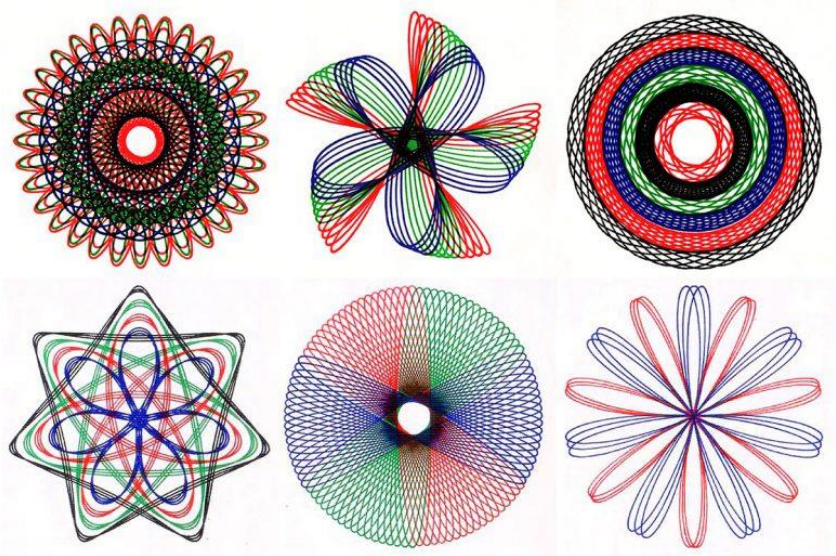
Project 1 - Spirographs

Project Goals

- Code a function to return lists of x, y points, for a given functional relation x(t), y(t) that generates a spirograph

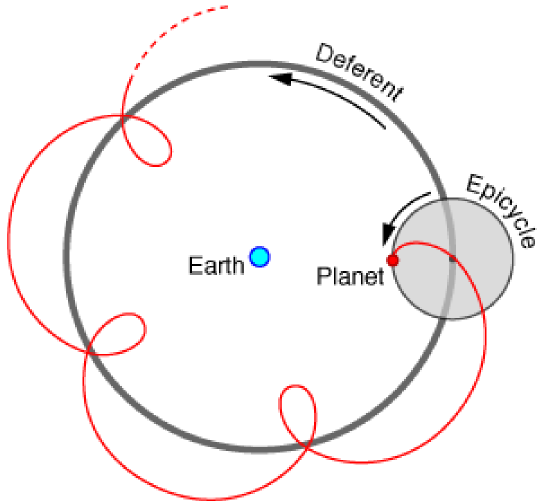
  For example: $x(t) = t$, $y(t) = \sin(t)$ will describe a sine wave



- Use `matplotlib.pyplot` to plot the curve described by the lists of x,y
- Explore different parameters to find 4 nice spirographs
- Arrange the plots in a single figure as a 2x2 array

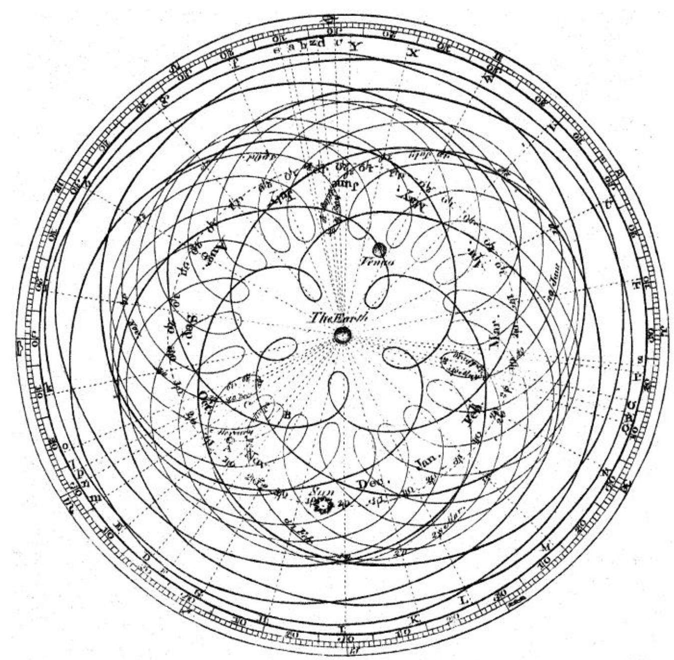
Background

The Geocentric model (ca. 500 BC) assumed a spherical earth, immobile and at the center of the solar system



Plato & Aristotle's geocentric model assumed circular orbits to explain the recurrent motions of planets

Their models did not explain apparent changes in distance of the planets or that they sometimes appear to move backwards (*retrograde motion*)



Ptolemy (140 CE) added epicycles – circles on circles to make charts of planetary motion that were used for 1500 years.

Epitrochoid

Epitrochoid:

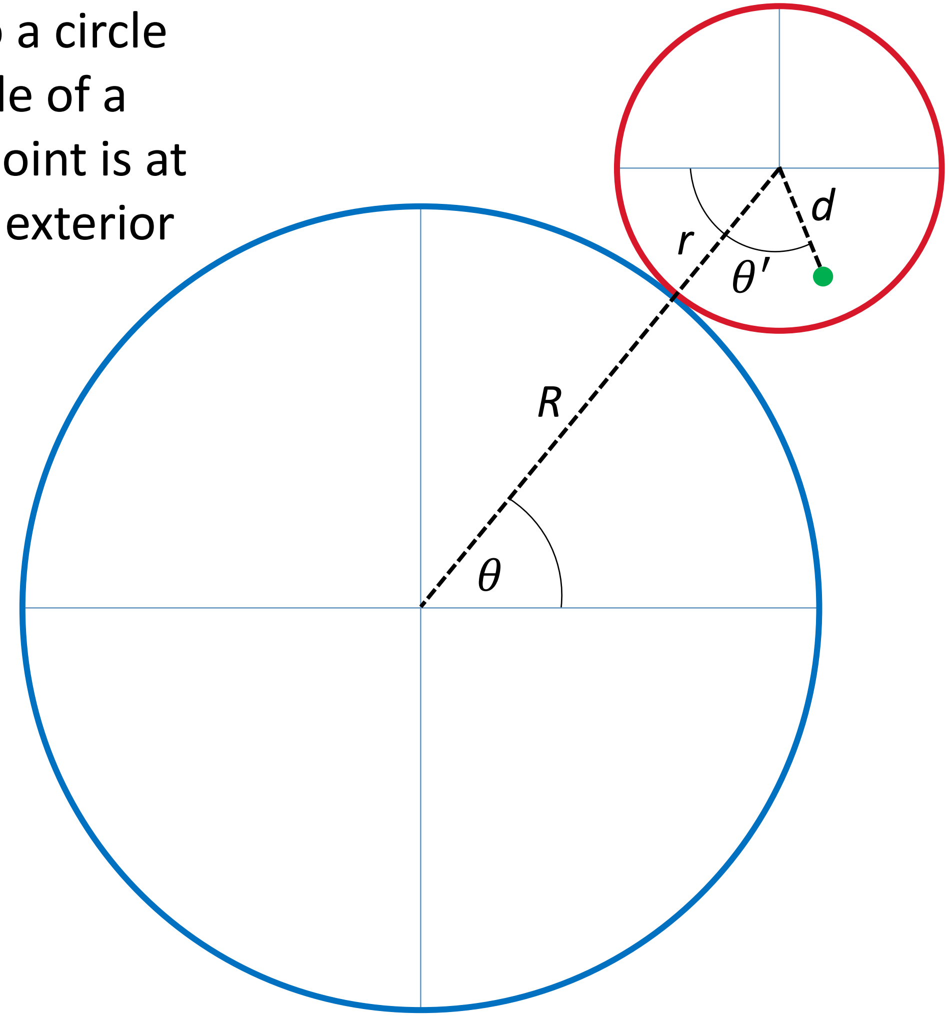
Curve traced by a point attached to a circle of radius *r* rolling around the outside of a fixed circle of radius *R*, where the point is at a distance *d* from the center of the exterior circle.

Parametric Equations:

$$x(\theta) = (R + r)\cos\theta - d\cos\theta'$$

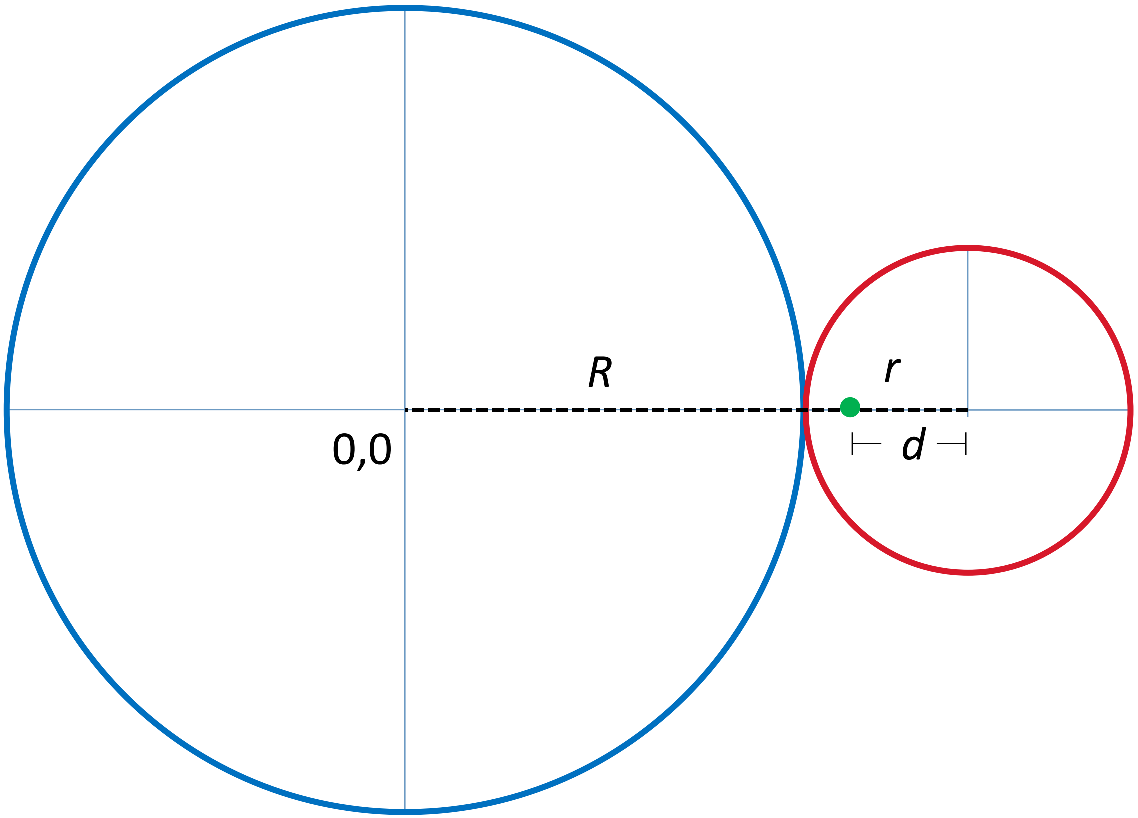
$$y(\theta) = (R + r)\sin\theta - d\sin\theta'$$

