applyBCs - non-uniform

Dirichlet

Direct (wall coincident)
trivial assignment

$$u = u_{bc}$$

Interpolated (wall incoincident)

Forward

$$\frac{u_1 - u_2}{h_{c1} - h_{c2}} = \frac{u_{wall} - u_2}{h_{n1} - h_{c2}}$$

Solving for u_1 yields

$$u_1 = u_2 + \frac{h_{c1} - h_{c2}}{h_{n1} - h_{c2}}(u_{wall} - u_2)$$

Backward

$$\frac{u_s - u_{s-1}}{h_{cs} - h_{cs-1}} = \frac{u_{wall} - u_{s-1}}{h_{ns} - h_{cs-1}}$$

Solving for u_s yields

$$u_1 = u_{s-1} + \frac{h_{cs} - h_{cs-1}}{h_{ns} - h_{cs-1}}(u_{wall} - u_{s-1})$$

Neumann

Direct (wall coincident) ~O(dh^2)

Forward differencing
Let

$$\alpha_k = h_{n2} - h_{n1} \qquad k = 1$$

$$\beta_j = h_{n3} - h_{n1} \qquad j = 2$$

$$f_i = \frac{\left(f_i^{'}(\beta_j - \alpha_k) - \frac{\beta_j}{\alpha_k} f_{i+k} + \frac{\alpha_k}{\beta_j} f_{i+j}\right)}{\left(\frac{\alpha_k}{\beta_j} - \frac{\beta_j}{\alpha_k}\right)}$$

Backward differencing

$$\alpha_k = -(h_{ns} - h_{ns-1}) \qquad k = -1$$

$$\beta_j = -(h_{ns} - h_{ns-2}) \qquad j = -2$$

$$f_i = \frac{\left(f_i^{'}\left(\beta_j - \alpha_k\right) - \frac{\beta_j}{\alpha_k} f_{i+k} + \frac{\alpha_k}{\beta_j} f_{i+j}\right)}{\left(\frac{\alpha_k}{\beta_j} - \frac{\beta_j}{\alpha_k}\right)}$$

Interpolated (wall incoincident) O(dh)

This BC only requires a simply Taylor expansion that leads to a simple derivative at the boundary.

Forward differencing

$$\frac{p_1 - p_2}{h_{c1} - h_{c2}} = p_{\frac{1}{2}}^{'}$$

Solving for the ghost point yields

$$p_1 = p_2 + (h_{c1} - h_{c2}) p_{\frac{1}{2}}^{'}$$

Backward differencing

$$\frac{p_s - p_{s-1}}{h_{cs} - h_{cs-1}} = p_{s-\frac{1}{2}}^{'}$$

Solving for the ghost point yields

$$p_s = p_{s-1} + (h_{cs} - h_{cs-1}) p_{s-\frac{1}{2}}^{'}$$

Periodic