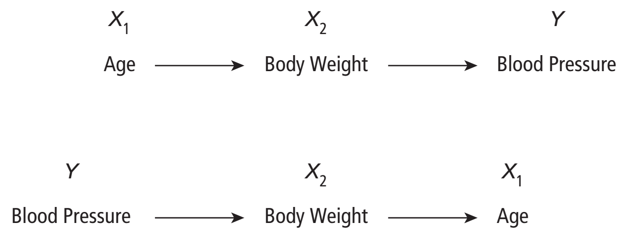


Figure 10.21 ♦ Completely Mediated Causal Models (in Which X_2 Completely Mediates a Causal Connection Between X_1 and Y)

NOTE: Top panel: a plausible mediated model in which age causes an increase in body weight, then body weight causes an increase in blood pressure. Bottom panel: an implausible mediated model.

(X_2); increases in body weight might cause increases in SBP (Y). If the relationship between X_1 and Y is completely mediated by X_2, then there is no direct path leading from X_1 to Y; the only path from X_1 to Y is through the mediating variable, X_2. Figure 10.21 illustrates two possible causal sequences in which X_2 is a mediating variable between X_1 and Y. Because of the temporal ordering of variables and our commonsense understandings about variables that can causally influence other variables, the first model ($X_1 \rightarrow X_2 \rightarrow Y$, i.e., age influences weight, and weight influences blood pressure) seems reasonably plausible. The second model ($Y \rightarrow X_2 \rightarrow X_1$, which says that blood pressure influences weight and then weight influences age) does not seem plausible.

What pattern of correlations and partial correlations would be consistent with these completely mediated causal models? If we find that $r_{Y1.2} = 0$ and $r_{Y1} \neq 0$, this outcome is logically consistent with the entire set of models that include paths (either causal or noncausal) between X_1 and X_2 and X_2 and Y but that do *not* include a direct path (either causal or noncausal) between X_1 and Y. Examples of these models appear in Figures 10.19, 10.20, and 10.21. However, models that do not involve mediated causal sequences (which appear in Figures 10.19 and 10.20) and mediated causal models (which appear in Figure 10.21) are equally consistent with the empirical outcome where $r_{1Y} \neq 0$ and $r_{1Y} = 0$. The finding that $r_{1Y} \neq 0$ and $r_{1Y} = 0$ suggests that we do not need to include a direct path between X_1 and Y in the model, but this empirical outcome does not tell us which among the several models that do not include a direct path between X_1 and Y is the "correct" model to describe the relationships among the variables.

We can make a reasonable case for a completely mediated causal model only in situations where it makes logical and theoretical sense to think that perhaps X_1 causes X_2 and then X_2 causes Y; where there is appropriate temporal precedence, such that X_1 happens before X_2 and X_2 happens before Y; and where additional statistical analyses yield results that are consistent with the mediation hypothesis (e.g., Baron & Kenny, 1986; also, see Chapter 11 in this textbook). When a mediated causal sequence does not make sense, it is more appropriate to invoke the less informative explanation that X_2 accounts for the X_1, Y correlation (this explanation was discussed in the previous section).

10.12.4 ♦ When You Control for X_2, the Correlation Between X_1 and Y Becomes Smaller (but Does Not Drop to 0 and Does Not Change Sign)

This may be one of the most common outcomes when partial correlations are compared with zero-order correlations. The implication of this outcome is that the association between X_1 and Y can be only partly accounted for by a (causal or noncausal) path via X_2. A direct path (either causal or noncausal) between X_1 and Y is needed in the model, even when X_2 is included in the analysis.

If most of the paths are thought to be noncausal, then the explanation for this outcome is that the relationship between X_1 and Y is "partly accounted for" or "partly explained by" X_2. If there is theoretical and empirical support for a possible mediated causal model, an interpretation of this outcome might be that X_2 "partly mediates" the effects of X_1 on Y, as discussed below. Figure 10.22 provides examples of general models in which an X_2 variable partly accounts for the X_1, Y relationship. Figure 10.23 provides examples of models in which the X_2 variable partly mediates the X_1, Y relationship.

10.12.4.1 ♦ X_2 *Partly Accounts for the* X_1, Y *Association, or* X_1 *and* X_2 *Are Correlated Predictors of* Y

Hansell, Sparacino, and Ronchi (1982) found a negative correlation between facial attractiveness (X_1) of high-school-aged women and their blood pressure (Y); that is, less attractive young women tended to have higher blood pressure. This correlation did not occur for young men. They tentatively interpreted this correlation as evidence of social stress; they reasoned that for high school girls, being relatively unattractive is a source of stress that may lead to high blood pressure. This correlation could be spurious, due to a causal or noncausal association of both attractiveness ratings and blood pressure with a third variable, such as body weight (X_2). If heavier people are rated as less attractive and tend to have higher blood pressure, then the apparent link between attractiveness and blood pressure might be due to the associations of both these variables with body weight. When Hansell et al. obtained a partial correlation between attractiveness and blood pressure, controlling for weight, they still saw a sizeable relationship between attractiveness and blood pressure. In other words, the correlation between facial attractiveness and blood pressure was apparently not completely accounted for by body weight. Even when weight was statistically controlled for, less attractive people tended to have higher blood pressure. The models in Figure 10.22 are consistent with their results.