$$\theta' = \frac{R + r}{r}\theta$$



Project 1 – Derivation

For $\boldsymbol{\theta = 0}$:



$x(0) =$

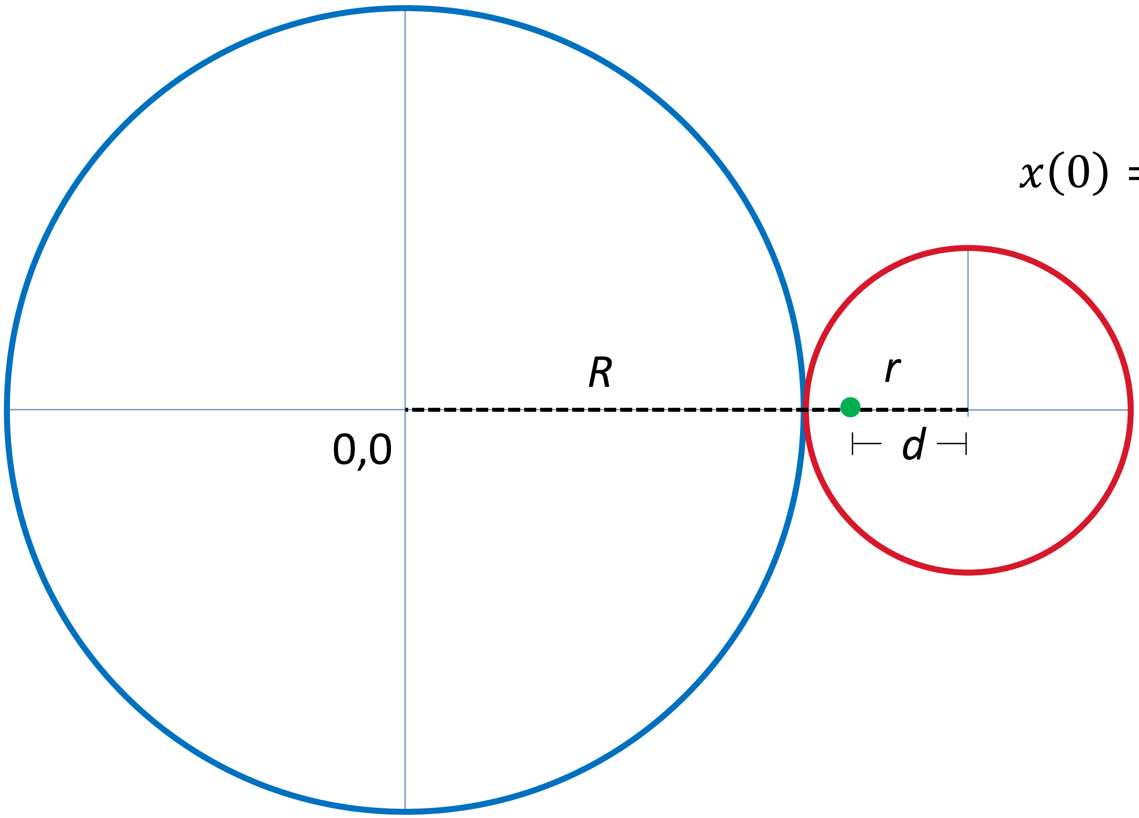
Project 1 – Derivation

For $\boldsymbol{\theta = 0}$:

Center of small circle

Center of small circle to point

$$x(0) = (R + r) - d$$



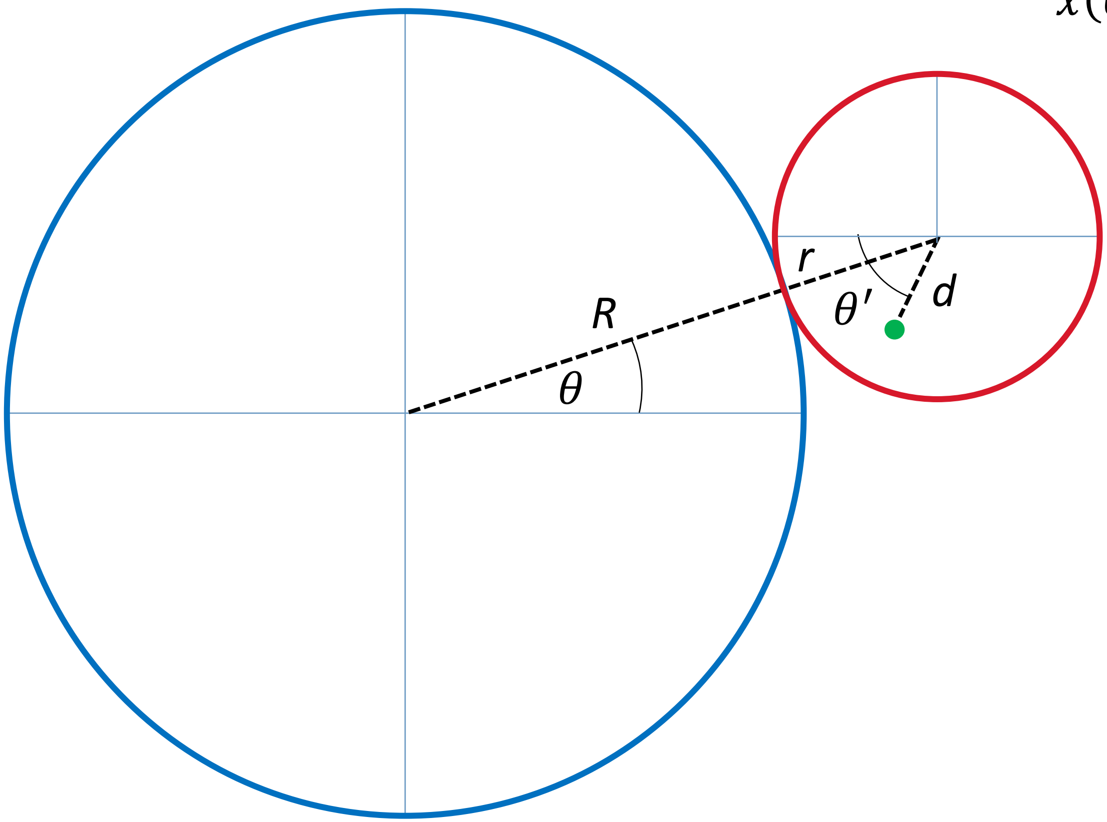
Project 1 – Derivation

For $\boldsymbol{\theta}$:

Center of small circle

Center of small circle to point

$$x(\theta) =$$



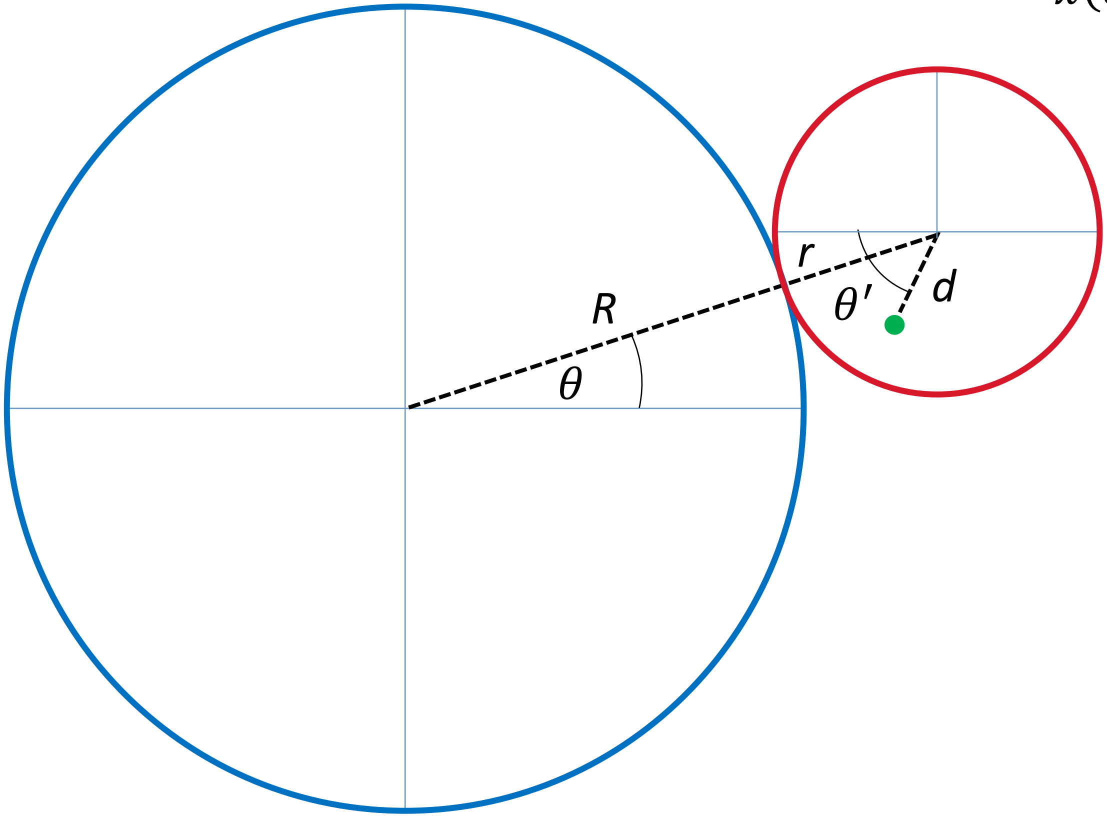
Project 1 – Derivation

For θ:

Center of small circle

Center of small circle to point

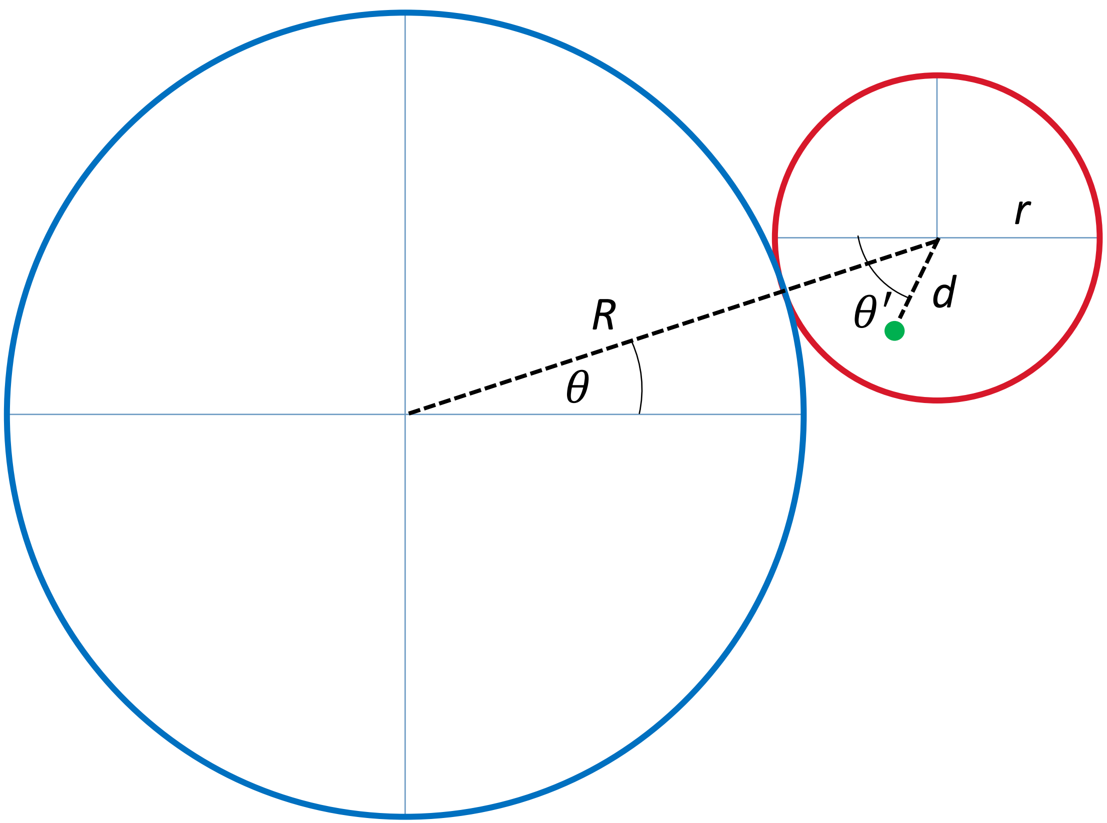
$$x(\theta) = (R + r)\cos\theta - d\cos\theta'$$



Project 1 – Derivation

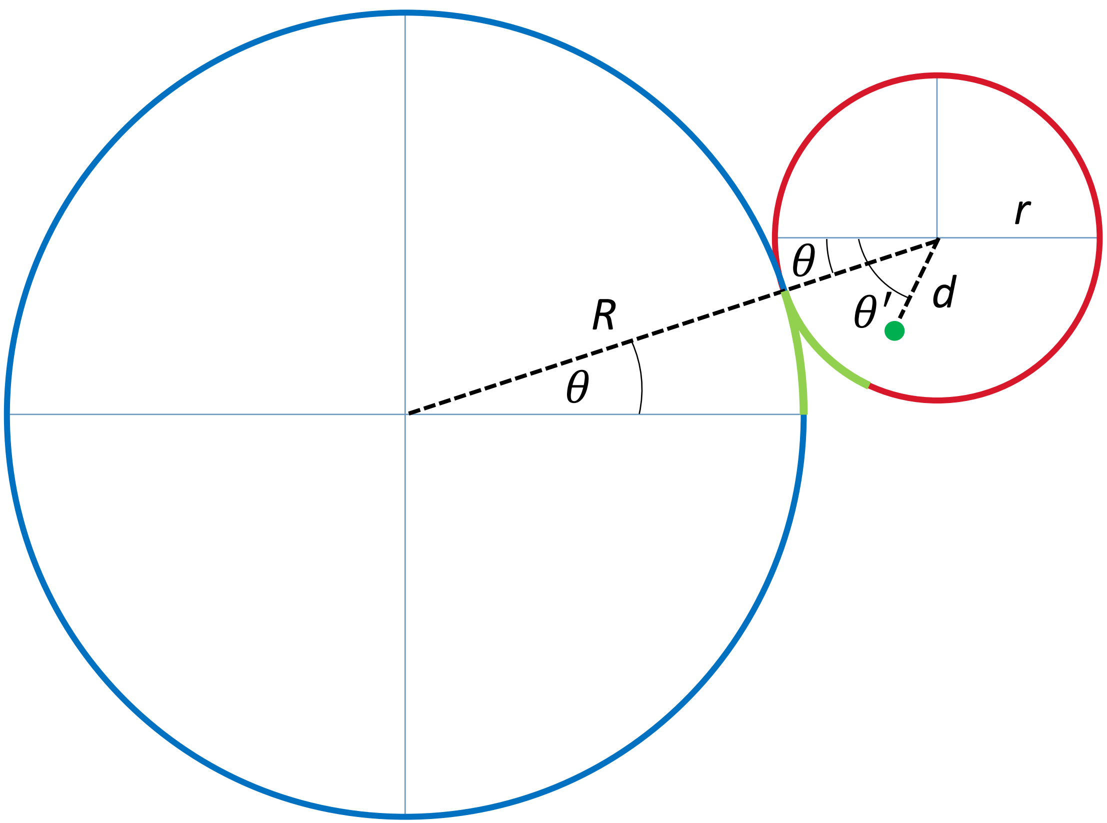
For $\boldsymbol{\theta}$:

Relate θ to θ':



Project 1 – Derivation

For $\boldsymbol{\theta}$:



Relate θ to θ':

Equal arclength (no slip):

$$\theta R = (\theta' - \theta) r$$

$$\frac{\theta(R + r)}{r} = \theta'$$

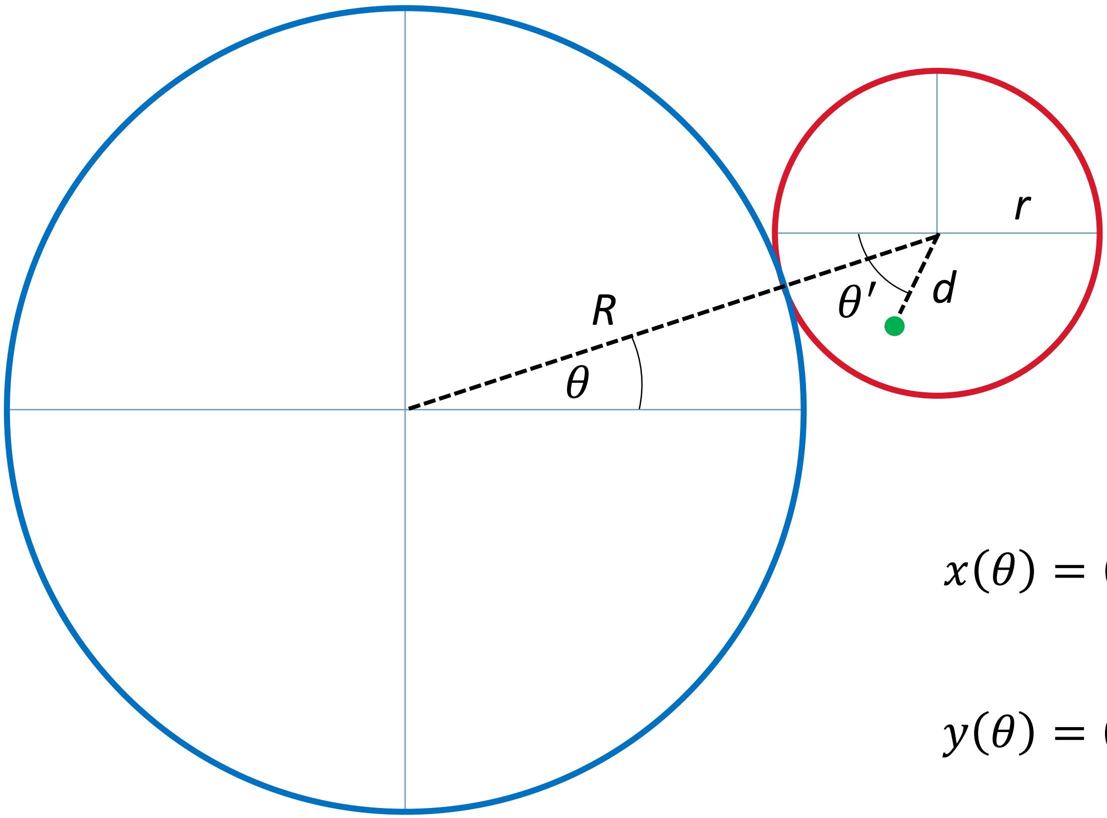
Project 1 – Derivation

Repeat for y:

$$x(\theta) = (R + r)\cos\theta - d\cos\theta'$$

$$y(\theta) = (R + r)\sin\theta - d\sin\theta'$$

$$\theta' = \frac{R + r}{r}\theta$$

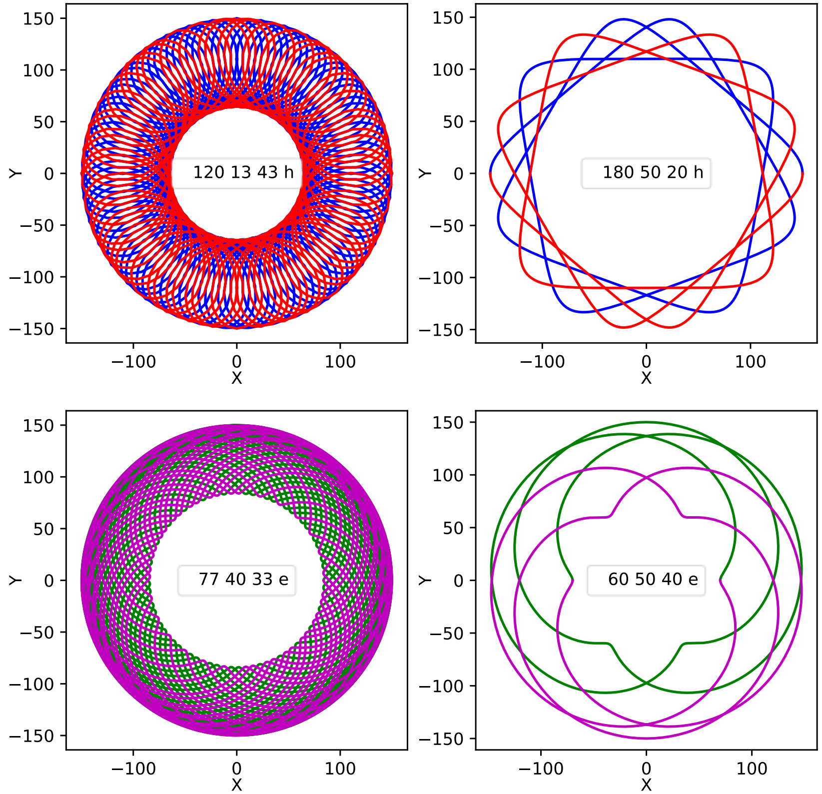


or

$$x(\theta) = (R + r)\cos\theta - d\cos\frac{R + r}{r}\theta$$

$$y(\theta) = (R + r)\sin\theta - d\sin\frac{R + r}{r}\theta$$

Project 1 - Example



4 plots (2x2) in 1 Figure

Square frame

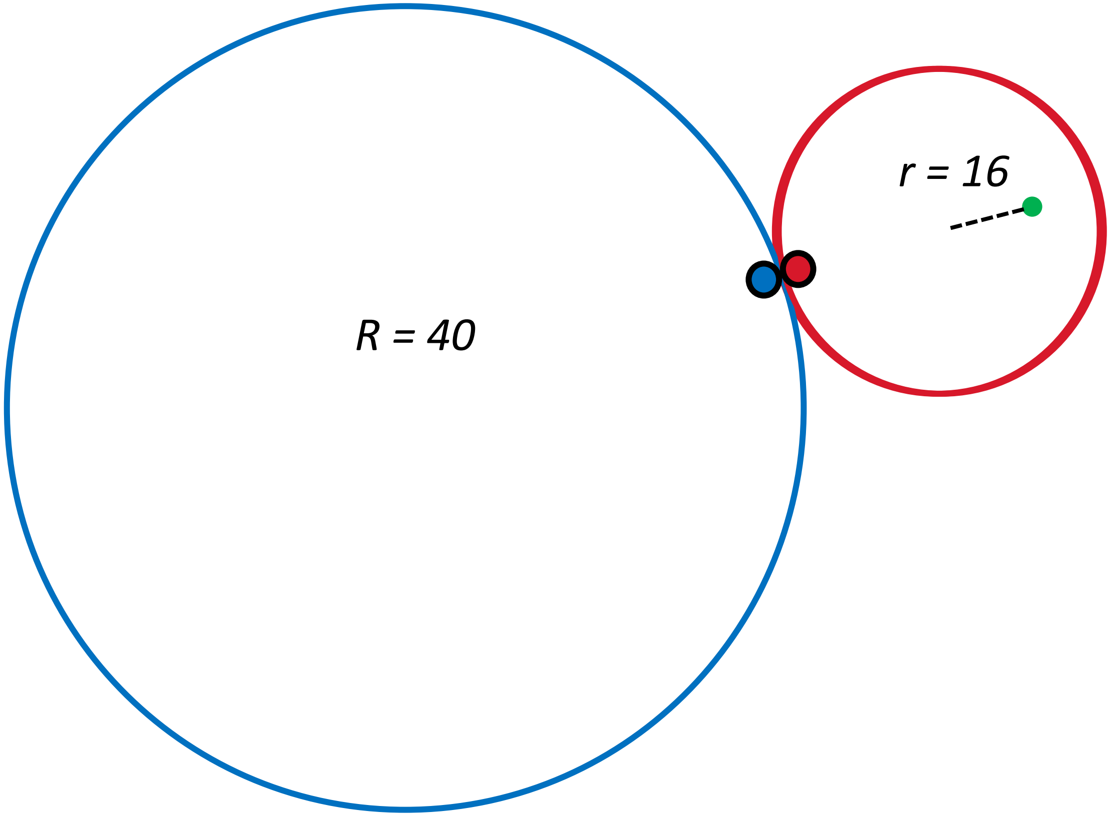
Axis labels and scales

Complete cycles

Different colors

Project 1 – How many repeating cycles?

$$\theta = 0$$



Project 1 – How many repeating cycles?

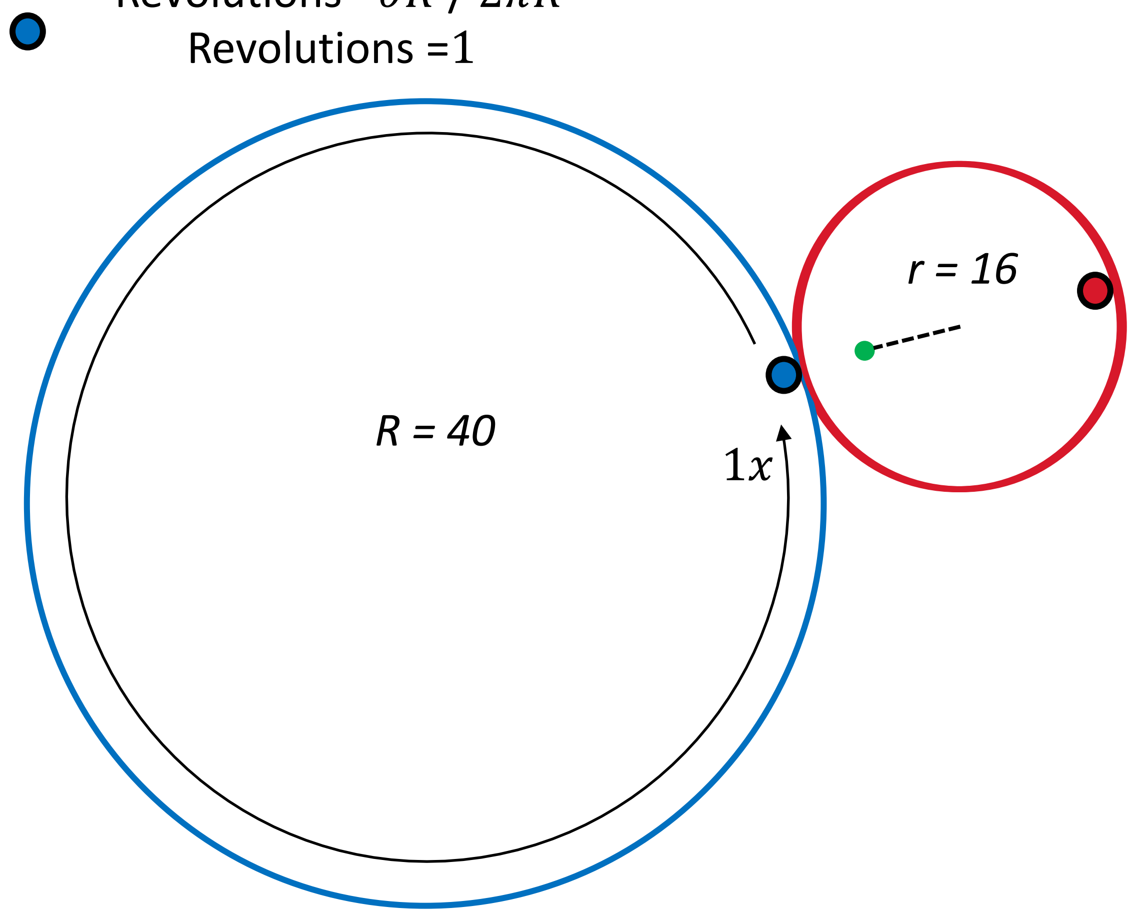
Revolutions = Distance / Circumference

Revolutions = $(\boldsymbol{\theta} \times \boldsymbol{Radius})$ **/** $(\boldsymbol{2\pi} \times \boldsymbol{Radius})$

$$\theta = 2\pi$$

Revolutions $=\theta R \;/\; 2\pi R$

Revolutions =1



Project 1 – How many repeating cycles?

