

Operational Deposit Model

LCR-ODM (2458)

Model Owner:	Rubing Qian, Head of ALM Liquidity, Corporate Treasury Ilona Moldavsky, Head of Liquidity Analytics, Corporate Treasury
Model Developer:	Yujie (Omar) Ma, Financial Analyst, Corporate Treasury
Model User(s):	Corporate Treasury
Documentation Date:	July 27, 2016

Purpose and Use

- A. Purpose of Model:** As part of the US Liquidity Coverage Ratio (LCR) compliance, BNY Mellon needs to determine the level of deposits held in operational accounts in excess of operational needs. The Operational Deposit Model calculates the operational core balances for a subset of business lines in BNY Mellon based on the level of outflows from operationally eligible clients.
- B. Areas of Use:** The core balance projections that are results of these calculations are used in US LCR determination, which in turn is also used downstream calculations, including Liquidity Stress Testing (and further downstream CCAR deposit models dependent on LST), G-SIB surcharge, Funds Transfer Pricing, Interest Rate Risk (IRR) Deposit Modeling and Resolution financial projections.
- C. Model Family:** IRR Deposit Model 2432, Internal Liquidity Stress Test 1885 are downstream users of the results of this model. Calculated core balances determined by this model undergo a specific logic in Model #1885 and 2432.
- D. Limitations:** Model uses historical client data to predict future client operational needs. To reflect the potential market or regulatory environment changes, a conservative ceiling is set on the model results prior to usage. The model is based on a set of operational accounts predates and may not fully align with the account list used for daily LCR calculation (see Assumptions for further explanation). The main limitation is that there is no objective measurement of operational deposit amount.

Table of Contents

Background	3
Model Specification	5
A. Methodology	5
<i>Approach</i>	5
<i>Formulation</i>	6
B. Input Data and Data Assumptions	12
<i>Data Sourcing and Application Process</i>	12
<i>Determination of Operational accounts</i>	12
<i>Operational Account List Extraction Process</i>	13
<i>Deposits and Transaction Alignment</i>	14
<i>Data Quality Test</i>	15
<i>Assumptions Used as Inputs</i>	15
C. Calculations	16
<i>Group length distribution and periodicity distribution tests</i>	16
Testing the Model	19
A. Analysis of the Model	19
<i>Result accuracy</i>	19
<i>Parameter sensitivity</i>	21
<i>Model fidelity (stability and behavior)</i>	22
B. Analysis of Implementation	25
<i>Model User</i>	25
<i>Automation</i>	25
<i>Control Environment</i>	26
C. Ongoing Performance Monitoring Plan	27
Appendix A: US Liquidity Coverage Ratio Detailed Background	32
Operational Deposit Eligibility	32
Operational Amount/Excess Guidance	33
Appendix B: Systems	35
Appendix C: Mathematical Terms Glossary/Terminology	36
Appendix D: Examples and Detailed Processes	38

Background

In order to comply with the US implementation of the Basel III Liquidity Coverage (“US LCR”), BNY Mellon must daily¹ (as of July 1, 2015) calculate the Liquidity Coverage Ratio (“LCR”) and maintain that ratio over a given percentage (80% in 2015, 90% in 2016, and 100% from 2017 on).

$$\text{US LCR} = \frac{\text{High Quality Liquid Assets}}{\text{Total Net Cash Outflows over 30 Calendar Days}}$$

High Quality Liquid Assets are assets which can be converted easily and immediately into cash in private markets. They are outside the scope of this model. Total net cash outflows are calculated based on assigned outflow and inflow rates, reflecting a standardized stress scenario, to different funding sources, obligations, and assets. Each category is divided into specific subcategories to account for the specific risk profile of each type of asset and liability.

As BNY Mellon is a liability driven bank, the US LCR calculation will be largely driven by the deposit run-off calculations, which are part of the Net Outflows denominator (Deposits accounted for ~70% of the consolidated entity balance sheet on 12/31/15). Deposits are run-off at the US LCR rule designated rates, as shown in Figure 1.

Figure 1: Deposit Run-Off Summary of US LCR Rules

FUNDING

Retail: Individuals, SME or Living/Testamentary Trust														
Non-Brokered				Brokered										Other
Stable	Other Retail	Deposits Owned by Retail Customer, Placed by a Third Party		Reciprocal Brokered		Brokered Sweep			Other Brokered Deposits in Transaction Account with No Maturity		Other Brokered (e.g., Time Deposits,)		Non-Deposit Funding	
[Established Relationship or Transaction Account] and Fully FDIC	[Neither Established Relationship nor Transaction Account] or Not Fully FDIC	Fully FDIC	Not Fully FDIC	Fully FDIC	Not Fully FDIC	Fully FDIC		Not Fully FDIC	Fully FDIC	Not Fully FDIC	Maturity <=30 Days	Maturity >30 Days		
						Contract between Retail Client and BNYM	No Contract between Retail Client and BNYM							
3%	10%	20%	40%	10%	25%	10%	25%	40%	20%	40%	100%	10%	40%	

Wholesale													
Collateralized		Non-Operational Deposits			Operational Deposits							Other	
Backed by Treasuries	Backed by Agencies	Financial Institution (or consolidated subsidiary of FI)	Non-Financial Institution		Prime Brokerage Service or Hedge Fund/ Private Equity	Escrow	Other					Unsecured Funding	
			Non-Brokered Fully FDIC	Either Brokered or Not Fully FDIC			Core Balances		Excess Balances				
							Fully FDIC	Not Fully FDIC	FI	Non-FI	Non-FI		
										Non-Brokered Fully FDIC	Either Brokered or Not Fully FDIC		
0%	15%	100%	20%	40%	100%	25%	5%	25%	100%	20%	40%	100%	

For operationally eligible wholesale deposits (definition and background in Appendix A), BNY Mellon must determine the split between the balances needed for operational purposes versus any excess in operationally eligible deposit accounts. Any balance that exceeds the amount needed for operational

¹ Prior to daily reporting, BNY Mellon calculated monthly (end of month spot) US LCR starting for January 31, 2015.

purposes will be considered as “Excess” and treated as non-operational (see red-circled section in Figure 1).

Per the US LCR rules, each institution subject must develop its own tailored methodology to determine the excess portions of an operational account. The US LCR rule does not provide any specific methodology except to “explicitly require a covered company to take into account the volatility of the average operational deposit balance when designing its methodology for identifying excess deposit amounts”. A major challenge of the process is that there is no observable data for operational balances and the concept might not be mathematically quantifiable by our client base as required by the regulatory agencies.

The firm faces two boundary constraints in determining the methodology.

- 1) If the approach developed by the bank is judged not to be conservative enough, all operationally eligible deposits will be treated as excess (that is, as non-operational). In such a scenario, the firm’s LCR will breach regulatory limits and it will fall under strict regulatory oversight during the remediation period.
- 2) If our approach to the methodology is too conservative by classifying too large a proportion of operational deposits as excess, BNY Mellon will keep need to keep excess liquidity on hand. This will lead to hoarding of liquid assets and prevent the firm from providing important intermediation during both BAU and crisis periods

Ultimately, BNY Mellon has determined the best approach is to begin by determining what operational balances are and consider as excess as anything above that amount. While that approach will inevitably classify some operational balances (as operational needs are constantly fluctuating) as excess, we believe this approach best aligns with the regulatory guidance to consider operational volatility in determination of operational need.

For this version of the model, only the major deposit generating businesses were modeled. That does not mean that operationally eligible deposits do not exist in other business lines, they were outside the scope of this iteration of the operational deposit model.

Model Specification

A. Methodology

The US LCR rule focused on four key items in determining the Operational Deposit Amount:

- *Requirement 1:* [The Bank must] demonstrate a methodology for identifying any deposits in excess of the amount necessary to provide the operational services, the amount of which would be excluded from the operational deposit amount
- *Requirement 2:* A covered company's methodology must also take into account the volatility of the average deposit balance to ensure the proper identification of excess balances.
- *Requirement 3:* The [bank] must demonstrate that the deposit is empirically linked to the operational services.
- *Guidance 1:* In all instances, a covered company's analysis of operational deposits must be conducted at a sufficiently granular level to adequately assess the risk of withdrawal in an idiosyncratic stress.

Approach

We have chosen to use client outflows as the indicator of operational need. Specifically, we have chosen to determine operational need based on client specific periodicity via the Activity Based Balance (“ABB”) approach utilized for the prior model. The underlying logic is that within an account identified as operational, a customer’s withdrawals of cash over a given period are the best measure of operational activity, as clients have more discretion over inflows.

Based on the periodicity (periodic payment structure) of a client relationship within a given business line, the ABB calculates the operational balances for said relationship. On any given day, if a client’s balance is lower than ABB threshold, it means that the amount of cash present in the account is lower than the historical level of cash maintained for operational services in this account. Therefore based on historical observation, this balance will be fully used for operational activity (aka “core”).

The level of analysis was set at client level (in a given business line), as that was the most granular level of comprehensive information available for analysis (satisfying **Guidance 1**). Client level aggregation was chosen for two reasons. First, account level data lineage was not consistent due to migration of client accounts from one cash/custody system to another (client level identifiers were not affected by the migration). Second, due to the structure of client relationship, clients manage their accounts in aggregate, with some accounts acting as an aggregation account at the end of day. Failing to consider that complexity of such a relationship structure would lead to an underestimate of true operational need.

Client relationships are looked at within the context of a given business line. There are structural differences in the services offered in different business lines, which means that economic activity tied to those services will differ (and so would periodicity for the same client across business lines).