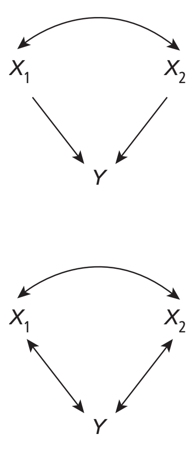


Figure 10.22 ♦ Two Possible Models in Which X_2 Partly Explains or Partly Accounts for the X_1, Y Relationship

NOTE: Top: X_1 and X_2 are correlated or confounded causes of Y (or X_1 and X_2 are partly redundant as predictors of Y). Bottom: All three variables are noncausally associated with each other. The pattern of correlations that would be consistent with this model is as follows: $r_{Y1} \neq 0$ and $r_{Y1.2} < r_{Y1}$ but with $r_{Y1.2}$ significantly greater than 0.

10.12.4.2 ♦ X_2 *Partly Mediates the* X_1, Y *Relationship*

An example of a research situation where a "partial mediation" hypothesis might make sense is the following. Suppose a researcher conducts a nonexperimental study and measures the following three variables: the X_1 predictor variable, age in years; the Y outcome variable, SBP (given in millimeters of mercury); and the X_2 control variable, body weight (measured in pounds or kilograms or other units). Because none of the variables has been manipulated by the researcher, the data cannot be used to prove causal connections among variables. However, it is conceivable that as people grow older, normal aging causes an increase in body weight. It is also conceivable that as people become heavier, this increase in body weight causes an increase in blood pressure. It is possible that body weight completely mediates the association between age and blood pressure (if this were the case, then people who do not gain weight as they age should not show any age-related increases in blood pressure). It is also conceivable that increase in age causes increases in blood pressure through other pathways and not solely through weight gain. For example, as people age, their arteries may become clogged with deposits of fats or lipids, and this accumulation of fats in the arteries may be another pathway through which aging could lead to increases in blood pressure.

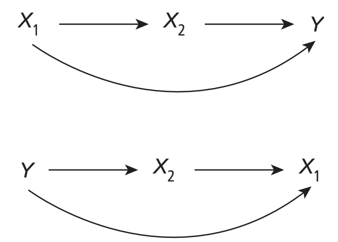


Figure 10.23 ♦ Two Possible Models for Partly Mediated Causation Between X_1 and Y (X_2 Is the Mediator)

NOTE: Top: X_1 causes or influences X_2, and then X_2 causes or influences Y; in addition, X_1 has effects on Y that are not mediated by X_2. Bottom: Y causes or influences X_2, and then X_2 causes or influences X_1; in addition, Y has effects on X_1 that are not mediated by X_2. The pattern of correlations that would be consistent with this model is as follows: $r_{Y1} \neq 0$ and $r_{Y1.2} < r_{1Y}$ but with $r_{Y1.2}$ significantly greater than 0. (Because this is the same pattern of correlations that is expected for the models in Figure 10.22, we cannot determine whether one of the partly mediated models in Figure 10.23 or the "partly redundant predictor" model in Figure 10.22 is a better explanation, based on the values of correlations alone.)

Figure 10.24 shows two hypothetical causal models that correspond to these two hypotheses. The first model in Figure 10.24 represents the hypothesis that the effects of age (X_1) on blood pressure (Y) are completely mediated by weight (X_2). The second model in Figure 10.24 represents the competing hypothesis that age (X_1) also has effects on blood pressure (Y) that are not mediated by weight (X_2).

In this example, the X_1 variable is age, the X_2 mediating variable is body weight, and the Y outcome variable is SBP. If preliminary data analysis reveals that the correlation between age and blood pressure $r_{1Y} = +.8$ but that the partial correlation between age and blood pressure, controlling for weight, $r_{1Y.2} = .00$, we may tentatively conclude that the relationship between age and blood pressure is completely mediated by body weight. In other words, as people age, increase in age causes an increase in SBP but only if age causes an increase in body weight.

We will consider a hypothetical example of data for which it appears that the effects of age on blood pressure may be only partly mediated by or accounted for by weight. Data on age, weight, and blood pressure for $N = 30$ appear in Table 10.3. Pearson correlations were performed between all pairs of variables; these bivariate zero-order correlations appear in Figure 10.25. All three pairs of variables were significantly

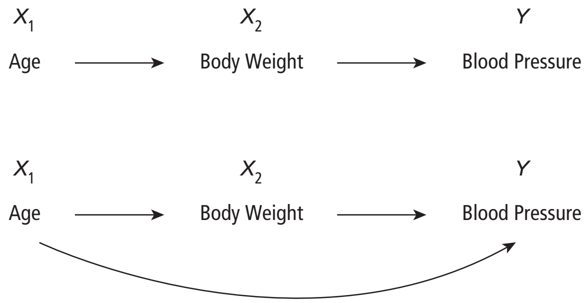


Figure 10.24 ♦ Hypothetical Mediated Models for Age, Weight, and Blood Pressure
Hypothetical Model Showing a Fully Mediated Relationship
Hypothetical Model Showing a Partially Mediated Relationship

NOTES: Preliminary evidence consistent with a fully mediated model is as follows: Partial correlation between age and blood pressure, controlling for weight ($r_{1Y.2}$), is less than the zero-order correlation between age and blood pressure (r_{1Y}), and $r_{1Y.2}$ is not significantly greater than 0.

Preliminary evidence consistent with a partial mediation model is as follows: Partial correlation between age and blood pressure, controlling for weight ($r_{1Y.2}$) is less than the zero-order correlation between age and blood pressure (r_{1Y}), but $r_{1Y.2}$ is significantly greater than 0.