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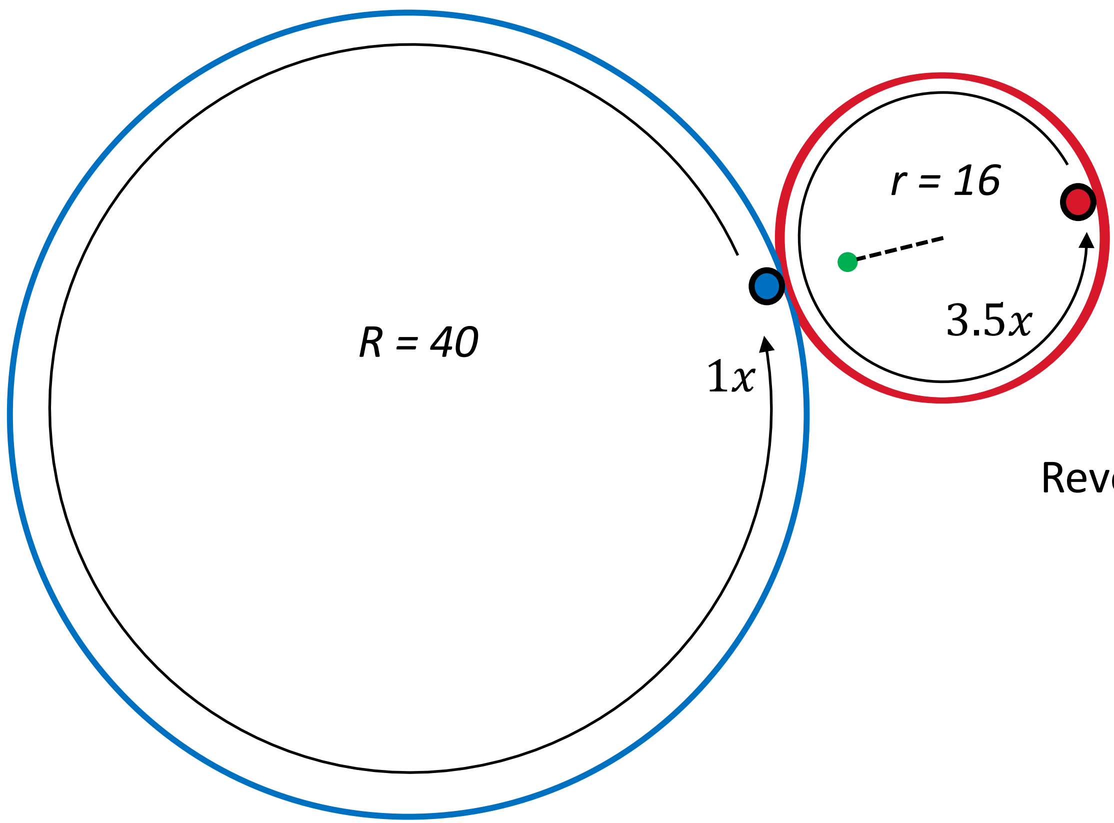
Revolutions $=1$

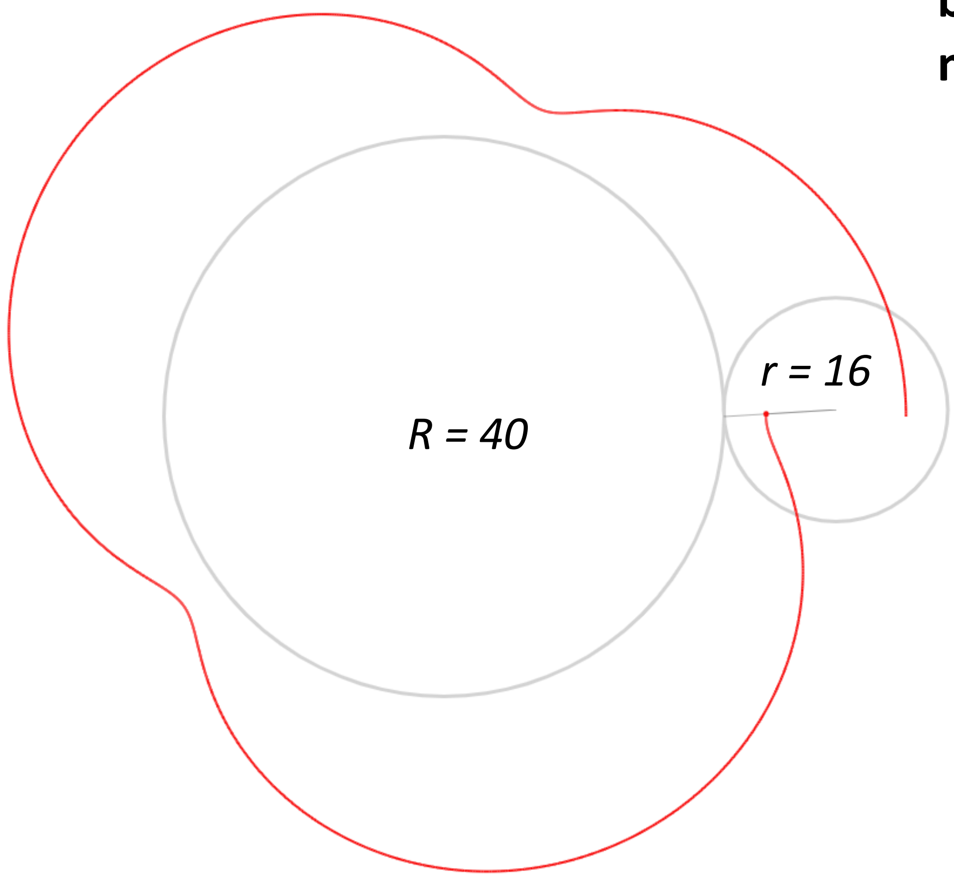


Revolutions $= \theta' r \;/\; 2\pi r$

$$\theta \frac{R+r}{r} r \;/\; 2\pi r$$

Revolutions = 3.5

Project 1 – How many repeating cycles



Spirograph is incomplete unless both circles travel an integral multiple number of rotations!

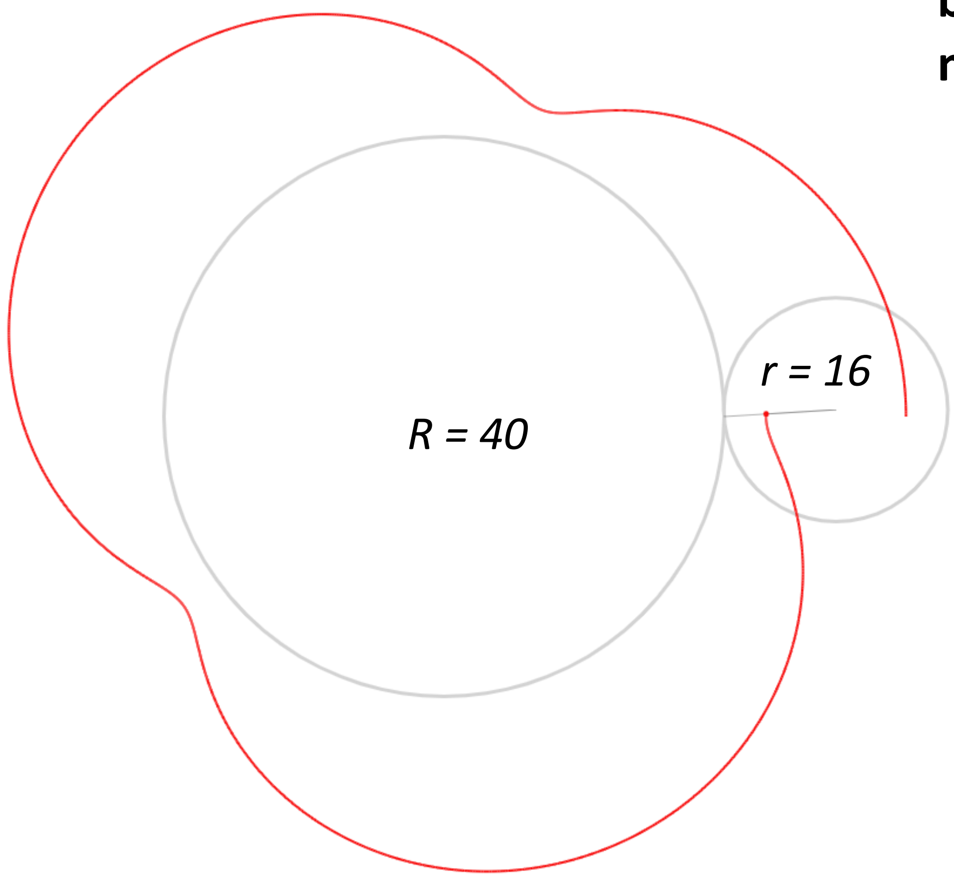
$$n2\pi R = n'2\pi r$$

$$nR = n'r$$

Find least common multiple

$$n40 = n'16 =$$

Project 1 – How many repeating cycles



Spirograph is incomplete unless both circles travel an integral multiple number of rotations!