One of the underpinnings of the concept of operational needs is that a one-day outflow might not accurately capture a client’s operational needs during that time. Rather clients’ payments over a distinct payment cycle time period describe their true operational need, while those payments might be spread over a number of days, the operational need and the way that clients financially plan for said need are based on the aggregated flow during that time cycle (aka “group”). The lack of absolute matching between daily outflows and operational need is due to transactional frictionality and the significant optionality in the timing of bill payment in the wholesale marketplace (e.g., “payment within 30 days of

receipt” provides a wide window for payment). Due to the nature of economic activity, there is an underlying recurrent payment interval (aka periodicity) for which clients maintain liquidity. This payment interval pattern might be explicitly managed by the client base or be part of their assumed structure of doing business. We gauge the flows of client activity by measuring grouped outflow transactions tied to deposit balances.

Within the concept of grouped payment structure, ABB is used to determinate the cash liquidity needed by our clients for their operational needs (specifically addressing **Requirement 1 and 3**).

We also consider the overall macro equilibrium of client behavior. Given the standard business planning horizon of one year, projected ABB is averaged over the past one year period to capture said equilibrium.

Formulation

As per the Methodology introduction, the ABB calculation is performed at the customer level in a given business line based on a client specific periodicity of payments.

Process Flow:

The operational balance calculation process follows the steps listed below:

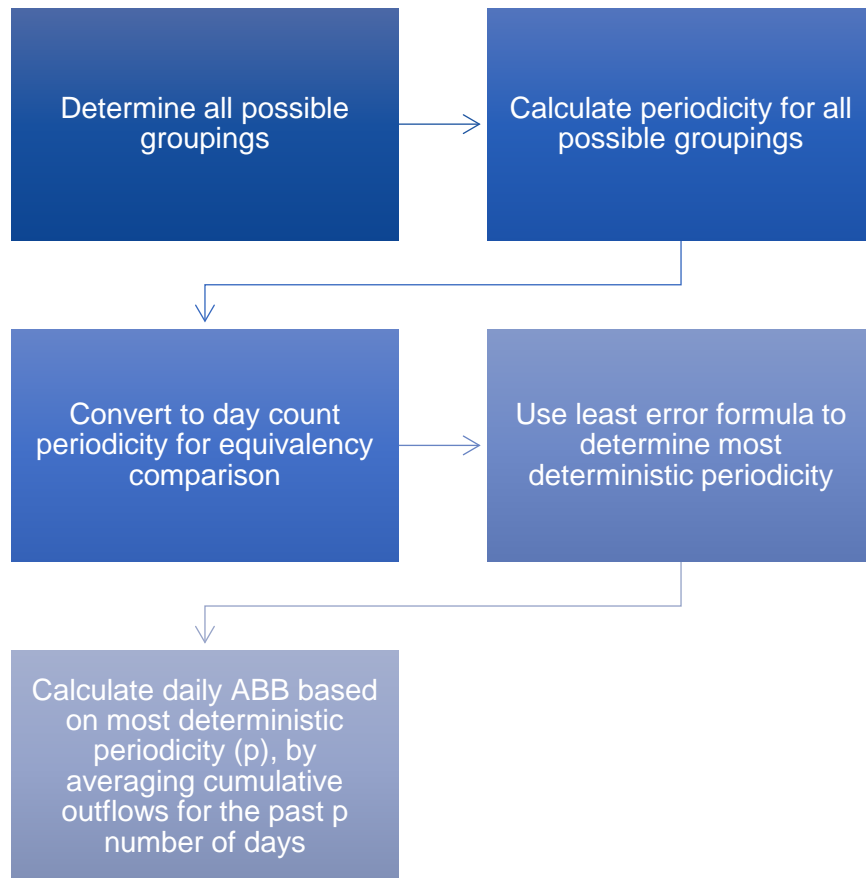
- 1) Data Preparation

Figure 2: Data Preparation Process



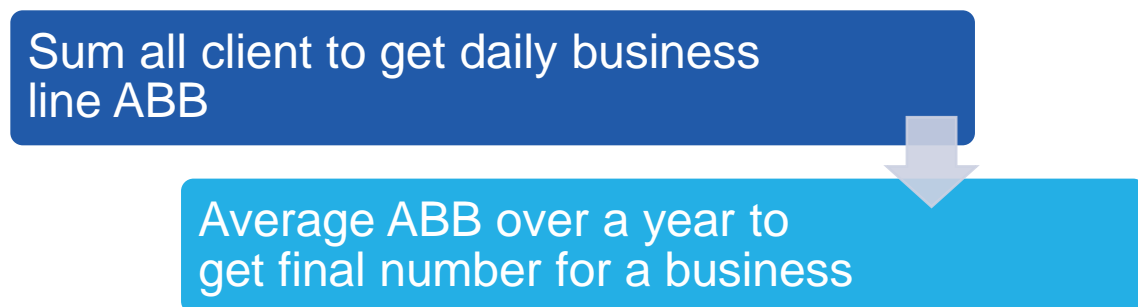
- 2) For each client in a business line determine daily ABB as per below

Figure 3: Client Daily ABB Calculation Process



3) Calculate ABB for business line

Figure 4: Business Line Final ODM Number



4) Market Shift Management Control

Structural changes in the marketplace, regulatory or economic landscape affect the operational needs of clients outside of their planning perspective. As that situation would affect the estimation of the ABB process, a control is embedded to ensure we do not overestimate operational deposit needs.

Final ODM = Minimum (ABB Result for Business, 3 month average of operationally eligible account balances)

A mathematical example of the ODM calculation flow is attached in Appendix D.

Detailed Process Flow is attached in Appendix D.

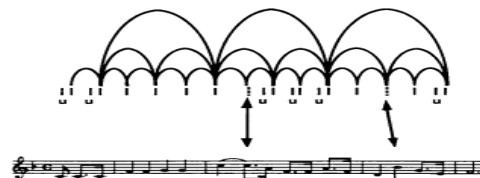
Mathematical Formulae for Periodicity Calculation

Using periodicity transform (see attached “Periodicity Transforms” (Sethares & Staley, 1999), we determine the periodicity of the client account for a given business line using Best Frequency Algorithm. Periodicity transformation is an enhanced Fourier transformation that allows us to prioritize the results of a Fourier transformation and determine what frequency best describes a given function. For mathematical background of Fourier and Periodicity transform, see “Periodicity Transform Background” in Appendix D.

1) The Weakness of Fourier Transform

One situation where periodicities play a key role is in musical rhythm. D. Rosenthal (see “Periodicity Transforms”) has created a rhythm parsing program called “Fa,” which searches for regularly spaced onset times in a Musical Instrument Digital Interface data stream. An example is given of the best rhythmic parsing found by Fa for the song “La Marseillaise” which is reproduced here:

Figure 5: La Marseillaise Musical Notes Rhythmic Parsing



The four measures of the melody “La Marseillaise” were coded into the binary sequence with 256 digits:

```

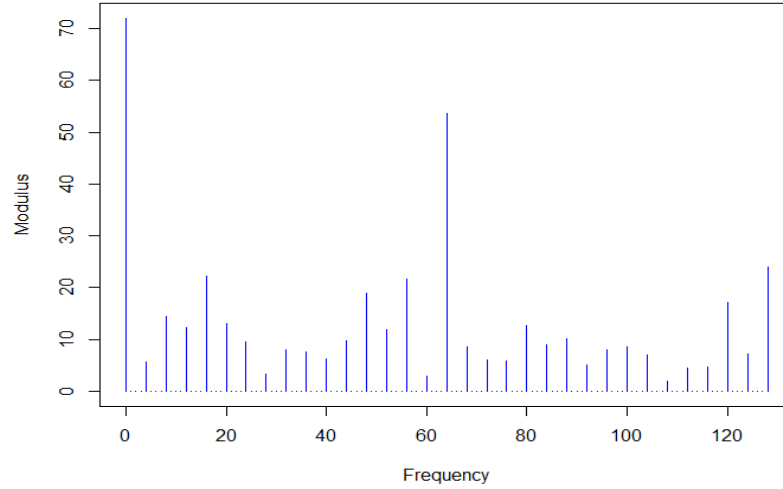
1 1 0 0 1 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0 0
0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0
1 1 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0
1 1 0 0 1 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0 0
0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0

```

Each digit represents a time equal to that of $1/16^{\text{th}}$ note. A 1 indicates that a note event occurred at that time; whereas a 0 means that no new note event occurred.

After careful investigation, Rosenthal found that the most significant period of the binary sequence above should be 4. Therefore, it is reasonable to speculate that the Fourier Transform of the binary sequence should have the maximum modulus when the period equals to 4, i.e. the frequency equals to 64 ($256/4 = 64$). Below is the output:

Figure 6: Fourier Transform (frequency domain) of La Marseillaise



Unfortunately, the frequency with the largest modulus is 0. When frequency equals to zero, the trigonometric function becomes a straight line, which means that the input data is most likely flat. However, to the human ear, La Marseillaise is lively rather than monotone.

Periodicity Transform Used

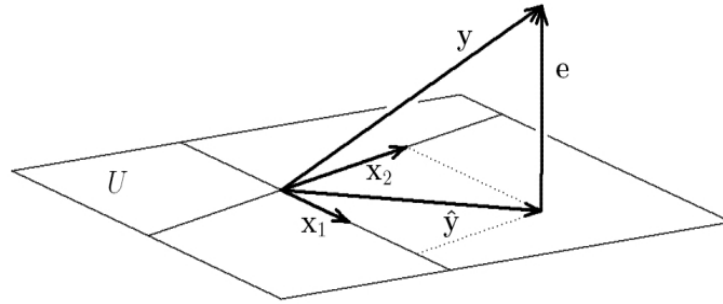
The Best Frequency Algorithm was used for modeling purposed.