See Baron and Kenny (1986) for a discussion of additional conditions that should be met before a mediated model is proposed as an explanation for correlational results.

positively correlated. A partial correlation was calculated between age and blood pressure, controlling for weight (the SPSS partial correlation output appears in Figure 10.26). The partial correlation between age and blood pressure, controlling for weight, was $r_{Y1.2} = +.659$, $p = .005$, two-tailed. The zero-order correlation between age and blood pressure (not controlling for weight) was $r_{1Y} = +.782$, $p < .001$. Because the partial correlation was smaller than the zero-order correlation between age and blood pressure but was still substantially greater than 0, a partially mediated model (Figure 10.24) could reasonably be considered. It is possible that age has some influence on blood pressure through its effects on weight and the effects of weight on blood pressure, but the partial correlation results suggest that age may also have effects on blood pressure that are not mediated by weight gain. For a researcher who wants to test mediated causal models, an examination of partial correlation provides only a preliminary test. Additional analyses of these data will be discussed in Chapter 11, to evaluate whether other conditions necessary for mediation are satisfied (Baron & Kenny, 1986).

10.12.5 ♦ When You Control for X_2, the X_1, Y Correlation Becomes Larger Than r_{1Y} or Becomes Opposite in Sign Relative to r_{1Y}

When $r_{Y1.2}$ is larger than r_{Y1} or when $r_{Y1.2}$ is opposite in sign relative to r_{1Y}, X_2 is described as a "suppressor" variable. In other words, the true strength or true sign of the X_1, Y association is "suppressed" by the X_2 variable; the true strength or sign of the X_1, Y correlation becomes apparent only when the variance associated with the X_2 suppressor variable is removed by partialling X_2 out. This corresponds to the regions marked Area *e* in Figure 10.15. In spite of the fact that Area *e* occupies a large amount of the space in

Table 10.3 ♦ Hypothetical Data for an Example of a Partly Mediated Relationship: Age, Weight, and Blood Pressure

Age	*Weight*	*Blood Pressure*
25	82	89
30	142	151
33	66	37
35	113	127
37	123	65
40	147	96
41	115	103
43	178	194
44	115	176
48	116	74
52	181	228
55	164	158
57	189	177
58	133	169
60	188	184
61	192	195
63	207	201
65	219	197
67	177	202
66	244	257
69	199	187
71	158	128
73	200	247
72	219	246
75	187	231
78	195	225
80	214	214
82	154	288
84	118	266
85	125	206

NOTE: $N = 30$ cases. Age in years, body weight in pounds, and SBP in millimeters of mercury.

Figure 10.15, suppression is a relatively uncommon outcome. The following sections describe two relatively simple forms of suppression. For a discussion of other forms that suppression can take, see Cohen, Cohen, West, and Aiken (2003).

10.12.5.1 ♦ *Suppression of Error Variance in a Predictor Variable*

Consider the following hypothetical situation. A researcher develops a paper-and-pencil test of "mountain survival skills." The score on this paper-and-pencil test is the X_1 predictor variable. The researcher wants to demonstrate that scores on this paper-and-pencil test (X_1) can predict performance in an actual mountain survival situation (the score for this survival test is the Y outcome variable). The researcher knows that, to some extent, performance on the paper-and-pencil test depends on the level of verbal ability (X_2). However, verbal ability is completely uncorrelated with performance in the actual mountain survival situation.

Correlations

		Age	Weight	Blood Pressure
Age	Pearson Correlation	1	.563**	.782**
	Sig. (2-tailed)		.001	.000
	N	30	30	30
Weight	Pearson Correlation	.563**	1	.672**
	Sig. (2-tailed)	.001		.000
	N	30	30	30
BloodPressure	Pearson Correlation	.782**	.672**	1
	Sig. (2-tailed)	.000	.000	
	N	30	30	30

**. Correlation is significant at the 0.01 level (2-tailed).

Figure 10.25 ♦ Zero-Order Correlations Among Age, Weight, and Blood Pressure (for the Hypothetical Data From Table 10.3)

Correlations

Control Variables			Age	Blood Pressure
Weight	Age	Correlation	1.000	.659
		Significance (2-tailed)	.	.000
		df	0	27
	BloodPressure	Correlation	.659	1.000
		Significance (2-tailed)	.000	.
		df	27	0

Figure 10.26 ♦ Partial Correlation Between Age and Blood Pressure, Controlling for Weight (for the Hypothetical Data From Table 10.3)

The hypothetical data in the file named papertest_suppress.sav show what can happen in this type of situation. The correlation between the score on the paper-and-pencil test (X_1) and actual mountain survival skills (Y) is $r = +.50$. The correlation between verbal ability (X_2) and actual mountain survival skills is not significantly different from 0 ($r = -.049$). We, thus, have a situation where the predictor variable of interest, X_1, is reasonably predictive of the outcome variable, Y. Note that the control variable, X_2 (verbal ability), is highly correlated with the predictor variable of interest, X_1 ($r_{12} = +.625$), but X_2 has a correlation close to 0 with the outcome or Y variable.

The partial correlation between X_1 and Y, controlling for X_2, is calculated (i.e., the partial correlation between performance on the paper-and-pencil test and actual mountain survival skills, controlling for verbal ability) and is found to be $r_{1Y.2} = +.75$. That is, when the variance in the paper-and-pencil test scores (X_1) due to verbal ability (X_2) is partialled out or removed, the remaining part of the X_1 scores become more predictive of actual mountain survival skills.