$$n2\pi R = n'2\pi r$$

$$nR = n'r$$

Find least common multiple

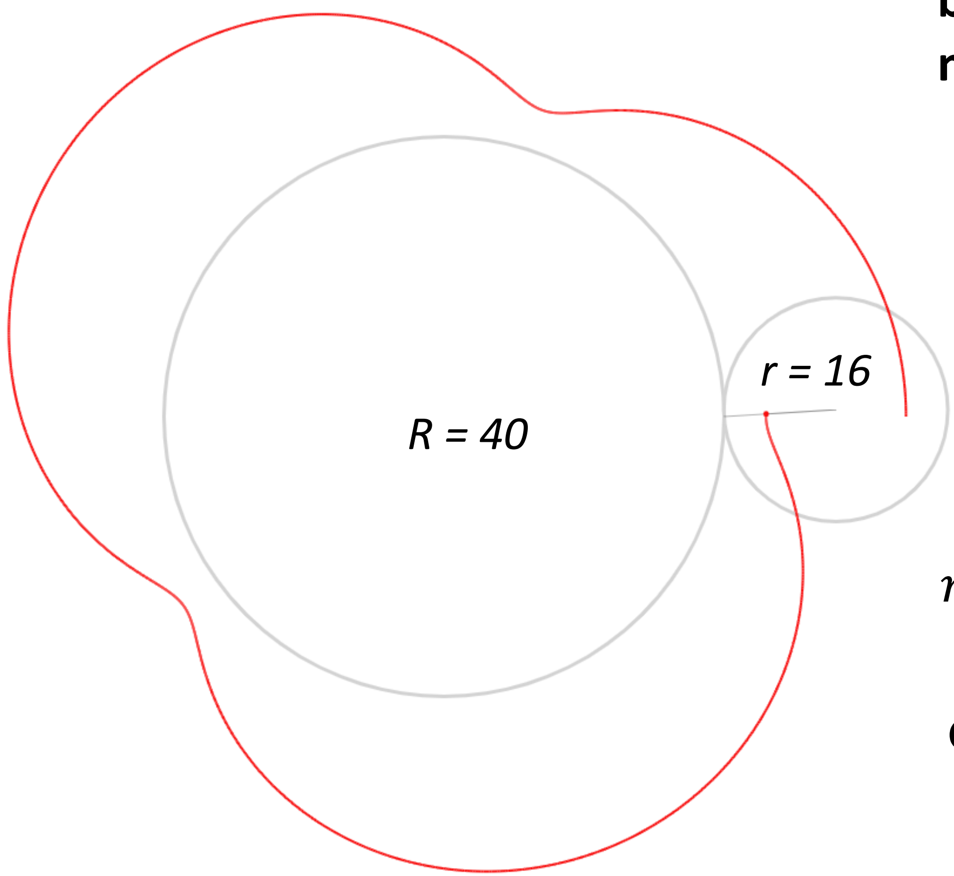
$$n40 = n'16 = 40 \times 16 = 640$$

This is not least common multiple

$$n40 = 640$$
$$n = 16$$

$$n'16 = 640$$
$$n' = 40$$

Project 1 – How many repeating cycles



Spirograph is incomplete unless both circles travel an integral multiple number of rotations!

$$n2\pi R = n'2\pi r$$

$$nR = n'r$$

Find least common multiple

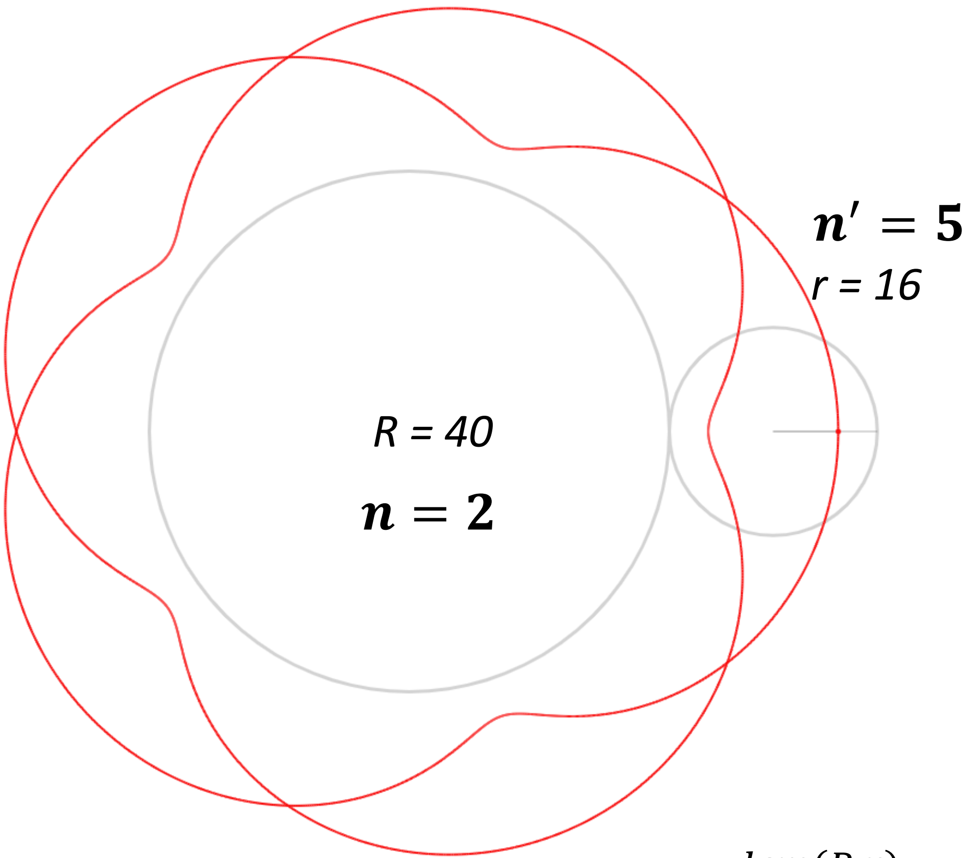
$$n40 = n'16 = \frac{40\times 16}{\mathrm{gcd}(40,16)} = \frac{40\times 16}{8} = 80$$

Greatest common factor

$$n40 = 80$$
$$n = 2$$

$$n'16 = 80$$
$$n' = 5$$

Project 1 – How many repeating cycles



For complete revolution, plot for:

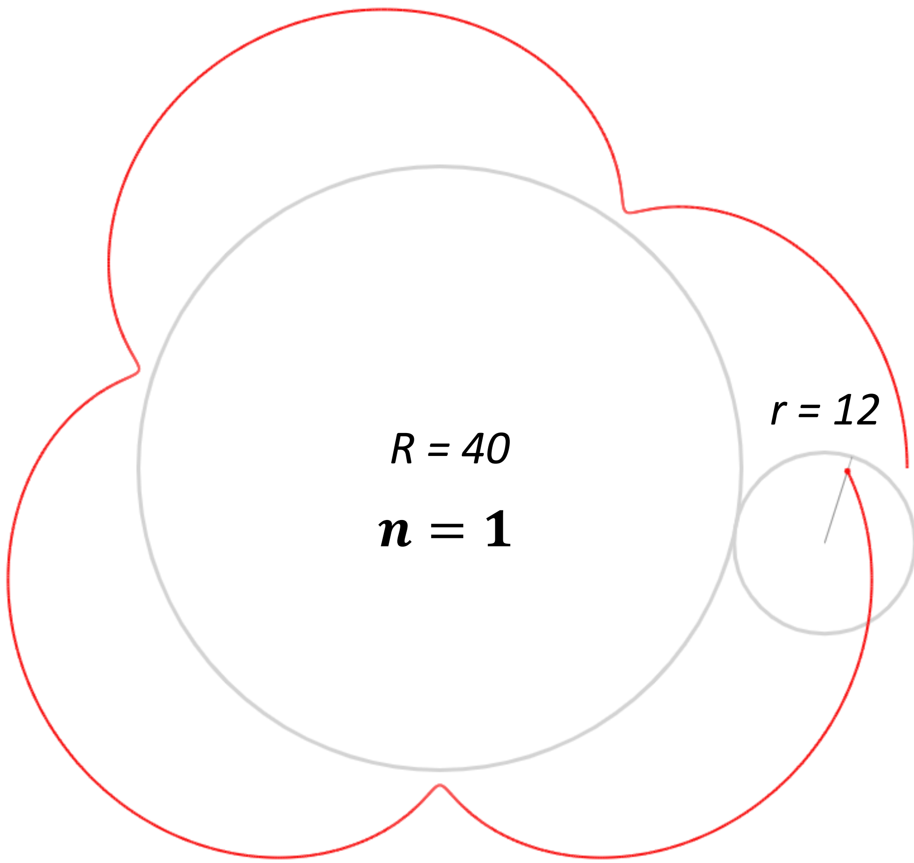
$$\theta = 4\pi$$

$$\frac{R \times r}{\gcd(R, r)} = lcm(R, r)$$

$$\frac{40 \times 16}{\gcd(40{,}16)} = lcm(40{,}16) = 80$$

$$n = \frac{lcm(R{,}r)}{R} = \frac{80}{40} = 2 \qquad n' = \frac{lcm(R{,}r)}{r} = \frac{80}{16} = 5$$

