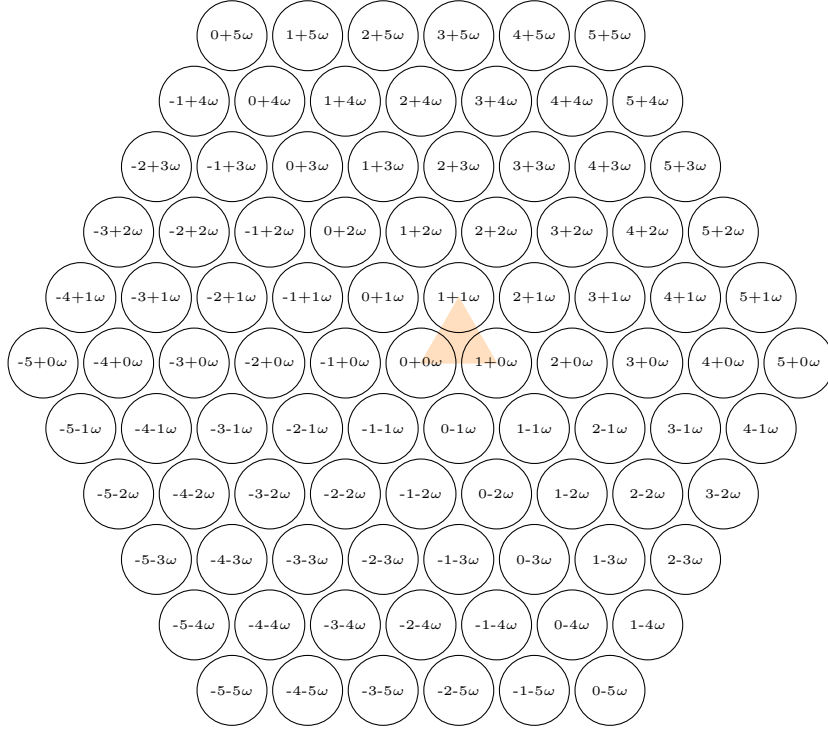


The Eisenstein integers are of the form  $a + b\omega$ , where  $\omega = \frac{-1+i\sqrt{3}}{2}$ . They form a triangular lattice and the ring  $\mathbb{Z}[\omega]$ .

A triangle can be represented as the set of three Eisenstein integers which surround its center. E.g.,  $\{0, 1, 1 + \omega\}$  corresponds to the triangle highlighted orange below.



The manhattan distance of an Eisenstein integer from the origin is<sup>1</sup>

$$d(a + b\omega) = \begin{cases} |a| + |b| & \text{if } ab \leq 0 \\ \max(|a|, |b|) & \text{if } ab > 0. \end{cases}$$

When  $a$  and  $b$  are of opposite signs,  $a$  and  $b$  individually contribute to the number of steps moved. But when  $a$  and  $b$  are the same sign, each multiple of  $1 + \omega$  corresponds to *just one step*, not two. After removing all the multiples of  $1 + \omega$ , one of  $a$  or  $b$  will be zero, and the absolute value of the nonzero component contributes the remaining steps. Putting this thought process into algebra reveals that the manhattan distance when  $a$  and  $b$  are the same sign is

$$\begin{aligned} d(a + b\omega) &= \min(|a|, |b|) && \text{(multiples of } 1 + \omega) \\ &\quad + \max(|a|, |b|) - \min(|a|, |b|) && \text{(remaining steps after factoring out all } 1 + \omega) \\ &= \max(|a|, |b|) \end{aligned}$$

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<sup>1</sup>It doesn't matter which one is the "or equal to" comparison. If one of  $a$  or  $b$  is zero  $|a| + |b| = \max(|a|, |b|)$ .