myDel - non-uniform

Central differencing

Let

$$\alpha_k = -(x_i - x_{i-1}) = -\Delta x_i \qquad k = -1$$

$$\beta_j = x_{i+1} - x_i = \Delta x_{i+1} \qquad j = 1$$

This results in

1st derivative

$$\boxed{f_i^{'} = \frac{1}{(\Delta x_{i+1} + \Delta x_i)}\left\{f_i\left(\frac{-\Delta x_i}{\Delta x_{i+1}} + \frac{\Delta x_{i+1}}{\Delta x_i}\right) - \frac{\Delta x_{i+1}}{\Delta x_i} f_{i-1} + \frac{\Delta x_i}{\Delta x_{i+1}} f_{i+1}\right\}}$$

or, in short

$$\boxed{f_i^{'} = \frac{1}{\Delta x_i(\gamma + 1)}\left\{f_i\left(\gamma - \frac{1}{\gamma}\right) - \gamma f_{i-1} + \frac{1}{\gamma} f_{i+1}\right\}}$$

where

$$\gamma = \frac{\Delta x_{i+1}}{\Delta x_i}$$

2nd derivative

$$\boxed{f_i^{''} = -\frac{2f_i}{\Delta x_i \Delta x_{i+1}} + \frac{2f_{i-1}}{\Delta x_i^{\ 2} + \Delta x_i \Delta x_{i+1}} + \frac{2f_{i+1}}{\Delta x_{i+1}^{\ 2} + \Delta x_{i+1} \Delta x_i}}$$

or, in short

$$\boxed{f_i^{''} = -\frac{2f_i}{\Delta x_i \Delta x_{i+1}} + \frac{2f_{i-1}}{\gamma} + \frac{2f_{i+1}}{\gamma}}$$

where

$$\gamma = \Delta x_i^{\ 2} + \Delta x_i \Delta x_{i+1}$$

Forward differencing

Let

$$\alpha_k = (x_{i+1} - x_i) = \Delta x_{i+1} \qquad k = 1$$

$$\beta_j = x_{i+2} - x_i = (x_{i+2} - x_{i+1}) + (x_{i+1} - x_i) = \Delta x_{i+1} + \Delta x_{i+2} \qquad j = 2$$

This results in

1st derivative

$$\boxed{f_i^{'} = \frac{1}{(\Delta x_{i+1} + \Delta x_{i+2} - \Delta x_{i+1})}\left\{f_i\left(\frac{\Delta x_{i+1}}{\Delta x_{i+1} + \Delta x_{i+2}} - \frac{\Delta x_{i+1} + \Delta x_{i+2}}{\Delta x_{i+1}}\right) + \frac{\Delta x_{i+1} + \Delta x_{i+2}}{\Delta x_{i+1}} f_{i+1} - \frac{\Delta x_{i+1}}{\Delta x_{i+1} + \Delta x_{i+2}} f_{i+2}\right\}}$$

Or

$$\boxed{f_i' = \frac{1}{(\Delta x_{i+2})}\left\{f_i\left(\frac{\Delta x_{i+1}}{\Delta x_{i+1}+\Delta x_{i+2}} - \frac{\Delta x_{i+1}+\Delta x_{i+2}}{\Delta x_{i+1}}\right) + \frac{\Delta x_{i+1}+\Delta x_{i+2}}{\Delta x_{i+1}} f_{i+1} - \frac{\Delta x_{i+1}}{\Delta x_{i+1}+\Delta x_{i+2}} f_{i+2}\right\}}$$

in short we have

$$\boxed{f_i' = \frac{1}{(\Delta x_{i+2})}\left\{f_i\left(\gamma - \frac{1}{\gamma}\right) + \frac{1}{\gamma} f_{i+1} - \gamma f_{i+2}\right\}}$$

where

$$\gamma = \frac{\Delta x_{i+1}}{\Delta x_{i+1}+\Delta x_{i+2}}$$

2nd derivative

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_{i+1}(\Delta x_{i+1}+\Delta x_{i+2})} + \frac{2f_{i+1}}{\Delta x_{i+1}{}^2 - \Delta x_{i+1}(\Delta x_{i+1}+\Delta x_{i+2})} + \frac{2f_{i+2}}{(\Delta x_{i+1}+\Delta x_{i+2})^2 - (\Delta x_{i+1}+\Delta x_{i+2})\Delta x_{i+1}}}$$

Or

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_{i+1}(\Delta x_{i+1}+\Delta x_{i+2})} + \frac{2f_{i+1}}{-\Delta x_{i+1}\Delta x_{i+2}} + \frac{2f_{i+2}}{\Delta x_{i+1}^2 + 2\Delta x_{i+2}\Delta x_{i+1} + \Delta x_{i+2}^2 - \Delta x_{i+1}^2 - \Delta x_{i+2}\Delta x_{i+1}}}$$