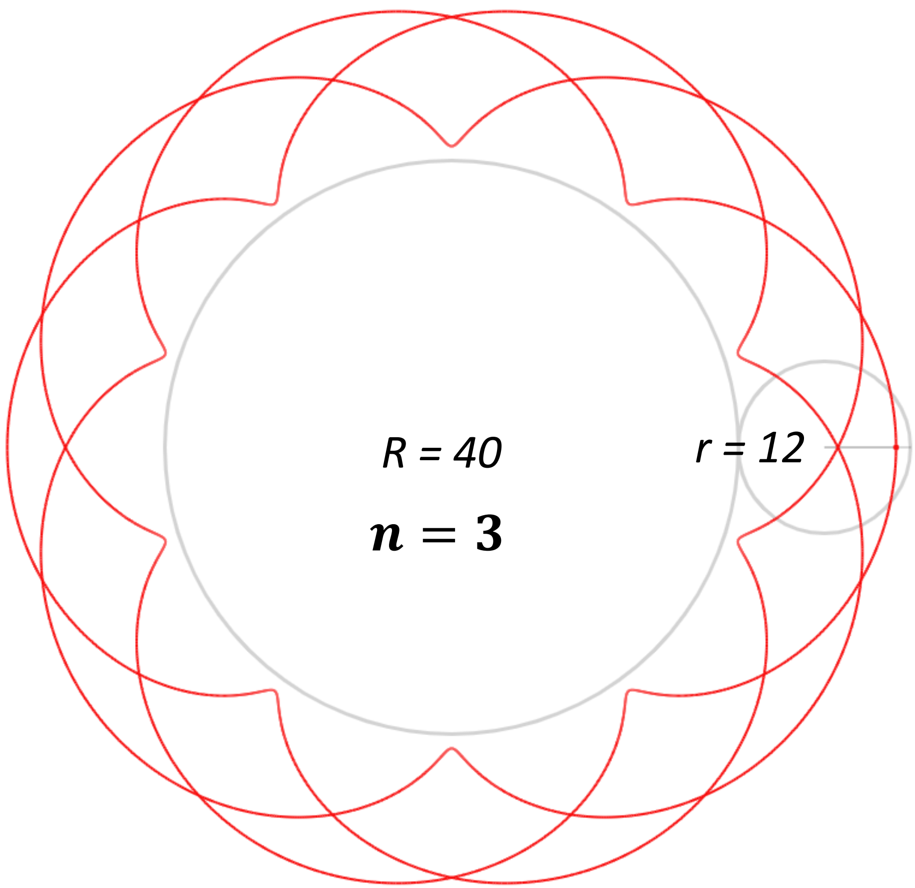
Project 1 – How many repeating cycles



$$\theta = 2\pi$$

What should θ be for complete pattern?

Project 1 – How many repeating cycles



What should $\boldsymbol{\theta}$ be for complete pattern?

$$lcm(12,40) = \frac{R \times r}{\gcd(R,r)} = 120$$

$$n40 = 120$$
$$n = 3$$

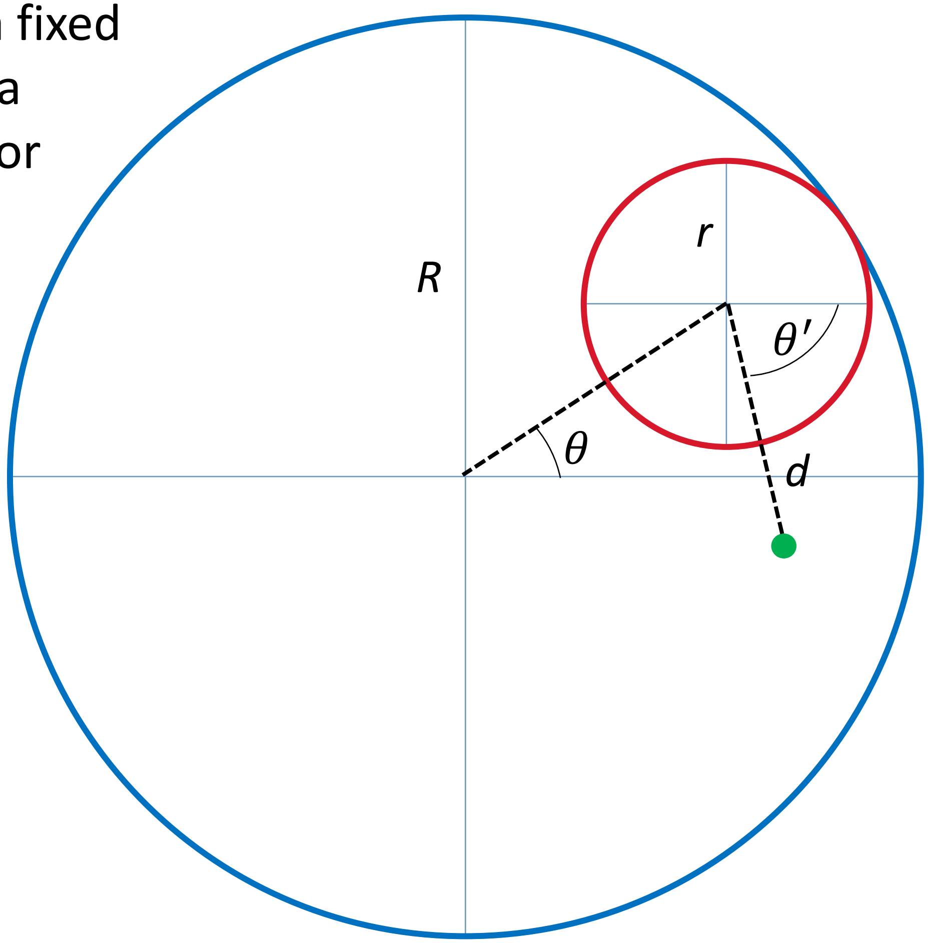
$$n'12 = 120$$
$$n' = 10$$

$$\theta = 6\pi$$

Project 1 - Hypotrochoid

Hypotrochoid :

Curve traced by a point attached to a circle of radius *r* rolling around the *inside* of a fixed circle of radius *R*, where the point is at a distance *d* from the center of the interior circle.



Parametric Equations:

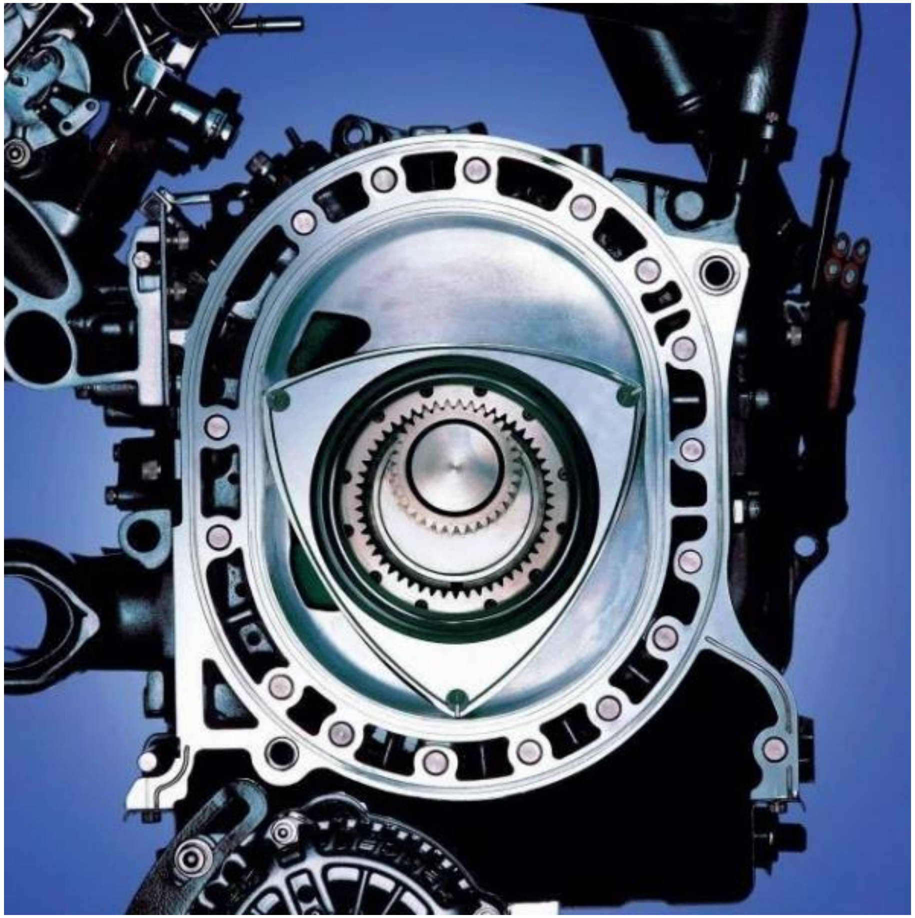
$$x(\theta) = (R - r)\cos\theta + d\cos\theta'$$

$$y(\theta) = (R - r)\sin\theta - d\sin\theta'$$

$$\theta' = \frac{R - r}{r}\theta$$

Hypotrochoid Example

Wankel Rotary Engine



Project 1 – Example…

```
Colors = ['b', 'r', 'g', 'm']
Inputs = [[120, 13, 43, 'h'], \
          [180, 50, 20, 'h'], \
          [77,  40, 33, 'e'], \
          [60,  50, 40, 'e']]               # Set up inputs for each case
plt.figure(1, figsize=(8,8))
plt.clf()
for case in [0, 1, 2, 3]:                   # Loop over cases
    plt.subplot(2,2,case+1)                 # Set subplot to case+1
    parms = spiro(*Inputs[case])            # Derefernce Inputs list in function call
    spiroPlot(*parms)                       # Dereference parms tuple
plt.savefig('spiro.svg')
plt.show()                                  # Show the plot from external call
```