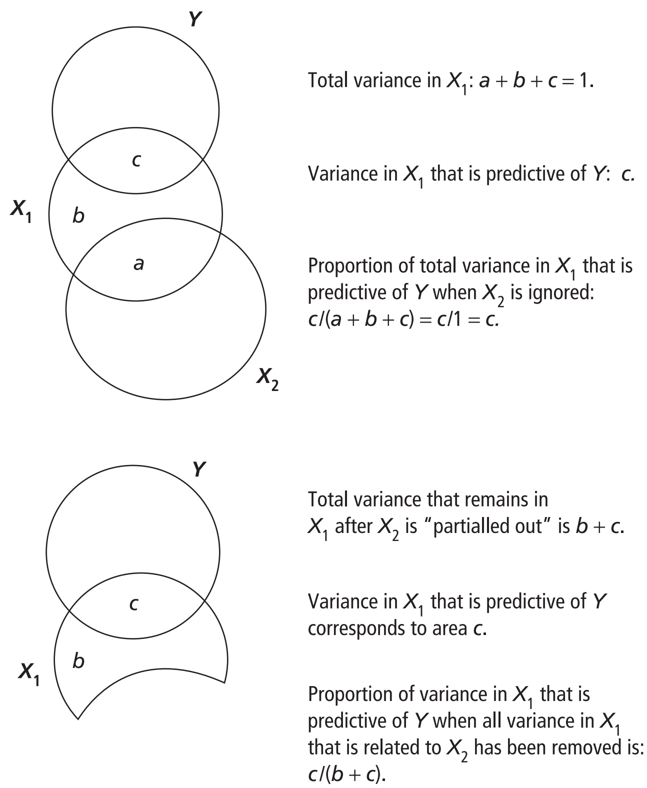


Figure 10.27 ♦ X_2 (a Measure of Verbal Ability) Is a Suppressor of Error Variance in the X_1 Predictor (a Paper-and-Pencil Test of Mountain Survival Skills); Y Is a Measure of Actual Mountain Survival Skills

The overlapping circle diagrams that appear in Figure 10.27 can help us understand what might happen in this situation. The top diagram shows that X_1 is correlated with Y and X_2 is correlated with X_1; however, X_2 is not correlated with Y (the circles that represent the variance of Y and the variance of X_2 do not overlap). If we ignore the X_2 variable, the squared correlation between X_1 and Y (r^2_{1Y}) corresponds to Area c in Figure 10.27. The total variance in X_1 is given by the sum of areas $a + b + c$. In these circle diagrams, the total area equals 1.00; therefore, the sum $a + b + c = 1$. The proportion of variance in X_1 that is predictive of Y (when we do not partial out the variance associated with X_2) is equivalent to $c/(a + b + c) = c/1 = c$.

When we statistically control for X_2, we remove all the variance that is predictable from X_2 from the X_1 variable, as shown in the bottom diagram in Figure 10.27. The second diagram shows that after the variance associated with X_2 is removed, the remaining variance in X_1 corresponds to the sum of areas $b + c$. The variance in X_1 that is predictive of Y corresponds to Area c. The proportion of the variance in X_1 that is predictive of Y after we partial out or remove the variance associated with X_2 now corresponds to $c/(b + c)$. Because $(b + c)$ is less than 1, the proportion of variance in X_1 that is associated with Y after removal of the variance associated with X_2 (i.e., $r^2_{Y1.2}$) is actually higher than the original proportion of variance in Y that was predictable from X_1 when X_2 was not controlled (i.e.,r^2_{Y1}). In this example, $r_{Y1.2} = +.75$, whereas $r_{Y1} = +.50$. In other words, when you control for verbal ability, the score on the paper-and-pencil test accounts for $.75^2 = .56$ or 56% of the variance in Y. When you do *not* control for verbal ability, the score on the paper-and-pencil test only predicted $.50^2 = .25$ or 25% of the variance in Y.

In this situation, the X_2 control variable suppresses irrelevant or **error variance** in the X_1 predictor variable. When X_2 is statistically controlled, X_1 is more predictive of Y than when X_2 is not statistically controlled. It is not common to find a suppressor variable that makes some other predictor variable a better predictor of Y in actual research. However, sometimes a researcher can identify a factor that influences scores on the X_1 predictor and that is not related to or predictive of the scores on the outcome variable Y. In this example, verbal ability was one factor that influenced scores on the paper-and-pencil test, but it was almost completely unrelated to actual mountain survival skills. Controlling for verbal ability (i.e., removing the variance associated with verbal ability from the scores on the paper-and-pencil test) made the paper-and-pencil test a better predictor of mountain survival skills.

10.12.5.2 ♦ *A Second Type of Suppression*

Another possible form of suppression occurs when the sign of $r_{Y1.2}$ is opposite to the sign of r_{Y1}. In the example we are going to discuss now, r_{1Y}, the zero-order correlation between crowding (X_1) and crime rate (Y_2) across neighborhoods, is large and positive. However, when you control for X_2 (level of neighborhood socioeconomic status or SES), the sign of the partial correlation between X_1 and Y, controlling for X_2, $r_{Y1.2}$, becomes negative. A hypothetical situation where this could occur is shown in Figure 10.28.

