Grid - non-uniform

The non-uniform grid was made using a Roberts stretching function, which was documented in MOONS as well.

Robert's stretching function

The Robert's stretching function is

$$y = h\frac{(\beta + 2\alpha)\gamma - \beta + 2\alpha}{(2\alpha + 1)\{1 + \gamma\}}$$

Where

$$\gamma = [(\beta + 1)/(\beta - 1)]^{(\bar{h}-\alpha)/(1-\alpha)}$$

$$\alpha = 0 \rightarrow \text{stretching at } h = h_{max} \text{ only}$$

$$\alpha = \frac{1}{2} \rightarrow \text{stretching at } h = h_{max} \text{ and } h = 0$$

$$0 \leq \bar{h} \leq 1$$

Scaled Robert's stretching function

In a slightly different notation, for a scaled grid, not starting at zero, we may write

$$h = h_{min} + (h_{max} - h_{min})\underbrace{\frac{(\beta + 2\alpha)\gamma - \beta + 2\alpha}{(2\alpha + 1)\{1 + \gamma\}}}_{(0-1)}$$

Where

$$\gamma = [(\beta + 1)/(\beta - 1)]^{(\bar{h}-\alpha)/(1-\alpha)}$$

$$\alpha = 0 \rightarrow \text{stretching at } h = h_{max} \text{ only}$$

$$\alpha = \frac{1}{2} \rightarrow \text{stretching at } h = h_{max} \text{ and } h = 0$$

$$0 \leq \bar{h} \leq 1$$

Matching β's

We would like to ensure that the first fictitious cell is the same size as the first interior cell in order to be able to linearly extrapolate from the interior cells to the exterior cell.

To do this, we must choose a β such that

$$h_N - h_{N-1} = \Delta h$$

Where

$$\Delta h = h_{interior}(2) - h_{interior}(1)$$

Note that this Δh is dimensional. Plugging this in, we have:

$$h_{min} + (h_{max} - h_{min})\frac{(\beta+2\alpha)\gamma_N - \beta + 2\alpha}{(2\alpha+1)\{1+\gamma_N\}} - \left\{h_{min} + (h_{max} - h_{min})\frac{(\beta+2\alpha)\gamma_{N-1} - \beta + 2\alpha}{(2\alpha+1)\{1+\gamma_{N-1}\}}\right\} = \Delta h$$

$$\underbrace{\frac{(\beta+2\alpha)\gamma_N - \beta + 2\alpha}{(2\alpha+1)\{1+\gamma_N\}}}_{dimensionless} - \underbrace{\left\{\frac{(\beta+2\alpha)\gamma_{N-1} - \beta + 2\alpha}{(2\alpha+1)\{1+\gamma_{N-1}\}}\right\}}_{dimensionless} = \frac{\Delta h}{(h_{max} - h_{min})}$$

Now, here comes the tricky part.

Note that $\bar{h}_N$ is normalized:

$$\gamma_N = \left(\frac{\beta+1}{\beta-1}\right)^{(\bar{h}_N - \alpha)/(1-\alpha)} = \frac{\beta+1}{\beta-1}$$

And the $\bar{h}_{N-1}$ is also normalized:

$$\gamma_{N-1} = \left(\frac{\beta+1}{\beta-1}\right)^{(\bar{h}_{N-1} - \alpha)/(1-\alpha)}$$

We may write this term as

$$\bar{h}_{N-1} = 1 - \Delta h^* = 1 - \left(\frac{1-0}{N_w}\right)$$

Where N_w is the number of cells in the wall.