In the paper “Periodicity Transform”, the authors suggested a new way to utilize Fourier Transform. Before introducing the algorithm, we need to clarify several definitions and concepts intuitively:

- A sequence of real numbers is called *p-periodic* if there is an integer p with $x(k + p) = x(k)$ for all integers. Let P_p be the set of all p -periodic sequences, and P be the set of all periodic sequences.
- Both P_p and P form linear vector spaces.
- If $1 < s < p$, any vectors x in P_p can be projected to the subspace P_s , the projection is called x^* .

The detailed mathematical proofs of linear spaces and the definition of projection can be found in the paper “Periodicity Transform”. The basic idea of projection can be shown in the following Figure 11:

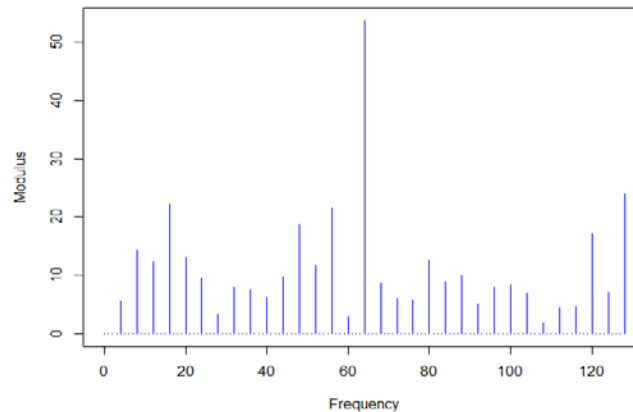
Figure 7: Projection Example



In the Figure 11, the vector y is in a higher dimensional space than the space U . \hat{y} is the projection of y on the subspace U , which means that if we only use the information in the space U , then \hat{y} is our best approximation of the vector y . The difference between y and \hat{y} is the vector e , which is the extra information that we failed to capture.

Now let's go back to the song "La Marseillaise". What if we project the original sequence x to the P_1 subspace with period 1 (let us denote the projection on the subspace as x^*), get the difference between x and x^* (let denote the difference as $x - x^*$), and do the Fourier Transform on $x - x^*$? The new output on the frequency domain can be shown as following:

Figure 8: Periodicity Transform (frequency domain) of La Marseillaise



We can see that the frequency with the largest modulus is 64, thus the period is 4, which is exactly the result we were looking for. This algorithm is called Best Frequency in the paper.

Strengths and Weaknesses of Approach used

1) The Strengths of the Best Frequency Algorithm

While other proposed algorithms are provided in the "Periodicity Transforms" paper, we chose the Best Frequency Algorithm because all the other algorithms need to set a threshold to determine the periodic pattern is significant enough. Some other specific weaknesses are:

- a) *Objectivity*. All the other algorithms (Small to Large, Best Correlation, M-Best) need to set a threshold on the norm of $\mathbf{x} - \mathbf{x}^*$ to determine if the periodic pattern is significant enough. The threshold is arbitrarily set, which hamper the objectivity of the algorithm.
- b) *Applicability*. The Best Correlation algorithm works the best with the spiky data; however, there is no guarantee that every client's data set fits that criterion.
- c) *Conservativeness*. The M-Best algorithm is sometimes fooled into returning multiples of the basic periodicities, which will largely overestimate the period and the amount of operational deposit. For example, if the output is 23, 31, 33, then M-Best might regard 23529 as the period.

2) The Weakness of the Best Frequency Algorithm

We compensate against the following identified weaknesses in the Best Frequency algorithm with the group done prior to utilizing the periodicity transform:

- a) A 1-day period will be generated when continuous zeros exist in the data. With the grouping exercise, the zero data becomes consolidated and allows the algorithm to work as intended.
- b) When the period is not a factor of the length of the input data, Best Frequency Algorithm tends to set the factor that is the closest to the “true” period. For example, for a 24-day period, with a “true” period of 7, the Best Frequency algorithm will output 6 because it is the factor of 24 which is the closest to the “true” period of 7.

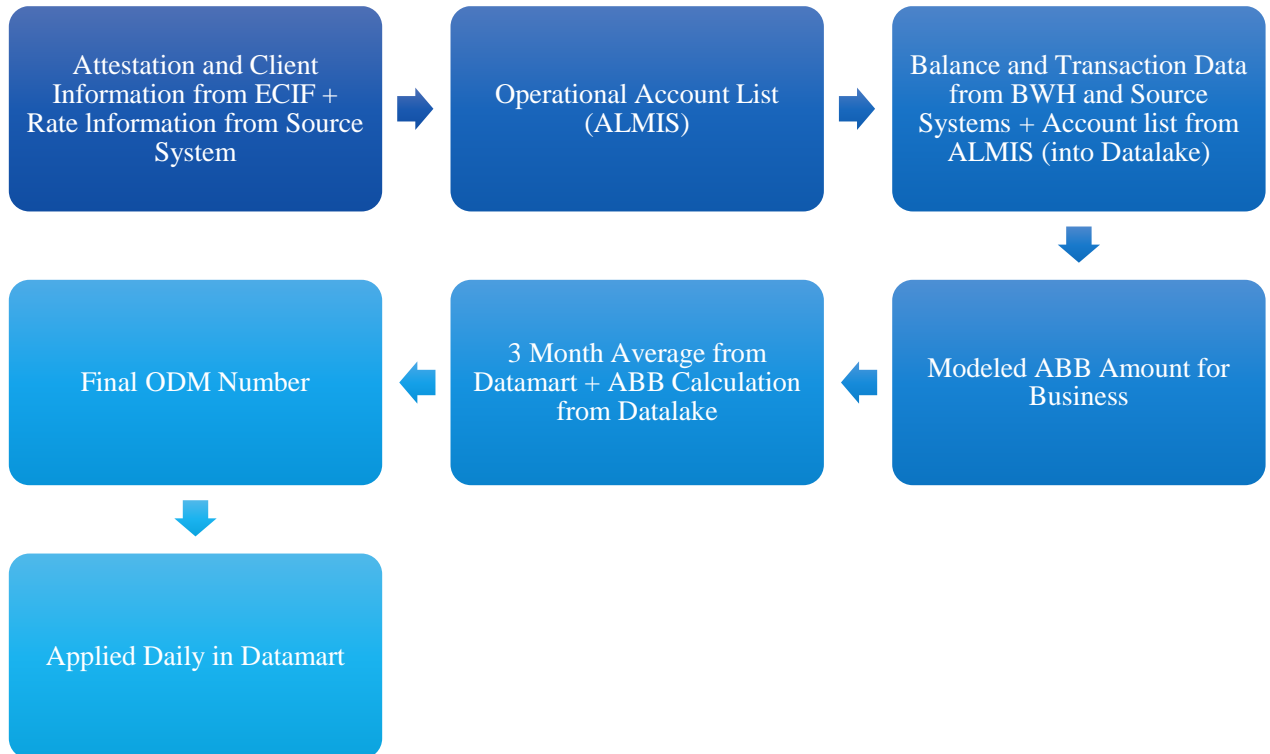
Mathematical Formulae for Market Shift Management Control

The Market Shift Management Control was chosen as the 3 calendar month business day average of operationally eligible operational accounts, with the last month being equal to the last month in the ABB data set. A 3 month average was chosen based on expert judgment as a monthly average would have been too sensitive to short-term fluctuations, while a longer period would not have captured the immediacy of market changes. The average data source was the total eligible operational balance daily excluding negative balances, weekends and holidays.

B. Input Data and Data Assumptions

Data Sourcing and Application Process

Figure 9: Process Flow for Data Sourcing and Modeling



Determination of Operational accounts

Operational accounts qualification was determined based on the criteria set forth by the US LCR rule. An operational deposit is defined by the rule as “unsecured wholesale funding or a collateralized deposit that is necessary for the [BANK] to provide operational services as an independent third-party intermediary, agent, or administrator to the wholesale customer or counterparty providing the unsecured wholesale funding or collateralized deposit.”

Three broad categories of client account information are necessary for Operational account designation (see detailed information in Appendix A).

- Account designation, usage and governing agreements. The business lines provide this data and the data accuracy is attested to by the business line.
- The rate paid on the deposit cannot be used to create an economic incentive. Corporate Treasury has chosen to set this hurdle as the target deposit rate set by the national regulators for a given currency. If no such rate exists, the most prevalent deposit rate in the currency is used.
- Non-Regulated funds (i.e., hedge funds and private equity funds) cannot be treated as operational, as per the US LCR.

Operational Account List Extraction Process

The operational account list used for modeling is sourced from tables in ALMIS (see Appendix B for definition). It is refreshed prior to model update in order to capture the most current business line and the customer identifier, as the customer/account list is constantly being updated/merged by the operational team in ECIF (see Appendix for definition).

Attestation of operational accounts was first done in November 2014, as such; some operational accounts that had closed prior to this might not have been included in the attestation process. Currently, attested account are fed daily from ECIF into ALMIS and new accounts are attested by the business line during the KYC process.

Deposit and transaction data sourcing

Deposit total and transaction data is sourced from the following systems (see definitions in Appendix B).

Table 1: Data Source Systems

Source System	Balance data	Transaction data
CAS	BWH	Data Lake from Source System
CDS	BWH	Data Lake from Source System
IMMS	BWH	BWH
TAS	BWH	BWH
CMS	BWH	BWH

Prior to moving the initial data set from BWH to the Data Lake, an overall data validation was performed (against March 2014 data) to ensure that data from BWH matched data from Datamart, which reconciles with the General Ledger on a monthly basis.