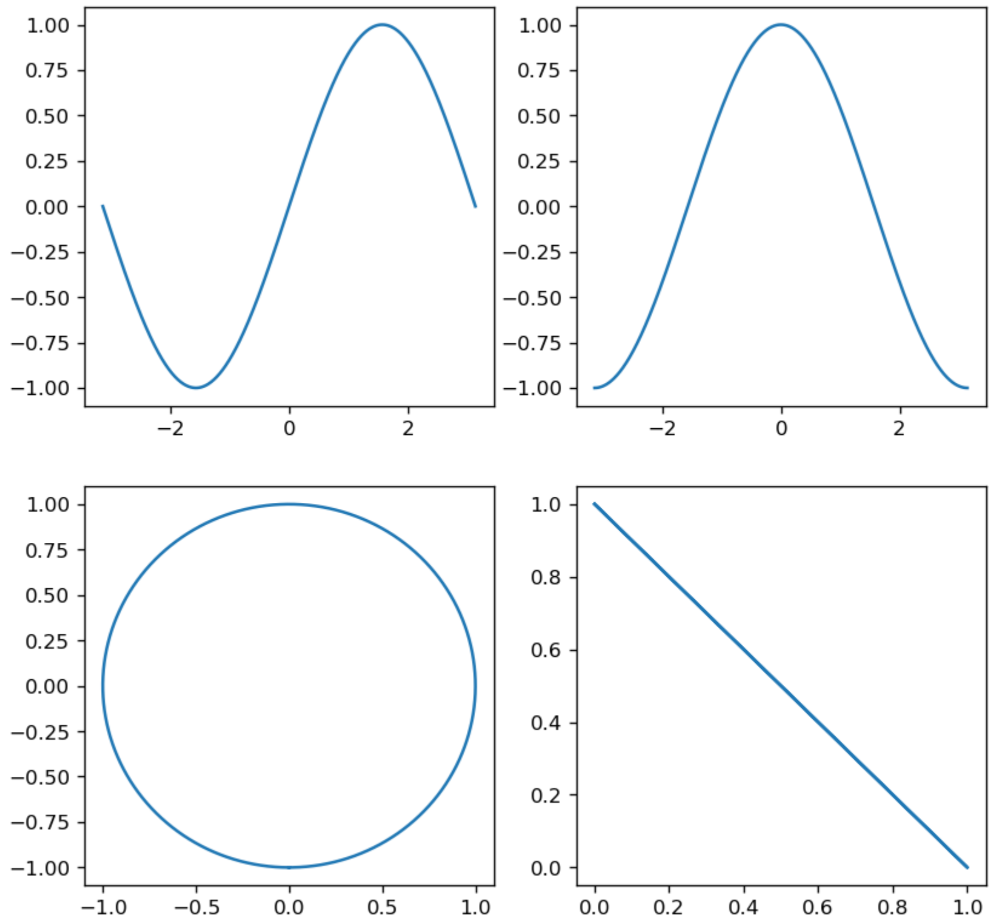
Subplots

Generate a single figure with multiple subplots by using:

`matplotlib.pyplot.subplot()`

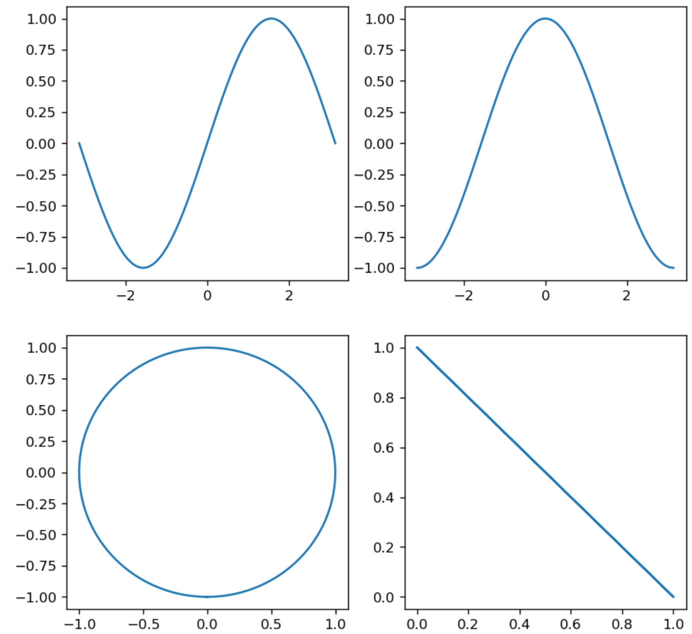
or

`plt.subplot()`



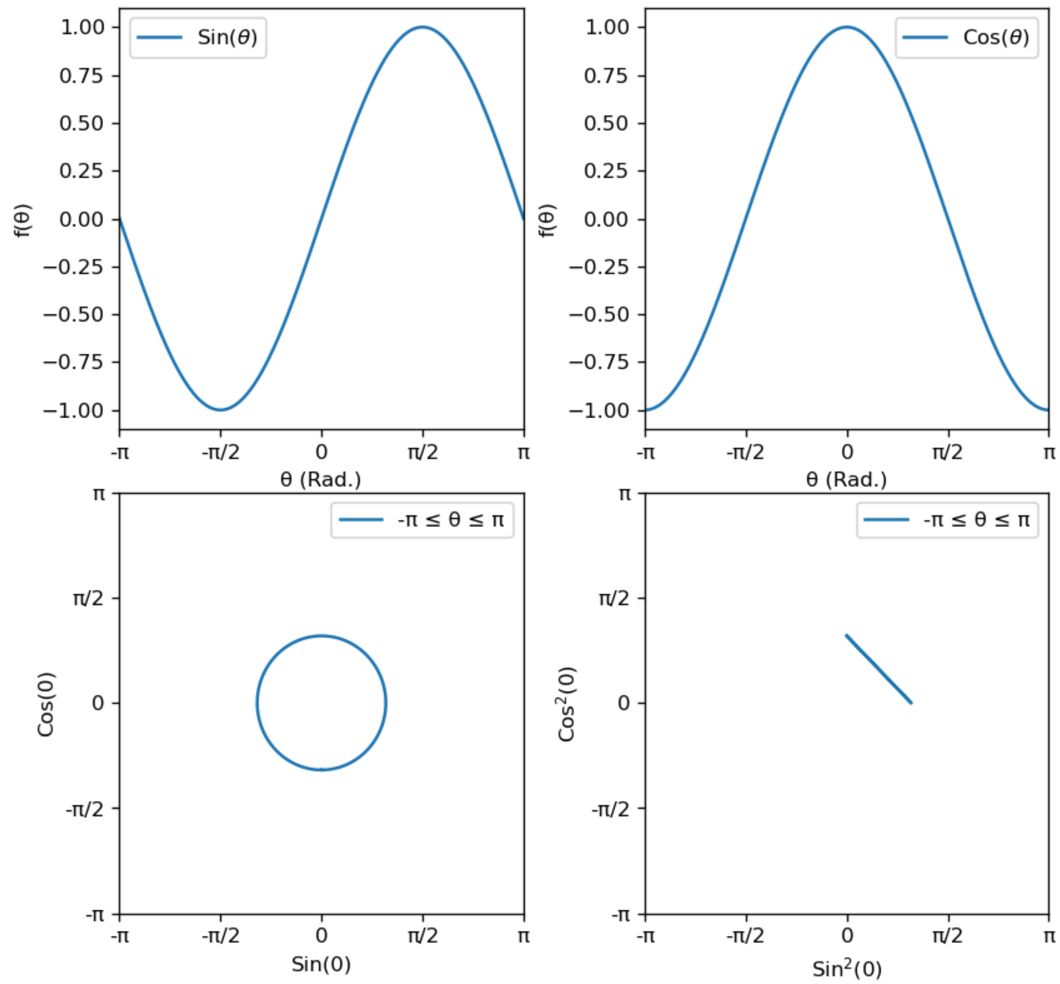
Subplots

```python
#import Modules & Functions
import math
import matplotlib.pyplot as plt
from Functions import linear_space

#Generate Data to Plot
theta_list = linear_space(-math.pi,math.pi,1000)

sin_list, cos_list, sin2_list, cos2_list = [], [], [], []
for i in theta_list:
    sin_list.append(math.sin(i))
    cos_list.append(math.cos(i))
    sin2_list.append(math.sin(i)**2)
    cos2_list.append(math.cos(i)**2)

#Generate a 2x2 Subplot
plt.figure(num=0,dpi=120,figsize=(8,8))

plt.subplot(2,2,1)
plt.plot(theta_list,sin_list)

plt.subplot(2,2,2)
plt.plot(theta_list,cos_list)

plt.subplot(2,2,3)
plt.plot(sin_list,cos_list)

plt.subplot(2,2,4)
plt.plot(sin2_list,cos2_list)
```



Scripting subplots

How do we make it look nice?



Scripting subplots

```python
#Generate a 2x2 Subplot
plt.figure(num=0,dpi=120,figsize=(8,8))
plt.subplots_adjust(hspace=0.15,wspace=0.3)

plt.subplot(2,2,1)
plt.plot(theta_list,sin_list,label=r"Sin($\theta$)")

plt.subplot(2,2,2)
plt.plot(theta_list,cos_list,label=r"Cos($\theta$)")

plt.subplot(2,2,3)
plt.plot(sin_list,cos_list,label="-π ≤ ϑ ≤ π")
plt.xlabel("Sin(θ)")
plt.ylabel("Cos(θ)")

plt.subplot(2,2,4)
plt.plot(sin2_list,cos2_list,label="-π ≤ ϑ ≤ π")
plt.xlabel("Sin$^2$(θ)")
plt.ylabel("Cos$^2$(θ)")


for i in range(4):
    plt.subplot(2,2,i+1)
    plt.legend()
    plt.xlim(-math.pi,math.pi)
    plt.xticks([-math.pi,-math.pi/2,0,math.pi/2,math.pi],labels=["-π","-π/2","0","π/2","π"])

for i in range(2):
    plt.subplot(2,2,i+1)
    plt.xlabel("ϑ (Rad.)")
    plt.ylabel("f(ϑ)")

for i in range(2,4):
    plt.subplot(2,2,i+1)
    plt.ylim(-math.pi,math.pi)
    plt.yticks([-math.pi,-math.pi/2,0,math.pi/2,math.pi],labels=["-π","-π/2","0","π/2","π"])
```