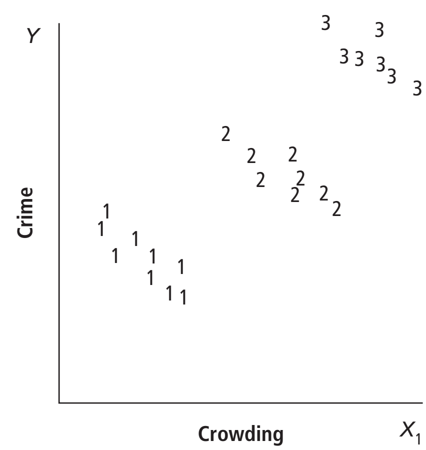


Figure 10.28 ♦ Example of One Type of Suppression

NOTE: On this graph, cases are marked by SES level of the neighborhood (1 = High SES, 2= Middle SES, 3 = Low SES). When SES is ignored, there is a large positive correlation between X_1 (neighborhood crowding) and Y (neighborhood crime). When the X_1, Y correlation is assessed separately within each level of SES, the relationship between X_1 and Y becomes negative. The X_2 variable (SES) suppresses the true relationship between X_1 (crowding) and Y (crime). Crowding and crime appear to be positively correlated when we ignore SES; when we statistically control for SES, it becomes clear that within SES levels, crowding and crime are actually negatively related.

In this hypothetical example, the unit of analysis or case is "neighborhood"; for each neighborhood, X_1 is a measure of crowding, Y is a measure of crime rate, and X_2 is a measure of income level (SES). X_2 (SES) is coded as follows: 1 = Upper class, 2 = Middle class, 3 = Lower class. The pattern in this graph represents the following hypothetical situation. This example was suggested by correlations reported by Freedman (1975), but it illustrates a much stronger form of suppression than Freedman found in his data. For the hypothetical data in Figure 10.28, if you ignore SES and obtain the zero-order correlation between crowding and crime, you would obtain a large positive correlation, suggesting that crowding predicts crime. However, there are two confounds present: Crowding tends to be greater in lower-SES neighborhoods (3 = Low SES), and the incidence of crime also tends to be greater in lower-SES neighborhoods.

Once you look separately at the plot of crime versus crowding within each SES category, however, the relationship becomes quite different. Within the lowest SES neighborhoods (SES code 3), crime is negatively associated with crowding (i.e., more crime takes place in "deserted" areas than in areas where there are many potential witnesses out on the streets). Freedman (1975) suggested that crowding, per se, does not "cause" crime; it just happens to be correlated with something else that is predictive of crime—namely, poverty or low SES. In fact, within neighborhoods matched in SES, Freedman reported that higher population density was predictive of *lower* crime rates.

Another example of this type of suppression, where the apparent direction of relationship reverses when a control variable is taken into account, was reported by Guber (1999). The unit of analysis in this study was each of the 50 states in the United States. The X_1 predictor variable was per-student annual state spending on education; the Y outcome variable was the mean Scholastic Aptitude Test (SAT) score for students in each state. The zero-order bivariate correlation, r_{Y1}, was large and negative; in other words, states that spent the largest amount of money per student on education actually tended to have lower SAT scores. However, there is a third variable that must be taken into account in this situation: the proportion of high school students in each state who take the SAT. When the proportion is low, it tends to mean that only the most capable students took the SAT and the mean SAT score tends to be high; on the other hand, in states where the proportion of students who take the SAT is high, the mean SAT score tends to be lower. When the proportion of high school students in each state who took the SAT (X_2) was statistically controlled, the partial correlation between state spending on education per pupil and mean SAT scores (controlling for X_2) became positive.

10.12.6 ♦ "None of the Above"

The interpretations described above provide an extensive, but not exhaustive, description of possible interpretations for partial correlation outcomes. A partial correlation can be misleading or difficult to interpret when assumptions such as linearity are violated or when the value of the partial r depends disproportionately on the outcomes for a few cases that have outlier scores.

A preliminary examination of scatter plots for the X_1, Y relationship separately within each value of X_2 can be helpful in evaluating whether correlation is or is not a good description of the association between X_1 and Y at each level of X_2. One possible outcome is that the slope or correlation for the X_1, Y relationship differs across levels of the X_2

control variable. If this occurs, the overall partial correlation $r_{Y1.2}$ can be misleading; a better way to describe this outcome would be to report different slopes or different correlations between X_1 and Y for each level of X_2. In this situation, we can say that there is an interaction between X_1 and X_2 as predictors of Y or that the X_2 variable moderates the predictive relationship between X_1 and Y. When a preliminary analysis of three-variable research situations suggests the presence of an interaction, data analyses that take interaction into account (as discussed in Chapters 12 and 13) are needed.

10.13 ♦ Mediation Versus Moderation

A classic paper by Baron and Kenny (1986) distinguished between mediating and moderating variables. X_2 is a *mediating* variable if it represents an intermediate step in a causal sequence, such that X_1 causes X_2 and then X_2 causes Y. Mediation should not be confused with **moderation**. X_2 is a *moderating* variable if the nature of the X_1, Y relationship is different for people who have different score values on the X_2 moderator variable; or, to say this in another way, X_2 is a moderator of the X_1, Y relationship if there is an interaction between X_1 and X_2 as predictors of Y. When moderation occurs, there are "different slopes for different folks"; the slope that predicts Y from X_1 differs depending on the score on the X_2 control variable. The analyses described in this chapter provide only preliminary information about the possible existence of mediation or moderation. If a preliminary examination of data suggests that mediation or moderation is present, then later analyses (such as multiple regression) need to take these issues into account.

10.13.1 ♦ Preliminary Analysis to Identify Possible Moderation

For example, suppose that X_2 corresponds to gender, coded as follows: 1 = Male, 2 = Female; X_1 is the need for power, and Y is job performance evaluations. Figure 10.29 shows hypothetical data for a sample of male and female managers. Each manager took a test to assess his or her level of need for power; this was the X_1 or predictor variable. After 6 months in a management position, each manager's employees completed self-report evaluations of the manager's job performance (Y). Let's suppose that back in the 1970s, employees responded more favorably to male managers who scored high on need for power but that employees responded less favorably to female managers who scored high on need for power. We can set up a scatter plot to show how evaluations of job performance (Y) are related to managers' need for power (X_1); different types of markers identify which scores belong to male managers (X_2 = m) and which scores belong to female managers (X_2 = f). Gender of the manager is the X_2 or "controlled for" variable in this example.