The different checks described below were performed on a given day for all systems:

1) Transactions data:

End of day balance on the previous day + Debit transaction amount today + Credit transaction amount today = End of day balance today

2) Cash balance: End of day balance from BWH = End of day balance from Datamart

Deposit and transaction data for the testing framework was truncated (in comparison to ODM 2014) to contain data from December 1, 2013 through May 31, 2016. This was done so that all business line analysis was done on a consistent time frame. The limiter in this decision is tied to the structural change in the Tri-Party Repo industry that was completed in 4th quarter 2013. Tri-Party Repo services are offered in Broker Dealer Services (see Deposit White Paper for a description of the services offered by the business lines).

Deposits and Transaction Alignment

Client account number (which is a unique key of deposit holding system, branch of said system and client account number) was used to link deposits and aggregated transaction inflows and outflows for a given day across data sources.

Any account that has no end of day balance record is set to have a balance of zero (this can be an artifact of systems dropping zero balance accounts from their record keeping for efficiency).

If a unique client account record contains end of day balance, end of day gross inflows and end of day gross outflows (where any of the prior variable is non-zero), further aggregation is performed against a combination of client identifier and business line. For a given client identifier on a given day, each client account record in a given business line is aggregated across end of day balance, gross outflows and gross inflows. See illustrative example below.

Table 2: Illustrative Example of Client Accounts across Business Lines

Client Identifier	Business	Client Account	End of Day Balance	Gross Outflows	Gross Inflows	Date
0000000000001	Asset Servicing	CMS01ABS123456	0	100,000	100,000	12/31/15
0000000000001	Asset Servicing	MMD101523878700	200,000	5,000,000	5,100,000	12/31/15
0000000000001	Asset Servicing	MMD101523878709	100,000	0	0	12/31/15
0000000000001	Treasury Services	CDS2088881235	5,000,000	100,000,000	104,000,000	12/31/15
0000000000001	Treasury Services	CDS2088881236	6,000,000	0	10,000,000	12/31/15

Table 3: Illustrative Example of Consolidated Client Balances in a Given Business Line

Client Identifier	Business	Date	End of Day Balance	Gross Outflows	Gross Inflows
0000000000001	Asset Servicing	12/31/15	300,000	5,100,000	5,200,000
0000000000001	Treasury Services	12/31/15	11,000,000	100,000,000	114,000,000

Note that due to the nature of a global cash/custody platform and data sourcing standards, there might exist a variance between end of day balance and net transaction flows. This exists for the following reasons:

- 1) Data used is USD, based on that day's system exchange rate

- a) Even for accounts that have no inflows/outflows on a given day might have different end of day balances day over day
- 2) Data sourcing that is needed for daily LCR reporting requires a spot feed from the cash/custody systems into ALMIS
 - a) Each system/system branch might have a different time at which they close the books (e.g., Tokyo branch on MMD will end their day at a different time than New York branch, in terms of absolute time). Once a system closes for the day, they send a data dump of their spot positions to ALMIS (and BWH)
 - b) Transactions still occur (or the reconciliation team is able to determine to which account a given transaction belongs to) between system official close time and next day's open time. These now allocated transactions lead to a situation that would cause a mismatch between previous day's close and today's open balance:
 - (i) Transaction date stamp is Day 1, but it is not reflected in Day 1's end of day balance, because it occurred after system close
 - (ii) The opening balance Day 2 is adjusted by the pending transactions, creating a variance between Day 1 end of day balance and Day 2 opening balance

However, given our grouping approach to capture operational needs, such situations do not affect our view of clients' operational needs.

Data Quality Test

In case of the loss of import information in the data sourcing process, we need to confirm the data quality before we implement the model. The quality of the data can be checked in following steps:

- 1) For every month, check the numbers of the clients, the end of day balance, or outflows on each day in the testing month, denote it as c
- 2) On day i , define the critical value as $x[i] = \frac{\log(c[i]) - \text{mean}(\log(c))}{\text{standard deviation}(\log(c))}$, the logarithm here is to normalized the client numbers
- 3) If on a certain day, the abstract value of $x[i]$ is bigger than 95% of standard normal distribution, then we are 95% percent sure that the data on this day may be in an abnormal situation which deserves further investigation

Assumptions Used as Inputs

Data is assumed to be complete and correct.

C. Calculations

Table 4 below shows the results of running the model for a selected set of business lines, along with overall operationally eligible balances. As can be seen in the “% Operational” column, the business lines reflect differing levels of operational deposit modeled amounts, which reflects the differing behaviors of clients in different business lines.

Table 4: Results by Business of Running ODM including 3 Month Control (May 2016)

<i>May-16</i>	Total Deposit	Operational Deposit	% Operational
Alternative Investment Services (AIS)	6,551	4,817	74%
Asset Servicing (AS)	77,215	68,461	89%
Broker Dealer Services (BDS)	5,888	4,788	81%
Corporate Trust (CT)	41,672	31,541	76%
Treasury Services (TS)	33,730	27,389	81%
Wealth Management (WM)	1,866	685	37%
Total	166,922	137,682	82%

Group length distribution and periodicity distribution tests

Table 5 below shows the distribution of the group lengths within the different lines. For example, 25% of TS clients have a group length of 1, which means they keep funds with the bank to make payments for that day. That reasoning makes sense when you add to the quantitative analysis that a large number of TS clients are using BNY Mellon for cash management purposes (and that has a high daily outflow component).

Table 5: Group Length Distribution by Business (May 2016)

Business	1	2-5	6-10	11-20	21-40	41-100	>100
AIS	24%	6%	3%	6%	8%	21%	32%
AS	34%	4%	3%	5%	9%	16%	29%
BDS	30%	27%	8%	9%	8%	5%	12%
CT	17%	4%	4%	6%	9%	23%	37%
TS	25%	14%	5%	6%	7%	11%	32%
WM	15%	3%	3%	3%	7%	16%	54%

Figure 13 below, takes the data displayed in Table 5 and provides a visual representation of the distribution of the group length, which shows concentrations at either side of the distribution length. This can be seen as the differences in the mix of clients across both businesses and within a business, with the client payment grouping distribution showing a bimodal pattern, which point to different product usages within business lines.

Figure 10: Group Length Distribution Graph (May 2016)

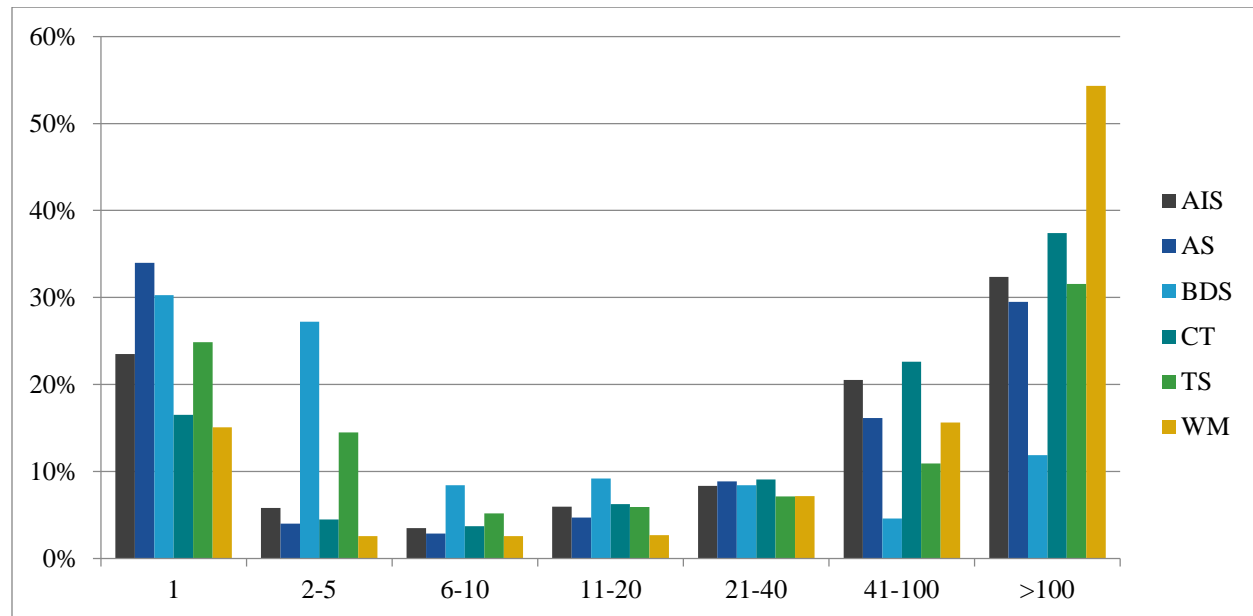


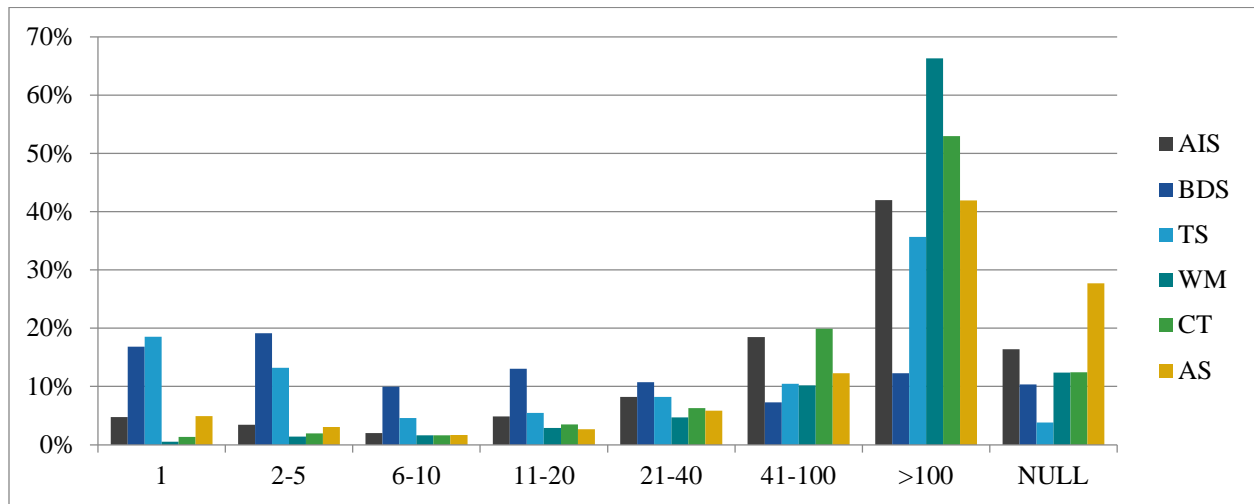
Table 6 below shows the distribution of the final client periodicity for each business line with NULL representing clients without any positive outflows during the measurement period.

