

Hello!  
Here is some math.

$$\alpha\beta\gamma\delta\Gamma\Delta\Lambda\Theta abcdABCD$$

Now here are some symbols.

$$\int_{-\infty}^{\infty} \sin \theta = \sqrt{\frac{e^{i\pi}}{\sum_{i=0} \epsilon \Gamma \Lambda \cdot i}}$$

Here is a compatibility test. Lowercase letters:

$$\alpha a \beta b \gamma y \delta d \epsilon e \epsilon \eta \theta \iota \kappa \lambda \ell \mu \nu \rho \sigma \tau \upsilon \phi \chi \omega \psi$$

Uppercase letters:

$$\Gamma F \Delta A \Theta O \Lambda T \Xi E \Sigma X \Upsilon Y O \Phi I \Psi U \Omega O$$

Large symbols, text-size:

$$[(\langle \{ \int^S \Sigma^E \} \rangle)]$$

Display style:

$$\left[ \left( \left\langle \left\{ \int^S \Sigma^E \right\} \right\rangle \right) \right]$$

Common math symbols:

$$a + \frac{2}{\pi} \neq 15 \implies A \in \Pi, \forall A \approx \nabla.$$

## Unbound Orbits: Deflection of Light by the Sun

Consider a particle or photon approaching the sun from very great distances. At infinity the metric is Minkowskian, that is,  $A(\infty) = B(\infty) = 1$ , and we expect motion on a straight line at constant velocity  $V$

$$\begin{aligned} b &\simeq r \sin(\phi - \phi_\infty) \simeq r(\phi - \phi_\infty) \\ -V &\simeq \frac{d}{dt}(r \cos(\phi - \phi_\infty)) \simeq \frac{dr}{dt} \end{aligned}$$

where  $b$  is the “impact parameter” and  $\phi_\infty$  is the incident direction. We see that they do satisfy the equations of motion at infinity, where  $A = B = 1$ , and that the constants of motion are:

$$J = bV^2 \quad (1)$$

$$E = 1 - V^2 \quad (2)$$

(Of course a photon has  $V = 1$ , and as we have already seen, this gives  $E = 0$ .) It is often more convenient to express  $J$  in terms of the distance  $r_0$  of closest approach to the sun, rather than the impact parameter  $b$ . At  $r_0$ ,  $dr/d\phi$  vanishes, so our earlier equations give:

$$J = r_0 \left( \frac{1}{B(r_0)} - 1 + V^2 \right)^{1/2}$$

The orbit is then described by

$$\phi(r) = \phi_\infty + \int_r^\infty \left\{ \frac{A^{\frac{1}{2}}(r) dr}{r^2 \left( \frac{1}{r_0^2} \left[ \frac{1}{B(r)-1+V^2} \right] \left[ \frac{1}{B(r)-1+V^2} \right]^{-1} - \frac{1}{r^2} \right)^{\frac{1}{2}}} \right\}.$$

The total change in  $\phi$  as  $r$  decreases from infinity to its minimum value  $r_0$  and then increases again to infinity is just twice its change from  $\infty$  to  $r_0$ , that is,  $2|\phi(r_0) - \phi'_\infty|$ . If the trajectory were a straight line, this would equal just  $\pi$ ;

$$\Delta\phi = 2|\phi(r_0) - \phi_\infty| - \pi.$$

If this is positive, then the angle  $\phi$  changes by more than  $180^\circ$ , that is, the trajectory is bent *toward* the sun; if  $\Delta\phi$  is negative then the trajectory is bent away from the sun.

The end.