If we look only at the scores for male managers (each case for a male is denoted by "m" in Figure 10.29), there is a positive correlation between need for power (X_1) and job performance evaluations (Y). If we look only at the scores for female managers (denoted by "f" in Figure 10.29), there is a negative correlation between need for power (X_1) and job performance evaluations (Y). We can test whether the r_{1Y} correlation for the male subgroup and the r_{1Y} correlation for the female subgroups are significantly different, using significance tests from Chapter 7. Later chapters show how interaction terms can be included in regression analyses to provide different estimates of the slope for the prediction of Y from X_1 for subgroups defined by an X_2 control variable.

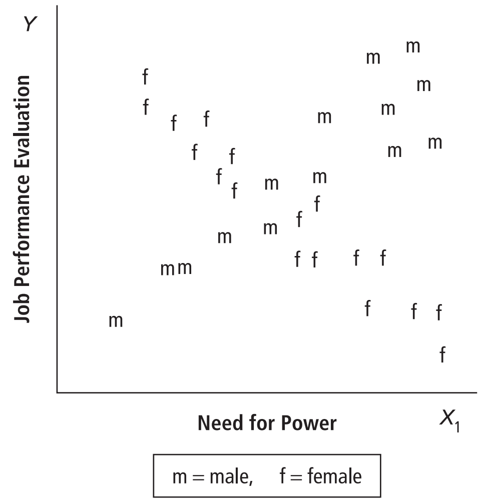


Figure 10.29 ♦ Interaction Between Gender (X_2) and Need for Power (X_1) as Predictors of Job Performance Evaluation (Y): For the Male Group, X_1 and Y Are Positively Correlated; Within the Female Group, X_1 and Y Are Negatively Correlated

In this example, we could say that gender and need for power interact as predictors of job performance evaluation; more specifically, for male managers, their job performance evaluations increase as their need for power scores increase, whereas for female managers, their job performance evaluations decrease as their need for power scores increase. We could also say that gender "moderates" the relationship between need for power and job performance evaluation.

In this example, the slopes for the subgroups (male and female) had opposite signs. Moderation or interaction effects do not have to be this extreme. We can say that gender moderates the effect of X_1 on Y if the slopes to predict Y from X_1 are significantly different for males and females. The slopes do not actually have to be opposite in sign for an interaction to be present. Another type of interaction occurs when there is no correlation between X_1 and Y for females and a strong positive correlation between X_1 and Y for males. Yet another kind of interaction is seen when the b slope coefficient to predict Y from X_1 is positive for both females and males but is significantly larger in magnitude for males than for females.

Note that partial correlations do not provide information about the existence of moderation or interaction, and in fact, the values of partial correlations can be quite misleading when an interaction or moderator variable is present. When we compute a partial correlation, it is essentially the mean of the within-group slopes between Y and X_1 when these slopes are calculated separately for each group of scores on the X_2 variable. If the slope to predict Y from X_1 differs across values of X_2, it is misleading to average these slopes together; to provide an accurate description of the relations among variables, you need to provide information about the way in which the X_1, Y relationship differs across levels or scores on the X_2 variable. For the scatter plot that appears in Figure 10.29, the zero-order correlation, r_{1Y}, between need for power and job performance evaluation, ignoring gender, would be close to 0. The partial r between need for power and job performance evaluation, controlling for gender, would be (approximately, not exactly) the mean of the r between these variables for men and the r between these variables for women; $r_{Y1.2}$ would also be close to 0. Neither the zero-order r nor the partial r would tell us that there is an interaction present. To detect the interaction, we need to look at a scatter plot or compute correlations between these variables separately for each gender (using the SPSS Split File command to examine male and female cases separately).

10.13.2 ♦ Preliminary Analysis to Detect Possible Mediation

One possible causal model that might be applicable in a three-variable research situation involves a mediated causal process. For example, consider a situation in which the X_1 variable first causes X_2, and then the X_2 variable causes Y. If the relationship between X_1 and Y is completely mediated by X_2, then the only path that leads from X_1 to Y is a path via X_2. If the relationship between X_1 and Y is only partly mediated by X_2, there may be two paths that lead from X_1 to Y. One path involves the indirect or **mediated relationship** via X_2, as shown above, but there is an additional direct path from X_1 to Y.

What evidence would be consistent with (but not proof of) a mediated causal process model? The first requirement for consideration of a mediated causal model is that it must be consistent with a theory that predicts that there could be a causal sequence (or at least with common sense), such that first X_1 causes X_2 and then X_2 causes Y. Temporal priority among variables should also be considered when asking whether a particular mediation model makes sense. Causes must precede effects in time. If we first measure X_1, then measure X_2, and then measure Y, the temporal order of measurements is consistent with a causal theory in which X_1 causes X_2, which then causes Y. Of course, obtaining measurements in this temporal sequence does not prove that the events actually occur in this temporal order. In many nonexperimental studies, researchers measure all three variables (X_1, X_2, and Y) at the same point in time. When all variables are measured at one point in time, it is difficult to rule out other possible sequences (such as Y causes X_2, which then causes X_1).