Table 6: Periodicity Distribution in Days (May 2016)

Business	1	2-5	6-10	11-20	21-40	41-100	>100	NULL
AIS	5%	3%	2%	5%	8%	18%	42%	16%
AS	5%	3%	2%	3%	6%	12%	42%	28%
BDS	17%	19%	10%	13%	11%	7%	12%	10%
CT	1%	2%	2%	4%	6%	20%	53%	12%
TS	19%	13%	5%	5%	8%	10%	36%	4%
WM	1%	1%	2%	3%	5%	10%	66%	12%

As can be seen in Figure 14, client periodicity has wide distribution, which can be tied to different product usages within business lines.

Figure 11: Periodicity Distribution by Business



Testing the Model

A. Analysis of the Model

Result accuracy

Two benchmarks were used to compare the results with the current model.

Benchmark 1

Substitute the Best Frequency Algorithm with the Best Correlation Algorithm under the same grouping procedure, and calculate the percentage of the situations where our methodology has a better performance than the Best Correlation Algorithm. The criterion of which one is better is the error term $\frac{\sum_{i=1}^{N-p} |x_{i+p} - x_i|}{N-p}$ we used before. Note that clients with no periodicity under either of the algorithms will not be calculated, so that the comparison is fair to both algorithms.

Table 7: Results of Benchmark 1 (May 2016)

<i>May-16</i>	Percentage
AIS	80%
AS	72%
BDS	81%
CT	87%
TS	89%
WM	93%
Total	80%

The comparison tells us that in about 80% of the consolidated level of the times when both algorithms generate a periodicity our clients' outflows, the Best Frequency Algorithm has a smaller error than the Best Correlation Algorithm.

Best Correlation is not a good choice in our situation. The Best Correlation Algorithm is the best when the dataset is spiky enough. Table 8 shows the percentage of "spikes" in the data set for various thresholds of "spikes" in our clients' outflows:

Table 8: Results of Best Correlation Spike Analysis:

Threshold (X)	\$10,000	\$100,000	\$1,000,000	\$5,000,000
AIS	6.4%	5.0%	3.0%	1.7%
AS	7.4%	5.2%	2.8%	1.5%
BDS	14.6%	13.9%	11.4%	8.9%

CT	5.0%	3.6%	2.0%	1.0%
TS	14.5%	11.4%	7.4%	4.7%
WM	3.0%	1.4%	0.5%	0.2%

In the table 8 above, we define a “spike” as an outflow that is X dollars bigger than the outflow on the previous day, and then calculate the percentage of the number of the spikes to the total time length of the outflows. By changing X from 10,000 to 5,000,000, BDS and TS have always been spiky regardless of the value of X. On the other hand, the other business lines do not have very spiky outflows. As a result, we chose to use the Best Frequency algorithm.

Table 9: Using the Best Correlation Algorithm on the Ungrouped Raw Data (May 2016)

<i>May-16</i>	Total Deposit	Operational Deposit	% Operational
AIS	6,551	619	9%
AS	77,215	42,592	55%
BDS	5,888	4,034	69%
CT	41,672	13,867	33%
TS	33,730	22,450	67%
WM	1,866	22	1%
Total	166,922	83,584	50%

As can be seen in Table 9 above, Best Correlation algorithm provides reasonable results for BDS and TS (which are “spiky”). Reasonableness measurement is based on the assumption that clients mostly utilize BNY Mellon’s balance sheet for operational purposes and seek to minimize excess. The other business lines do not have spiky outflows, thus the Best Correlation Algorithm does not perform as well as the Best Frequency Algorithm.

Benchmark 2

Benchmark 2 compares the results of using the least error method (from 1 day to half time frame, choose the periodicity p that minimizes the error term $\frac{\sum_{i=1}^{N-p} |x_{i+p} - x_i|}{N-p}$) on the raw data without grouping, and compare the periodicity distribution to that of the model.

Figure 12: Periodicity Distribution of Benchmark 2 – No Grouping (May 2016)

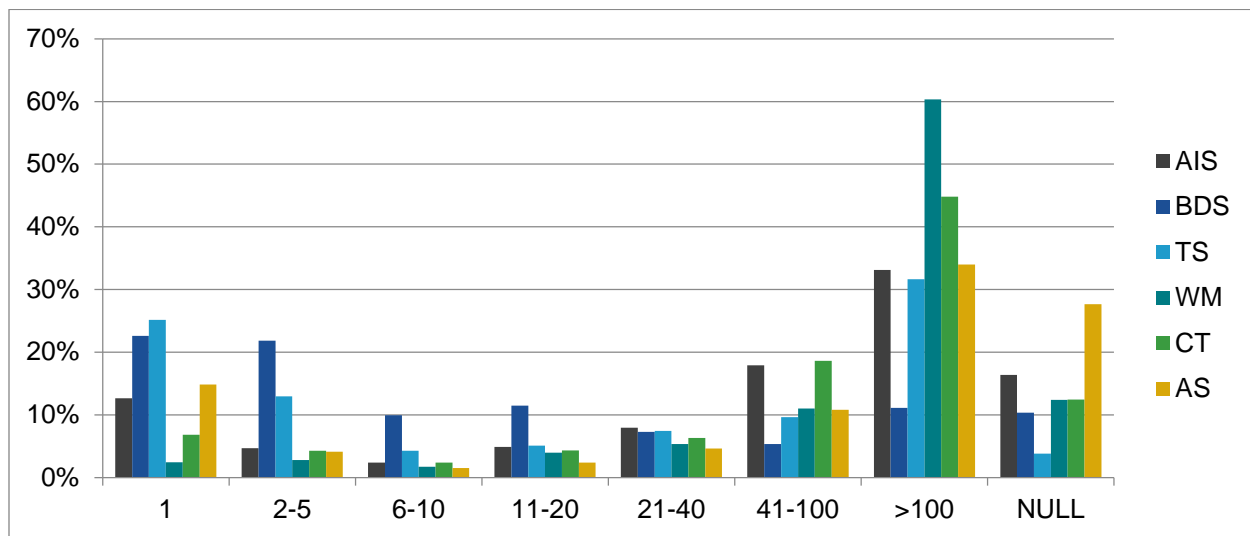
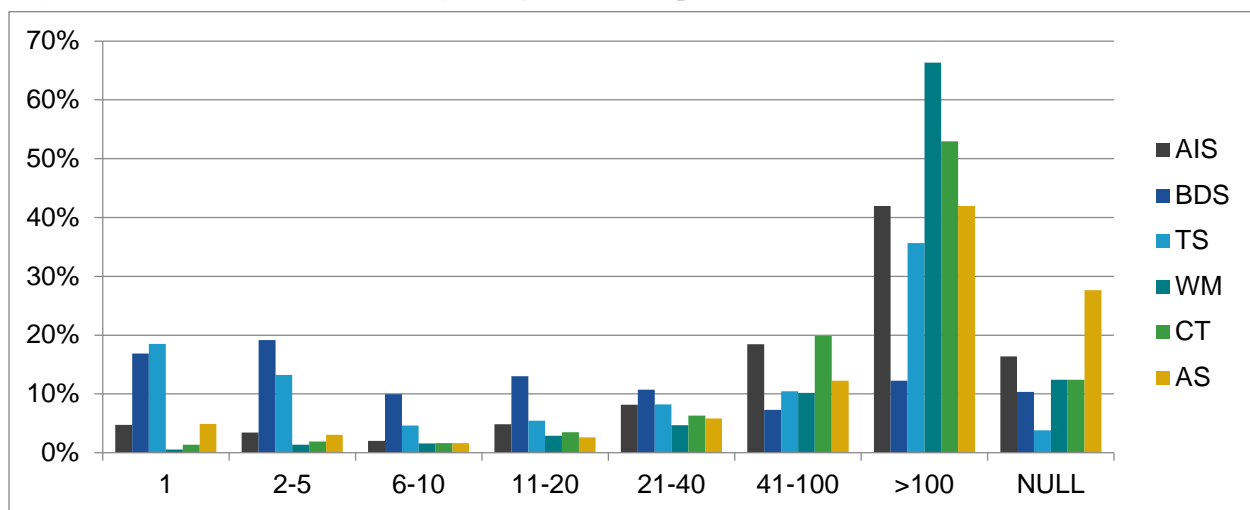


Figure 13: Periodicity Distribution of Grouped Best Frequency



The distributions are very similar. However, we think that simply using the least error method contains vulnerability in detecting periodicity that the Best Frequency Algorithm solves, as it is a more comprehensive method that checks the Fourier Transform's output sine functions' amplitude, modulus, and phase (see Appendix D for example of vulnerability).

