RunData Module

Characteristic scales

$$U_c \rightarrow reference\ velocity$$

$$L_c \rightarrow reference\ length$$

$$\rho_c \rightarrow reference\ density$$

$$\sigma_c \rightarrow reference\ electrical\ conductivity$$

Non-dimensionalizing

Dimensionless transformations			
$x_i^* = x_i/L_c$	$\boldsymbol{u}^* = \boldsymbol{u}/U_c$	$p^* = p/(0.5\rho_c U_c^2)$	$\rho^* = \rho/\rho_c$ Let $\rho_c = 1$ (w/o loss of generality)
$t^* = t/t_c$	$t_c = \min(t_\nu, t_u, t_\nu)$	$t_\nu = L_c^2/\nu$	$t_u = L_c/U_c$
$t_\eta = L_c^2/\eta$	$Re = U_c L_c/\nu$	$Re_m = \frac{t_\eta}{t_u} = \frac{L_c^2/\eta}{L_c/U_c}$	$Ha = B_0 \frac{L_{\|\|}}{2}\sqrt{\frac{\sigma_c}{\rho_c \nu}} = B_0 L_c \sqrt{\frac{\sigma}{\rho_c \nu}}$
	$\nu = \frac{U_c L_c}{Re}$	$\eta = \frac{U_c L_c}{Re_m}$	

Sequence of Calculations

GIVEN:

$$(t^*, Fo, Co, Re, Re_m, gd, \boldsymbol{u})$$

Preliminary Characteristic values:

$$L_c = \max(\max(x) - \min(x), \max(y) - \min(y), \max(z) - \min(z))$$

$$U_c = \max(\boldsymbol{u})$$

$$dh = \min(dx, dy, dz)$$

Time Scales

$$t_u = \frac{L_c}{U_c}$$

$$t_\nu = \frac{L_c^2}{\nu} = \frac{L_c^2}{\left(\frac{U_c L_c}{Re}\right)} = t_u Re$$

$$t_\eta = \frac{L_c^2}{\eta} = \frac{L_c^2}{\left(\frac{U_c L_c}{Re_m}\right)} = t_u Re_m$$

Chosen characteristic Time

The characteristic time is the convective time because that is how time was non-dimensionalized in the B-formulation:

$$t_c = \mathrm{t_u}$$

Real Time Step

$$\nu = \frac{U_c L_c}{Re}, \qquad \eta = \frac{U_c L_c}{Re_m}$$

$$\Delta t = \min\left(\frac{Co\ dh}{U_c}, \frac{Fo\ dh^2}{\nu}, \frac{Fo\ dh^2}{\eta}\right) = \min\left(\frac{Co\ dh}{U_c}, \frac{Fo\ dh^2}{\nu}, \frac{Fo\ dh^2}{\eta}\right)$$

Characteristic Time Step

The characteristic time step is divided by the convective time because that is how time was non-dimensionalized in the B-formulation:

$$\Delta t^* = \frac{\Delta t}{t_c} = \frac{\Delta t}{t_u}$$

Characteristic Time

Again, the characteristic time step depends on the smallest characteristic time:

$$t^* = \frac{t}{t_c} = \frac{t}{t_u}$$

CFL Condition

Nothing is allowed to flow more than 1 grid spacing within one time step, i.e.

$$\Delta t < \frac{\Delta x}{u}\ (time\ to\ travel\ 1\ grid\ spacing)$$

What is usually chosen is

$$\Delta t = C \min\left(\frac{\Delta x}{u}\right)$$

Where, if $C = 0.5$, then Δt is half the time required for flow to cross one grid spacing. The CFL condition is a necessary (but not sufficient) condition for the stability of ANY explicit differencing method. Note that 4th order accurate derivatives reach TWO grid spaces away. This may change the CFL condition.