If a theory includes the hypothesis that X_2 mediates a causal sequence from X_1 to Y, then examining the partial correlation $r_{Y1.2}$ provides preliminary evidence as to whether any mediation that is present is complete or only partial. If $r_{Y1.2}$ is not significantly different from 0, it is possible that the X_1, Y relationship is completely mediated by X_2. On the other hand, if $r_{Y1.2} < r_{1Y}$, and $r_{Y1.2}$ is significantly different from 0, then perhaps the X_1, Y relationship is only partly mediated by X_2. However, several additional conditions should be satisfied before a researcher interprets an outcome as evidence of a mediated causal process (Baron & Kenny, 1986). Chapter 11 in this textbook will demonstrate statistical procedures (including a regression equation in which Y is predicted from both X_1 and X_2) to evaluate whether these additional conditions are satisfied. However, correlation analysis of data obtained in nonexperimental research situations does not provide a strong basis for making causal inferences. If a researcher has a hypothesis about a mediated causal sequence, there may be ways to test this model using experimental methods.

10.13.3 ♦ Experimental Tests for Mediation Models

The best evidence to support a mediated causal model would come from an experiment in which the researcher can manipulate the presumed causal variable X_1, control the mediating variable X_2 (either by experimental means or statistically), control other variables that might influence the variables in the study, and measure the outcome variable Y. For example, Levine, Gordon, and Fields (1978) theorized that placebo pain relief is mediated by the release of endorphins (naturally occurring chemicals in the body that mimic the effects of narcotic drugs). They tested this mediated process model experimentally. The pain induction involved medically necessary dental surgery. After surgery, half the patients received an intravenous dose of naloxone (a drug that blocks the pain-relieving effects of both artificial narcotic drugs and endogenous opioids). The other half of the patients received an intravenous dose of an inert substance. Subsequently, all patients were given a placebo pain reliever that they were told should reduce their pain. Patients who had received naloxone (which blocks the pain-relieving effects of opiates and endorphins) did not report any pain relief from the placebo injection. In contrast, many of the patients who had not received naloxone reported substantial pain relief in response to the placebo. The theoretical model was as follows: Administration of a placebo (X_1) causes the release of endorphins (X_2) in at least some patients; the release of endorphins (X_2) causes a reduction in pain (Y). Note that in most studies, only about 30% to 35% of patients who receive placebos experience pain relief.

Administration of placebo → Release of endorphins → Pain relief

By administering naloxone to one group of patients, these investigators were able to show that placebos had no effect on pain when the effects of endorphins were blocked. This outcome was consistent with the proposed mediated model. It is possible that the effects of placebos on pain are mediated by the release of endorphins.

10.14 ♦ Model Results

The first research example introduced early in the chapter examined whether height (X_1) and vocabulary (Y) are related when grade level (X_2) is statistically controlled. The results presented in Figures 10.3 through 10.14 can be summarized briefly.

Results

The relation between height and vocabulary score was assessed for $N = 48$ students in three different grades in school: Grade 1, Grade 5, and Grade 9. The zero-order Pearson r between height and vocabulary was statistically significant: $r(46) = .72$, $p < .001$, two-tailed. A scatter plot of vocabulary scores by height (with individual points labeled by grade level) suggested that both vocabulary and height tended to increase with grade level (see Figure 10.3). It seemed likely that the correlation between vocabulary and height was spurious—that is, entirely attributable to the tendency of both these variables to increase with grade level.

To assess this possibility, the relation between vocabulary and height was assessed controlling for grade. Grade was controlled for in two different ways. A first-order partial correlation was computed for vocabulary and height, controlling for grade. This partial r was not statistically significant: $r(45) = -.01$, $p = .938$. In addition, the correlation between Height and Vocabulary was computed separately for each of the three grade levels. For Grade = 1, $r = .067$; for Grade = 5, $r = .031$; for Grade = 9, $r = -.141$. None of these correlations was statistically significant, and the differences among these three correlations were not large enough to suggest the presence of an interaction effect (i.e., there was no evidence that the nature of the relationship between vocabulary and height differed substantially across grades).

When grade was controlled for, either by partial correlation or by computing Pearson r separately for each grade level, the correlation between vocabulary and height became very small and was not statistically significant. This is consistent with the explanation that the original correlation was spurious. Vocabulary and height are correlated only because both variables increase across grade levels (and not because of any direct causal or noncausal association between height and vocabulary).

10.15 ♦ Summary

Partial correlation can be used to provide preliminary exploratory information that may help the researcher understand relations among variables. In this chapter, we have seen that when we take a third variable, X_2, into account, our understanding of the nature and strength of the association between X_1 and Y can change in several different ways.

This chapter outlines two methods to evaluate how taking X_2 into account as a control variable may modify our understanding of the way in which an X_1 predictor variable is related to a Y outcome variable. The first method involved dividing the dataset into separate groups, based on scores on the X_2 control variable (using the Split File command in SPSS), and then examining scatter plots and correlations between X_1 and Y separately for each group. In the examples in this chapter, the X_2 control variables had a small number of possible score values (e.g., when gender was used as a control variable, it had just two values, male and female; when grade level in school and SES were used as control variables, they had just three score values). The number of score values on X_2 variables was kept small in these examples to make it easy to understand the examples. However, the methods outlined here are applicable in situations where the X_2 variable has a larger number of possible score values, as long as the assumptions for Pearson correlation and partial correlation are reasonably well met. Note, however, that if the X_2 variable has 40 possible different score values, and the total number of cases in a dataset is only $N = 50$, it is quite likely that when any one score is selected (e.g., $X_2 = 33$), there may be only one or two cases with that value of X_2. When the ns within groups based on the value of X_2 become very small, it becomes impossible to evaluate assumptions such as linearity and normality within the subgroups, and estimates of the strength of association between X_1 and Y that are based on extremely small groups are not likely to be very reliable. The minimum sample sizes that were suggested for Pearson correlation and bivariate regression were on the order of $N = 100$. Sample sizes should be even larger for studies where an X_2 control variable is taken into account, particularly in situations where the researcher suspects the presence of an interaction or moderating variable; in these situations, the researcher needs to estimate a different slope to predict Y from X_1 for each score value of X_2.