Parameter sensitivity

Instead of using 1 year average in the last step of the methodology, we can also use the 6 months average and 3 month average. The results of such a parameter adjustment can be seen below in Tables 11 and 12.

Table 10: Result of Using 6 Months Average

May-16	Total Deposit	Operational Deposit	% Operational
--------	---------------	---------------------	---------------

AIS	6,551	4,914	75%
AS	77,215	66,428	86%
BDS	5,888	4,831	82%
CT	41,672	30,441	73%
TS	33,730	28,322	84%
WM	1,866	1,209	65%
Total	166,922	136,146	82%

Table 11: Result of Using 3 Months Average

<i>May-16</i>	Total Deposit	Operational Deposit	% Operational
AIS	6,551	4,788	73%
AS	77,215	65,337	85%
BDS	5,888	4,586	78%
CT	41,672	30,410	73%
TS	33,730	27,360	81%
WM	1,866	1,123	60%
Total	166,922	133,604	80%

As can be seen in Tables 11 and 12 above, the results are close to the proposed model. However, we believe that the 12 month timeframe is more appropriate as that is the standard financial planning period for most firms.

Model fidelity (stability and behavior)

Below is a result of running the latest operational account data set through the ABB model from December 2013 through the end of the month listed below, which shows the stability of the model based on the data of different time frames. As can be seen in the results in Table 13 below, the overall levels of operational core exhibits a variance of 8.9% between the maximum and minimum results for the monthly calculations; which is well within the parameters of variability the LCR calculation can handle.

If the model was to prove to be too unstable, that could create a large fluctuation in the bank's LCR calculations leading to possible regulatory limit breaches,

Table 12: Stability Testing - Model Monthly Results (only ODM)

May-16	Apr-16	Mar-16	Feb-16	Jan-16	Dec-15	Nov-15	Oct-15	Sep-15
--------	--------	--------	--------	--------	--------	--------	--------	--------

AIS	4,817	4,852	4,822	4,804	4,716	4,555	4,448	4,296	4,448
AS	68,461	68,347	69,363	71,438	71,158	70,990	71,190	69,604	69,237
BDS	4,788	5,131	5,345	5,101	5,225	5,118	5,233	5,154	5,138
CT	31,541	29,416	29,601	30,515	28,882	27,644	27,695	27,067	25,814
TS	27,389	29,422	29,487	29,452	29,596	29,059	27,824	26,787	25,315
WM	685	1,201	1,221	1,255	1,219	1,198	1,162	1,053	1,012
TOTAL	137,682	138,367	139,838	142,565	140,795	138,564	137,552	133,961	130,964

The model results was also compared against previous year's ABB and Regression model, as can be seen in the results in Table 14 and 15 below.

Table 13: Results of ODM versus the last year's ABB

	May-16	Apr-16	Mar-16	Feb-16	Jan-16	Dec-15	Nov-15	Oct-15	Sep-15
AIS	3,340	3,428	3,499	3,479	3,624	3,367	3,370	3,269	3,276
AS	68,766	66,111	66,315	65,576	65,565	72,640	74,969	73,768	69,843
BDS	4,643	5,270	5,366	5,319	5,360	6,324	5,800	5,722	6,060
CT	34,400	33,871	31,977	31,984	33,187	31,956	31,870	34,385	34,876
TS	25,748	26,085	26,082	26,074	27,503	29,935	28,575	30,741	31,149
WM	538	673	597	639	831	859	862	822	842
TOTAL	131,651	135,438	133,836	133,073	136,071	145,083	145,451	148,707	146,046

Table 14: Results of ODM versus the last year's Regression

	May-16	Apr-16	Mar-16	Feb-16	Jan-16	Dec-15	Nov-15	Oct-15	Sep-15
AIS	5,148	5,755	6,321	5,686	6,046	6,581	6,759	6,547	5,148
AS	70,396	69,766	71,336	72,077	78,505	80,605	69,111	69,907	70,396
BDS	4,677	5,419	4,526	5,507	6,508	5,384	5,886	6,191	4,677
CT	34,567	34,469	35,564	35,730	36,549	35,397	37,997	39,230	34,567
TS	30,539	30,573	31,309	32,063	35,152	33,440	34,540	34,499	30,539
WM	1,712	1,657	1,705	1,822	1,959	1,941	1,779	1,783	1,712
TOTAL	150,359	147,040	147,639	150,761	152,886	164,720	163,348	156,072	158,157

Figure 14: Comparison of Historical and Present Models

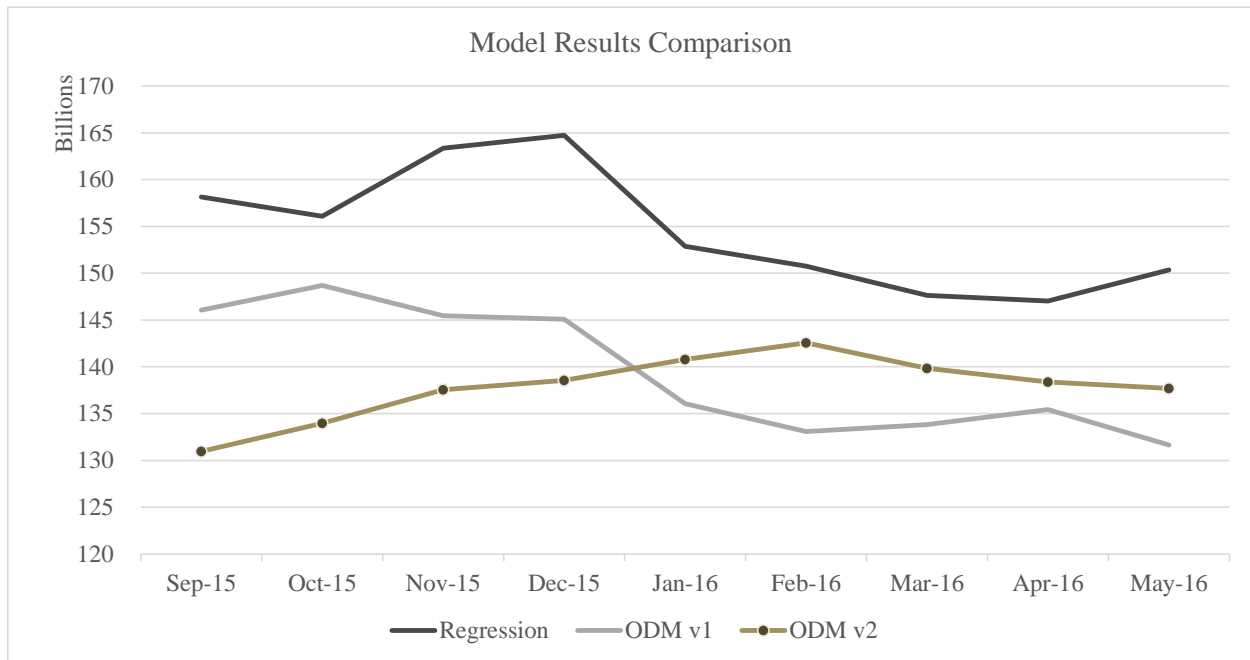


Figure 14 above includes the results of the 3 Month Market Control for May 2016 data ODM v2.

B. Analysis of Implementation

Model User

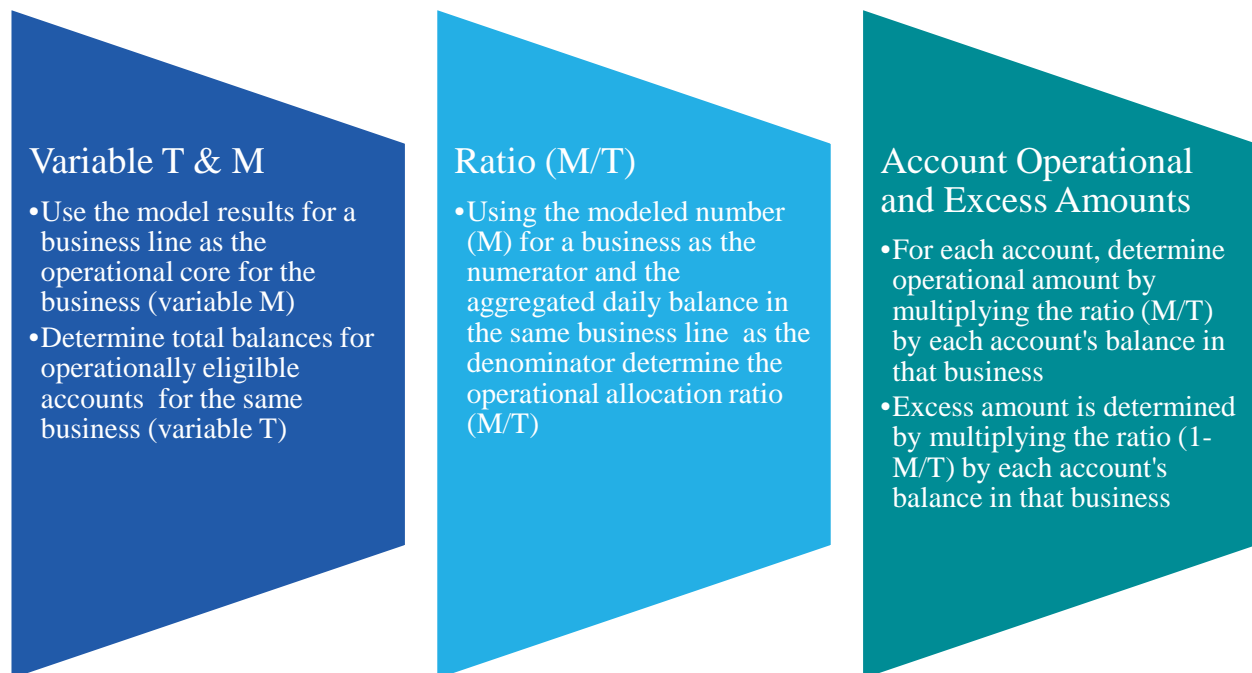
End users of the ALMIS Liquidity Tagging process – Asset & Liability Management, LCR/5G Regulatory Reporting.

