University of Toronto

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Assignment 3
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Part 2: Functional Dependencies, Decompositions, and Normal Forms

1. (a) To determine which of the given FDs violate BCNF, check for closure.

$L^+ = LNQSOPRM$	L is a superkey.
$MNR^+ = MNRO$	MNR is NOT a superkey. Therefore, $MNR \to O$ violates BCNF.
$O^+ = OM$	O is NOT a superkey. Therefore, $O \to M$ violates BCNF.
$NQ^+ = NQLSOPRM$	NQ is a superkey.
$S^+ = SOPRM$	S is NOT a superkey. Therefore, $S \to OPR$ violates BCNF.

   (b) In A, $MNR \to O$ violates BCNF.
   Set $A_1 = MNRO$.
      Projecting the FDs, we get $MNR \to O$ and $O \to M$.
   Set $A_2 = LMNPQRS$.
      Projecting the FDs, we get $L \to NQSPRM$, $NQ \to LSPRM$, and $S \to PRM$.

   In A_1, $O \to OM$ violates BCNF.
   Set $A_3 = OM$.
      Projecting the FDs, we get $O \to M$.
   Set $A_4 = ONR$.
      Projecting the FDs, we get no FDs.

   In A_2, $S \to SPRM$ violates BCNF.
   Set $A_5 = SPRM$.
      Projecting the FDs, we get $S \to PRM$.
   Set $A_6 = LNQS$.
      Projecting the FDs, we get $L \to NQS$ and $NQ \to LS$.

   The final decomposition consists of the following four relations and their respective FDs:
   $A_6 = LNQS$ and FDs $L \to NQS$ and $NQ \to LS$
   $A_3 = OM$ and FD $O \to M$.
   $A_4 = ONR$ and no FDs.
   $A_5 = SPRM$ and FD $S \to PRM$.

2. (a) **Step 1:** Split the RHSs to get our initial set of FDs, T1:

i. $AB \to C$
ii. $C \to A$
iii. $C \to B$
iv. $C \to D$
v. $CFD \to E$
vi. $E \to B$
vii. $BF \to E$
viii. $BF \to C$
ix. $B \to D$
x. $B \to A$

Step 2: For each FD, try to reduce the LHS:
i. $B^+ = BDAC$, so reduce it to $B^+ \to C$.
ii. Cannot reduce this FD any further.
iii. Cannot reduce this FD any further.
iv. Cannot reduce this FD any further.
v. $CF^+ = CFABDE$, so reduce it to $CF^+ \to E$.
vi. Cannot reduce this FD any further.
vii. $B^+ = BDAC$ and $F^+ \to F$. Cannot reduce this FD any further.
viii. $B^+ = BDAC$, so reduce it to $B^+ \to C$. This is same as i.
ix. Cannot reduce this FD any further.
x. Cannot reduce this FD any further.

Our new set of FDs, let's call it T2, is
i. $B \to C$
ii. $C \to A$
iii. $C \to B$
iv. $C \to D$
v. $CF \to E$
vi. $E \to B$
vii. $BF \to E$
viii. $B \to D$
ix. $B \to A$

Step 3: Try to eliminate each FD.
i. $B^+_{T2-(i)} = BAD$. Therefore, we need (i).
ii. $C^+_{T2-(ii)} = CBDA$. Therefore, we can remove (ii).
iii. $C^+_{T2-(ii)-(iii)} = CD$. Therefore, we need (iii).
iv. $C^+_{T2-(ii)-(iv)} = CBDA$. Therefore, we can remove (iv).
v. $CF^+_{T2-(ii)-(iv)-(v)} = CFBDAE$. Therefore, we can remove (v).
vi. $E^+_{T2-(ii)-(iv)-(v)-(vi)} = E$. Therefore, we need (vi).
vii. $BF^+_{T2-(ii)-(iv)-(v)-(vii)} = BFCDA$. Therefore, need (vii).
viii. $B^+_{T2-(ii)-(iv)-(v)-(viii)} = BCA$. Therefore, need (viii).
ix. $B^+_{T2-(ii)-(iv)-(v)-(ix)} = BCD$. Therefore, need (ix).

Therefore, the minimal basis for P is

i. $B \to A$
ii. $B \to C$
iii. $B \to D$
iv. $BF \to E$
v. $C \to B$
vi. $E \to B$

(b) Since F, G, and H are in are not in the RHS of any FD, they are in every key.
Since A and D are not in the LHS and in the RHS, they are in no key.
Since B, C, and E are in the LHS and in the RHS, they may or may not be in a key. We need to check.

$FGHB^+ = FGHBACDE$. Therefore, it is a key.
$FGHC^+ = FGHCBADE$. Therefore, it is a key.
$FGHE^+ = FGHEBACD$. Therefore, it is a key.

Since all the other combinations of B, C, and E are a supersets of one of these three keys, BFGH, CFGH, and EFGH are the only three keys of the relation P.

(c) Note that the minimal basis of P is the following as shown from part (a):

i. $B \to ACD$
ii. $BF \to E$
iii. $C \to B$
iv. $E \to B$

For each FD $X \to Y$ in the minimal basis, define a new relation with schema $X \cup Y$.

i. $BACD^+ = BACD$
ii. $BFE^+ = BFEACD$
iii. $CB^+ = CB$ is already a part of $BACD^+$.
iv. $EB^+ = EB$is already a part of BFE^+.

Since no relation is a superkey of P, add relation BFGH.
$BFGH^+ = BFGHEACD$. Therefore, its schema is a key.

Therefore, the collection of relations that are in 3NF are ABCD, BEF, and BFGH.

(d) If one of the above relations contain any redundancy which can be reduced with BCNF, we know that the schema allows redundancy.
Checking to see if any projection of FDs violate any BCNF, we find that for BEF, the FD, $E \to B$, is violated.
In other words, BEF can be reduced into the following:
BE with FD, $E \to B$, and EF with no FDs.
Therefore, my schema allows for redundancy.