Or

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_{i+1}(\Delta x_{i+1}+\Delta x_{i+2})} + \frac{2f_{i+1}}{-\Delta x_{i+1}\Delta x_{i+2}} + \frac{2f_{i+2}}{\Delta x_{i+2}\Delta x_{i+1} + \Delta x_{i+2}^2}}$$

Backward differencing

Let

$$\alpha_k = -(x_i - x_{i-1}) = -\Delta x_i \qquad k = -1$$

$$\beta_j = -(x_i - x_{i-2}) = -(\Delta x_i + \Delta x_{i-1}) \qquad j = -2$$

$$i = s$$

This results in

1st derivative

$$\boxed{f_i' = \frac{1}{(-(\Delta x_i + \Delta x_{i-1}) + \Delta x_i)}\left\{f_i\left(\frac{\Delta x_i}{(\Delta x_i + \Delta x_{i-1})} - \frac{(\Delta x_i + \Delta x_{i-1})}{\Delta x_i}\right) + \frac{(\Delta x_i + \Delta x_{i-1})}{\Delta x_i} f_{i+k} - \frac{\Delta x_i}{(\Delta x_i + \Delta x_{i-1})} f_{i+j}\right\}}$$

Or

$$\boxed{f_i' = \frac{1}{(-\Delta x_{i-1})}\left\{f_i\left(\frac{\Delta x_i}{(\Delta x_i + \Delta x_{i-1})} - \frac{(\Delta x_i + \Delta x_{i-1})}{\Delta x_i}\right) + \frac{(\Delta x_i + \Delta x_{i-1})}{\Delta x_i} f_{i+k} - \frac{\Delta x_i}{(\Delta x_i + \Delta x_{i-1})} f_{i+j}\right\}}$$

Or, in short

$$\boxed{f_i' = \frac{1}{(-\Delta x_{i-1})}\left\{f_i\left(\gamma_i - \frac{1}{\gamma_i}\right) + \frac{1}{\gamma_i}f_{i+k} - \gamma_i f_{i+j}\right\}}$$

where

$$\gamma_i = \frac{\Delta x_i}{(\Delta x_i + \Delta x_{i-1})}$$

NOTE that the indexes for Δx range from only $1\!:s-1$. Therefore we have:

$$\boxed{f_s' = \frac{1}{(-\Delta x_{s-2})}\left\{f_s\left(\gamma_{s-1} - \frac{1}{\gamma_{s-1}}\right) + \frac{1}{\gamma_{s-1}}f_{s-1} - \gamma_{s-1} f_{i-2}\right\}}$$

where

$$\gamma_{s-1} = \frac{\Delta x_{s-1}}{(\Delta x_{s-1} + \Delta x_{s-2})}$$

$$\boxed{f_s' = \frac{1}{(-\Delta x_{s-2})}\left\{f_s\left(\frac{\Delta x_{s-1}}{(\Delta x_{s-1} + \Delta x_{s-2})} - \frac{(\Delta x_{s-1} + \Delta x_{s-2})}{\Delta x_{s-1}}\right) + \frac{(\Delta x_{s-1} + \Delta x_{s-2})}{\Delta x_{s-1}}f_{s-1} - \frac{\Delta x_{s-1}}{(\Delta x_{s-1} + \Delta x_{s-2})}f_{s-2}\right\}}$$

2nd derivative

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_i(\Delta x_i + \Delta x_{i-1})} + \frac{2f_{i-1}}{\Delta x_i{}^2 - \Delta x_i(\Delta x_i + \Delta x_{i-1})} + \frac{2f_{i-2}}{(\Delta x_i + \Delta x_{i-1})^2 - (\Delta x_i + \Delta x_{i-1})\Delta x_i}}$$

Or

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_i(\Delta x_i + \Delta x_{i-1})} + \frac{2f_{i-1}}{-\Delta x_i \Delta x_{i-1}} + \frac{2f_{i-2}}{\Delta x_i^2 + 2\Delta x_i \Delta x_{i-1} + \Delta x_{i-1}^2 - \Delta x_i^2 - \Delta x_{i-1}\Delta x_i}}$$

Or

$$\boxed{f_i'' = \frac{2f_i}{\Delta x_i(\Delta x_i + \Delta x_{i-1})} + \frac{2f_{i-1}}{-\Delta x_i \Delta x_{i-1}} + \frac{2f_{i-2}}{\Delta x_i \Delta x_{i-1} + \Delta x_{i-1}^2}}$$

Finally

$$\boxed{f_s'' = \frac{2f_s}{\Delta x_{s-1}(\Delta x_{s-1} + \Delta x_{s-2})} + \frac{2f_{s-1}}{-\Delta x_{s-1}\Delta x_{s-2}} + \frac{2f_{s-2}}{\Delta x_{s-1}\Delta x_{s-2} + \Delta x_{s-2}^2}}$$