Automation

The results of the model are applied to operational accounts in the ALMIS data set to determine each account's share of operational core deposits daily (see tagging logic for Operational Deposit Amount attachment).

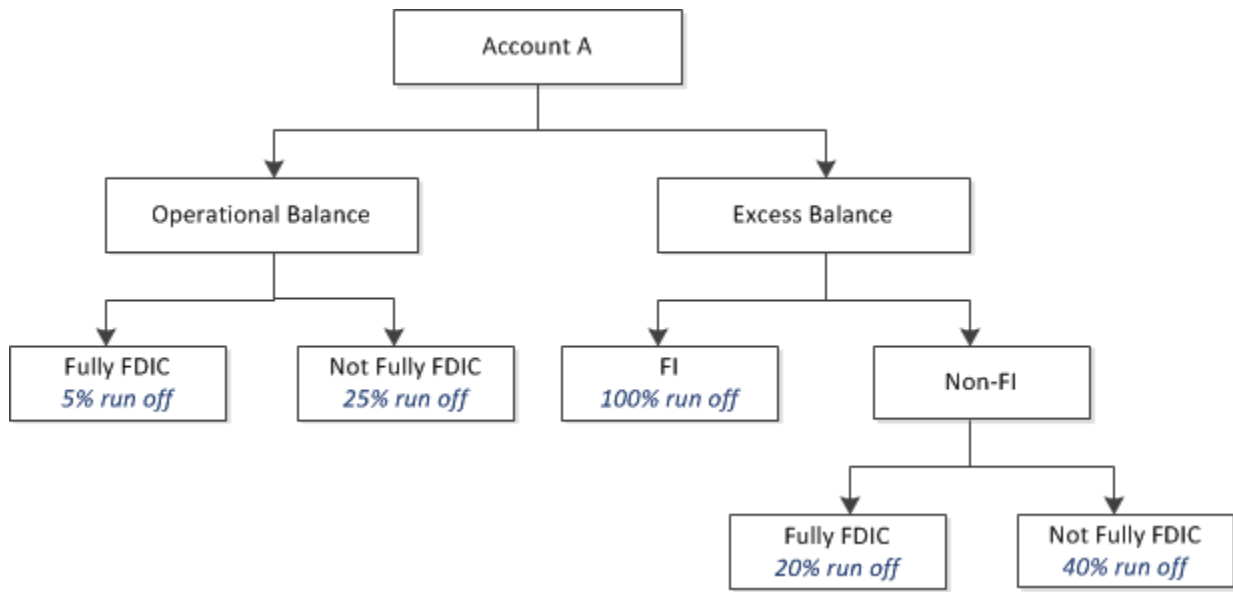
First, the account operational amount in a given business line is determined according to Figure 17 in ALMIS.

Figure 15: Daily Operational Amount Allocation in a Business Line



Then a reporting or analysis platform consumes the allocation data and assigns specific outflow rates (as defined by the US LCR rules) to account operational deposits amounts and the excess deposit amounts as can be seen in Figure 18.

Figure 16: LCR Run-Off Logic



Illustrative Example of the daily allocation process is attached in Appendix D.

Control Environment

Testing data set and applied data set are kept on separate servers and the data is not comingled.

- 1) Testing data refers to the data contained in the Datalake, which includes the transaction information on which the model is run.
- 2) The results of the model are then applied to the applied data set in ALMIS, which is used to run daily reporting on the LCR.

C. Ongoing Performance Monitoring Plan













The model is refreshed on at least a quarterly basis to capture the flow of operational client accounts and the most updated data set. The model is only used in conjunction with the market shift control mechanism. Additionally, a number of tests/controls are maintained to detect any changes in business patterns/economic environment that might indicate a fundamental change in the transaction/deposit relationship.










The following analyses are conducted post model generation to determine the applicability of the results:

- 1) End of day balances are compared to the previous month's ODM, with five key questions for each business line. If three out of five answers are yes, then an anomalous pattern exists.
 - Is current month coefficient of variation > historical trend?
 - Is current month mean balance < (historical minimum - one standard deviation)?
 - Is current month median balance < (historical median - one standard deviation)?
 - Is current month mean balance > (historical maximum + one standard deviation)?
 - Is current month median balance > (historical maximum + one standard deviation)?
- 2) Monitoring of operational eligible balances to detect if they were continuously below the OD estimate in the previous 3 months (at the firm level) on a daily basis. The test checks whether the frequency of such an occurrence is greater than 10% of the time.
- 3) On an individual business line level, a pattern of balance trends that is increasing or decreasing would incite further investigation. The detection methodology is tied to a limit that is set to be the 5th and 95th percentile of the distribution of the daily percentage changes of the weighted average (by account number) daily balances per business. Said distribution is calculated at the beginning of each year using the data from the previous year. A continuous breach would indicate an anomalous pattern.
- 4) Given the dependency of the model on periodicity, a large shift in the periodicity patterns would indicate an anomalous pattern. Monitoring of the distribution dispersal for a given business line is done by comparing the distribution percentage for a given intersection on a month over month basis (e.g., the distribution of AIS's 1-day periodicity between July and June decreased .6%). An anomaly is detected if any distribution point's (except NULL) difference is more than 5% in any direction.

If any of the above 4 tests/controls detect an anomalous pattern, it will be considered a breach. As such, an overlay of the previous month's average balance will be used as the Operational Deposit Amount, if and only if approval is obtained from both Model Risk Management Group and Treasury Risk Committee. Otherwise, the model will be used as is.

References for Model Documentation

Periodicity Transforms	 Periodicity Transforms
US LCR Rules	 LCR Fed Register 2014.pdf
BNY Mellon White Paper	 Deposit White Paper June 2015.pdf
ODM Documentation 2014	 Operational Deposit Methodolo
Deposit Tagging Logic	 Operational Deposit Logic v6.ppt
Operational Account Tagging Logic	 ALM_US LCR_Operational Cl
Operational Deposit Amount Tagging Logic	 ALM_US LCR_Operatioal Dep
Operational Deposit Data Sourcing and Aggregation Procedures	 Operational Deposit Data Sourci
R-Code for ABB Stability Test	 ABB_Stability_Test.R
R-Code for Benchmark 1	 benchmark1.R
R-Code for Benchmark 2	 benchmark2.R
R-Code for Data Quality Test	 Data_quality_test.R

R-Code for Group Length Distribution	 group_length_distribution.R
R-Code for Grouped Best Frequency	 grouped_best_frequency.R
R-Code for Grouped Best Frequency on 3 Month Average	 grouped_best_frequency_3M.R
R-Code for Grouped Best Frequency on 6 Month Average	 grouped_best_frequency_6M.R
R-Code for Least Error	 least_error.R
R-Code for Periodicity Distribution	 periodicity_distribution.R
R-Code for Regression Stability Test	 Regression_Stability_Test.R
R-Code for Spike Test	 spike_test.R
Model Monitoring Plan	 ODM v2 Monitoring Plan.xls

Change Log

Revision History of Model

Date	Section	Description of Change	Validation of Change	Validation Date
12/02/15	All	Initial draft.		
12/17/15	All	Updated information and results.		
04/28/16	All	Updated information.		
05/03/16	Calculations & Results	Updated information,		
06/05/16	Added grouping section and updated	Updated information and results.		

	results.			
06/13/16	All	Updated information and results.		
6/19/16	All	Updated information.		
7/01/16	All	Updated information and results.		
7/07/16	All	Responses to MRMG, plus update on testing results.		
07/15/16	Descriptions	Cleaned up language, moved items into attachments in Appendix D		
07/21/16	All	Standardized formatting		
07/27/16	Data	Updated results to include 3 month average, changed 3 month control calculation logic, moved factual/structural assumptions into description of model.		
11/03/16	Ongoing Performance Monitoring Plan References for Model Documentation	Updated Ongoing Performance Monitoring Plan. Updated the code file. Added monitoring plan file.		
11/04/16	Appendix D	Updated Mathematical example of ODM calculation (both periodicity and ABB)		

Access Controls

Ilona Moldavsky, Head of Business Liquidity Reviews and Analytics, Corporate Treasury

Yujie (Omar) Ma, Financial Analyst, Corporate Treasury

Appendix A: US Liquidity Coverage Ratio Detailed Background

Operational Deposit Eligibility

The Federal Reserve Board, the Federal Deposit Insurance Corporation and the Office of the Comptroller of the Currency adopted a final rule on September 3, 2014 implementing the Basel III Liquidity Coverage Ratio (“LCR”). This final rule (“US LCR”) establishes an ultimate quantitative minimum LCR of 100% and requires companies to maintain an amount of high quality liquid assets at least equal to the total net cash outflows over a prospective 30 calendar-day period.