We can use partial correlation to statistically control for an X_2 variable that may be involved in the association between X_1 and Y as a rival explanatory variable, a confound, a mediator, a suppressor, or in some other role. However, statistical control is generally a less effective method for dealing with extraneous variables than experimental control. Some methods of experimental control (such as random assignment of participants to treatment groups) are, at least in principle, able to make the groups equivalent with respect to hundreds of different participant characteristic variables. However, when we measure and statistically control for one specific X_2 variable in a nonexperimental study, we have controlled for only one of many possible rival explanatory variables. In a nonexperimental study, there may be dozens or hundreds of other variables that are relevant to the research question and whose influence is not under the researcher's control; when we use partial correlation and similar methods of statistical control, we are able to control statistically for only a few of these variables.

The next three chapters continue to examine the three-variable research situation, but they do so using slightly different approaches. In Chapter 11, X_1 and X_2 are both used as predictors in a multiple regression to predict Y. Chapter 12 discusses the use of dummy-coded or dichotomous variables as predictors in regression and demonstrates the use of product terms in regression to represent interaction or moderation. Chapter 13 reviews two-way factorial ANOVA; this is a version of the three-variable situation where both the X_1 and X_2 predictor variables are categorical and the outcome variable Y is quantitative. From Chapter 14, we begin to consider multivariate analyses that may include more than three variables. Multivariate analyses can include multiple predictors and/or multiple outcomes and/or multiple control variables or **covariates**.

In this chapter, many questions were presented in the context of a three-variable research situation. For example, is X_1 confounded with X_2 as a predictor? When you control for X_2, does the partial correlation between X_1 and Y drop to 0? In multivariate analyses, we often take several additional variables into account when we assess each X_1, Y predictive relationship. However, the same issues that were introduced here in the context of three-variable research situations continue to be relevant for studies that include more than three variables.

Note

1. This assumes that Pearson r is an appropriate statistical analysis to describe the strength of the relationship between X_1 and Y; if the assumptions required for Pearson r are violated; for example, if the relation between X_1 and Y is nonlinear, then Pearson r is not an appropriate analysis to describe the strength of the association between these variables. In this case, data transformations may be applied to scores on one or both variables to make the relationship between them more linear, or an entirely different statistical analysis may be required to describe the strength of the association between X_1 and Y.

Comprehension Questions

1. When we assess X_1 as a predictor of Y, there are several ways in which we can add a third variable (X_2) and several "stories" that may describe the relations among variables. Explain what information can be obtained from the following two analyses:
   I. Assess the X_1, Y relation separately for each group on the X_2 variable.
   II. Obtain the partial correlation (partial r of Y with X_1, controlling for X_2).
   a. Which of these analyses (I or II) makes it possible to detect an interaction between X_1 and X_2? Which analysis assumes that there is no interaction?
   b. If there is an interaction between X_1 and X_2 as predictors of Y, what pattern would you see in the scatter plots in Analysis I?

2. Discuss each of the following as a means of illustrating the partial correlation between X_1 and Y, controlling for X_2. What can each analysis tell you about the strength and the nature of this relationship?
   I. Scatter plots showing Y versus X_1 (with X_2 scores marked in plot)
   II. Partial r as the correlation between the residuals when X_1 and Y are predicted from X_2

3. Explain how you might interpret the following outcomes for partial r:
   a. $r_{1Y} = .70$ and $r_{1Y.2} = .69$
   b. $r_{1Y} = .70$ and $r_{1Y.2} = .02$
   c. $r_{1Y} = .70$ and $r_{1Y.2} = -.54$
   d. $r_{1Y} = .70$ and $r_{1Y.2} = .48$

4. What does the term *partial* mean when it is used in connection with correlations?

5. Suppose you correlate age with SBP and find a strong positive correlation. Compute a first-order partial correlation between age and SBP, controlling for weight (body weight).
   a. What would you conclude if this partial r were almost exactly 0?
   b. What would you conclude if this partial r were a little smaller than the zero-order r between age and SBP but still substantially larger than 0?
   c. What would you conclude if the zero-order r and the partial r for these variables were essentially identical?

6. Suppose you want to predict job performance by a police officer (Y) from scores on the Police Skills Inventory, a paper-and-pencil test (X_1). You also have a measure of general verbal ability (X_2). Is it possible that $r_{1Y.2}$ could actually be larger than r_{1Y}? Give reasons for your answer.

7. Give a plausible example of a three-variable research problem in which partial correlation would be a useful analysis; make sure that you indicate which of your three variables is the "controlled for" variable. What results might you expect to obtain for this partial correlation, and how would you interpret your results?

8. Describe two (of the many possible) problems that would make a partial correlation "nonsense" or uninterpretable.

9. Choose three variables. The predictor and outcome variables should both be quantitative. The control variable for this exercise should be dichotomous (although, in general, the control variable can be quantitative, that type of variable doesn't lend itself to the analysis of separate groups). For this set of three variables, do the following. Run Pearson *r*s among all three variables. Make a scatter plot of scores on the predictor and outcome variables (with cases labeled by group membership on the control variable). Determine the partial correlation.