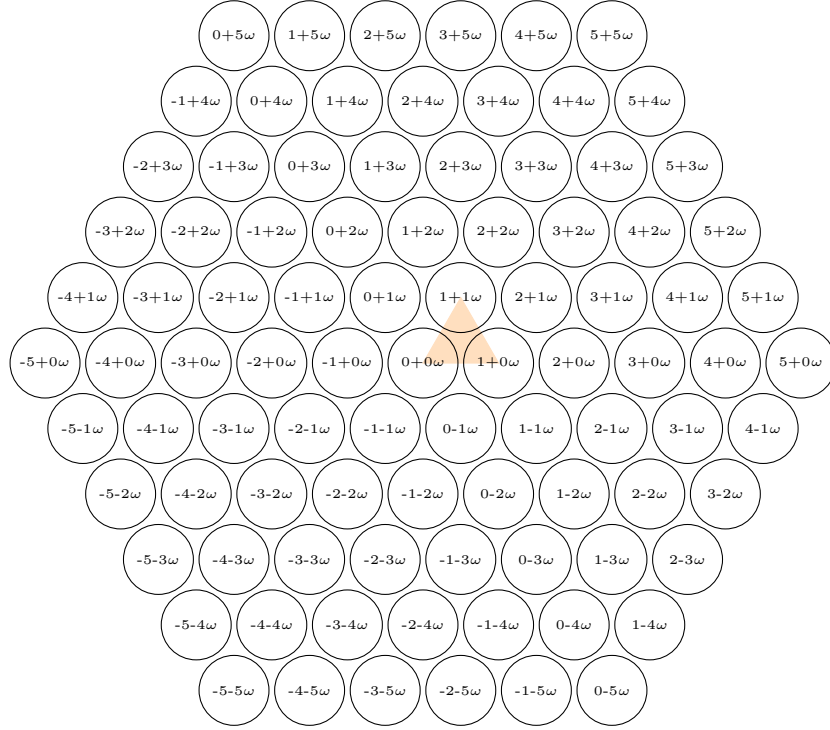


The Eisenstein integers are of the form $a + b\omega$, where $\omega = \frac{-1+i\sqrt{3}}{2}$. They form a triangular lattice and the ring $\mathbb{Z}[\omega]$.

A triangle can be represented as the set of three Eisenstein integers which surround its center. E.g., $\{0, 1, 1 + \omega\}$ corresponds to the triangle highlighted orange below.



The manhattan distance of an Eisenstein integer from the origin is¹

$$d(a + b\omega) = \begin{cases} |a| + |b| & \text{if } ab \leq 0 \\ \max(|a|, |b|) & \text{if } ab > 0 \end{cases}.$$

When a and b are of opposite signs, a and b individually contribute to the number of steps moved. But when a and b are the same sign, each multiple of $1 + \omega$ corresponds to *just one step*, not two. After removing all the multiples of $1 + \omega$, one of a or b will be zero, and the absolute value of the nonzero component contributes the remaining steps. Putting this thought process into algebra reveals that the manhattan distance when a and b are the same sign is

$$\begin{aligned} d(a + b\omega) &= \min(|a|, |b|) && \text{(multiples of } 1 + \omega) \\ &+ \max(|a|, |b|) - \min(|a|, |b|) && \text{(remaining steps after factoring out all } 1 + \omega) \\ &= \max(|a|, |b|) \end{aligned}$$

¹It doesn't matter which one is the "or equal to" comparison. If one of a or b is zero $|a| + |b| = \max(|a|, |b|)$.