Total net cash outflows are calculated based on assigned outflow and inflow rates, reflecting a standardized stress scenario, to different funding sources, obligations, and assets. Each category is divided into specific subcategories to account for the specific risk profile of each type of assets and liabilities.

A subset of deposits is tied to the provision of operational services by a covered company (as defined by the US LCR). As a result, these “operational deposits” are less likely to run off during a stress event. In order to account for the nature of the relationship between a customer, the covered company, and the operational services provided, a lower run off rate has been assigned to this category of deposits.

An operational deposit is defined by the rule as: “unsecured wholesale funding or a collateralized deposit that is necessary for the [BANK] to provide operational services as an independent third-party intermediary, agent, or administrator to the wholesale customer or counterparty providing the unsecured wholesale funding or collateralized deposit.”

The deposit must meet the following criteria to be recognized as operational:

- 1) The customer must hold the deposit for the primary purpose of obtaining specified operational services, which include:
 - a) Payment remittance
 - b) Administration of payments and cash flows related to the safekeeping of investment assets, not including the purchase or sale of assets
 - c) Payroll administration and control over the disbursement of funds
 - d) Transmission, reconciliation, and confirmation of payment orders
 - e) Daylight overdraft
 - f) Determination of intra-day and final settlement positions
 - g) Settlement of securities transactions
 - h) Transfer of capital distributions and recurring contractual payments
 - i) Customer subscriptions and redemptions
 - j) Scheduled distribution of customer funds

- k) Escrow, funds transfer, stock transfer, and agency services, including payment and settlement services, payment of fees, taxes, and other expenses
- l) Collection and aggregation of funds
- 2) Deposit must be held in an account designated as an operational account;
- 3) Deposit balance must be empirically linked to the operational services;
- 4) Operational services must be performed pursuant to a legally binding written agreement, and the termination of the agreement must be subject to a minimum 30 calendar-day notice period; or incur significant contractual termination costs or switching costs;
- 5) The deposit account must not be designed to create an economic incentive for the customer to maintain excess funds;
- 6) The deposit must not be provided in connection with the provision of prime brokerage services and operational services provided to a non-regulated fund (defined as any hedge fund or private equity fund whose investment adviser is required to file SEC Form PF – “Reporting Form for Investment Advisers to Private Funds and Certain Commodity Pool Operators and Commodity Trading Advisors”), other than a small business investment company;
- 7) The deposit must not be needed for primarily correspondent banking arrangements.

Based on the criteria set forth above, different parts of BNY Mellon were responsible for creating the checks and balances to determine if the overall account was operational.

- For items 1-4, each business line provided attestation to the operational eligibility of each of their client’s accounts.
- For item 5, Corporate Treasury set the threshold rates of “economic incentive” test as the central bank target rate for each currency (e.g., ECB for EUR, Fed Funds Target for USD).
- For item 6, non-regulated funds, both domestic and international (i.e., those funds who fit the criteria of being a hedge fund but do not need to file SEC Form PF due to jurisdictional rules) were determined based on the client categorization in ECIF.
- For item 7, it was determined that BNY Mellon does not provide correspondent banking arrangements as a primary service, rather those arrangement come around as secondary services in additional to traditional operational services utilized by the client base.

Operational Amount/Excess Guidance

The US LCR rules provide some direction in terms of determining what is an operational deposit versus excess, as seen in section (i) below along with guidance on the requirement to strip out volatility from the data set.

“(i). Exclusion of “Excess” Amounts Section 1.4(b)(6) of the proposed rule would have required that a covered company demonstrate that an operational deposit is empirically linked to an

operational service and that the covered company has a methodology for identifying any deposits in excess of the amount necessary to provide the operational services, the amount of which would be excluded from the operational deposit amount ... operational deposits are afforded a lower outflow rate due to their perceived stability arising from the nature of the relationship between a customer and covered company and the operational services provided, as well as factors, such as the switching costs associated with moving such deposits ... In contrast, excess deposits are not necessary for the provision of operational services and therefore do not exhibit these characteristics. The agencies are of the view that there is no single methodology for identifying excess deposits that will work for every covered company, as there is a range of operational deposit products offered and covered company data systems processing those products. Aggregation may be undertaken on a customer basis, a service basis, or both, but in all instances, a covered company's analysis of operational deposits must be conducted at a sufficiently granular level to adequately assess the risk of withdrawal in an idiosyncratic stress. The agencies expect covered companies to be able to provide supporting documentation that justifies the assumptions behind any aggregated calculations of excess deposits and expect that the higher (that is, the further from the individual account or customer) the level of aggregation, the more conservative the assumptions related to excess deposit amounts will be. A covered company's methodology must also take into account the volatility of the average deposit balance to ensure the proper identification of excess balances ... explicitly require a covered company to take into account the volatility of the average operational deposit balance when designing its methodology for identifying excess deposit amounts."

Appendix B: Systems

- 1) **ALMIS:** Asset and Liability Management Information System (ALMIS) is the data warehouse used for Asset and Liability Management in Corporate Treasury. The current version of ALMIS is 3.0 and contains data around all assets and liabilities that feed into the balance sheet, as well as non-balance sheet items. Daily data for most systems exist in ALMIS from November 22, 2013 (April 22, 2015 for CMS data).
- 2) **BWH:** One of the data warehouses maintained by the Bank, which contains daily end of the day cash, security and transaction data for the majority of the bank's major cash/custody systems.
- 3) **Data Lake:** Colloquial name for large data storage space maintained by BNY Mellon that is structured to take advantage of new large data set analysis tools (e.g., managed via Hadoop, allowing analysis by R, Tableau, etc.).
- 4) **ECIF:** ENTERPRISE CLIENT INFORMATION FACILITY is a facility for maintaining comprehensive customer profile information, relative to BNY Mellon's business. Its purpose is to provide consistent and comprehensive identification of BNY Mellon's customers, allowing for a "Single Customer View" from both The Bank of New York Mellon perspective and the customer's perspective. Unique client legal entities are Identified Party Identification Data (IPID), that information is searchable on CMT (Client Management Tool). Client legal structure is maintained via immediate parent and ultimate parent hierarchical structures. Client data is sourced from CIF, and KYC (Know Your Customer) process. The KYC process utilized external vendors such as Kingland and Goldtier to get enhanced information on clients. IPIDs are constantly being cleaned up and merged. ECIF data is used by Risk, Corporate Treasury and Regulatory Reporting for reporting to US Regulators.

Appendix C: Mathematical Terms Glossary/Terminology

Amplitude: For a sinusoidal function, $y = A \sin(B(x - C)) + D$, A is called the amplitude of the function.

Basis Matrix: The matrix whose column vectors can span the whole vector space.

Discrete Fourier Transformation:

If a matrix S has following column vectors:

$$s_k[n] = \frac{e^{i\frac{2\pi}{N}kn}}{\sqrt{N}} = \frac{1}{\sqrt{N}} \left(\cos\left(\frac{2\pi}{N}kn\right) + i \cdot \sin\left(\frac{2\pi}{N}kn\right) \right), \quad 1 \leq n, k \leq N$$

$X = S^H x$ is called the discrete Fourier Transform of the vector x . S^H is the inverse matrix or the conjugate matrix of S .

Domain: In mathematics, and more specifically in naive set theory, the domain of definition (or simply the domain) of a function is the set of "input" or argument values for which the function is defined.

Factor: A positive integer a is a factor of a positive integer b if and only if $b \div a$ is also a positive integer.

Frequency Domain: The input of the function is frequency.

Function: In mathematics, a function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

Linearly Independent: For a set of vectors v_1, v_2, \dots, v_n , if none of them can be represented as the linear combination of the other vectors in the set, then v_1, v_2, \dots, v_n are called linearly independent.

Linear vector space: A vector space (also called a linear space) is a collection of objects called vectors, which may be added together and multiplied ("scaled") by numbers, called scalars in this context. Scalars are often taken to be real numbers, but there are also vector spaces with scalar multiplication by complex numbers, rational numbers, or generally any field.

Matrix: In mathematics, a matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

Modulus: For a complex number $a + bi$, its modulus is defined as $\sqrt{a^2 + b^2}$

Orthogonal basis matrix: For a set of vectors v_1, v_2, \dots, v_n , if the inner product of any of the two vectors are 0, the basis matrix with v_1, v_2, \dots, v_n as column vectors are called orthogonal basis.

Orthonormal basis matrix: For a set of orthogonal vectors v_1, v_2, \dots, v_n , if their Euclidean norms are all equal to one, the basis matrix with v_1, v_2, \dots, v_n as column vectors are called orthonormal basis.

Real Matrix: A matrix with only real numbers as its elements.

Scalar: A number having only magnitude, not direction.

Scalar multiplication: To multiply every element of a vector or a matrix with a scalar number.






Space: In mathematics, a space is a set (sometimes called a universe) with some added structure. Different structures define different spaces.

Transformation: if \mathbf{v} is a vector, \mathbf{B} is a matrix, then $\mathbf{w} = \mathbf{vB}$ is called the transformation of \mathbf{v} under the matrix \mathbf{B} .

Synthesis: The inverse of transformation, i.e. $\mathbf{v} = \mathbf{wB}^{-1}$

Vector: a series of elements satisfying certain common properties.

Appendix D: Examples and Detailed Processes

Mathematical example of ODM calculation (both periodicity and ABB)	 Mathematical Example of ODM Ca
Detailed Process flow for ODM Calculation	 Detailed Process Flow for ODM Calcu
Simple Least Error Vulnerability Example	 Example of Simple Least Error Inefficier
Illustrative Example of Modeled Amount Allocation	 Illustrative Example of Modeled Amount
Periodicity Transforms Background	 Periodicity Transform Backgrou